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Chained-Form Design and Motion Planning for a Nonholonomic Manipulator

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Abstract

This paper provides a method for a controllable nonholonomic mechanical system. By use of the nonholonomy of the disk's rolling constraint in the plane and the condition of chained form conversion, a multi-joint nonholonomic manipulator, which uses only two actuators and can be converted into chained form and achieves control simplicity, is presented. The Frobenius and Chow theorems prove that the nonholonomic manipulator is controllable and satisfies the condition of chained form transformation. For further verification, the manipulator prototype with two actuators and three joints is fabricate. The validity of motion planning of the manipulator prototype is demonstrated by simulation and experimental results. The conclusion indicates that the three joints nonholonomic manipulator prototype can move to terminal configuration smoothly with singularity avoidance method and it is feasible to design a controllable and nonholonomic mechanical system.

Key words

Nonholonomic constraint, Motion planning, Nonholonomic manipulator, Chained form

1. Introduction

With the extensive application of robot, the conflict between complexity of mechanism and effectiveness of control reflects to the requirement of integrated design between mechanical structure and control scheme. Therefore, it's significant to explore the way of integration of robotic mechanics and cybernetics. This paper attempts to bridge over gap between mechanical design and nonlinear control theory of nonholonomic manipulator.

Yoshihiko Nakamura proposes nonholonomic manipulator which combines friction ball mechanism of decomposition of vector with nonholonomic control theory[1]. SØrdalen.O.J exploits the steering mechanism of trailer system based on motion planning conditions to achieve the motion control of the wheeled mobile robot with n trailers [2, 3]. Inspired by above studies, we try to design a novel nonholonomic manipulator whose kinematic model possesses a triangular structure for conversion into chained form [4].

There are many works involved in designing new mechanism using nonholonomic constraints. [5] presents a chained-form transformable WMR with four inputs. A nonholonomic parallel locating platform, which can be converted into chained form with two inputs and four outputs, is proposed in [6]. Sun hanxu develops spherical robot which uses counter weight driven by motor to change the position of center of gravity and achieve the rolling on the plane [7]. The bilinear model of the system is obtained by the rational installation of the actuator position and Euler parameter description of the configuration to convert into chained form and achieve control simplicity [8, 9]. Two sets of active steering systems are introduced to convert the system into four-control three-chain (Single-generator) [10].

On the basis of Frobenius and Chow theorems, the manipulator is controllable and the kinematic model can be diffeomorphically convertible to the chained form. The motion planning simulation and experiment results prove the validity and usefulness of the integrated design of the nonholonomic manipulator.

2. Analysis of Controllable Nonholonomic System

Generally, the nonholonomic system, whose kinematic model can express non-integrable velocity constraint, can be considered as nonholonomic system, in which the numbers of control input are less than dimensionality of reachable configuration space. Furthermore, chained form system is a standard controllable nonholonomic system with existing controllers. As long as a nonholonomic system is equal to the chained form or can be transformed into chained form, it means that existing control laws can be applied. The condition of conversion into chained form is presented as the following theorem.

Theorem 1[11, 12]**:** Let a driftless, two-input system with the triangular structure is given by:

$$\begin{cases} \dot{q}_1 = v_1 \\ \dot{q}_2 = v_2 \\ \dot{q}_i = f_i(\underline{q_{i-1}})v_1 \end{cases} \qquad (i = 3,4,\cdots,n)$$

(1)

where $f_i(\underline{q_{i-1}})$ is a smooth function in the neighborhood of $q = q_0$, q_i is generalized coordinates.

$$\frac{\partial f_i(\underline{q_{i-1}})}{\partial q_{i-1}}\Big|_{q=q_0} \neq 0$$

(2)

Then, a coordinate transformation and an input feedback transformation converting into the chained form in the neighborhood of $q = q_0$ can be expressed as follows:

$$\begin{cases} z_n = h_n(q_n) \\ z_i = h_i(\underline{q_i}) \equiv \dfrac{\partial h_{i+1}(\underline{q_{i+1}})}{\partial \underline{q_{i+1}}} \cdot \underline{f_{i+1}}(\underline{q_i}) \\ z_1 = h_1(q_1) = q_1 \end{cases}$$

(3)

$$\begin{aligned} v_1 &= u_1 \\ v_2 &= \frac{\partial h_2(\underline{q_2})}{\partial \underline{q_3}} \underline{f_3}(\underline{q_2})\, u_1 + \frac{\partial h_2(\underline{q_2})}{\partial q_2} u_2 \end{aligned} \tag{4}$$

where $i \in \{2, \cdots, n-1\}$, $\underline{q_i} = [q_i \ \cdots \ q_n]^T$, $\underline{f_i}(\underline{q_{i-1}}) = [f_i(q_{i-1}) \ \cdots \ f_n(q_{n-1})]^T$.

The nonholonomic system expressed eqs.(1) can be converted into chained form system as follows:

$$\begin{aligned} \dot{z}_1 &= v_1 \\ \dot{z}_2 &= v_2 \\ \dot{z}_i &= z_{i-1} v_1 \end{aligned} \tag{5}$$

where $i = 3, \cdots, n$.

The eqs. (3) and (4) are called coordinate transformation and feedback transformation for chained form conversion, respectively. Obviously, the nonholonomic mechanical system can be converted into chained form as long as it is designed from viewpoint of kinematic model of satisfying condition of chained form conversion, then many existing controllers can be applied to motion planning of the nonholonomic system. The method for design of controllable nonholonomic mechanical system is shown in Fig. 1.

Following the way of design as shown in Fig. 1, the first step is to mechanism design subjecting to nonholonomic constraint on the basis of theory of nonholonomic mechanics and mechanism. On the other hand, the conditions of chained form conversion should be established in the light of nonholonomic motion planning and control theory. These two independent basic designs are related to the complexity of structure of nonholonomic mechanical system and the extent of difficulty of control. After that, the designed mechanism with nonholonomic constraint constitutes motion transmission chain which is conformed to the condition of chained form conversion. And the nonholonomic mechanical system can be obtained according to the model of motion transmission chain. Evidently, the key of this method is to build the velocity constraint model which fulfills the condition of chained form conversion. Actually, the nonholonomic mechanical system is not only independently designed from the viewpoint of mechanism, but also from the integration of conditions of control. And the nonholonomic mechanical system designed will be controllable and carry out motion planning with existing control laws for chained form.

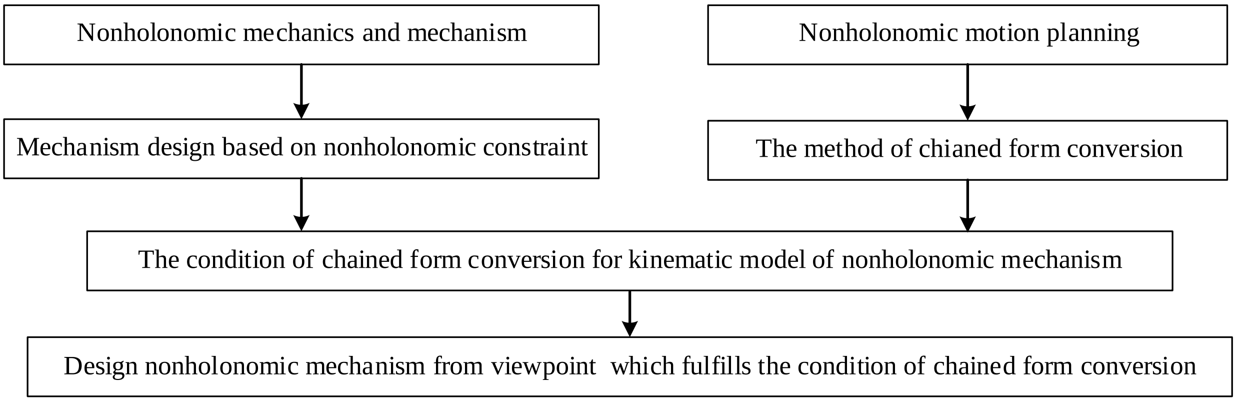


Fig. 1. Design of Controllable Nonholonomic Mechanical System

3. Nonholonomic Manipulator and Characteristic Analysis

3.1 Design of Nonholonomic Manipulator

According to the nonholonomic constraints of disk's rolling without slipping contact in the plane and the approach mentioned above, we present a nonholonomic manipulator which consists of n joints using two actuators[13]. Fig. 2 shows the structural diagram of joint i of the nonholonomic manipulator. There are two sets of friction disk mechanism in each of joint except the last one. P_1, P_2 and P_3 represent the rolling without slipping contact points between disk A and disk B. The disk B_i, with radius r_1, rotates around the fixed axis with a given angular velocity $\dot{\varphi}_i$ which makes disk A_i rotate. The angular velocity $\dot{\alpha}_i$ of disk A_i is divided into two parts, one is taken as velocity input of disk B_{i+1} through a set of bevel gear and spur gear; another makes the next joint i+1 rotate through a set of friction disk mechanism and timing belt. $\dot{\phi}_i$ and $\dot{\beta}_i$ represent the angular velocity of disk B'_i with radius r_2 and the angular velocity of disk A'_i, respectively. R and r_3 express the distance from rotate axle of disk A to rolling contact points. The manipulator's nonholonomic constrains are due to the rolling contact between disk A_i and disk B_i. The kinematic model of the manipulator with n joints and two actuators can be described as follows:

$$\begin{cases} \dot{\varphi}_1 = u_1 \\ \dot{\theta}_1 = u_2 \\ \dot{\theta}_i = k_i \cos^3 \theta_{i-1} \prod_{j=1}^{i-2} \cos \theta_j \cdot u_1 \\ \dot{\varphi}_{n-1} = k_g \prod_{j=1}^{n-2} \cos \theta_j \cdot u_1 \end{cases}$$

(6)

where $k_g = \left(\frac{r_1}{\lambda R}\right)^2$, $k_i = \frac{\lambda R}{\eta r_3}\left(\frac{r_1}{\lambda R}\right)^{i-1}, (i = 2,3,\cdots,n)$, supposed $\dot{\beta}_i / \dot{\theta}_{i+1} = \eta$ and $\dot{\alpha}_i / \dot{\varphi}_{i+1} = \lambda$.

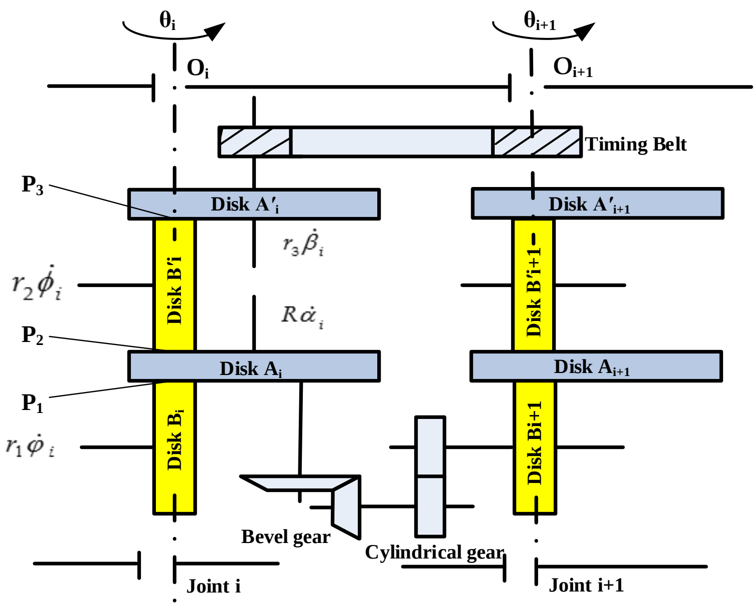


Fig.2. Structure Diagram of Joint i Mechanical System

It's noteworthy that this kinematic model has a similar structure as eqs.(3), but not identical. By setting configuration variables $q = [\varphi_{n-1}, \theta_1, \theta_2, \cdots, \theta_n]^T$, the kinematic model can be rewritten as following equations:

$$\begin{cases} \dot{q}_1 = \dot{\varphi}_{n-1} = k_g \prod_{j=1}^{n-2} \cos \theta_j \cdot u_1 \stackrel{\Delta}{=} v_1 \\ \dot{q}_2 = \dot{\theta}_1 = u_2 \stackrel{\Delta}{=} v_2 \\ \qquad \vdots \\ \dot{q}_{n+1} = \dot{\theta}_n = \frac{k_n \cos^2 \theta_{n-1}}{p_n(\underline{\theta_n})} v_1 \stackrel{\Delta}{=} f_{n+1}(\underline{q_n}) v_1 \end{cases}$$

(7)

where $p_i(\underline{\theta_i}) \stackrel{\Delta}{=} k_g \prod_{j=i}^{n-2} \cos\theta_j$, $\dot{\varphi}_n \stackrel{\Delta}{=} 0$, $(i = 2,3,\cdots,n)$. This formulation of the kinematic model has the same mathematical structure with eqs.(2). The following section will prove eqs.(7) of nonholonomy, controllability and diffeomorphism for conversion into chained form.

3.2 The Properties of Nonholonomic Manipulator

The demonstration of nonholonomy and controllability for nonholonomic manipulator will be shown in [11]. Although the Frobenius and Chow can prove it, the difficulty is how to find a good family of vector fields and give a proof for n dimensional system and not only when n is specified. The eqs.(6) can be expressed as follows:

$$\dot{q} = g_1(q) \cdot u_1 + g_2(q) \cdot u_2$$

(8)

where $g_1(q)$ and $g_2(q)$ are vector fields along with $\dot{\varphi}_1$ and $\dot{\theta}_1$ direction. This system is said to be driftless, meaning to say that the state of the system does not drift when the controls are set to zero.

Theorem 2 [11] (Frobenius) A regular distribution is integrable if and only if it is involutive.

Theorem 3 (Chow) The control system (8) is locally controllable at $q \in R^n$ if $\Delta = T_q R^n$. Δ is a distribution defined by vector fields $g_1(q)$, $g_2(q)$ and Lie products which $g_1(q)$, $g_2(q)$ span.

Let $q = [\theta_1, \theta_2, \cdots, \theta_n, \varphi_1]^T$, $g_2(q) \stackrel{\Delta}{=} X_1$, $g_1(q) \stackrel{\Delta}{=} Y_1$, we introduce the following vector fields

$$X_1 = [1, \mathbf{0_n}]^T$$

(9)

$$Y_1 = [0, k_2 c_1^3, k_3 c_2^3 C_1^1, \cdots k_n c_{n-1}^3 C_1^{n-2}, k_g C_1^{n-2}]^T$$

(10)

$$Z_1 = [X_1, Y_1] = \frac{\partial Y_1}{\partial q} X_1 - \frac{\partial X_1}{\partial q} Y_1 = \frac{\partial Y_1}{\partial \theta_1}$$

(11)

where $\mathbf{0_n} \stackrel{\Delta}{=} [0, \cdots, 0]$, $\dim \mathbf{0_n} = n$, $s_i^j = \sin^j \theta_i$, $c_i^j = \cos^j \theta_i$. And the following equations can be concluded from eqs.(6):

$$X_{i+1} = (c_i Y_i - s_i Z_i)/(c_i^2 k_{i+1})$$

(12)

$$Y_{i+1} = (s_i Y_i + c_i Z_i)$$

(13)

where $X_i = [\mathbf{0}_{i-1}, 1, \mathbf{0}_{n-i+1}]^T$, $Y_i = [\mathbf{0}_i, \mathbf{y}_i]^T$, $Z_i = [0_i, \mathbf{z}_i]^T$, $\mathbf{y}_i \overset{\Delta}{=} [k_{i+1}c_i^3, s_i\mathbf{y}_{i+1}]$, $\mathbf{z}_i \overset{\Delta}{=} [-k_{i+1}c_i^2 s_i, c_i\mathbf{y}_{i+1}]$

The vector fields that will constitute the basis for IR^{n+1} are built iteratively from the above eqs.(9)-(13).

$$X_{i+1} = \frac{([\mathbf{0}_i, c_i k_{i+1} c_i^3, c_i s_i \mathbf{y}_{i+1}]^T - [\mathbf{0}_i, -s_i k_{i+1} c_i^2 s_i, s_i c_i \mathbf{y}_{i+1}]^T)}{(c_i^2 k_{i+1})} = ([\mathbf{0}_i, 1, \mathbf{0}_{n=i}]^T \tag{14}$$

$$Y_{i+1} = ([\mathbf{0}_i, s_i k_{i+1} c_i^3, s_i \mathbf{y}_{i+1}]^T + [\mathbf{0}_i, -c_i k_{i+1} c_i^2 s_i, c_i \mathbf{y}_{i+1}]^T) = [\mathbf{0}_i, 0, (s_i^2 + c_i^2)\mathbf{y}_{i+1}]^T = [\mathbf{0}_{i+1}, \mathbf{y}_{i+1}]^T$$

(15)

$$Z_{i+1} = [X_{i+1}, Y_{i+1}] = \frac{\partial Y_{i+1}}{\partial q} X_{i+1} - \frac{\partial X_{i+1}}{\partial q} Y_{i+1} = \frac{\partial Y_{i+1}}{\partial \theta_{i+1}}$$

(16)

From eqs.(9)-(16) the determinant of vector fields for eq.(8) is:

$$\det(\Delta) \neq 0$$

(17)

where $\Delta = \{X_1, X_2, \cdots, X_n, Y_n\}$.

$$\forall X_i, X_j \in \Delta_{sub}, \quad \exists X_k \notin \Delta_{sub}$$

(18)

where $k \in \{1,2,3, \cdots n\}$ and $k \neq i, j$.

Actually, the element of set $\{X_1, X_2, \cdots, X_n, Y_n\}$ constitutes a set of basis for R^{n+1} and take the span over R^{n+1}.

Since the determinant does not vanish everywhere and the distribution which vector fields $g_1(q)$ and $g_2(q)$ span is not involutive. We have the following conclusion that the nonholonomic manipulator defined in Section 3.1 is controllable and nonholonomy.

3.3 The Diffeomorphism for Transformation

In section 3.1, eqs.(7) can be converted into chained form according Theorem 1. Consider a map:

$$h(q) : IR^{n+1} \Rightarrow IR^{n+1}$$

which takes values in the state space $q = [\varphi_{n-1}, \theta_1, \theta_2, \cdots, \theta_n]^T$ and maps them in the chained form space $z = [z_1, z_2, \cdots, z_n]^T$. We will show that $h(q)$ is a local diffeomorphism by demonstrating that the Jacobian of $h(q)$ is nonsingular.

$$J(x) \overset{\Delta}{=} \frac{\partial h(q)}{\partial q}$$

where $q = [\varphi_{n-1}, \theta_1, \theta_2, \cdots, \theta_n]^T$, since eqs.(7) has the triangular structure, we see form the map $h_i(\underline{q_i})$ in Theorem 1 that

$$J_{i,j}(\underline{q_i}) = 0 \qquad (i > j)$$

The matrix $J(x)$ is upper triangular. $J(x)$ is nonsingular if and only if the diagonal elements satisfy the condition of $J_{i,i} \neq 0$. From eqs.(3) we have that for $(i = 2, 3, \cdots, n-1)$

$$J_{i,i}(\underline{q_i}) = \frac{\partial h_{i+1}(\underline{q_{i+1}})}{\partial q_{i+1}} \frac{\partial f_{i+1}(\underline{q_i})}{\partial q_i} = J_{i+1,i+1}(\underline{q_{i+1}}) \frac{\partial f_{i+1}(\underline{q_i})}{\partial q_i}$$

(19)

$J_{i,i}(\underline{q_i}) = \dfrac{\partial h_i(q)}{\partial q_i}$ is assumed to be non-zero in a neighborhood of $q = p$. It then follows by induction that $J_{i,i}(\underline{q_i}) \neq 0$ in a neighborhood of $q = p$ if and only if

$$\left. \frac{\partial f_{i+1}(\underline{q_i})}{\partial q_i} \right|_{q=p} \neq 0 \qquad (i = 2, 3, \cdots, n-1)$$

(20)

From eqs.(7) we see that $J_{1,1}(q) \neq 0$ and $J_{n,n}(q) \neq 0$. By noting from eqs.(3), (7) and (19), we have the following equation that

$$\left. \frac{\partial f_{i+1}(\underline{q_i})}{\partial q_i} \right|_{\theta_{i-1} \neq 0} = - \left. \frac{k_i \sin(2\theta_{i-1})}{p_i(\underline{\theta_i})} \right|_{\theta_{i-1} \neq 0} \neq 0$$

(21)

Inverse Function Theorem guarantees there is a smooth inverse map:

$$h^{-}(z) : IR^{n+1} \Rightarrow IR^{n+1}$$

which maps chained form space into the state space if and only if $\theta_i \neq 0$. From eqs.(7) it can be easily checked that the transformation eqs.(4) is diffeomorfic since

$$\left. J_{2,2}(\underline{q_2}) \right|_{\theta_1 \neq 0} \neq 0$$

(22)

3.4 Prototyping of Nonholonomic Manipulator

Obviously, the configuration of the nonholonomic manipulator in section 3 has $n+1$ dimensions, which are determined by angular displacement θ_i $(i = 2,3 \cdots n)$ of joint and the angular velocity $\dot{\varphi}_{n-1}$ of disk B in the joint n-1. This multi-joint manipulator is a kind of nonholonomic manipulator, which not only has nonholonomy and controllability but also can be converted into chained form and guarantee diffeomorphic mapping for conversion. Fig. 3 shows the structural diagram of the prototype: three joints nonholonomic manipulator with two actuators. Thus, as long as it follows the control algorithm given by motion planning of chained form, the manipulator can reach desired position with velocity inputs of two actuators over the time period.

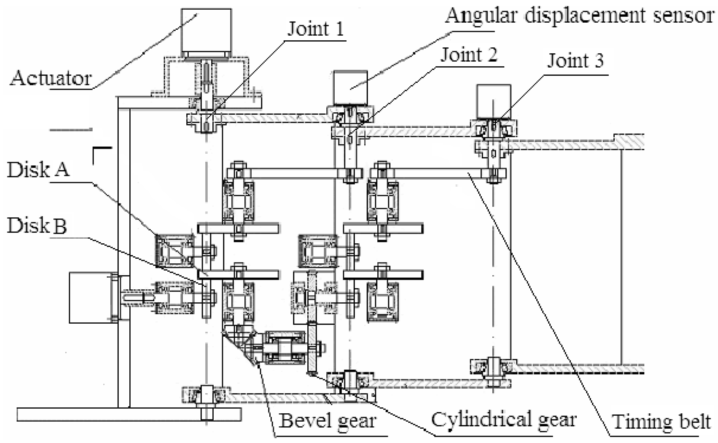


Fig. 3. Structural Diagram of Three Joints Nonholonomic Manipulator

4. Steering Nonholonomic Manipulator

4.1 Conversion into Chained Form

The nonholonomic manipulator is designed to convert into chained form and achieve control simplicity, although a little complication should be added to mechanical structures. The advantage of mathematical properties of kinematic model for nonholonomic manipulator is to satisfy conditions of chained form conversion. By setting the general coordinate for three joints nonholonomic manipulator mentioned section 3.3. the kinematic model with generalized coordinates $q = [q_1, q_1, q_1, q_1]^T = [\varphi_2, \theta_1, \theta_2, \theta_3]^T$ in eq.(21) is valid in the subspace $\theta_i \in (-\pi/2, 0) \cup (0, \pi/2)$. From theorem 1, nonlinear coordinate transformation and input feedback transformation of kinematics model of three joints nonholonomic manipulator can be expressed as follows (see [13] for details):

$$\begin{cases} z_4 = \theta_3 \\ z_3 = \dfrac{\partial z_4}{\partial q_4} f_4(q_3) = (\dfrac{k_3}{k_g}) \cos^3 \theta_2 \\ z_2 = \dfrac{\partial z_3}{\partial q_3} f_3(q_2) = \dfrac{-3k_3k_2}{k_g^2} \cos^2 \theta_1 \cos^2 \theta_2 \sin \theta_2 \\ z_1 = \varphi_2 \end{cases} \tag{23}$$

$$\begin{cases} v_1 = u_1 = \varphi_2 \\ v_2 = \dfrac{3k_2^2k_3}{k_g^3} c_1^4(2c_2s_2^2 - c_2^3)u_1 + \dfrac{6k_2k_3}{k_g^2} c_1s_1c_2^2s_2u_2 \end{cases} \tag{24}$$

4.2 Motion Planning

Nonholonomic control theory approaches are based on differential geometric framework for general nonholonomic systems. Murray and Sastry named a simple nonlinear system the chained form and proposed to use it as a canonical form for a class of driftless systems [12]. Many open loop controllers have been proposed for the driftless nonholonomic systems [14-15].

There are two major control schemes for chained form. One is open loop control, the other is feedback control. A major advantage of open loop control is that solutions for practical applications with low computational cost can be provided. Furthermore, chained form of feedback control has two drawbacks: one is that stabilizing chained system to the nonzero configuration is extremely difficult in practice, another is that obstacle or singularity avoidance problem cannot be solved some form of feedback control because of no specified extent of overshoot [16, 17]

There are many existing open loop controllers for the chained form, such as sinusoidal inputs, time polynomials inputs and piecewise constant inputs. Any of the control laws can be applied to control the nonholonomic manipulator. Time polynomial inputs are applied since inputs are easily obtained by solving simple algebraic equations. The control laws equation for three joints nonholonomic manipulator are given as follows:

$$\begin{cases} v_1 = b_0 \\ v_2 = c_0 + c_1t + c_2t^2 \end{cases}$$

(25)

This approach has advantage of a constant input on v_1 with the added simplicity of computational costs on v_2. However, one of the important things that must be considered is coordinate transformation singularities. Although the control law polynomial inputs can steer control variables z to their desired configuration, there is no guarantee that this path, when mapped back into the state variables q, will avoid the transformation singularities. That means we must check every path and ensure the existence of every variable in state space. If a singularity does really happen, some measures should be taken to find a valid path. The eq.(21) shows that transformation singularity happens at $\theta_i = 0$. The singularity in state space can be considered as obstacle. In order to solve an obstacle avoidance problem, the resultant path should remain the collision-free space. Singularity avoidance is enlightened from parking of car, the motion planning is established as follows.

Step 1 The boundary value $z(0)$ and $z(T)$ of chained form can be obtained by initial configuration $q(0)$ and desired configuration $q(T)$ from eqs.(23).

Step 2 The open loop controller, time polynomials inputs, are applied to steer control variables $z_i(t)$ form $z(0)$ to $z(T)$ over the finite time T in the first coordinate, then the coefficient b_0 is

$$b_0 = \frac{z_1(T) - z_1(0)}{T}$$

(26)

Step 3 Once T has been determined, the eqs.(25) can be integrated using the initial condition $z(0)$

$$\begin{cases} z_1(t) = z_1(0) + b_0 t \\ z_2(t) = z_2(0) + c_0 t + \frac{1}{2} c_1 t^2 + \frac{1}{3} c_2 t^3 \\ z_3(t) = z_3(0) + z_2(0)t + \frac{1}{2} c_0 t^2 + \frac{1}{6} c_1 t^3 + \frac{1}{12} c_2 t^4 \\ z_4(t) = z_4(0) + z_3(0)t + \frac{1}{2} z_2(0)t^2 + \frac{1}{6} c_0 t^3 + \frac{1}{24} c_1 t^4 + \frac{1}{60} c_2 t^5 \end{cases}$$

Step 4 Desired values $z(T)$ is taken into the following equation, and the other coefficients can be calculated

$$M\begin{bmatrix} c_0 \\ c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} z_2(T) \\ z_3(T) \\ z_4(T) \end{bmatrix} - D\begin{bmatrix} z_2(0) \\ z_3(0) \\ z_4(0) \end{bmatrix} \tag{27}$$

where $M = \begin{bmatrix} T & \frac{T^2}{2} & \frac{T^3}{3} \\ \frac{b_0 T^2}{2} & \frac{b_0 T^3}{6} & \frac{b_0 T^4}{12} \\ \frac{b_0 T^3}{6} & \frac{b_0 T^4}{24} & \frac{b_0 T^5}{60} \end{bmatrix}$, $D = \begin{bmatrix} 1 & 0 & 0 \\ b_0 T & 1 & 0 \\ \frac{b_0{}^2 T^2}{2} & b_0 T & 1 \end{bmatrix}$.

M can be always made nonsingular if and only if $z_1(0) \neq z_1(T)$. It is worth noting that if $z_1(0) = z_1(T)$, the coefficients will fail to yield a solution. The solution is that the path is divided into two parts. The first path steers state variable $z_1(t)$ from initial position $z_1(0)$ to intermediate position $z_1(\xi)$ ($z_1(0) \neq z_1(\xi)$); the second path is from $z_1(\xi)$ to desired position $z_1(T)$. An offset value from the initial position can be chosen as follows:

$$z_1(\xi) = z_1(0) + offset$$

$$offset = \frac{\sum_{i=1}^{n} z_i}{n}$$

Step 5 The path is dealt with by establishing constraint points in order to ensure the range of $z_i(t_c)$ value when it is found to pass through singularities. Eqs.(25) is integrated using the initial condition $z_i(0)$ and t_c instead of T.

$$z_i(t_c) = z_i(0) + \sum_{j=2}^{i-1} \frac{t_c^{i-j}}{(i-j)!} z_j(0) + \sum_{j=0}^{n-2} \frac{j!\, c_j t_c^{i+k-1}}{(i+k-1)!} \tag{28}$$

where $t_c \in [t_{si}, t_{sj}] \subset [0, T]$, t_{si} and t_{sj} represent boundary value at the time of entering and leaving singularities , respectively. The degree of polynomials and the number of the undetermined coefficients of v_2 expression in eqs.(25) will be increased with the increase of the numbers of constraint points. For example, v_2 in eqs.(25) expression will be quartic polynomial for three joints nonholonomic manipulator with two constraint points.

4.3 Simulation and Experimental Results

In simulation, boundary value is given as $q(0) = [1°,1°,1°]^T$, $q(T) = [30°,30°,30°]^T$ in the state space, which corresponds to $z(0) = [-0.0324, 0.7113, 0.0175]^{\mathrm{T}}$, $z(T) = [-0.532, 0.4622, 0.5236]^{\mathrm{T}}$ in chained form space. Fig.5 indicates the trajectory of Z without any constraint points. Although control parameter Z can move to setting values $z(T)$, $z_2(t)$ path in Fig.5 has two zero crossings against the conditions of keeping negative. The inverse chained form transformation singularities happen in interval between two zero crossings, as shown in Fig.5 cyan marker of $z_2(t)$ trajectory. Fig.5 indicates a constraint point (5,-0.2) is chosen in the coordinate system. The boundary value of control parameter Z is re-defined as

$z(0) = [-0.0324, 0.7113, 0.0175, -0.0324]^T$ and $z(T) = [-0.532, 0.4622, 0.5236, -0.2]^T$. It can be seen that $z_2(t)$ can still move to target values $z(T)$ smoothly and avoid the singularities.

The joints and actuator paths are computed. In Fig.6, three joints angular displacement (θ_1, θ_2, θ_3) versus time is plotted. It is clarity that the joint angles can reach the desired configuration smoothly without singularities. Fig.7 illustrates input angular (u_1, u_2) velocity of two actuators. The three joints nonholonomic manipulator motion in IR2 is presented in Fig.8. The length of each connecting rob is set to 1(unit length). It's clear that the motion is quite smooth and continuous.

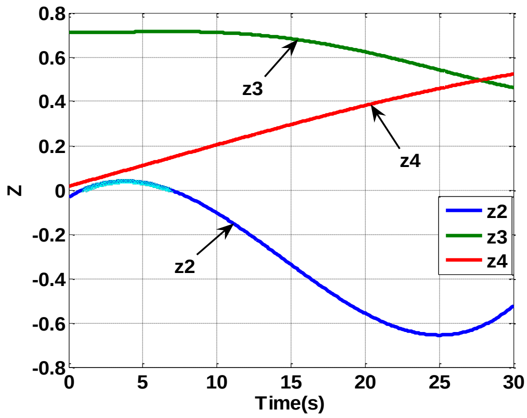


Fig.4. Control parameters z versus time

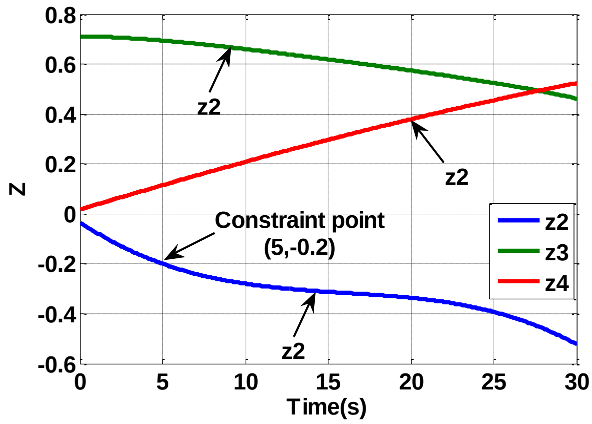


Fig.5. Control parameters z with one constraint point versus time

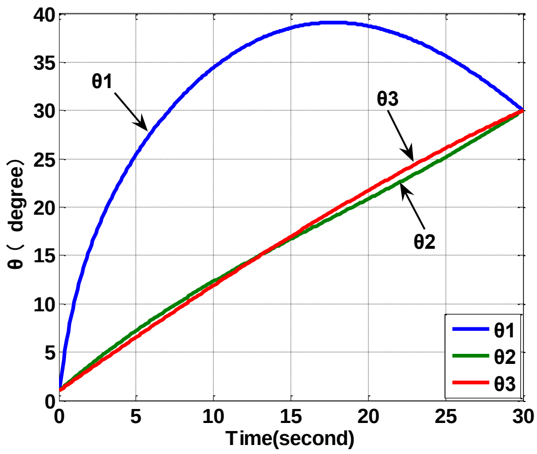


Fig.6. Joint Angles Displacement (Simulation)

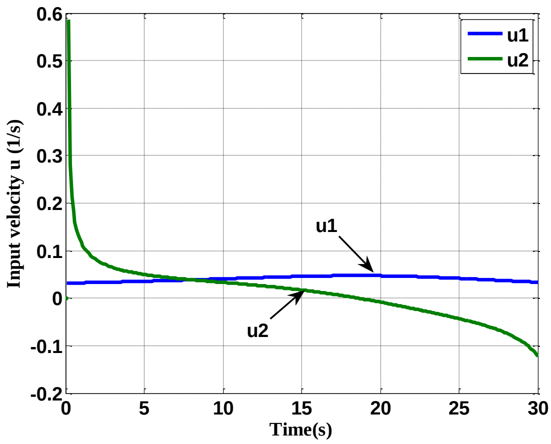


Fig.7. Input Angular Velocity (Simulation)

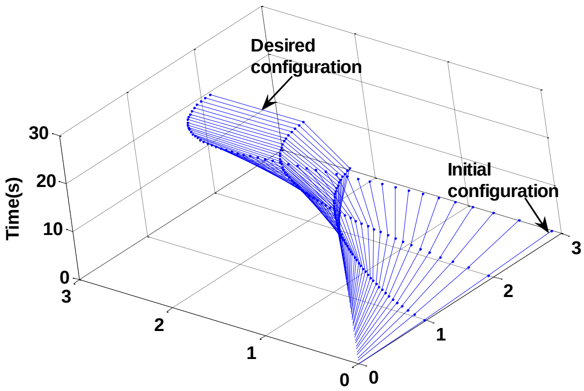
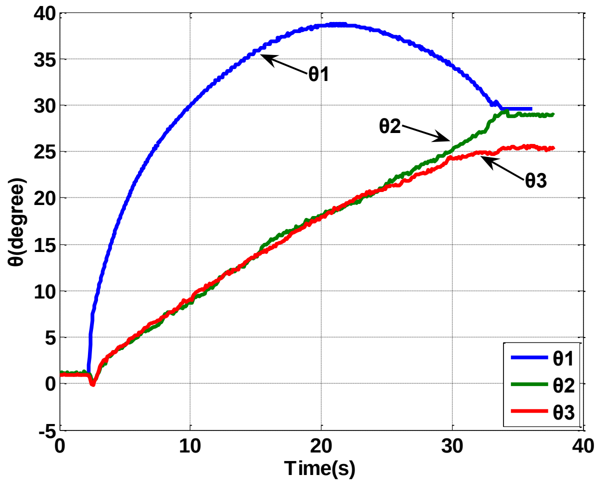


Fig.8. Three Nonholonomic Manipulator Motion in IR2

The time polynomials control law with constraint point method is experimentally tested with prototype of the three joints nonholonomic manipulator. Fig.10 illustrates actual input velocity of two servo motors, and it matches well with Fig.7. It is obvious that the velocity of two motors is precisely controlled. Fig.9 shows experimental results of joints trajectories.



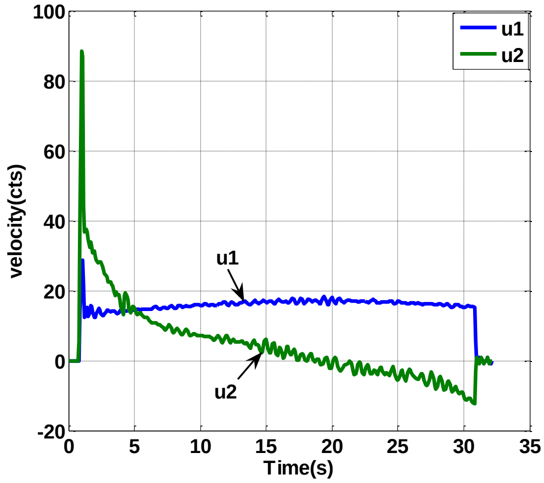


Fig.9. Joint Angles Displacement (Experiment) Fig.10. Input Angular Velocity (Experiment)

The index of error of the joint angular displacement is defined as follows:

$$E = \max(e|e = |\theta_{i,r} - \theta_{i,d}|, i \in 1,2,3) \tag{29}$$

where $\theta_{i,r}$ is experimental data of joint angular displacement, $\theta_{i,d}$ is a desired value. There is existence of 5 degree error on the θ_3 curve of Fig.9. The backlash at the bevel gear, the low stiffness of the long shaft, transmission parts with low resolution and the lack of drive torque under guaranteeing rolling without slipping condition would have caused the error.

5. Conclusion and Further Works

In this paper, we present the multi-joint nonholonomic manipulator which is designed from the viewpoint of kinematics and velocity constraints. The kinmatic model is designed focusing on meeting driftless nonholonomic constraint with triangular structure which can be converted chained form. Based on motion planning simulation and experimental results by use of prototype of three joints c manipulator, the validity and usefulness of the nonholonomic system design method.

Although the motion planning scheme in the previous section shows satisfactory results, actual motion performances are greatly affected by various error. For example, the path will be very difference when input control changes slightly. Reducing the sensitivity of input control parameter is following work.

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