

Economic Order Quantity Model for Non-Instantaneously Deteriorating Items under Order-Size-Dependent Trade Credit for Price-Sensitive Quadratic Demand

¹Nita H. Shah and ²Mrudul Y. Jani

¹Department of Mathematics, Gujarat University, Ahmedabad-380009, Gujarat, India

²Dept of Applied Sciences, Faculty of Engineering & Technology,
Parul University, Vadodara-391760, Gujarat, India

Corresponding Author: ¹Nita H. Shah

(¹nitahshah@gmail.com, ²janimrudul07@gmail.com)

Abstract

This article focuses on inventory system in which a supplier gives different credit periods linked to order quantity to the retailer. In this model, deterioration rate is non-instantaneous. Here, price-sensitive quadratic demand is discussed; which is suitable for the products for which demand increases initially and afterward it starts to decrease with the new version of the item. The objective is to maximize the total profit of retailer with respect to selling price and cycle time. Scenarios are established and illustrated with numerical examples. Moreover, best possible scenario is discussed. Through, sensitivity analysis important inventory parameters are classified. Graphical results, in two and three dimensions, are offered and supervisory decision.

Keywords: Inventory model, non-instantaneous deterioration, price-sensitive quadratic demand, order-size-dependent trade credit

1. Introduction

Harris-Wilson's economic order quantity model is based on the hypothesis that a trader practices cash-on supply policy. Nevertheless, in the market, if the purchaser has a choice to pay later on without interest charges then he gets attracted to buy which may be demonstrated to be an outstanding approach in today's cutthroat scenario. During this permissible period, the retailer can earn interest on sold items. Goyal (1985) prepared decision strategy by incorporating concept of acceptable delay in payments to settle the accounts due against purchases in the classical EOQ model. Jamal *et al.* (2000) developed optimal payment time for retailer under acceptable delay by

supplier. Chang *et al.* (2001) modelled linear trend demand in inventory model under the form of permissible delay in payment. Jaggi *et al.* (2008) analyzed retailer's optimal policy with credit linked demand under allowable delay in payments. Subsequently, credit period and its variants were given by numerous researchers. Shah *et al.* (2010) contributed review article on inventory modeling with trade credits. Lou and Wang (2013) considered the seller's decision about setting delay payment period. They used deterministic constant demand. Almost all the researchers established that the span of trade credit increases the demand rate.

Most of the previous studies dealing with inventory problems in circumstances of acceptable delay in payments discuss a case in which the delay in payments is independent of the quantity ordered. Conversely, in today's business dealings, in order to inspire the retailer to order large quantities, the supplier may offer an allowable delay of payment for large quantities but require instant payment for small quantities. Thus, the supplier may set a predetermined order quantity below which delay in payment is not allowed and payments must be made instantly. For order quantities above this inception, the trade credit period is permitted. Khouja and Mehrez (1996) examined the effect of supplier credit policies on the optimal order quantity. They provided two types of supplier credit policies: the first type is one in which credit terms are independent of the quantity ordered, and the second type is one in which the credit terms are linked to the order quantity. Shinn and Hwang (2003) studied the problem of the retailer who has to decide his/her sale price and order quantity simultaneously in the case of an order-size-dependent delay in payments. Chang *et al.* (2003) established an EOQ model with deteriorating items where suppliers link credit to order quantity. Chung and Liao (2004) discussed the optimal replenishment cycle time for an exponentially deteriorating product under the condition that the delay in payments depends on the quantity ordered. Some interesting articles by Chang (2003), Chung *et al.* (2005), Liao (2007), Ouyang *et al.* (2008,2009), Chang *et al.* (2010), and Yang *et al.* (2010) , Cárdenas-Barrón *et al.* (2014), Wu *et al.* (2014,2016), address this topic.

Because of radical environmental changes, most of the items losses its efficiency over time, termed as deterioration. Ghare and Schrader (1963) considered consequence of deterioration in inventory model. The review articles on deteriorating items for inventory system by Raafat (1991), Shah and Shah (2000), Goyal and Giri (2001), Bakker *et al.* (2012), Sarkar *et al.* (2015) throw light on the role of deterioration. The citations in the review articles include constant rate of deterioration, weibull distributed deterioration etc. In the present study, all the models assume that the deterioration of items in an inventory starts from the moment of their arrival in stock. Though, in real life there is a time length during which most suppliers maintain their quality or original

condition, that is, during which no deterioration occurs. Outside this period, however, some of the items will start deteriorating. Wu *et al.* (2006) defined this phenomenon as “non-instantaneous deterioration.” It exists usually among medicines, first-hand vegetables, and fruits, all of which can preserve their freshness for a little span of time. During this time limit, there is almost no deterioration. For these kinds of items, the supposition that the deterioration occurs immediately on the arrival of the items may lead retailers to adopt unsuitable replenishment policies resulting overvaluing the total relevant inventory cost. Chang *et al.* (2010) offered best replenishment policies for non-instantaneously deteriorating items with stock-dependent demand. Their model set a maximum inventory level to reflect the limited shelf space of most retail outlets. Geetha and Uthayakumar (2010) studied an economic order quantity (EOQ) model for non-instantaneously deteriorating items with allowable delay in payments in which model shortages are allowed and partially backlogged. Maihami and Kamalabadi (2012) offered a joint pricing and inventory model for non-instantaneously deteriorating items with a price-and-time-dependent demand function. Our study shows the significance of taking into consideration the inventory problems related with non-instantaneously deteriorating items in the inventory management system.

In above mentioned articles, constant demand rate is considered. Though, the market analysis says that the demand hardly remains constant. Shah and Mishra (2010) developed inventory model for deteriorating items with salvage value under retailer partial trade credit and stock-dependant demand in supply chain. Shah *et al.* (2014) studied optimal pricing and ordering policies for deteriorating items with two-level trade credits under price-sensitive trended demand. In this paper, we considered demand to be price-sensitive time quadratic. Quadratic demand initially increases with time for some time and then decreases. In this article we study a suitable inventory model for non-instantaneous deteriorating items, in which dealer offers order-quantity dependent credit period to his clients for payment under the consideration of price sensitive quadratic demand. Main focus is to maximize the total profit per unit time for the retailer. Numerical examples and graphical analysis are provided to discuss the outcomes. Lastly, we do sensitivity analysis to study the consequences on optimal solution with respect to one inventory parameter at a time. For the retailer, managerial insights are furnished.

2. Notations and assumptions

We shall use following notations and assumptions to build up the mathematical model of the problem under consideration.

2.1 Notations

A	Ordering cost per order
C	Purchase cost per unit
P	Selling price per unit (a decision variable) ($P > C$)
h	Inventory holding cost (excluding interest charges) per unit per unit time
θ	Constant deterioration rate, $0 \leq \theta < 1$.
I_e	Interest earned per \$ per year
I_c	Interest charged per \$ for unsold stock per annum by the supplier
	Note: $I_c > I_e$
$R(P, t)$	Price - sensitive time dependent demand
t_d	Constant time from which deterioration starts
	$j = 1, 2, \dots, n$
M_j	Permissible delay period (decision variable)
T_j	Length of replenishment cycle when the permissible delay period is M_j (decision variable)
$I(t)$	Inventory level at any instant of time t , $0 \leq t \leq T_j$
Q	Order quantity (units/order)
SR	Sales Revenue
PC	Purchasing Cost
OC	Ordering Cost
HC	Holding Cost
IC	Interest Charged

IE

Interest Earned

$Z_{i,j}(T_j, P)$

Total profit per unit time

2.2 Assumptions

1. The system under review deals with single item
2. The demand rate, (say) $R(P, t) = a \cdot P^{-\eta} \cdot (1 + b \cdot t - c \cdot t^2)$ is function of time; where P is selling price per unit, $a > 0$ is scale demand, $0 \leq b < 1$ denotes the linear rate of change of demand with respect to time, $0 \leq c < 1$ denotes the quadratic rate of change of demand and $\eta > 1$ is mark up for selling price.
3. The supplier offers the order dependent credit period as follows:

$$M = \begin{cases} M_1 & Q_1 \leq Q < Q_2 \\ M_2 & Q_2 \leq Q < Q_3 \\ \vdots & \vdots \\ M_k & Q_k \leq Q < Q_{k+1} \end{cases} \quad (1)$$

where, $0 < Q_1 < Q_2 < \dots < Q_k < Q_{k+1}$ and $0 \leq M_1 < M_2 < \dots < M_k$.

4. The non-instantaneous deterioration is considered.
5. Planning horizon is infinite.
6. Lead time is zero or negligible.
7. The capital opportunity cost incurred only if $T_j > M_j$ and the interest earned (I_e) from sales revenue during the interval $[0, M_j]$.

3. Mathematical Model

We analyze one inventory cycle. The following two situations are to be discussed:

(i) $T_j \leq t_d$, and (ii) $T_j \geq t_d$, for a given permissible delay period M_j , to determine the inventory level, $I(t)$ at any instant of t .

When $T_j \leq t_d$, the replenishment cycle is shorter than or equal to the length of time in which the product does not deteriorate; thus, no deterioration occurs during the replenishment cycle. In this situation, the order quantity per order is

$$Q_1 = \int_0^{T_j} R(P, t) dt, \quad (2)$$

and the inventory level decreases only owing to the demand during the time interval $[0, T_j]$. Hence, the inventory level, $I(t)$, at time $t \in [0, T_j]$ is given by

$$\begin{aligned} I(t) &= Q_1 - \int_0^t R(P, t) dt \\ &= -\frac{1}{6} aP^{-\eta} (2cT_j^3 - 3bT_j^2 - 6T_j + 6t + 3bt^2 - 2ct^3), \quad 0 \leq t \leq T_j. \end{aligned} \quad (3)$$

When $T_j \geq t_d$, during the time interval $[0, t_d]$, the inventory level decreases only due to demand. Hence, the inventory level, $I_1(t)$, at time $t \in [0, t_d]$ is given by

$$I_1(t) = Q_j - \int_0^t R(P, t) dt, \quad 0 \leq t \leq t_d \quad (4)$$

and during the time interval $[t_d, T_j]$, the inventory level, $I_2(t)$, decreases due to demand and deterioration. Hence, the change of inventory level can be represented by the following differential equation:

$$\frac{dI_2(t)}{dt} + \theta I_2(t) = -R(P, t), \quad t_d < t < T_j, \quad (5)$$

with the boundary condition $I_2(T_j) = 0$. The solution of Eq. (5) is

$$I_2(t) = aP^{-\eta} \left(\frac{(1+bT_j - cT_j^2)e^{\theta(T_j-t)}}{\theta} - \frac{(b-2cT_j)e^{\theta(T_j-t)}}{\theta^2} - \frac{2ce^{\theta(T_j-t)}}{\theta^3} - \frac{(1+bt - ct^2)}{\theta} + \frac{(b-2ct)}{\theta^2} + \frac{2c}{\theta^3} \right), \quad t_d \leq t \leq T_j. \quad (6)$$

Considering continuity of $I_1(t)$ and $I_2(t)$ at time $t = t_d$, i.e. $I_1(t_d) = I_2(t_d)$, it follows from

Eq. (4) and

Eq. (6) that

$$Q_j - \int_0^{t_d} R(P, t) dt = aP^{-\eta} \left(\frac{(1+bT_j - cT_j^2)e^{\theta(T_j-t_d)}}{\theta} - \frac{(b-2cT_j)e^{\theta(T_j-t_d)}}{\theta^2} - \frac{2ce^{\theta(T_j-t_d)}}{\theta^3} - \frac{(1+bt_d - ct_d^2)}{\theta} + \frac{(b-2ct_d)}{\theta^2} + \frac{2c}{\theta^3} \right),$$

which implies that the order quantity for each cycle is

$$Q_j = aP^{-\eta} \left(\begin{array}{l} t_d + \frac{bt_d^2}{2} - \frac{ct_d^3}{3} + \frac{(1+bT_j - cT_j^2)e^{\theta(T_j-t_d)}}{\theta} \\ - \frac{(b-2cT_j)e^{\theta(T_j-t_d)}}{\theta^2} - \frac{2ce^{\theta(T_j-t_d)}}{\theta^3} \\ - \frac{(1+bt_d - ct_d^2)}{\theta} + \frac{(b-2ct_d)}{\theta^2} + \frac{2c}{\theta^3} \end{array} \right). \quad (7)$$

Substituting Eq.(7) into Eq.(4), we obtain

$$I_1(t) = aP^{-\eta} \left(\begin{array}{l} t_d + \frac{bt_d^2}{2} - \frac{ct_d^3}{3} + \frac{(1+bT_j - cT_j^2)e^{\theta(T_j-t_d)}}{\theta} \\ - \frac{(b-2cT_j)e^{\theta(T_j-t_d)}}{\theta^2} - \frac{2ce^{\theta(T_j-t_d)}}{\theta^3} \\ - \frac{(1+bt_d - ct_d^2)}{\theta} + \frac{(b-2ct_d)}{\theta^2} + \frac{2c}{\theta^3} \\ - \left(t + \frac{1}{2}bt^2 - \frac{1}{3}ct^3 \right) \end{array} \right), 0 \leq t \leq t_d \quad (8)$$

Next, we compute relevant cost components and total profit per time unit, depending upon different scenarios in table 1.

Table 1 Cost Components and Total Profit	
Scenarios	Costs and Total Profit
$T_j \leq M_j \leq t_d$	$SR = \frac{P}{T_j} Q_1$ $PC = \frac{C}{T_j} Q_1$ $OC = \frac{A}{T_j}$ $HC = \frac{h}{T_j} \int_0^{T_j} I(t) dt$

	$IE = \frac{PI_e}{T_j} \left(\int_0^{T_j} R(P,t) t dt + aP^{-\eta} (1 + bT_j - cT_j^2) T_j (M_j - T_j) \right)$ $IC = 0$ $Z_{1,j}(T_j, P) = SR - CP - OC - HC - IC + IE$
$M_j \leq T_j \leq t_d$	$SR = \frac{P}{T_j} Q_1$ $PC = \frac{C}{T_j} Q_1$ $OC = \frac{A}{T_j}$ $HC = \frac{h}{T_j} \int_0^{T_j} I(t) dt$ $IE = \frac{PI_e}{T_j} \int_0^{M_j} R(P,t) t dt$ $IC = \frac{CI_c}{T_j} \int_{M_j}^{T_j} I(t) dt$ $Z_{2,j}(T_j, P) = SR - CP - OC - HC - IC + IE$
$M_j \leq t_d \leq T_j$	$SR = \frac{P}{T_j} Q_j$ $PC = \frac{C}{T_j} Q_j$ $OC = \frac{A}{T_j}$ $HC = \frac{h}{T_j} \left[\int_0^{t_d} I_1(t) dt + \int_{t_d}^{T_j} I_2(t) dt \right]$

	$IE = \frac{PI_e}{T_j} \int_0^{M_j} R(P,t) t dt$ $IC = \frac{CI_c}{T_j} \left[\int_{M_j}^{t_d} I_1(t) dt + \int_{t_d}^{T_j} I_2(t) dt \right]$ $Z_{3,j}(T_j, P) = SR - CP - OC - HC - IC + IE$
$T_j \leq t_d \leq M_j$	$SR = \frac{P}{T_j} Q_1$ $PC = \frac{C}{T_j} Q_1$ $OC = \frac{A}{T_j}$ $HC = \frac{h}{T_j} \int_0^{T_j} I(t) dt$ $IE = \frac{PI_e}{T_j} \left(\int_0^{T_j} R(P,t) t dt + aP^{-\eta} (1 + bT_j - cT_j^2) T_j (M_j - T_j) \right)$ $IC = 0$ $Z_{4,j}(T_j, P) = SR - CP - OC - HC - IC + IE$
$t_d \leq T_j \leq M_j$	$SR = \frac{P}{T_j} Q_j$ $PC = \frac{C}{T_j} Q_j$ $OC = \frac{A}{T_j}$ $HC = \frac{h}{T_j} \left[\int_0^{t_d} I_1(t) dt + \int_{t_d}^{T_j} I_2(t) dt \right]$

	$IE = \frac{PI_e}{T_j} \left(\int_0^{T_j} R(P,t)t dt + aP^{-\eta} (1+bT_j - cT_j^2) T_j (M_j - T_j) \right)$ $IC = 0$ $Z_{5,j}(T_j, P) = SR - CP - OC - HC - IC + IE$
$t_d \leq M_j \leq T_j$	$SR = \frac{P}{T_j} Q_j$ $PC = \frac{C}{T_j} Q_j$ $OC = \frac{A}{T_j}$ $HC = \frac{h}{T_j} \left[\int_0^{t_d} I_1(t) dt + \int_{t_d}^{T_j} I_2(t) dt \right]$ $IE = \frac{PI_e}{T_j} \int_0^{M_j} R(P,t)t dt$ $IC = \frac{CI_c}{T_j} \int_{M_j}^{T_j} I_2(t) dt$ $Z_{6,j}(T_j, P) = SR - CP - OC - HC - IC + IE$

Clearly, $Z_{1,j}(M_j, P) = Z_{2,j}(M_j, P)$ and $Z_{2,j}(t_d, P) = Z_{3,j}(t_d, P)$. Moreover, $Z_{4,j}(t_d, P) = Z_{5,j}(t_d, P)$ and $Z_{5,j}(M_j, P) = Z_{6,j}(M_j, P)$. Hence, $Z_{i,j}(T_j, P)$, $i = 1..6$ are well-defined for $T_j > 0$.

Now, the necessary conditions to make total profit maximum are

$$\frac{\partial Z_{i,j}(T_j, P)}{\partial P} = 0, \quad \frac{\partial Z_{i,j}(T_j, P)}{\partial T_j} = 0. \quad (9)$$

Here, we use following algorithm for the optimal solution.

Step 1: Assign hypothetical values to the inventory parameters.

Step 2: Solve equations for P and T_j in (9) simultaneously satisfying Eq.(1), by mathematical software Maple XIV.

Step 3: Verify second order (sufficiency) conditions

$$\text{i.e. } \left| \begin{array}{cc} \frac{\partial^2 Z_{i,j}(T_j, P)}{\partial P^2} & \frac{\partial^2 Z_{i,j}(T_j, P)}{\partial P \partial T_j} \\ \frac{\partial^2 Z_{i,j}(T_j, P)}{\partial T_j \partial P} & \frac{\partial^2 Z_{i,j}(T_j, P)}{\partial T_j^2} \end{array} \right| > 0 \text{ and } \frac{\partial^2 Z_{i,j}(T_j, P)}{\partial P^2} < 0, \frac{\partial^2 Z_{i,j}(T_j, P)}{\partial T_j^2} < 0.$$

Step 4: Compute profit $Z_{i,j}(T_j, P)$ per unit time from table 1 and ordering quantity Q from equations (2) or (7) depending upon different scenarios.

The objective is to make total profit per unit time maximum with respect to selling price and cycle time. For that point of view, we consider order dependent credit limit M_j as per table 2.

Table 2 Credit Limit depending upon order quantity

Credit Limit (years)	Order quantity (units/order)
$M_1 = 0.082$	$1 \leq Q < 100$
$M_2 = 0.123$	$100 \leq Q < 200$
$M_3 = 0.164$	$200 \leq Q$

Next, we analysis the working of the model with numerical values for the inventory parameters as shown in table 3.

Table 3 Numerical Examples

Parameters	Examples					
	1	2	3	4	5	6
a	175000	175000	175000	175000	175000	175000

b	0.05	0.15	0.15	0.15	0.15	0.15
c	0.5	0.5	0.5	0.5	0.5	0.5
η	1.5	1.5	1.5	1.5	1.5	1.5
θ	0.05	0.05	0.05	0.05	0.05	0.05
h	8	8	8	8	8	8
A	83	78	100	46	56.6	110
C	10	10.5	10	10.5	10.35	10
I_c	0.15	0.15	0.15	0.15	0.15	0.15
I_e	0.08	0.08	0.08	0.08	0.08	0.08
T_d	0.137	0.137	0.137	0.110	0.110	0.110
Second Order (Sufficiency) Conditions						
$\left \frac{\partial^2 Z_{i,j}(T_j, P)}{\partial P^2} \frac{\partial^2 Z_{i,j}(T_j, P)}{\partial P \partial T_j} \right > 0$	1592571.08	1031615.98	1095924.00	190.49	830420.00	935996.00
$\frac{\partial^2 Z_{i,j}(T_j, P)}{\partial P^2} < 0$	-16.04	-14.17	-15.64	-14.58	-14.94	-15.49
$\frac{\partial^2 Z_{i,j}(T_j, P)}{\partial T_j^2} < 0$	-102057.42	-75361.10	-73000	-82868.41	-58000	-63300
Optimal Solution						
M_j	0.123	0.123	0.123	0.123	0.123	0.123
P	31.31	32.98	31.69	32.61	32.32	31.83
Q	122.02	125.78	148.26	101.03	114.55	157.86
T_j	0.122	0.135	0.150	0.107	0.119	0.161
Total Profit $Z_{i,j}(T_j, P)$	20286	19975	20297	20238	20277	20235
Feasible Scenario	$T_j \leq M_j \leq t_d$	$M_j \leq T_j \leq t_d$	$M_j \leq t_d \leq T_j$	$T_j \leq t_d \leq M_j$	$t_d \leq T_j \leq M_j$	$t_d \leq M_j \leq T_j$

From the table 3 it is seen that the dealer's total profit is maximum when $M_j \leq t_d \leq T_j$. The concavity of the profit function is validated in fig. 1

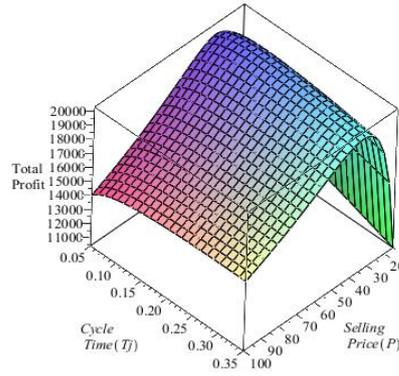


Figure. 1 Concavity of total profit w. r. t. cycle time (T_j) and selling price (P)

4. Sensitivity analysis:

Now, for example 3, we scrutinise the effects of various inventory parameters on total profit, decision variables selling price and cycle time by changing them as -20%, -10%, 10% and 20%.

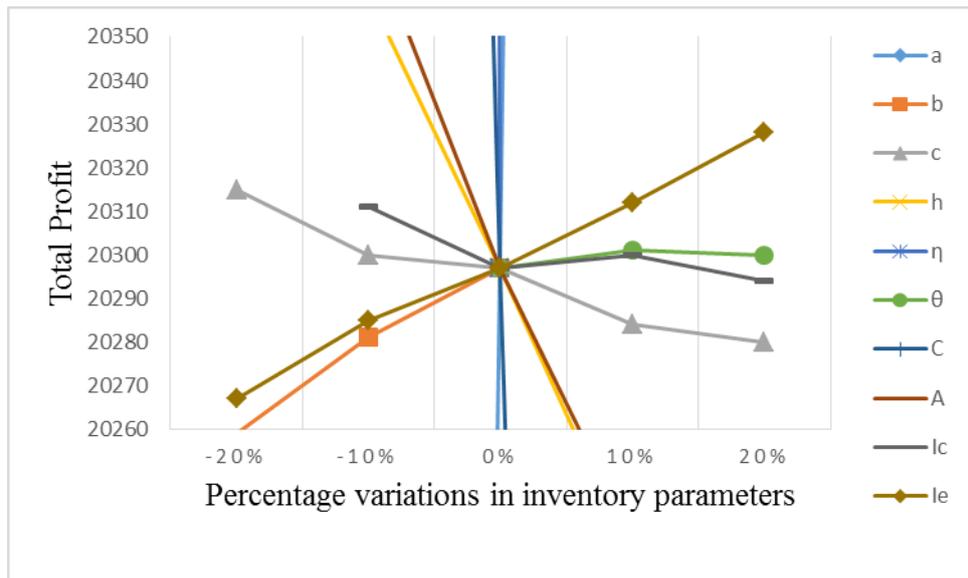


Figure. 2 Variations in total profit w. r. t. inventory parameters

From Figure. 2, it is detected that linear rate of change of demand (b) and interest earned (I_e) has huge positive impact on profit whereas scaled demand (a) and deterioration rate(θ) increases profit slowly. If holding cost (h) increases then clearly profit decreases. Mark up for selling price(η) has very large negative effect on profit. Purchase cost(C) decreases profit slightly. By growing quadratic rate of

change of demand(c), ordering cost(A) and interest charged for unsold stock by the supplier (I_c) total profit gets decreased gradually.

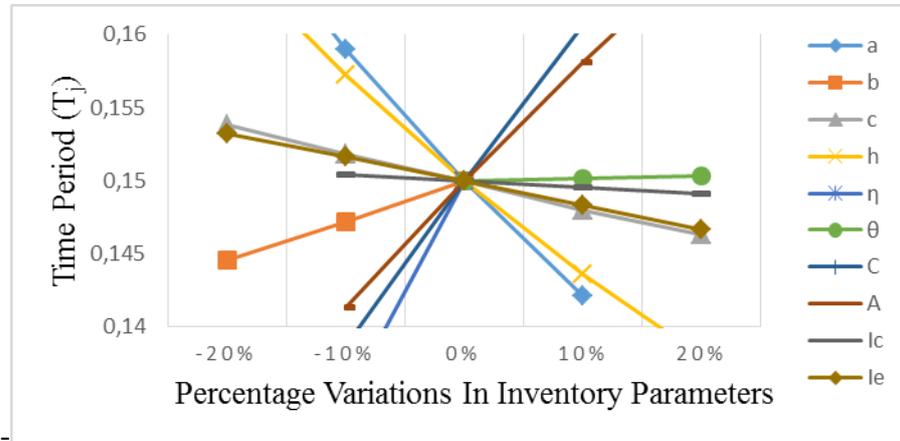


Figure. 3 Variations in time period(T_j) w. r. t. inventory parameters

From Figure. 3, it is marked that linear rate of change of demand(b), ordering cost(A), purchase cost(C), mark up for selling price(η) and deterioration rate(θ) effect positively on time period. whereas time period(T_j) has negative effect of scaled demand (a), quadratic rate of change of demand(c), holding cost (h), interest earned (I_e) and interest charged for unsold stock by the supplier (I_c).

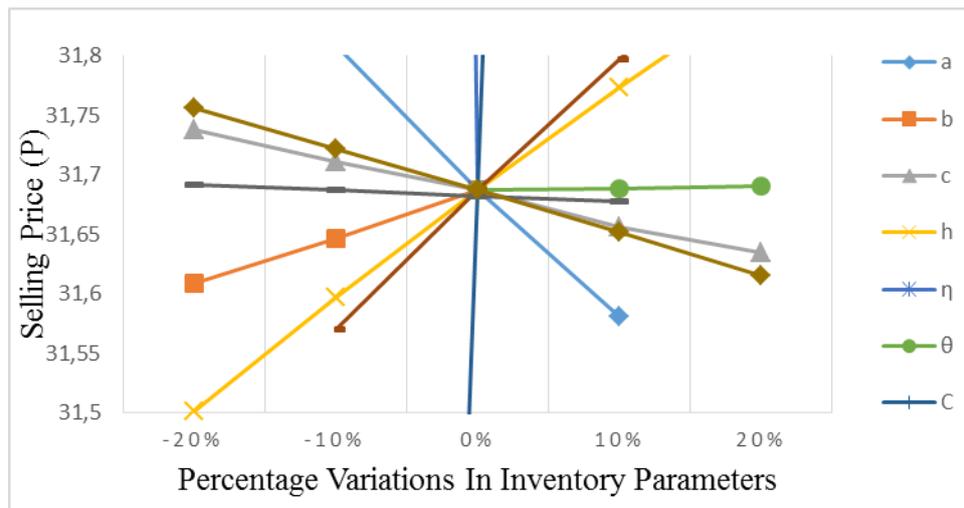


Figure. 4 Variations in selling price(P) w. r. t. inventory parameters

From Figure. 4, it is observed that selling price (P) has positive impact of linear rate of change of demand(b), holding cost(h), deterioration rate(θ), ordering cost(A), purchase cost(C), while selling price (P) has negative result of scaled demand (a), quadratic rate of change of demand(c), mark up for selling price(η), interest earned (I_e) and interest charged for unsold stock by the supplier (I_c).

5. Conclusion

A few researchers discuss the fact that there is a time length during which items preserve their quality or original condition. To reflect the real-life situation, it is therefore important to consider non-instantaneously deteriorating items in the inventory system. Moreover, use of a trade credit is a mutual payment policy in B2B (business-to-business) and B2C (business-to-customer) transactions. In this article, we develop an appropriate inventory model for non-instantaneously deteriorating items in conditions where the supplier offers the retailer several trade credits linked to order quantity. In this paper, we considered demand to be price-sensitive time quadratic. Quadratic demand initially increases with time for some time and then decreases. Some mathematical results and algorithms are established to detect the optimal pricing and ordering policies for maximizing the retailer's total profit. Furthermore, we provide numerical examples and conduct a sensitivity analysis to illustrate the proposed model. Current research have several possible extension like, model can be further generalized by taking maximum fixed-life deterioration rate. Consideration of stochastic demand instead of deterministic one would also be a worthwhile contribution.

Acknowledgement:

The authors thank anonymous reviewer and editor for their constructive comments. The first author is thankful to DST-FIST File # MSI-097 for support to carry out this research.

References:

1. A.M.M. Jamal, B.R. Sarker and S. Wang; Optimal payment time for a retailer under permitted delay of payment by the wholesaler, 2000; *International Journal of Production Economics*, 66 (1), 2000, 59–66.

2. B. Sarkar, S. Saren and L.E. Cárdenas-Barrón; An inventory model with trade-credit policy and variable deterioration for fixed-life time products, 2015; *Annals of Operations Research*, 229(1), 677–702.
3. C.K. Jaggi, S.K. Goyal and S.K. Goel; Retailer's optimal replenishment decisions with credit-linked demand under permissible delay in payments, 2008; *European Journal of Operational Research*, 190 (1), 130-135.
4. C.T. Chang and S.J. Wu, A note on optimal payment time under permissible delay in payment for products with deterioration, 2003; *Production Planning & Control*, 14, 478–482.
5. C.T. Chang, J.T. Teng and S.K. Goyal; Optimal replenishment policies for non-instantaneous deteriorating items with stock-dependent demand, 2010; *International Journal of Production Economics*, 123, 62–68.
6. C.T. Chang, L.Y. Ouyang and J.T. Teng; An EOQ model for deteriorating items under supplier credits linked to ordering quantity, 2003; *Applied Mathematical Modelling*, 27, 983–996.
7. C.T. Yang, L.Y. Ouyang, K.S. Wu and H.F. Yen; An inventory model with temporary price discount when lead time links to order quantity, 2010; *Journal of Scientific and Industrial Research*, 69, 180–187.
8. F. Raafat; Survey of literature on continuously deteriorating inventory models, 1991; *Journal of the Operational Research Society*, 42 (1), 27-37.
9. H.C. Chang, C.H. Ho, L.Y. Ouyang and C.H. Su; The optimal pricing and ordering policy for an integrated inventory model when trade credit linked to order quantity, 2010; *Applied Mathematical Modelling*, 33, 2978–2991.
10. H.J. Chang, C.H. Hung and C.Y. Dye; An inventory model for deteriorating items with linear trend demand under the condition of permissible delay in payments, 2001; *Production Planning & Control*, 12 (3), 274-282.

11. J. Wu, F.B. Al-khateeb, J.T. Teng and L.E. Cárdenas-Barrón; Inventory models for deteriorating items with maximum lifetime under downstream partial trade credits to credit-risk customers by discounted cash-flow analysis, 2016; *International Journal of Production Economics*, 171(Part 1) 105-115.
12. J. Wu, L.Y. Ouyang, L.E. Cárdenas-Barrón and S.K. Goyal; Optimal credit period and lot size for deteriorating items with expiration dates under two-level trade credit financing, 2014; *European Journal of Operational Research*, 237(3), 898-908.
13. J.J. Liao; A note on an EOQ model for deteriorating items under supplier credit linked to ordering quantity, 2007; *Applied Mathematical Modelling*, 31, 1690–1699.
14. K.J. Chung and J.J. Liao; Lot-sizing decisions under trade credit depending on the ordering quantity, 2004; *Computers & Operations Research*, 31, 909–928.
15. K.J. Chung, S.K. Goyal and Y.F. Huang; The optimal inventory policies under permissible delay in payments depending on the ordering quantity, 2005; *International Journal of Production Economics*, 95, 203–213.
16. K.R. Lou and W.C. Wang; Optimal trade credit and order quantity when trade credit impacts on both demand rate and default risk, 2013; *Journal of the Operational Research Society*, 64 (10), 1551-1556.
17. K.S. Wu, L.Y. Ouyang and C.T. Yang; An optimal replenishment policy for non-instantaneous deteriorating items with stock-dependent demand and partial backlogging, 2006; *International Journal of Production Economics*, 101, 369–384.
18. K.V. Geetha and R. Uthayakumar; Economic design of an inventory policy for non-instantaneous deteriorating items under permissible delay in payments, 2010; *Journal of Computer Applications and Mathematics*, 233, 2492–2505.
19. L.E. Cárdenas-Barrón, K.J. Chung and G. Treviño-Garza; Celebrating a century of the economic order quantity model in honor of Ford Whitman Harris, 2014; *International Journal of Production Economics*, 155, 1-7.

20. L.Y. Ouyang, C.H. Ho and C.H. Su; Optimal strategy for an integrated system with variable production rate when the freight rate and trade credit are both linked to the order quantity, 2008; *International Journal of Production Economics*, 115, 151–162.
21. L.Y. Ouyang, J.T. Teng, S.K. Goyal and C.T. Yang; An economic order quantity model for deteriorating items with partially permissible delay in payments linked to order quantity, 2009; *European Journal of Operational Research*, 194, 418–431.
22. M. Bakker, J. Riezebos and R. Teunter; Review of inventory system with deterioration since 2001, 2012; *European Journal of Operational Research*, 221 (2), 275-284.
23. M. Khouja and A. Mehrez; Optimal inventory policy under different supplier credit policies, 1996; *Journal of Manufacturing Systems*, 15, 334–339.
24. N.H. Shah and P. Mishra; Inventory model for deteriorating items with salvage value under retailer partial trade credit and stock-dependant demand in supply chain, 2010; *AMSE Journals–2010-Series: Modelling D*, 31 (2), 38-54.
25. N.H. Shah and Y.K. Shah; Literature survey on inventory models for deteriorating items, 2000; *Economic Annals*, 44 (145), 221-237.
26. N.H. Shah, D.G. Patel and D.B. Shah; Optimal pricing and ordering policies for deteriorating items with two-level trade credits under price-sensitive trended demand, 2014; *AMSE Journals–2014-Series: Modelling D*, 35 (1), 1-8.
27. N.H. Shah, H. Soni and C.K. Jaggi; Inventory models and trade credit: a review, 2010; *Control and Cybernetics*, 30 (3), 867-882.
28. P.M. Ghare and G.H. Schareder; A model for exponentially decaying inventory system, 1963; *Journal of Industrial Engineering*, 14 (5), 238-243.
29. R. Maihami and I.N. Kamalabadi; Joint pricing and inventory control for non-instantaneous deteriorating items with partial backlogging and time and price

- dependent demand, 2012; *International Journal of Production Economics*, 136, 116–122.
30. S.K. Goyal and B.C. Giri; Recent trends in modeling of deteriorating inventory, 2001; *European Journal of Operational Research*, 134 (1), 1-16.
 31. S.K. Goyal; Economic order quantity under conditions of permissible delay in payments, 1985; *Journal of the Operational Research Society*, 36 (4), 335-338.
 32. S.W. Shinn and H. Hwang; Optimal pricing and ordering policies for retailers under order-size-dependent delay in payments, 2003; *Computers and Operations Research*, 30, 35-50.