AMSE JOURNALS-AMSE IIETA publication-2017-Series: Modelling B; Vol. 86; N°1; pp 1-13
Submitted Jan. 2017; Revised March 15, 2017, Accepted April 15, 2017

Effects of Variable Viscosity and Thermal Conductivity on Steady Magnetohydrodynamic Flow of a Micropolar Fluid through a Specially Characterized Horizontal Channel

*G.C. Hazarika, **B. Phukan

[*] Dibrugarh University, Dibrugarh, Assam, India

[**] Dibrugarh University, Dibrugarh, Assam, India (banditaphukan90@gmail.com)

Abstract

An investigation is made on effects of temperature dependent viscosity and thermal conductivity on MHD flow and heat transfer of a micropolar fluid through a horizontal channel, lower being a stretching sheet and upper being a permeable plate bounded by porous medium. It is assumed that both the fluid viscosity and thermal conductivity vary as inverse linear functions of temperature. Similarity transformations are used to transform the governing partial differential equations of motion into ordinary differential equations which are solved numerically with the help of shooting method. Numerical results are shown by plotting of graphs for the velocity, temperature and micro-rotation profiles. The values of coefficient of skin friction and Nusselt number are presented in the form of tables for different values of parameters which reflects the flow and heat transfer characteristics of the fluid. It is seen that viscosity parameter and thermal conductivity parameter as well as other parameters have significant effects on the flow and heat transfer of the flow.

Keywords

Magnetohydrodynamic, micropolar fluid, permeable plate, shooting method.

1. Introduction

The theory of micropolar fluids, first discussed by Eringen [1] has become a field of active research for few decades due to its increasing practical importance. The study of micropolar fluid flowing between parallel porous and non-porous boundaries is of practical importance in the design of thrust bearings, oil exploration, radial diffusers, lubrication of porous bearings etc.

Micropolar fluids are those fluids which contain micro-constituents that can go through rotation. These fluids can be defined as viscous, non-Newtonian fluid whose fluid elements exhibit rotation affecting the flow and heat transfer of the fluid.

Ishak et al.[2] studied the magnetohydrodynamic flow of a micropolar fluid towards a stagnation point on a vertical surface. Hady [3] discussed on the heat transfer to a micropolar fluid from a non-isothermal stretching sheet with injection. Numerical solution of free convection MHD micropolar fluid flow between two parallel porous vertical plates was investigated by Bhargava et al.[4]. Ojjela and Josyula [5] studied about the micropolar fluid flow between two porous parallel plates with suction and injection. Borgohain and Hazarika [6] also studied the effects of variable viscosity and thermal conductivity on hydromagnetic boundary layer micropolar fluid flow over a stretching surface embedded in a non-Darcian porous medium with radiation sheet. Borthakur and Hazarika [7] analyzed the effects of variable viscosity and thermal conductivity on flow and heat transfer of a stretching surface of rotating micropolar fluid with suction and blowing. The effects of magnetohydrodynamic on thin films of unsteady micropolar fluid through a porous medium were investigated by Rahman [8]. Hazarika and Phukan [9] examined the effects of variable viscosity and thermal conductivity on magnetohydrodynamic free convection flow of a micropolar fluid past a stretching plate through porous medium with radiation, heat generation and Joule dissipation. studied the effects of variable viscosity and thermal conductivity on MHD flow through a horizontal channel, lower being a stretching sheet and upper being a permeable plate.

In most of the existing studies on micropolar fluid the fluid properties are assumed to be constant including or excluding MHD effects for various flow situations. However, it is known from the work of Herwig and Gersten [11] that these physical properties may change with temperature especially for fluid viscosity and thermal conductivity. Khound and Hazarika [10] considered a MHD flow of general viscous incompressible fluid. The present work is an extended work of Khound and Hazarika [10] for micropolar fluid and the purpose of this work is to investigate the effect of temperature dependent viscosity and thermal conductivity on boundary layer flow and heat transfer of micropolar fluid through a specially characterized horizontal channel consisting of two plates, lower being a stretching sheet and upper being a permeable plate with temperature dependent viscosity and thermal conductivity. The fluid viscosity and thermal conductivity are assumed to vary as an inverse linear function of temperature. The governing partial differential equations of motion are reduced to ordinary differential equations using similarity transformations and then solved numerically for certain boundary conditions applying shooting method.

2. Mathematical formulation of the problem

Consider a steady MHD flow of a viscous incompressible micropolar fluid in the presence of transverse magnetic field between two horizontal plates, lower one is taken as stretching sheet and upper is a solid porous plate. The fluid motion is due to stretching of the lower plate with injection applied at the upper plate. Let, (u,v) be the velocity component along (x,y) direction respectively where x-axis is taken along the lower plate and y-axis is normal to the plates as shown in Figure 1. The plates are situated at y = 0 and y = h with different temperatures at $T = T_w$ and $T = T_\infty$ respectively. The lower plate is stretched by introducing two equal and opposite forces so that the position of the plate remains same. The fluid is injected through the upper solid plate with constant velocity V_0 and let, N be the micro-rotation component. The induced magnetic field is negligible which is valid for low magnetic Reynolds number. The fluid properties are assumed to be constant, except for the fluid viscosity and thermal conductivity which are assumed to be inverse linear functions of temperature. It is also assumed that the external electric field is zero and the electric field due to polarization of charges is negligible.

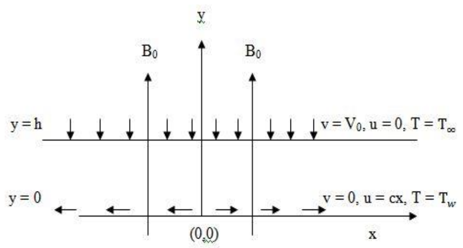


Fig.1. Flow configuration

Under the above consideration, the governing equations become:

The equation of continuity:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

The momentum equation in x-direction:

$$\rho\left[u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right] = -\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) + 2\frac{\partial \mu}{\partial x}\frac{\partial u}{\partial x} + \frac{\partial \mu}{\partial y}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) + k\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial N}{\partial y}\right) - \sigma B_0^2 u - \frac{\mu}{m^*} \tag{2}$$

The momentum equation in y-direction:

$$\rho\left[u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right] = -\frac{\partial p}{\partial y} + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) + 2\frac{\partial \mu}{\partial y}\frac{\partial v}{\partial y} + \frac{\partial \mu}{\partial x}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) + k\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial N}{\partial x}\right) - \frac{\mu}{m^*}v \tag{3}$$

The angular momentum equation:

$$\rho j\left[u\frac{\partial N}{\partial x}+v\frac{\partial N}{\partial y}\right]=-k\left(2N-\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)+\gamma\left(\frac{\partial^2 N}{\partial x^2}+\frac{\partial^2 N}{\partial y^2}\right) \quad (4)$$

The energy equation:

$$\rho C_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=\frac{\partial \lambda}{\partial y}\frac{\partial T}{\partial y}+\lambda\frac{\partial^2 T}{\partial y^2}+(\mu+k)\left[2\left(\frac{\partial u}{\partial x}\right)^2+2\left(\frac{\partial u}{\partial y}\right)^2+\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)^2\right]+\sigma(uB_0)^2 \quad (5)$$

The boundary conditions are:

$$\left.\begin{aligned} &u=cx,\ v=0,\ T=T_w,\ N=-\frac{1}{2}\frac{\partial u}{\partial y}\quad at\ y=0\\ &u=0,\ v=-V_0,\ T=T_\infty,\ N=0\quad at\ y=h\end{aligned}\right\} \quad (6)$$

where ρ is the fluid density, μ is the co-efficient of dynamic viscosity, j is the micro-inertia per unit mass, λ is the thermal conductivity, C_p is the specific heat at constant pressure, m^* is the coefficient of permeability, B_0 is the magnetic field, σ is the electrical conductivity, γ is the spin gradient viscosity, κ is the kinematic micro-rotation viscosity and c is a stretching parameter.

Following Lai and Kulacki [12], the fluid viscosity is assumed as:

$$\left.\begin{aligned} &\frac{1}{\mu}=\frac{1}{\mu_\infty}[1+\delta(T-T_\infty)]\\ &\text{or}\ \ \frac{1}{\mu}=a\ (T-T_r)\end{aligned}\right\} \quad (7)$$

$$a=\frac{\delta}{\mu_\infty}\ \text{ and }\ T_r=T_\infty-\frac{1}{\delta}$$

where, μ_∞ is the viscosity at infinity, a and T_∞ are constants and their values depend on the reference state and thermal property of the fluid. T_r is transformed reference temperature related to viscosity parameter, δ is a constant based on thermal property of the fluid and $a<0$ for gas, $a>0$ for liquid.

Similarly, we assume (Khound and Hazarika, [10]) the thermal conductivity as:

$$\left.\begin{aligned} &\frac{1}{\lambda}=\frac{1}{\lambda_\infty}[1+\xi(T-T_\infty)]\\ &\frac{1}{\lambda}=b(T-T_k)\end{aligned}\right\} \quad (8)$$

$$b=\frac{\xi}{\lambda_\infty},\ \text{and}\ T_k=T_\infty-\frac{1}{\xi}$$

where b and T_k are constants and their values depend on the reference state and thermal properties of the fluid, i.e. on ξ.

Let us introduce the following similarity transformations and parameters:

$$\left.\begin{aligned} &u=cxf'(\eta),\ v=-chf(\eta)\\ &\eta=\frac{y}{h}\ \text{ and }\ \beta=\frac{x}{h}\end{aligned}\right\} \quad (9)$$

$$\theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty},\ N=\frac{cx}{h}g(\eta)$$

Using the transformations the equation of continuity (1) is satisfied identically and rest of the equations (2), (3), (4) and (5) respectively reduced to:

$$-\frac{1}{\rho}\frac{1}{h}\frac{\partial p}{\partial \beta} = c^2\beta h[f'^2 - ff'' + \frac{1}{Re}\frac{\theta_r}{(\theta-\theta_r)}f''' - \frac{1}{Re}\frac{\theta_r}{(\theta-\theta_r)^2}\theta' f'' - \frac{K}{Re}(f''' + g') + \frac{M^2}{Re}f' - \frac{1}{mRe}\frac{\theta_r}{(\theta-\theta_r)}f'] \tag{10}$$

$$-\frac{1}{\rho}\frac{1}{h}\frac{\partial p}{\partial \eta} = c^2 h\left[ff' - \frac{1}{Re}\frac{\theta_r}{(\theta-\theta_r)}f'' + \frac{2}{Re}\frac{\theta_r}{(\theta-\theta_r)^2}\theta' f' + \frac{K}{Re}(f'' - g') + \frac{1}{mRe}\frac{\theta_r}{(\theta-\theta_r)}f\right] \tag{11}$$

$$g'' = \frac{1}{G}(2g + f'') + \frac{1}{\Delta}(f'g - fg') \tag{12}$$

$$\theta'' = \frac{\theta'^2}{(\theta-\theta_k)} + PrRe\left(\frac{\theta-\theta_k}{\theta_k}\right)f\theta' + PrEc\left(\frac{\theta-\theta_k}{\theta_k}\right)\left(K - \frac{\theta_r}{(\theta-\theta_r)}\right)\left(\frac{4h^2}{x^2}f'^2 + f''^2\right) + PrM^2Ecf'^2 \tag{13}$$

where,

$\theta_r = \frac{T_r - T_\infty}{T_w - T_\infty} = \frac{1}{\delta(T_w - T_\infty)}$ and $\theta_k = \frac{T_k - T_\infty}{T_w - T_\infty} = \frac{1}{\xi(T_w - T_\infty)}$ are dimensionless reference temperature corresponding to viscosity and thermal conductivity respectively. It is to be noted that these values are negative for liquids and positive for gases when $(T_w - T_\infty)$ is positive (Lai and Kulacki [12]).

Here the dimensionless parameters are defined as:

$Re = \frac{\rho c h^2}{\mu_\infty}$ is the stretching Reynolds number

$G = \frac{c\gamma}{\kappa \nu_\infty}$ is the micro-rotation parameter

$K = \frac{\kappa}{\mu_\infty}$ is the coupling constant parameter

$\Delta = \frac{\gamma}{\mu_\infty j}$ is the material constant

$M = \left(\frac{\sigma}{\mu}\right)^{\frac{1}{2}} B_0 h$ is the Hartmann number

$m = \frac{m^*}{h^2}$ is the permeability number

$Pr = \frac{\mu_\infty C_p}{\lambda_\infty}$ is the Prandtl number

$Ec = \frac{U^2}{C_p(T_w - T_\infty)}$ is the Eckart number

Differentiating equation (10) w.r.t. η, we get:

$$-\frac{1}{\rho}\frac{1}{h}\frac{\partial}{\partial \eta}\left(\frac{\partial p}{\partial \beta}\right) = c^2\beta h[f'f'' - ff''' + \frac{1}{Re}\frac{\theta_r}{(\theta-\theta_r)}f^{iv} - \frac{2}{Re}\frac{\theta_r}{(\theta-\theta_r)^2}\theta' f''' + \frac{2}{Re}\frac{\theta_r}{(\theta-\theta_r)^3}(\theta')^2 f'' - \frac{1}{Re}\frac{\theta_r}{(\theta-\theta_r)^2}\theta'' f'' - \frac{K}{Re}(f^{iv} + g'') + \frac{M^2}{Re}f'' + \frac{1}{mRe}\frac{\theta_r}{(\theta-\theta_r)^2}\theta' f' - \frac{1}{mRe}\frac{\theta_r}{(\theta-\theta_r)}f''] \tag{14}$$

Again, differentiating equation (11) w.r.t. β, we get:

$$-\frac{1}{\rho}\frac{1}{h}\frac{\partial}{\partial\beta}\left(\frac{\partial p}{\partial\eta}\right)=0 \qquad (15)$$

Now, eliminating '*p*' from equation (14) and (15), we get:

$$f^{iv}=\frac{1}{\left[1-K\left(\frac{\theta-\theta_r}{\theta_r}\right)\right]}\left[Re\frac{\theta-\theta_r}{\theta_r}(ff'''-f'f'')+\frac{2\theta' f'''}{(\theta-\theta_r)}-\frac{2(\theta')^2 f''}{(\theta-\theta_r)^2}+\right.$$

$$\left.\frac{\theta'' f''}{(\theta-\theta_r)}+K\frac{\theta-\theta_r}{\theta_r}g''-M^2\frac{\theta-\theta_r}{\theta_r}f''+\frac{1}{m}f''-\frac{1}{m}\frac{\theta' f'}{(\theta-\theta_r)}\right] \qquad (16)$$

The boundary conditions (6) reduce to:

$$\left.\begin{array}{l}\text{at } \eta=0,\ f=0,\ f'=1,\ \theta=1,\ g=-\frac{1}{2}f'' \\ \text{at } \eta=1,\ f'=\psi,\ \theta=0,\ g=0\end{array}\right\} \qquad (17)$$

where, $\psi=\frac{V_0}{ch}$ is a constant for a given flow.

The important physical quantities of interest in this problem are skin friction coefficient C_f and the Nusselt number *Nu* which are the rate of plate shear stress and the rate of heat transfer for the surface respectively. These are defined as:

$$C_f=\frac{2\tau_w}{\rho_\infty U^2}$$

where, τ_w is the shear stress which is given by

$$\tau_w=\left[(\mu+\kappa)\frac{\partial u}{\partial y}+\kappa N\right]_{y=0}$$

and, $Nu=\frac{xq_w}{\lambda_\infty(T_w-T_\infty)}$

where, q_w is the heat transfer from the surface given by,

$$q_w=-\left(\lambda\frac{\partial T}{\partial y}\right)_{y=0}$$

Thus, $C_f Re^{\frac{1}{2}}=\frac{1}{\beta}\left(\frac{2\theta_r}{\theta_r-1}+K\right)f''(0)$.

$Nu=-\beta\frac{\theta_k}{\theta_k-1}\theta'(0)$.

3. Results and discussion

3.1 Discussion

The differential equations (12), equation (13) and equation (16) together with the boundary conditions (17) are solved numerically applying shooting technique in conjunction with Runge-Kutta fourth order method. The numerical values of different parameters are taken as Re = 1, M = .5, Pr = .7, Ec = .01, K =.1, $\theta_r=-10$, $\theta_k=-10$, G = 1, Δ= .5, m =1 unless otherwise

mentioned.

The variations in velocity profile, temperature profile and micro-rotation profile are presented in figure 2 to figure 13 for the variation of different parameters. Variations in velocity distribution are shown in figure 2 to figure 5. From the figure 2 and figure 4 it is clear that velocity (f') increases near the lower stretching sheet but it decreases towards the upper plate and velocity (f) increases with the increase of viscosity parameter θ_r and coupling constant parameter K but its variation is not so significant. Figure 3 shows that there is no significant variation in velocity ($f'$and f) with the variation of thermal conductivity parameter θ_k. In figure 5 it can be seen that velocity (f') first decreases up to a certain point after that point it begins to increase and velocity (f) decreases with the increasing values of Hartmann number M. The applied magnetic field produces the Lorentz force due to the interaction with flowing fluid particles. This force acts opposite to the motion of the fluid as a result the velocity of the fluid decreases.

The variation of dimensionless temperature profile for various values of viscosity parameter θ_r, thermal conductivity parameter θ_k, Hartmann number M and coupling constant parameter K are shown in the figure 6 to figure 9. It is seen from the figure 6, figure 8 and figure 9 that the temperature increases with the increasing values of viscosity parameter θ_r, Hartmann number M and coupling constant parameter K. Increase of viscosity parameter θ_r and M lead to increase of viscous force and Lorentz force respectively. These forces give resistance to the flow of the fluid so the fluid has to work done to overcome these resistances. These energies transformed into thermal energy resulting the temperature of the fluid increases. Figure 7 shows that temperature of the fluid decreases as the thermal conductivity parameter increases. Due to the increase of thermal conduction the transportation of heat from a region of higher temperature to the region of lower temperature increases so the temperature of the fluid within the boundary layer decreases.

Figure 10 to figure 13 display the graphs obtained for micro-rotation profile with the variation of viscosity parameter θ_r, thermal conductivity parameter θ_k, Hartmann number M, coupling constant parameter K. From the figure 10, figure 12 and figure 13 it is observed that micro-rotation increases significantly as the viscosity parameter θ_r, Hartmann number M and coupling constant parameter K while in figure 11 it can be seen that there is no significant variation on micro-rotation profile with the variation of θ_k. Due to the increase of Hartmann number M and viscosity parameter θ_r the temperature of the fluid increases and when temperature increases the bondings holding the fluid molecules break down so they can move easily hence the micro-rotation increases.

The missing values $f''(0)$, $f'''(0)$, $g'(0)$, $\theta'(0)$ and the coefficient of skin friction C_f which is proportional to the wall shear stress and the Nusselt number *Nu* which represents the heat transfer rate are estimated for various combinations of parameters and presented in table 1 and 2. From these tables we have seen that $f''(0)$ and $g'(0)$ increase and $f'''(0)$ and $\theta'(0)$ decrease with the increasing values of viscosity parameter θ_r and thermal conductivity parameter θ_k whereas $f''(0)$ and $g'(0)$ decrease and $f'''(0)$ and $\theta'(0)$ increase on increasing coupling constant K and micro-rotation parameter G. From the same tables it is found that increasing values of coupling constant parameter K and thermal conductivity parameter θ_k reduce the Nusselt number *Nu* but enhance the skin friction coefficient C_f and both skin friction coefficient C_f and Nusselt number *Nu* decrease with the increase of micro-rotation parameter G. From the tables we have also found that skin friction coefficient C_f decreases and Nusselt number *Nu* increases as viscosity parameter θ_r increases.

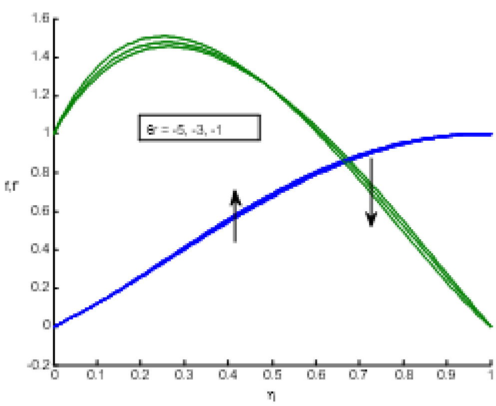


Fig. 2. Velocity profile for different θ_r

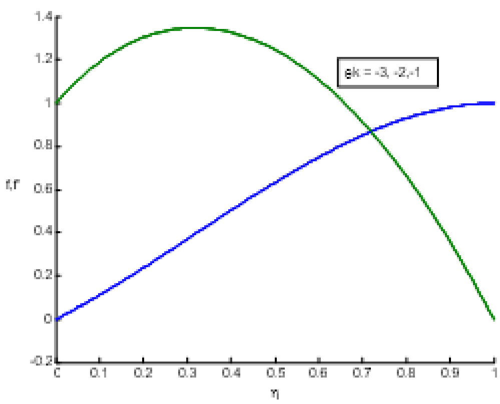
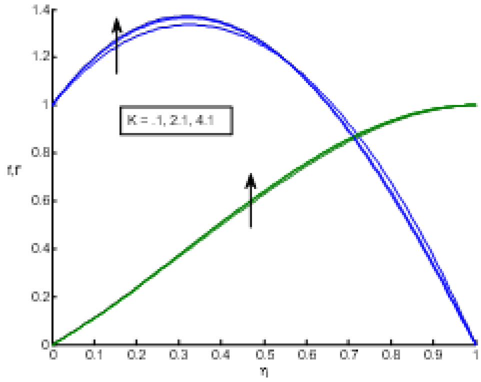


Fig. 3. Velocity profile for different θ_k



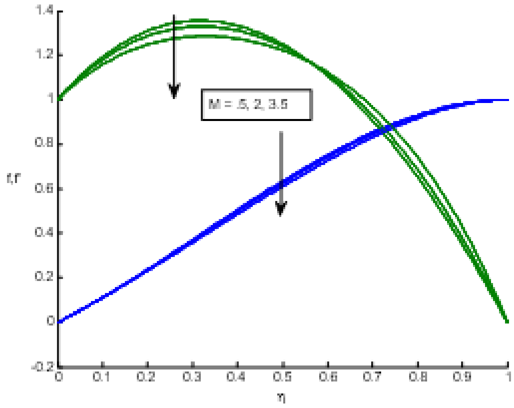
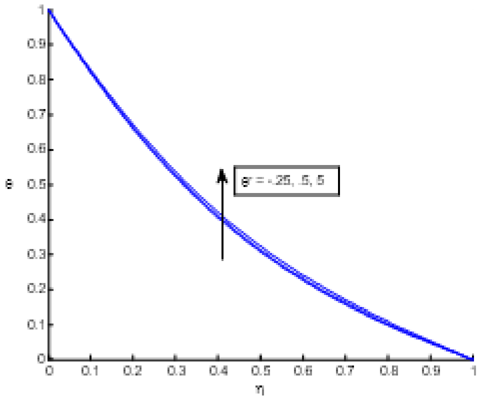


Fig. 4. Velocity profile for different K Fig. 5. Velocity profile for different M



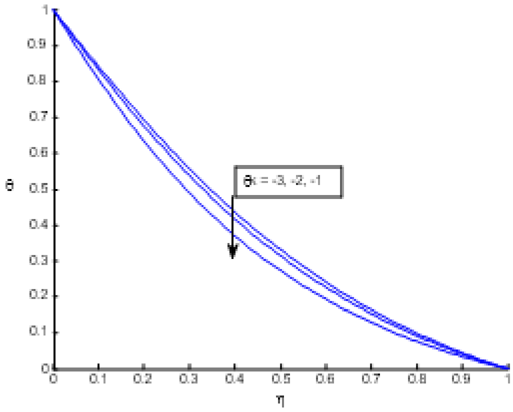
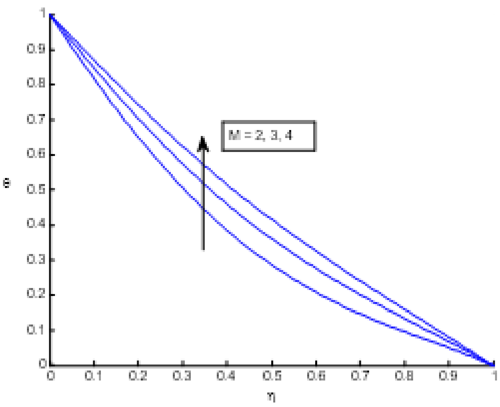


Fig. 6. Temperature profile for different θ_r Fig. 7. Temperature profile for different θ_k



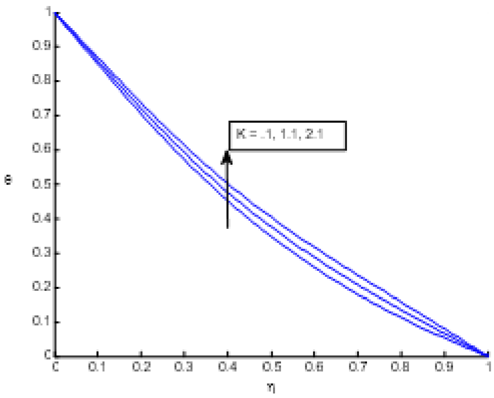


Fig. 8. Temperature profile for different M Fig. 9. Temperature profile for different K

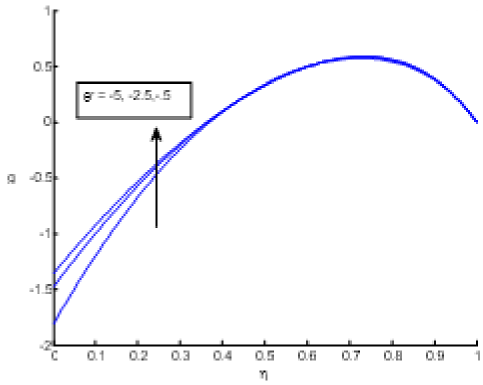


Fig. 10. Micro-rotation profile for different θ_r

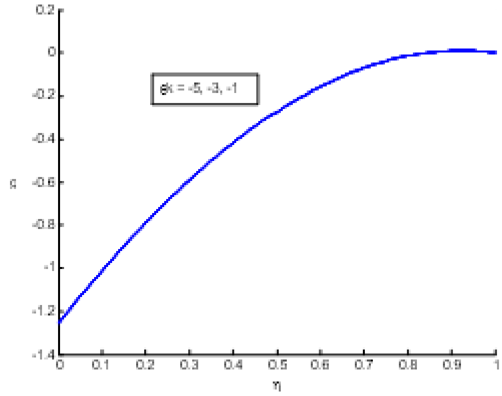


Fig. 11. Micro-rotation profile for different θ_k

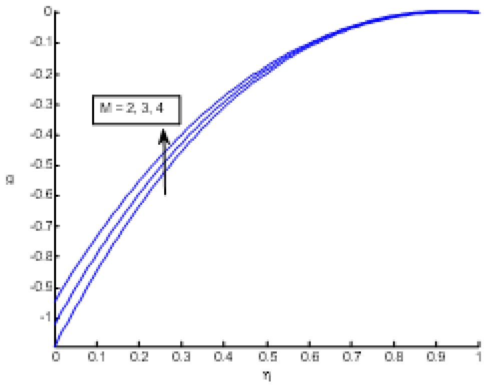


Fig. 12. Micro-rotation profile for different M

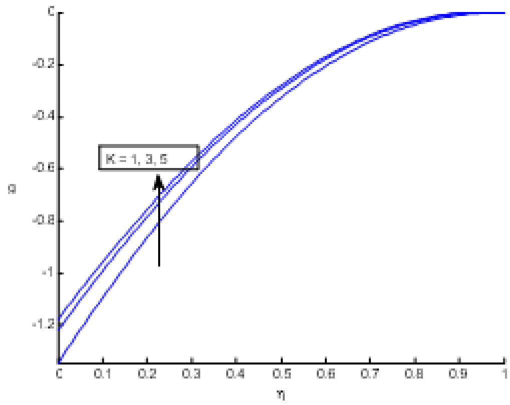


Fig. 13. Micro-rotation profile for different K

Table 1 Effect of coupling constant K

K	θ_r	$f'''(0)$	$f''(0)$	$g'(0)$	$\theta'(0)$	cf	Nu
.2	-10	2.3007	-7.9505	2.2087	-1.1961	4.6432	1.087
	-6	2.3630	-8.3921	2.2703	-1.1965	4.5235	1.087
	-2	2.6295	-10.273	2.5345	-1.1984	4.0320	1.089
.3	-10	2.2916	-7.8723	2.1997	-1.1951	4.8541	1.086
	-6	2.3487	-8.2729	2.256	-1.1955	4.731	1.086
	-2	2.588	-9.9369	2.4941	-1.1973	4.2283	1.088
.4	-10	2.2838	-7.8056	2.1920	-1.1942	5.066	1.085
	-6	2.3365	-8.172	2.2441	-1.1946	4.9401	1.086
	-2	2.5547	-9.6625	2.46	-1.1962	4.428	1.087

Table 2 Effect of coupling constant G

G	θ_k	$f'''(0)$	$f''(0)$	$g'(0)$	$\theta'(0)$	cf	Nu
.5	-6	2.3198	-8.0926	2.648	-1.2363	4.4499	1.05975
	-4	2.3203	-8.1043	2.6485	-1.2836	4.45081	1.02691
	-2	2.3215	-8.136	2.65	-1.414	4.45310	0.9427
1	-6	2.3131	-8.06	2.2916	-1.2363	4.43703	1.05973
	-4	2.3136	-8.071	2.292	-1.2836	4.43791	1.02689
	-2	2.3147	-8.1032	2.2932	-1.414	4.44019	0.94268
1.5	-6	2.31	-8.0462	2.1432	-1.2363	4.43163	1.05972
	-4	2.3107	-8.0578	2.1436	-1.2836	4.43251	1.02688
	-2	2.3119	-8.0893	2.1447	-1.414	4.43479	0.94267

4. Conclusions

From the above analysis made on this paper it is found that the viscosity and thermal conductivity parameter along with the other parameters such as magnetic parameter M, coupling constant parameter K, micro-rotation parameter G etc. have significant effects on velocity, micro-rotation, and temperature profile within the boundary layer. We can draw the following observation:

1. Increasing values of viscosity, magnetic field and coupling constant parameter (K) enhance the micro-rotation of the fluid element.
2. When viscosity, coupling constant parameter (K) and magnetic field increase the temperature increases but it decreases with the increase of thermal conductivity.
3. Velocity (f) increases with the increase of viscosity and coupling constant parameter K.
4. Skin friction decreases and Nusselt number increases when viscosity increases.
5. Micro-rotation parameter G enhances Both skin friction and Nusselt number.

References

1. A.C. Eringen, Simple micro fluids, 1964, Int. J. Eng. Sci., vol. 2, pp. 205-217.
2. A. Ishak, R. Nazar, I. Pop, Magnetohydrodynamic flow of a micropolar fluid towards a stagnation point on a vertical surface, 2008, Computers and Mathematics with applications, vol. 56, no. 12, pp. 3188-3194.

3. F.M. Hady, Heat transfer to a micropolar fluid from a non-isothermal stretching sheet with injection, 1996, Int. J. Num. Meth. Heat Fluid Flow, vol. 6, no. 6, pp. 99-104.
4. R. Bhargava, L. Kumar, H.L. Takhar, Numerical solution of free convection MHD micropolar fluid flow between two parallel porous vertical plate, 2003, Int. J. Eng. Sci., vol. 41, pp. 123-136.
5. O. Ojjela, R. M. Josyula, Micropolar fluidflow between two porous parallel plates with suction and injection, 2014, Int. J. of Appl. Math. and Mech., vol. 10, no. 5, pp. 47-57.
6. B. Borgohain, G. C. Hazarika, Effects of variable viscosity and thermal conductivity on hydromagnetic boundary layer micropolar fluid flow over a stretching surface embedded in a non-Darcian porous medium with radiation sheet, 2010, Far East Journal of Appl. Math., vol. 39, No. 2, pp. 139-149.
7. P.J. Borthakur, G.C. Hazarika, Effects of variable viscosity and thermal conductivity on flow and heat transfer of a stretching surface of rotating micropolar fluid with suction and blowing, 2006, Bull. Pure and Appl. Sci., vol. 25, No. 2, pp. 361-370.
8. G. M. A. Rahman, Effects of magnetohydrodynamic on thin films of unsteady micropolar fluid through a porous medium, 2011, J. Mod. Phys., vol. 2, No. 11, pp. 1290-1304.
9. G.C. Hazarika, B. Phukan, Effects of variable viscosity and thermal conductivity on magnetohydrodynamic free convection flow of a micropolar fluid past a stretching plate through porous medium with radiation, heat generation and Joule dissipation, 2016, Turk. J. Phys., vol. 40, No. 1, pp. 40-51.
10. P.K. Khound, G.C. Hazarika, Effects of variable viscosity and thermal conductivity on MHD flow through a specially characterized horizontal channel, 2002, In Proceedings: 47th Congress of the ISTAM, Indian Institute of Technology, Kharagpur, India.
11. H. Herwig, K. Gersten, The effect of variable properties on laminar boundary layer flow, 1986, Warme and Staffubertr ,vol. 20, pp. 47.
12. F.C. Lai, F.A. Kulacki, The Effect of Variable Viscosity on Convective Heat and Mass Transfer along a Vertical Surface in Saturated Porous Media, 1990, Int.J.Heat Mass Transfer, vol. 33, pp. 1028-1031.

Nomenclature

$Re = \frac{\rho c h^2}{\mu_\infty}$	stretching Reynolds number
$G = \frac{c\gamma}{\kappa \nu_\infty}$	micro-rotation parameter

$K = \frac{\kappa}{\mu_{\infty}}$	coupling constant parameter
$\Delta = \frac{\gamma}{\mu_{\infty} j}$	the material constant
Nu	local Nusselt number
$M = \left(\frac{\sigma}{\mu}\right)^{\frac{1}{2}} B_0 h$	Hartmann number
$m = \frac{m^*}{h^2}$	permeability number
$Pr = \frac{\mu_{\infty} C_p}{\lambda_{\infty}}$	Prandtl number
G	gravitational acceleration, m.s-2
λ	thermal conductivity, W.m-1. K-1
T	Temperature of the fluid
T_{∞}	Temperature of the fluid at infinity
Θ	dimensionless temperature
μ_{∞}	dynamic viscosity, kg. m-1.s-1
J	Microinertia per unit mass
σ	Electrical conductivity
m*	Coefficient of permeability
κ	kinematic micro-rotation viscosity
c	stretching parameter