

An exact solution for the propagation of cylindrical shock waves in a rotational axisymmetric non-ideal gas with axial magnetic field and radiative heat flux

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ABSTRACT

Propagation of cylindrical shock wave in a rotational axisymmetric non-ideal gas with axial magnetic field, radiation heat flux and the components of vorticity vectors are investigated. The axial magnetic field and the fluid velocity in the ambient medium are assumed to vary and obey the power laws. An exact similarity solution is obtained. The total energy of the shock wave is not constant but increases with time. The effects of variation of parameter of non-idealness of the gas, the Alfvén-Mach number and the adiabatic exponent of the gas are investigated. It is shown that an increase in the non-idealness of the gas or the ratio of specific heats of the gas or strength of initial magnetic field decreases the shock strength but increases the shock velocity. Further it is observed an increase in the value of parameter of non-idealness of the gas and adiabatic exponent of the gas have same behavior on the flow variables and the shock strength.

1. INTRODUCTION

The problem of propagation of magnetogasdynamics shock waves in a rotating interplanetary atmosphere assumes special significance in the study of astrophysical phenomena. The experimental studies and astrophysical observations show that due to rotation of the planets and stars the outer atmosphere of the planets and stars rotates. Macroscopic motion with supersonic speed occurs in an interplanetary atmosphere with rotation and shock waves are generated. Thus, the rotation of planets or stars considerably affects the process taking place in their outer layers, therefore question connected with the explosions in rotating gas atmospheres are of definite astrophysical interest.

Chaturani [1] studied the propagation of cylindrical shock waves through a gas with solid body rotation and obtained solutions using the similarity method adopted by Sakurai [2]. Nath et al. [3] derived similarity solutions for the flow behind the shock waves propagating in a non-uniform rotating interplanetary atmosphere with increasing energy. Gangualy and Jana [4] studied a theoretical model for the propagation of strong spherical shock waves in a self-gravitating atmosphere with a radiation flux in the presence of a magnetic field. They also considered that the medium behind the shock rotated, but neglected rotation of the undisturbed medium. In all the works mentioned, the medium is taken to be a gas satisfying the equation of state of a perfect gas.

In recent years the problems of radiative energy transfer in fluids have received extensive attention as a consequence of the increasing speed of bodies through the atmosphere and the very high temperatures attained by gases in motion. The influence of radiation on a shock wave and on the flow-field behind the shock front has always been of great interest, for instance, in the field of nuclear power and space research.

Similarity solutions in radiation-gas-dynamics have been given by Marshak [5], Elliott [6] and Wang [7] in which the flow is headed by a shock wave. Elliot [6] considered the explosion problem, solved earlier by Taylor [8] by introducing the radiation flux in its diffusion approximation. Wang [7] has discussed 'piston problem' with radiation energy transfer in the thick limit and thin limit, and also the general case with the idealized 'two direction' approximation. Ashraf and Sachdev [9] did not explicitly use the radiation transfer equation but evaluated the radiation flux from the conservation equations. Their solutions, therefore, hold without any restrictions imposed on optical properties of the medium. Verma and Vishwakarma [10] extended the problem treated by Ashraf and Sachdev [9] to take into account the effects of transverse magnetic field.

Because of high pressure and density that usually occur behind a shock wave, produced by an explosion, the belief that the gas is perfect is no longer valid. In recent years, several studies are performed regarding the problem of shock waves in non-ideal gases, specifically, by Anisimov and Spiner [11], Ranga Rao and Purohit [12], Vishwakarma and Nath [13-14], Nath [15] and many others. The popular alternative to the perfect gas might be a simplified van der Waals model. Wu and Roberts [16], Roberts and Wu [17] adopted this model to discuss the shock wave theory of sonoluminescence.

In the present work, we also adopt this model of a non-ideal gas to obtain a self-similar solution for the flow behind a magnetogasdynamics cylindrical shock wave. Shock wave is assumed to be propagating in a non-ideal gas permeated by an axial magnetic field in a rotational axisymmetric flow of a gas with constant density, which has variable azimuthal and axial fluid velocities [18-20]. The fluid velocities and the axial magnetic field in the ambient medium are assumed to vary as some power of the distance from the symmetry axis. Also, the

angular velocity of rotation of the ambient medium is assumed to be obeying a power law and to be decreasing as the distance from the axis increases. It is expected that such an angular velocity may occur in the atmospheres of rotating planets and stars.

In the present problem we have extended the problem treated by Vishwakarma and Patel [21] by taking into account the axial component of fluid velocity and components of vorticity vectors. Also, we have study the effect of adiabatic exponent of the gas on the flow variables. The exact similarity solutions are obtained with the general shock conditions in place of strong shock conditions and the shock is assumed to be transparent. We have taken the similarity form for energy and radiative heat flux [9] and the ‘Product Solutions’ of Mc. Vittie [22] is used to evaluate it. We have evaluated radiation flux from conservation equations.

The effects of variation of parameter of non-idealness of the gas, the Alfvén-Mach number, and the ratio of specific heats of the gas are investigated. It is shown that an increase in the value of adiabatic exponent of the gas or in the parameter of non-idealness of the gas or the presence of magnetic field have decaying effect on shock strength. An increase in the value of Alfvén-Mach number and parameter of non-idealness of the gas have opposite behaviour on pressure and axial magnetic field; whereas they have same behaviour on radial component of fluid velocity, density, radiation heat flux and the azimuthal component of vorticity vector.

2. FUNDAMENTAL EQUATIONS OF MOTIONS AND BOUNDARY CONDITIONS

The fundamental equations governing the one-dimensional, unsteady adiabatic and cylindrically symmetric rotational flow of a non-ideal gas in the presence of an axial magnetic field in which the effect of radiation heat flux may be significant, in Eulerian coordinates, can be expressed as [15, 18-20, 23-25]

$$\frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial r} + \rho \frac{\partial u}{\partial r} + \frac{u \rho}{r} = 0, \quad (1)$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} + \frac{1}{\rho} \left[\frac{\partial p}{\partial r} + \mu h \frac{\partial h}{\partial r} \right] - \frac{v^2}{r} = 0, \quad (2)$$

$$\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial r} + \frac{u v}{r} = 0, \quad (3)$$

$$\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial r} = 0, \quad (4)$$

$$\frac{\partial h}{\partial t} + u \frac{\partial h}{\partial r} + h \frac{\partial u}{\partial r} + \frac{h u}{r} = 0, \quad (5)$$

$$\frac{\partial e}{\partial t} + u \frac{\partial e}{\partial r} - \frac{p}{\rho^2} \left[\frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial r} \right] + \frac{1}{\rho r} \frac{\partial (F r)}{\partial r} = 0, \quad (6)$$

where r and t are the independent space and time coordinates; u , v , and w are the radial, azimuthal and axial components of the fluid velocity \vec{q} in the cylindrical coordinates (r, θ, z) ; ρ , p , e , h and F are the density, the pressure, the internal energy per unit mass, the axial magnetic field and the radiation heat flux respectively.

The general figure showing the velocity components in Eulerian coordinate in the case of rotational axisymmetric one-dimensional flow is shown in Figure A [19].

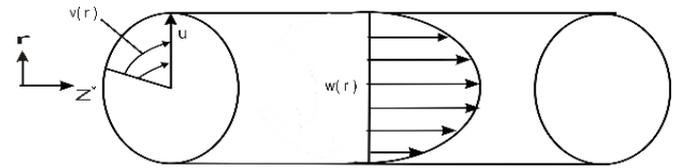


Figure A. Show the directions of velocity vector

The relation between angular velocity of the medium and the azimuthal component of fluid velocity is given by

$$v = C r, \quad (7)$$

where ‘ C ’ is the angular velocity of the medium at radial distance r from the axis of symmetry. In this case the vorticity vector $\vec{\zeta} = \frac{1}{2} \text{Curl} \vec{q}$ has the components

$$\zeta_r = 0, \quad \zeta_\theta = -\frac{1}{2} \frac{\partial w}{\partial r}, \quad \zeta_z = \frac{1}{2r} \frac{\partial}{\partial r} (r v), \quad (8)$$

The system of equations (1) - (6) should be supplemented with an equation of state. Most of the phenomena associated with shock wave arise in extreme conditions under which the ideal gas is not a sufficiently accurate description. To discover how deviations from the ideal gas can affect the flow behind a shock wave, we adopt a simple model. We assume that the gas obeys a simplified van der Waal equation of state of the form (Nath [15], Wu and Roberts [16], Roberts and Wu [17], Nath and Vishwakarma [26])

$$p = \frac{\Gamma T}{(v^* - b)}, \quad e = C_v T = \frac{p(v^* - b)}{(\gamma - 1)}, \quad (9)$$

where Γ is the gas constant, γ is the ratio of specific heats, T is the temperature of the gas, $v^* = 1/\rho$ is the specific volume, C_v is the specific heat at constant volume, constant b is the van der Waals excluded volume.

The relation between the shock velocity $W (= \frac{dR}{dt})$ and shock radius R is assumed to be given as (Ashraf and Sachdev [9])

$$W^2 = a_1^2 R^{-\lambda}, \quad (10)$$

where ‘ a_1 ’ and ‘ λ ’ are constants.

A diverging cylindrical shock wave is assumed to be propagating outwards from the axis of symmetry in the undisturbed non-ideal gas with constant density in the presence of an axial magnetic field. In order to obtain the similarity solution, it is assumed that initial angular velocity of the medium varies as $C_0 = C_a R^\delta$, where C_a and δ are constants. The flow variables immediately ahead of the shock front are

$$\mathbf{u} = \mathbf{u}_0 = \mathbf{0}, \quad \rho = \rho_0 = \text{constant}, \quad \mathbf{v} = \mathbf{v}_0 = C_a R^{\delta+1}, \quad (-1 < \delta < 0), \quad \mathbf{w} = \mathbf{w}_0 = \mathbf{w}_a R^\alpha, \quad \mathbf{h}_0 = \mathbf{h}_a R^{-\sigma},$$

$$p_0 = -\frac{\mu h_a^2}{2} R^{-2\sigma} + \frac{\rho_0 C_a^2}{(2\delta+2)} R^{2\delta+2}, \quad (11)$$

where w_a , h_a , α and σ are constants; $\sigma = -(\delta + 1)$, and subscript '0' refers the conditions immediately ahead of the shock front. The initial pressure p_0 is positive if $1 + \delta > 0$.

Ahead of the shock, the components of the vorticity vector, therefore vary as

$$\zeta_{r_0} = 0, \quad \zeta_{\theta_0} = -\frac{1}{2} \frac{\alpha w_0}{R}, \quad \zeta_{z_0} = \frac{1}{2} \frac{(\delta+2)v_0}{R}. \quad (12)$$

We assume the shock surface to be transparent; therefore, the radiation heat flux is continuous across it. Thus, the jump conditions across the magnetogasdynamics shock propagating into rotating non-ideal gas are given by (Vishwakarm and Patel [21], Vishwakarma et al. [25])

$$u_1 = (1-\beta)W, \quad \rho_1 = \frac{\rho_0}{\beta}, \quad p_1 = L \rho_0 W^2, \quad h_1 = \frac{h_0}{\beta}, \quad F_1 = F_0, \quad v_1 = v_0, \quad w_1 = w_0, \quad (13)$$

where $L = [(1-\beta) + \frac{1}{\gamma M^2} + \frac{M_A^{-2}}{2} (1 - \frac{1}{\beta^2})]$, $M = (\frac{\rho_0 W^2}{\gamma p_0})^{\frac{1}{2}}$ is the shock Mach number referred to the frozen speed of sound $(\frac{\gamma p_0}{\rho_0})^{\frac{1}{2}}$ and $M_A = (\frac{\rho_0 W^2}{\mu h_0^2})^{\frac{1}{2}}$ is the Alfvén-Mach number, and the subscript '1' denotes the condition immediately behind the shock front. The density ratio $\beta (0 < \beta < 1)$ across the shock is obtained by the

$$\beta^3 - \beta^2 \left[\frac{2}{(\gamma+1)M^2} + \frac{2\bar{b} + \gamma(1+M_A^{-2}) - 1}{(\gamma+1)} \right] + \beta \left[\frac{\bar{b} + \gamma - 2}{(\gamma+1)} \right] M_A^{-2} + \frac{\bar{b} M_A^{-2}}{(\gamma+1)} = 0, \quad (14)$$

where $\bar{b} = b\rho_0$.

Following Levin and Skopina [18] and Nath [15, 19, 20], we get the jump conditions for the components of vorticity vector across the shock front as

$$\zeta_{\theta_1} = \frac{\zeta_{\theta_0}}{\beta}; \quad \zeta_{z_1} = \frac{\zeta_{z_0}}{\beta}. \quad (15)$$

3. SELF-SIMILARITY TRANSFORMATIONS

To obtain the similarity solutions, the field variables describing the flow pattern can be written in terms of the dimensionless functions of ξ such that (Vishwakarma et al. [25], Vishwakarm and Patel [21])

$$\begin{aligned} u &= WU(\xi), \quad v = W\phi(\xi), \quad w = WV(\xi), \quad \rho = \rho_0 G(\xi), \\ p &= \rho_0 W^2 P(\xi), \quad \sqrt{\mu} h = \sqrt{\rho_0} W H(\xi), \quad e = W^2 E(\xi), \\ F &= \rho_0 W^3 \bar{F}(\xi), \end{aligned} \quad (16)$$

where U, ϕ, V, G, P, H, E , and \bar{F} are the function of ξ only, $\xi = r/R$ is the dimensionless quantity.

For existence of similarity solutions M and M_A should be constants, therefore,

$$-\lambda = 2\delta + 2, \quad \text{and} \quad \lambda = 2\sigma \quad (17)$$

Thus,

$$M^2 = \gamma^{-1} \left[-\frac{1}{2} + \frac{\rho_0 C_a^2}{(2\delta+2)\mu h_a^2} \right]^{-1} M_A^2. \quad (18)$$

By using similarity transformations from equation (16), equations (1)-(6) can be transformed and simplified to

$$(U - \xi) \frac{dG}{d\xi} + G \frac{dU}{d\xi} + \frac{GU}{\xi} = 0, \quad (19)$$

$$(U - \xi) \frac{dU}{d\xi} - \frac{\lambda U}{2} + \frac{1}{G} \left[\frac{dP}{d\xi} + H \frac{dH}{d\xi} \right] - \frac{\phi^2}{\xi} = 0, \quad (20)$$

$$(U - \xi) \frac{d\phi}{d\xi} + \left(\frac{U}{\xi} - \frac{\lambda}{2} \right) \phi = 0, \quad (21)$$

$$(U - \xi) \frac{dV}{d\xi} - \frac{\lambda}{2} V = 0, \quad (22)$$

$$\frac{dU}{d\xi} H + (U - \xi) \frac{dH}{d\xi} + \frac{UH}{\xi} - \frac{\lambda H}{2} = 0, \quad (23)$$

$$(U - \xi) \frac{dE}{d\xi} - \lambda E + \frac{P}{G} \left(\frac{dU}{d\xi} + \frac{U}{\xi} \right) + \frac{1}{G\xi} \frac{d}{d\xi} (\xi \bar{F}) = 0. \quad (24)$$

Using the similarity transformations (16), the shock conditions (13), transforms into

$$U(1) = (1-\beta), \quad G(1) = \frac{1}{\beta}, \quad P(1) = L, \quad H(1) = \frac{1}{\beta M_A}, \quad (25)$$

$$\phi(1) = \sqrt{\frac{\rho_0}{\mu}} \frac{C_a}{h_a} M_A^{-1}, \quad V(1) = \sqrt{\frac{\rho_0}{\mu}} \frac{w_a}{h_a} M_A^{-1},$$

where $\alpha = (\delta + 1) = -\sigma$ was necessary to use to obtain the similarity solution.

We assume the product solution of the 'progressive wave' in the form (c.f. Mc. Vittie [22])

$$u = \frac{a(t)}{t} r, \quad \rho = (d+1)f(t) t^{-2\epsilon} x^{d-2}, \quad p = \epsilon^2 f(t) t^{-2} b_0(t) x^d,$$

$$h = \epsilon f^{\frac{1}{2}}(t) t^{-1} c(t) x^{\frac{d}{2}}, \quad v = \frac{m(t)}{t} r, \quad w = \frac{g(t)}{t} r, \quad (26)$$

where $x = \mathbf{r}t^{-\epsilon}$, d and ϵ are constants, and $\mathbf{a}, \mathbf{f}, \mathbf{b}_0$ and \mathbf{c} are functions of t that satisfy the following equations:

$$a(t) = \frac{\epsilon d - t f'}{d} = \frac{1}{2} - \frac{tc'}{2c} = \frac{1}{2} - \frac{tm'}{2m} = 1 - \frac{tg'}{g}, \quad (27)$$

$$b_0(t) + \frac{\mu}{2} c^2(t) = \frac{(d+1)}{d\epsilon^2} (a - a^2 - ta' + m^2(t)). \quad (28)$$

It can be easily seen that these equations satisfy the equations (1) to (5) identically. After changing this solution to a similarity one, from which it follows that a should be constant $a = \left(\frac{2(1-\beta)}{\lambda+2} \right)$, we apply boundary conditions (25) and obtain the solution of the considered problem

$$U(\xi) = (1-\beta)\xi, \quad G(\xi) = \frac{1}{\beta}\xi^{d-2}, \quad P(\xi) = L\xi^d, \quad H(\xi) = \frac{1}{\beta M_A} \xi^{d/2},$$

$$\phi(\xi) = \sqrt{\frac{\rho_0 C_a}{\mu h_a} M_A^{-1}} \xi, \quad V(\xi) = \sqrt{\frac{\rho_0 w_a}{\mu h_a} M_A^{-1}} \xi. \quad (29)$$

Substituting equations (9) – (10) and (29) in equation (24), we evaluate the value of $\bar{F}(\xi)$ as

$$\bar{F}(\xi) = \frac{L\xi^{d+1}}{(\gamma-1)(d+2)} [2\beta + \lambda - 2(1-\beta)(\gamma-1)] - \frac{L\bar{b}\xi^{2d-1}}{2d(\gamma-1)} \left(d + \frac{\lambda}{\beta} \right). \quad (30)$$

Substituting equations (29) into equations (19) and (20) we obtain

$$d = \frac{2}{\beta} \quad \text{and} \quad \frac{\rho_0 C_a^2}{\mu h_a^2} M_A^{-2} = \beta(1-\beta)(d-1) - \frac{\lambda}{2}(1-\beta) + \frac{\beta d}{\gamma M^2} + \frac{M_A^{-2}}{2} d \beta. \quad (31)$$

From equations (21) – (23) and (29), we obtain

$$\lambda = 2(1-2\beta) \quad \text{and} \quad \lambda = -2\beta \quad (32)$$

It is obvious that a contradiction arises and therefore it is impossible for the solutions (29) to satisfy (19) – (24), though they can satisfy (19), (20) and (24) (i.e. conservation of mass, linear momentum and energy (c.f. Mc. Vittie [22])). For the existence of solution, we consider the physical conditions of the problem to avoid the contradiction. Since $\lambda < 0$ and $0 < \beta < 1$; therefore for the existence of solution we have taken $\lambda = 2(1-2\beta)$, thus β should be greater than $1/2$. Then, from (17) we have

$$\sigma = (1-2\beta), \quad \delta = -2(1-\beta). \quad (33)$$

Total energy E_T of the flow field behind the shock front is given as

$$E_T = 2\pi \int_0^R \left\{ \frac{\rho(1-b\rho)}{(\gamma-1)\rho} + \frac{\mu h^2}{2\rho} + \frac{1}{2}(u^2 + v^2 + w^2) \right\} \rho r dr. \quad (34)$$

Using equations (16) and (29), equation (34) becomes

$$E_T = 2\pi \rho_0 a^2 J R^{2-\lambda} = B t^{\frac{2(2-\lambda)}{2+\lambda}}, \quad (35)$$

where

$$J = \int_0^1 \left\{ \frac{L}{(\gamma-1)} \left(1 - \frac{\bar{b}}{\beta} \xi^{d-2} \right) + \frac{1}{2\beta^2 M_A^2} + \frac{(1-\beta)^2}{2\beta} + \frac{\rho_0 C_a^2}{\mu h_a^2} M_A^{-2} + \frac{\rho_0 w_a^2}{\mu h_a^2} M_A^{-2} \right\} \xi^{d+1} d\xi,$$

and B is constant. This equation shows that the total energy behind the shock increases with time. Such an increase may be obtained by time-dependent energy release from an explosive material along the symmetry axis.

Applying the similarity transformations (16) on equation (8) and using equations (19) – (24), (29), we obtain the non-dimensional components of vorticity vector $l_r = \frac{\zeta_r}{W}$, $l_\theta =$

$\frac{\zeta_\theta}{W}$, $l_z = \frac{\zeta_z}{R}$ in the flow-field behind the shock as

$$l_r = 0, \quad l_\theta = \sqrt{\frac{\rho_0}{\mu} \frac{W_a \lambda}{4h_a \beta}} M_A^{-1}, \quad l_z = \sqrt{\frac{\rho_0}{\mu} \frac{C_a (2-\lambda)}{4h_a \beta}} M_A^{-1}. \quad (36)$$

4. RESULTS AND DISCUSSION

For the density to remain finite at the center and the radiative heat flux not to be negative anywhere, the inequalities obtained from equations (26) and (30) should hold:

$$1/2 < \beta < 1, \quad (37)$$

$$\frac{\xi^{d+1}}{(\gamma-1)(d+2)} [2\beta + \lambda - 2(1-\beta)(\gamma-1)] - \frac{\bar{b}\xi^{2d-1}}{2d(\gamma-1)} \left(d + \frac{\lambda}{\beta} \right) > 0 \quad (38)$$

Inequality (37) is not only a necessary condition for the density to remain finite at the center but must also be satisfied for the existence of a shock wave. We have calculated the values of flow variables from equations (29), (30), (36) – (38) and the results are shown in figures 1, 2 and Table 1. The values of the physical parameters for calculations are taken to be $\gamma = 4/3, 5/3$; $M_A^{-2} = 0.3, 0.35$; and $\bar{b} = 0, 0.05, 0.1$; ([14, 15, 20, 26-33]). For fully ionized gas $\gamma = 5/3$ and for relativistic gases $\gamma = 4/3$. These two values of γ mark the most general range seen in stars. Therefore, it is applicable to stellar medium. The above values of M_A^{-2} are taken for calculations in the present problem as the effects of magnetic field on the flow-field behind the shock are significant when $M_A^{-2} \geq 0.01$ ([29]). The value $\bar{b} = 0$ corresponds to the ideal gas case. Our solution is the generalization of the solution obtained by Vishwakarma and Patel [21] by considering the components of vorticity vectors and the axial component of fluid velocity (see Figs. 1 (c, h, i)). Also, we have study the effect of adiabatic exponent of the gas on the flow variables.

The obtained solutions show that the component of fluid velocity, density, pressure, axial magnetic field and radiation heat flux tend to zero as the symmetry axis is approached. Also, the values of all this physical quantities decrease from the highest at the shock to zero at the symmetry axis, except for the radiation heat flux in the case of $\gamma = 5/3$ and $\bar{b} = 0.1$. For $\gamma = 5/3, \bar{b} = 0.1$ the radiation heat flux first increases, attains a maximum, and then decreases and tends to zero as the symmetry axis is approached. But the azimuthal and axial components of vorticity vector remain constant throughout the flow field.

From Table 1 and Figs. 1 and 2, it is found that the effects of an increase in the value of M_A^{-2} (i.e., in the strength of the initial magnetic field) are

(i) to increase significantly the value of β (see Table 1), i.e., decreases the shock strength;

(ii) to decrease the flow variables $U(\xi)$, $P(\xi)$, $F(\xi)$ and l_θ ; whereas to increase the flow variables $\phi(\xi)$, $V(\xi)$, $H(\xi)$ and l_z at any point in the flow field behind the shock (see Figures. 2 (a-c, e-i));

(iii) the density $G(\xi)$ decrease near shock but increase near inner boundary surface (see Fig.1(d));

(iv) increases the shock velocity W/a_1 (see Figure. 1).

The effects of an increase in the value of the parameter of non-idealness of the gas \bar{b} are

(i) to increase the value of β (see Table 1), i.e., decreases the shock strength;

(ii) to decrease the flow variables $U(\xi)$, $H(\xi)$, $F(\xi)$ and l_θ ; whereas to increase the pressure $P(\xi)$ at any point in the flow field behind the shock (see Figs. 1 (a, e, f-h));
 (iii) the density $G(\xi)$ decreases near shock but increases near inner boundary surface (see Fig. 1(d));

(iv) the flow variables $\phi(\xi)$, $V(\xi)$ and l_z remain unaffected (see Fig. 1 (b, c, i));
 (v) increases the shock velocity W/a_1 (see Figure. 1 and table 1).

Table 1. Variations of the density ratio β across the shock front, indices of shock velocity λ , total energy $2(2 - \lambda)/(2 + \lambda)$ and initial angular velocity indices δ for different values of M_A^{-2} , γ and \bar{b} .

M_A^{-2}	γ	\bar{b}	β	$\lambda = 2(1 - 2\beta)$	$2(2 - \lambda)/(2 + \lambda)$	$\delta = -2(1 - \beta)$
0.3	4/3	0	0.5296	-0.1187	2.2524	-0.9406
		0.1	0.5362	-0.1448	2.3123	-0.9275
	5/3	0	0.5440	-0.1762	2.3864	-0.9119
		0.1	0.5522	-0.2091	2.4670	-0.8954
0.35	4/3	0	0.5608	-0.2433	2.5540	-0.8783
		0.1	0.5708	-0.2833	2.6602	-0.8583
	5/3	0	0.5714	-0.2857	2.6666	-0.8571
		0.1	0.5770	-0.3082	2.7287	-0.8458
0.35	5/3	0	0.5837	-0.3349	2.8046	-0.8325
		0	0.5898	-0.3592	2.8756	-0.8203
	0.1	0.5970	-0.3880	2.9628	-0.8059	
		0.1	0.6054	-0.4216	3.0684	-0.7891

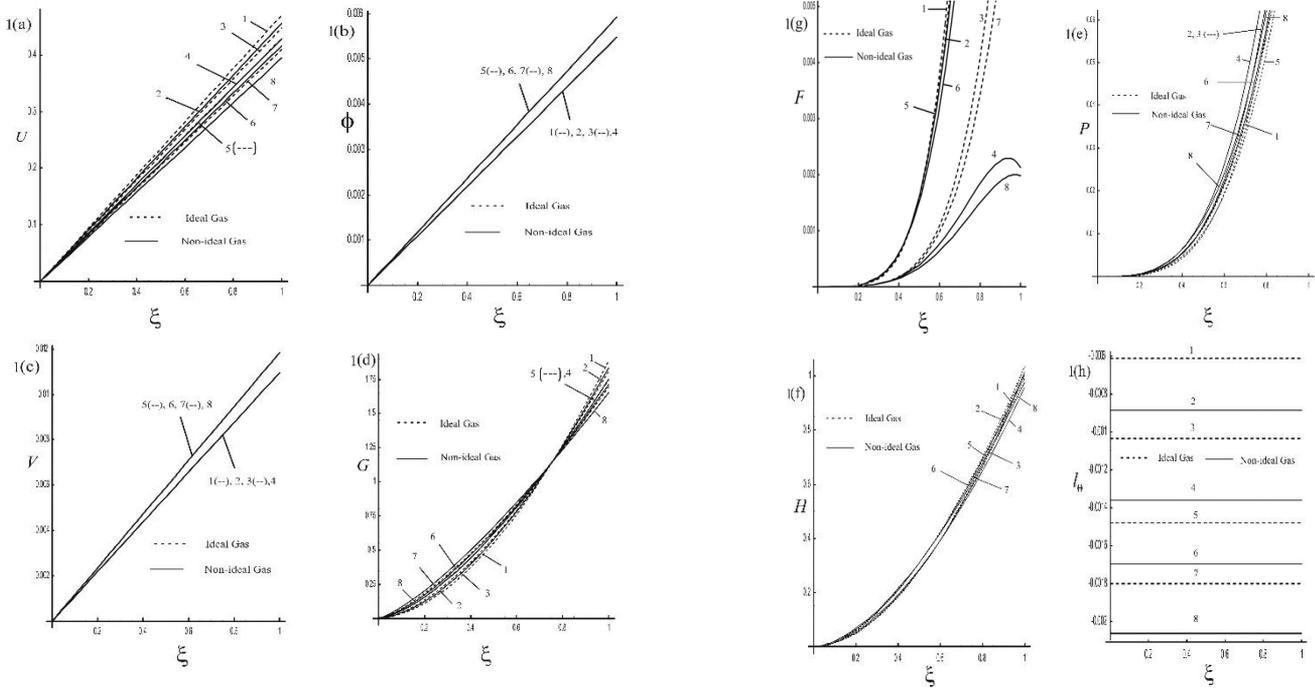


Figure 1. Variation of the flow variables with the distance in the region behind the shock front

Figure 1. (Continued)

(a) radial component of fluid velocity $U(\xi)$, (b) azimuthal component of fluid velocity $\phi(\xi)$, (c) axial component of fluid velocity $V(\xi)$, (d) density $G(\xi)$, (e) pressure $P(\xi)$, (f) axial magnetic field $H(\xi)$, (g) radiation flux $F(\xi)$, (h) non-dimensional azimuthal component of vorticity vector l_θ , (i) non-dimensional axial component of vorticity vector l_z :1. $M_A^{-2} = 0.3$, $\gamma=4/3$, $\bar{b} = 0$; 2. $M_A^{-2} = 0.3$, $\gamma=4/3$, $\bar{b} = 0.1$; 3. $M_A^{-2} = 0$, $\gamma=5/3$, $\bar{b} = 0$; 4. $M_A^{-2} = 0.3$, $\gamma=5/3$, $\bar{b} = 0.1$; 5. $M_A^{-2} = 0.35$, $\gamma=4/3$, $\bar{b} = 0$; 6. $M_A^{-2} = 0.35$, $\gamma=4/3$, $\bar{b} = 0.1$; 7. $M_A^{-2} = 0.35$, $\gamma=5/3$, $\bar{b} = 0$; 8. $M_A^{-2} = 0.35$, $\gamma=5/3$, $\bar{b} = 0.1$.

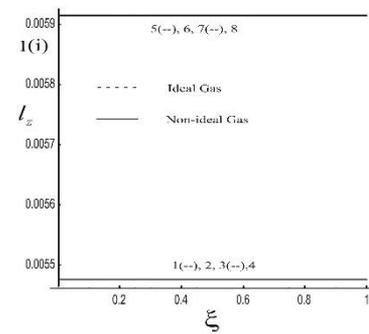


Figure 1. (Continued)

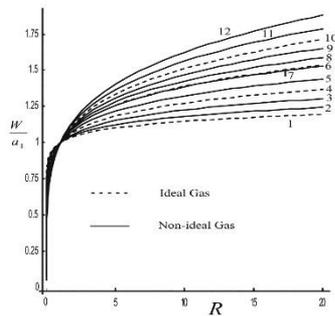


Figure 2. Variation of the shock velocity with the shock radius

1. $M_A^{-2} = 0.3, \gamma=4/3, \bar{b} = 0$; 2. $M_A^{-2} = 0.3, \gamma=4/3, \bar{b} = 0.05$;
3. $M_A^{-2} = 0.3, \gamma=4/3, \bar{b} = 0.1$; 4. $M_A^{-2} = 0.3, \gamma=5/3, \bar{b} = 0$;
5. $M_A^{-2} = 0.3, \gamma=5/3, \bar{b} = 0.05$; 6. $M_A^{-2} = 0.3, \gamma=5/3, \bar{b} = 0.1$;
7. $M_A^{-2} = 0.35, \gamma=4/3, \bar{b} = 0$; 8. $M_A^{-2} = 0.35, \gamma=4/3, \bar{b} = 0.05$;
9. $M_A^{-2} = 0.35, \gamma=4/3, \bar{b} = 0.1$; 10. $M_A^{-2} = 0.35, \gamma=5/3, \bar{b} = 0$;
11. $M_A^{-2} = 0.35, \gamma=5/3, \bar{b} = 0.05$; 12. $M_A^{-2} = 0.35, \gamma=5/3, \bar{b} = 0.1$.

The effects of an increase in value of adiabatic exponent γ are

(i) increases the value of β , i.e., decreases the shock strength (see Table 1);

(ii) to increase the pressure $P(\xi)$; whereas to decrease the flow variables $U(\xi)$, $H(\xi)$, $F(\xi)$ and l_θ at any point in the flow field behind the shock (see Figs. 1 (a, e, f-h));

(iii) the density $G(\xi)$ decreases near shock but increases near inner boundary surface (see Fig. 1(d)); (iv) the flow variables $\phi(\xi)$, $V(\xi)$ and l_z remain unaffected (see Fig. 1 (b, c, i));

(v) increases the shock velocity W/a_1 (see Fig.2).

5. CONCLUSION

The similarity solutions for the propagation of magnetogasdynamics cylindrical shock waves in a rotational axisymmetric non-ideal gas with radiation heat flux have been obtained. The radiation heat flux and other flow variables in the flow field behind the shock have been evaluated from the equations of motion without using the radiation heat transfer equations explicitly. On the basis of this work, we may draw the following conclusions:

(i) The shock strength decreases and the shock velocity increases with an increase in the strength of the ambient magnetic field or the adiabatic exponent or the non-idealness of the gas.

(ii) An increase in the value of the parameter of non-idealness of the gas \bar{b} decrease the radial component of fluid velocity, the axial magnetic field, the radiation flux and the azimuthal component of vorticity vector; whereas reverse behaviour is observed for the pressure. The density increases near shock but decreases near inner boundary surface with an increase in the value of parameter \bar{b} . Moreover, azimuthal, axial components of fluid velocity and axial component of vorticity vector remains unaffected with an increase in \bar{b} .

(iii) An increase in the strength of the ambient magnetic

field M_A^{-2} and value of parameter of non-idealness of the gas \bar{b} have same behaviour on radial component of fluid velocity, density, the radiation flux and the azimuthal component of vorticity vector; whereas they have opposite behaviour for the pressure and the axial magnetic field.

(iv) An increase in the value of adiabatic exponent γ and parameter of non-idealness of the gas \bar{b} have same behavior on all the flow variables.

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NOMENCLATURE

a	function of t
a_1	constant
B	constant
b	Van der Waals excluded volume
b_0	function of t
C	angular velocity of the medium
C_a	constant
C_v	specific heat at constant volume
c	function of t
d	constant
e	internal energy per unit mass
\bar{E}	non-dimensional internal energy per unit mass
E_T	total energy of the flow-field behind shock front
F	radiation flux
\bar{F}	non-dimensional radiation flux
f	function of t
G	non-dimensional density
g	function of t
H	non-dimensional azimuthal magnetic field
h	azimuthal magnetic field
h_a	constant
J	abbreviation
L	abbreviation
M	shock Mach number
M_A	Alfven- Mach number
m	function of t
P	non-dimensional fluid pressure
p	pressure
\vec{q}	fluid velocity
R	Shock radius
r	independent space coordinate
T	temperature of the gas
t	independent time coordinate
U	non-dimensional radial component of fluid velocity

u	radial component of fluid velocity	Γ	gas constant
V	non-dimensional axial component of fluid velocity	γ	ratio of specific heats
v	azimuthal component of fluid velocity	σ	magnetic field variation index.
v^*	the specific volume	μ	magnetic permeability
W	shock velocity	x	arbitrary function of r and t
w	axial component of fluid velocity	λ	constant
w_a	constant	ε	constant
(r, θ, z)	cylindrical coordinates	ξ	similarity variable
		ζ	vorticity vector
		$(\zeta_r, \zeta_\theta, \zeta_z)$	components of vorticity vector

Greek Letters

ρ	the fluid density
δ	constant
ϕ	non-dimensional azimuthal component of fluid velocity
α	constant
β	ratio of density across the shock front

Subscripts

0	immediately ahead the shock
1	immediately behind the shock

Superscript

'	derivative with respect to t
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