

Analysis and Application of Water Inrush Mechanism in Tunnel Based on Thin Plate Model

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Abstract

The problem of water inrush is commonplace in underground constructions. This paper using the thin plate model studied the critical thickness of tunnel aquifuge. According to the extreme equilibrium principle, the formula for calculating the minimum safe thickness of aquifuge was deduced. It further argued that the calculated plate thickness is just the minimum safe thickness of aquifuge, where the stress from underground water pressure is equal to the extreme stress supported by the extreme stress of the aquifuge. Through applying the formula to the practical cases, the authors concluded that the derived formula relatively conforms to the real constructions and can be applied into real tunnel engineering. Then after analysis and summary, the conditions and ranges of formula application were also obtained.

Key words

water inrush, thin plate model, the minimum safe thickness of aquifuge

1. Introduction

Water inrush disasters in underground constructions are very common (Zheng G, Dai X, Diao Y., 2015; Chen J, Yang R 2011; Tani M E., 2003), posing great threats on people's lives and property. In recent years, many such accidents happened amidst mining excavation, tunnel construction and underground space excavation in China (Li D, et al 2009; Li S C et al 2013; Wu

Q, et al). For instance, due to the thinning of the exposed aquifuge when Yuanlianshan Tunnel of Yuhuai Railway in China excavated at the mileage DK354+726 , a large body of water flowed into the tunnel, with the highest level of 145000m³/d, bringing about great difficulties on the construction(Ma S W., 2009; Kong W K., 2011). Besides, several accidents of water inrush occurred in China's Maluqing Tunnel of the Yiwan Railway when the tunnel excavation cut through the Karst areas, thus interrupting the construction and increasing the cost (Liu Z W., 2004; Ma S W., 2009;). The recent water inrush in the China Sichuan province Micangshan tunnel that illustrated by the Figure 1 below, suspending the work two months.

Globally, such hazards caused by water inrush are numerous. Recently, geologic researchers summarized the characteristics of water inrush occurred in China's tunnel and underground constructions. And some progresses in understanding water inrush disasters have been achieved. They proposed some geo-mechanics models from the perspective of mechanics to characterize water inrush in different conditions. In 2009, for instance, Doctor Ma Shiwei, at China Academy of Railway Sciences, studied the water inrush problem at tunnel face by proposing the thin plate model(Ma S W., 2009). What's more, Professor Li Tianbin categorized tunnel water inrush mechanisms into the following, cutting through and gushing free, hydraulic splitting of fissures, hydraulic fracturing of impermeable rock wall, and the static- dynamic disturbance of the discontinuities of rock masses (Li et al., 2015). And he gave the detailed analysis of hydraulic splitting of fissures. These greatly enhanced our understanding of the hazards from the view of mechanics. Nevertheless, drawing upon the achievements from the predecessors, many geo-mechanic models proposed by them are relatively conceptual and have not yet been put into the practices. This paper thus summarized the characteristics of several water inrush disasters in China's tunnels caused by the thinning of aquifuge. Moreover, by introducing the thin plate model in elastic mechanics, the problem of calculating the minimum safe thickness of aquifuge under the effect of relatively large hydraulic pressure in underground constructions was analyzed. Finally, the deduced formula was be verified by applying it into the actual cases.



(a) Early stage of water inrush (b) Late stage of water inrush

Fig.1. Water inrush phenomenon in tunnel

2. Analysis of Thin Plate Model for Water Inrush

Amidst the construction of tunnels and other underground projects, so frequent are the great pressure of underground water and the thin aquifuge. For instance, when the Qinling Tunnel excavated at mileage K0+298 section, water inrush happened. It was found that aquifuge was parallel to the axis of the tunnel excavation, the integrity of surrounding rock is well, the underground water pressure reached 1Mpa, and the thickness of aquifuge was less than 2m. When Mingyueshan Tunnel progressed at the range section of K5+300~ K5+410, the surrounding rock of the tunnel face was relatively vertical to the direction of the tunnel excavation. Water inrush accidents thus took place many times since the aquifuge was so thin and hard to drain the underground water. Such cases are not rare both in China and other countries. The underground water exists in above cases can be characterized in the following: aquifuge either parallel or vertical to tunnel excavation axis. Hence the thin plate model can be introduced to approximately study the minimum safe thickness of aquifuge

2.1 Analysis of Thin Plate Model for Water Inrush

In this paper, the water inrush at the tunnel vault parallel to the axis of the excavation is taken as a case. According to the thin plate theory, the formula for the minimum safe thickness of tunnel aquifuge is deduced. The Figure 2 below is the contour line of tunnel excavation. According to the contour line, the aquifuge that we choice to study is between the tunnel secondary lining and the tunnel face. The model can be selected and analyzed as seen in the Figure 3 below. The thickness of plate (h) is the smallest one, as seen in the Figure 2 above. The length (L) is the distance from the tunnel secondary lining to the tunnel face.

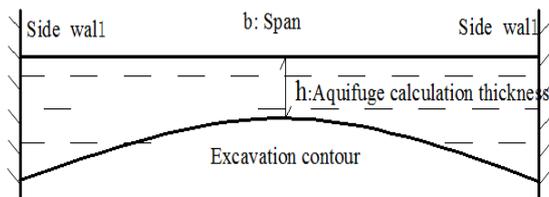


Fig.2 Excavation Contour

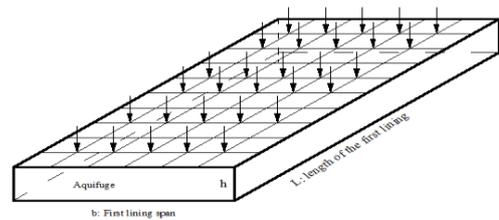


Fig3. Plate Model under Uniform Force

2.2 External Force Calculation

According to the practical tunnel construction, external forces affecting aquifuge mainly include hydrostatic pressure, aquifuge self-gravity, and surrounding rock pressure.

1) Hydrostatic Pressure

Aquifuge is affected by a certain hydrostatic pressure. Due to the complexity of tunnel geologic structure, underground or surface water supply channels exist in any underground water area. The hydrostatic pressure will offer mechanic conditions for tunnel water inrush. This must be factored into the geo-mechanic analysis modeling beam. The hydrostatic pressure is vertical to the aquifuge. And the aquifuge is deformed by the hydrostatic pressure. So the hydrostatic pressure thus must be included in pressure calculation.

2) Surrounding Rock Pressure

Due to the effect of tunnel excavation, the stress of primary rock has been almostly released after a period of time. The stress on the aquifuge is relative small and thus can be excluded from the calculation.

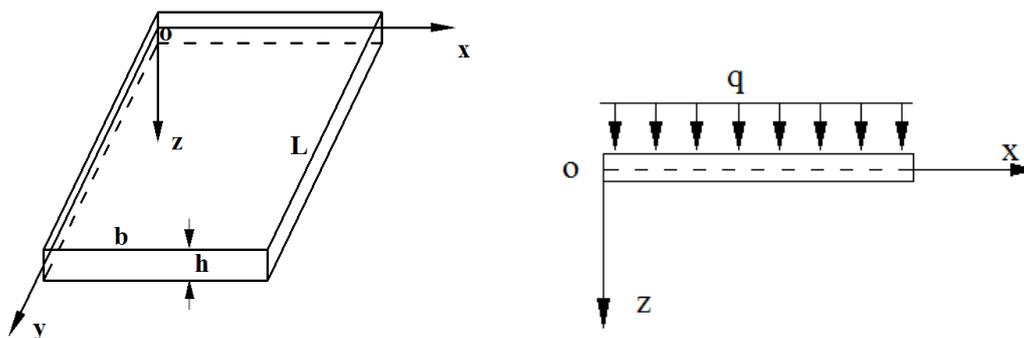
3) Aquifuge Self-gravity

Since the research subject selected here is to analyze the aquifuge stability, so the gravity of a certain thickness of aquifuge must be considered.

To sum up above, in the geo-mechanic model analysis based on the thin plate model, chief factors affecting the stability of aquifuge contain hydrostatic pressure and self gravity effect of aquifuge. Moreover, the two forces overlapped on the surface of aquifuge uniformly.

2.3 The Minimum Safe Thickness of Aquifuge

A certain thickness of plate was made. The analysis of its force under uniform force is seen in Figure 4.



(a)

(b)

Fig.4 Fixed end rectangular thin plate under uniform load

According to elastic mechanics, fixed end rectangular thin plate under uniform load(q) which is vertical to plate face down, has the deflection function below(Li et al., 2008; Xu Z L., 2006).

$$\omega = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A_{mn} \sin^2\left(\frac{m\pi x}{b}\right) \sin^2\left(\frac{n\pi y}{L}\right) \quad (1)$$

Where, L is length of the plate; b is the width of the plate; m and n are arbitrary positive integers; A_{mn} is the coefficient of the deflection function, and ω meets the boundary conditions of fixed end rectangular thin plate. According to the principle of the minimum potential energy, excluding the body stress of rectangular thin plate, the deflection coefficient A_{mn} is

$$A_{mn} = \frac{q}{D\pi^4} \frac{1}{\left(\frac{3m^4}{b^4} + \frac{2m^2n^2}{b^2L^2} + \frac{3n^4}{L^4}\right)} \quad (2)$$

Where,

$D = E\Gamma^3 / [12(1 - \mu^2)]$ is the flexural rigidity of the rectangular thin plate. E is the elasticity of rock, and μ is Poisson's ratio. Plug ω into the expressions about the stress of elastic rectangular thin plate, internal force, and deflection function. Further denote $m=n=1$, the expression of the fixed end rectangular thin plate stress and internal force under uniform load is obtained:

$$\sigma_x = -\frac{EzA_1}{1 - \mu^2} \left[\frac{2\pi^2}{b^2} \cos(2\pi x) \sin^2\left(\frac{\pi y}{b}\right) + \mu \frac{2\pi^2}{L^2} \sin^2\left(\frac{\pi x}{b}\right) \cos\left(\frac{2\pi y}{L}\right) \right] \quad (3)$$

$$\sigma_y = -\frac{EzA_1}{1 - \mu^2} \left[\frac{2\pi^2}{L^2} \sin^2\left(\frac{\pi x}{b}\right) \cos\left(\frac{2\pi y}{L}\right) + \mu \frac{2\pi^2}{b^2} \cos\left(\frac{2\pi x}{b}\right) \sin^2\left(\frac{\pi y}{L}\right) \right] \quad (4)$$

The thin plate failure is mainly caused by the bending, specifically when the bending moment of the plate at one point under the external force is over its extreme bending strength. Since the bending moment of the arbitrary cross section and the stress corresponds respectively, the formula for the minimum thickness of aquifuge is thus established. Since both σ_x and σ_y are proportional to Z , then under the effect of certain external force when the stress is maximum, the $z=-h/2$ must exist. Thus denote: $z=-h/2$.

Plug it into the above expression, then

$$\sigma_{x=-h/2} = \frac{EhA_1}{2(1 - \mu^2)} \left[\frac{2\pi^2}{b^2} \cos(2\pi x) \sin^2\left(\frac{\pi y}{b}\right) + \mu \frac{2\pi^2}{L^2} \sin^2\left(\frac{\pi x}{b}\right) \cos\left(\frac{2\pi y}{L}\right) \right] \quad (5)$$

$$\sigma_{y=-h/2} = \frac{EhA_1}{2(1-\mu^2)} \left[\frac{2\pi^2}{L^2} \sin^2\left(\frac{\pi x}{b}\right) \cos\left(\frac{2\pi y}{L}\right) + \mu \frac{2\pi^2}{b^2} \cos\left(\frac{2\pi x}{b}\right) \sin^2\left(\frac{\pi y}{L}\right) \right] \quad (6)$$

When, $\sigma_{x-max} \geq [\sigma]$ or $\sigma_{y-max} \geq [\sigma]$ the plate is destroyed.

When

$$A_{mn} h^3 = \frac{q[12(1-\mu^2)]}{E\pi^4} \frac{1}{\left(\frac{3m^4}{b^4} + \frac{2m^2n^2}{b^2L^2} + \frac{3n^4}{L^4}\right)} \quad (7)$$

Denote

$$\lambda = \frac{q[12(1-\mu^2)]}{E\pi^4} \frac{1}{\left(\frac{3m^4}{b^4} + \frac{2m^2n^2}{b^2L^2} + \frac{3n^4}{L^4}\right)} \quad (8)$$

Then

$$A_{mn} h^3 = \lambda \quad (9)$$

Combine the equations 5, 6, 7, 8 and 9, the minimum safe aquifuge thickness from the directions of x and y is obtained. It meets

$$h_x = \sqrt{\frac{\lambda E \left[\frac{2\pi^2}{b^2} \cos\left(\frac{2\pi x}{b}\right) \sin^2\left(\frac{\pi y}{L}\right) + \mu \frac{2\pi^2}{L^2} \sin^2\left(\frac{\pi x}{b}\right) \cos\left(\frac{2\pi y}{L}\right) \right]}{2[\sigma](1-\mu^2)}} \quad (10)$$

And

$$h_y = \sqrt{\frac{\lambda E \left[\frac{2\pi^2}{L^2} \cos\left(\frac{2\pi y}{L}\right) \sin^2\left(\frac{\pi x}{b}\right) + \mu \frac{2\pi^2}{b^2} \sin^2\left(\frac{\pi y}{L}\right) \cos\left(\frac{2\pi x}{b}\right) \right]}{2[\sigma](1-\mu^2)}} \quad (11)$$

Then, the minimum safe thickness aquifuge must meet

$$h = \max\{h_x, h_y\} \quad (12)$$

The above expression 12 is the formula for calculating the minimum safe thickness of aquifuge considering underground water and aquifuge gravity, by using thin plate as geomechanics model.

3. Actual Cases Application

In this section, two cases will be cited to verify the formula for calculation of the minimum safe thickness of aquifuge. Each case comes from the actual project and has happened water inrush during the tunnel construction, therefore, it has great research significance.

3.1 Qinling Tunnel Water Inrush Case

As one of tunnels in the Xihan Highway, the Qinling Tunnel, representative of the super long tunnels, is a critical and controlling project along the highway. The tunnel construction was pretty difficult, with complicated construction conditions. Due to the rich and widely distributed surface water through the tunnel, coupled with the relatively rich annual rainfall, the underground water thus has plenty of supply sources. It is thus imperative to prevent the water inrush disasters during the construction. In section 3.2, the mileage K0+298 where water inrush occurred, and the mileage around K0+395 where water inrush did not in Qinling Tunnels are taken as cases verification. Combining the actual surrounding rock conditions, the relevant construction parameters were collected to verify the formula for the minimum safe thickness of aquifuge.

3.2 Case Verification at Mileage K0+298 and K0+395 of Qinling Tunnel

Several water inrush disasters occurred at mileage K0+298. The aquifuge rock layer here is the mudstone, and the aquifer is sandstone. The aquifuge is relatively parallel to the tunnel axis, as seen in the Figure 5 below. By collecting the relevant information from the real tunnel construction, as well as from other related information sources, below is the classified relevant data related to the calculation, as seen in table 1.

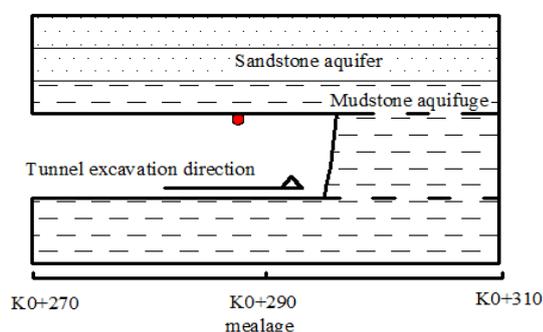


Fig. 5. Surrounding rock at mileage K0+298

According to the surveyed information, the underground water pressure is 1.02MPa. The comprehensive rock grade is III. Due to the existence of underground water, the local rock areas

were consolidated by bolt. The comprehensive tensile strength of the aquifuge takes the 1/10 of compressive strength of the aquifuge, equal to 9.2Mpa. The first lining span is about 10m.

Table 1. List of calculation parameters Qinling tunnel

Surrounding Rock Grades	q_0 (M Pa)	γ (KN/m ³)	σ (MPa)	E (GPa)	μ	b (m)	L (m)
III	1.0 2	26	9.2	15.6	0.3	10	80

Plug the above data into the expression 8 to calculate λ For

$$\lambda = \frac{q[12(1 - \mu^2)]}{E\pi^4} \frac{1}{\left(\frac{3m^4}{b^4} + \frac{2m^2n^2}{b^2L^2} + \frac{3n^4}{L^4}\right)}$$

Where $q = q_0 + \gamma h \approx q_0$,

$$\lambda \approx \frac{1.1 \times 10^7}{1.5 \times 10^{12}} \cdot \frac{10^4}{3} + \frac{(2.8 \times 10^5) \cdot h}{1.5 \times 10^{12}} \cdot \frac{10^4}{3} \approx 2.4 \times 10^{-2}$$

For $L^2 ? b^2$ So expression 10 and 11 can be expressed as follow:

$$h_x \approx \sqrt{\frac{\lambda E \frac{2\pi^2}{b^2} \cos\left(\frac{2\pi x}{b}\right) \sin^2\left(\frac{\pi y}{L}\right)}{4\sigma(1 - \mu^2)}} \quad (13)$$

$$h_y \approx \sqrt{\frac{\lambda E \mu \frac{2\pi^2}{b^2} \sin^2\left(\frac{\pi y}{L}\right) \cos\left(\frac{2\pi x}{b}\right)}{4\sigma(1 - \mu^2)}} \quad (14)$$

Thus, when $(x_l, y_l)=(0, 2/L)=(0, 40)$, h is the biggest. Plug λ and relevant parameters respectively

into the expression 13 and 14, then:

$$h_x = \sqrt{4.7} = 2.17m$$

$$h_y = \sqrt{1.37} = 1.17m$$

According to the expression 12, then,

$$h = \max\{h_x, h_y\} = 2.17m$$

Nevertheless, the actual thickness of the aquifuge at the K0+298 section is about 1.8m. And several water inrush disasters occurred at this section. This indicates the instability of the whole aquifuge, proving thus the deduced formula conforms to the real conditions. Besides, in the K0+395 section of the Qinling tunnel, the conditions are relatively the same as the above. The rock grade is the level III. The reserved excavation aquifuge became thicker, around 2.6m. Aquifuge is also mudstone, and the aquifer is still sandstone. But no water inrush disaster occurred in this place, and the whole surrounding rock is relatively stable. This also proves the conformity of the formula to the actual case.

3.3 Case Verification at Mileage DK257+600 of Maluqing Tunnel

Maluqing tunnel length is 7879m, one of the tunnels along the Yiwan railway in China. The river system and strongly developed karst along tunnel area underground is very obvious. During the tunnel construction design phase the normal discharge was 170254m³/d, the maximum discharge was 823961m³/d. Water inrush disaster occurred at the mileage DK257+600 during tunnel excavation, The maximum inflow was 720 thousand m³/h, the total water consumption was about 180 thousand m³.

Table 2: List of calculation parameters of Maluqing tunnel

Surrounding Rock Grades	q_0 (MPa)	γ (KN/m ³)	σ (MPa)	E (GPa)	μ	b (m)	L (m)
II	0.8	30	10.2	24	0.28	8.5	70

Plug the above data into the expression 8 to calculate λ

$$\lambda = \frac{q[12(1 - \mu^2)]}{E\pi^4} \frac{1}{\left(\frac{3m^4}{b^4} + \frac{2m^2n^2}{b^2L^2} + \frac{3n^4}{L^4}\right)}$$

Where $q=q_0+\gamma h \approx q_0$ Taking the data of Table 2 into Equation above

$$\lambda \approx \frac{0.8 \times 10^6 \times 12(1 - 0.28^2)}{24 \times 3.14^4 \times 10^9} \times \frac{8.5^4}{3} = \frac{4.6 \times 10^{10}}{7 \times 10^{12}} = 6.6 \times 10^{-3}$$

For

$L^2 \gg b^2$, so taking the data of Table 2 into expression 13 and 14

$$h_x \approx \sqrt{\frac{\lambda E \frac{2\pi^2}{b^2} \cos\left(\frac{2\pi x}{b}\right) \sin^2\left(\frac{\pi y}{L}\right)}{4\sigma(1-\mu^2)}} \quad (13)$$

$$h_y \approx \sqrt{\frac{\lambda E \mu \frac{2\pi^2}{b^2} \sin^2\left(\frac{\pi y}{L}\right) \cos\left(\frac{2\pi x}{b}\right)}{4\sigma(1-\mu^2)}} \quad (14)$$

Thus, when $(x, y)=(0, 2/L)=(0, 35)$, h is the biggest. Plug λ and relevant parameters respectively into the expression 13 and 14, then

$$h_x = \sqrt{2.28} = 1.5m$$

$$h_y = \sqrt{0.64} = 0.8m$$

According to the expression 12, then,

$$h = \max\{h_x, h_y\} = 1.5m$$

Nevertheless, the actual thickness of the aquifuge at the DK257+600 section is about 1.4m. And water inrush happened at this section. This also proving thus the deduced formula conforms to the real conditions.

4. Discussion

Actually, the real tunnel construction environment is very complex. The model selected in this paper is somewhat idealized. For instance, the excavated contour line of the tunnel is arc-shaped, as seen in Figure 1. The bearing capacity of the arc-shaped plate is stronger. So the calculated results tend to be conservative. Moreover, the tunnel construction is a dynamic process. In this paper, the thin plate was taken as an example to analyze the water inrush above the first lining of aquifuge. Since the study considered the whole length of the first lining, it was relative idealized, and somewhat deviated the real construction condition. It also should be noted that the taking of 1/10 of the tensile strength of that of the surrounding rock also is somewhat experience-based. Nevertheless, given the calculated result of the case tunnel, the deduced formula is correct. With regard to its application, the formula can be applied into the arc-shaped aquifuge parallel to the axis of tunnel excavation. It can also be used in the stability of tunnel sidewall, tunnel face and even any rectangular-shaped like thin rock mass under the certain external forces. It therefore can be used to analyze and deal with the such many underground

space problems. In view of taking the thin plate as the geo-mechanic model, the geologic body should be integrity and wholeness. The recommended surrounding rock grade thus should be I~III.

5. Conclusion

Based on the above analysis, the paper concluded as followings.

1) The thin plate model was used as the geo-mechanical model to analysis the deformation characteristics of the aquifuge. And The formula of the minimum safe thickness of aquifuge is obtained through limit equilibrium analysis

2) It is proved that the formula for calculating the minimum thickness of the aquifuge is correct through the two field cases study.

3) The derived formula can be used in the stability of the rectangular-shaped like thin rock mass of any aquifuge rock with relative even thickness under external forces. The recommended surrounding rock grade that the thin plate model be used to should be I~III.

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