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The Structural Parameters Optimization of Asphalt Foaming Cavity by Response Surface Analyzing

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Abstract

In order to obtain foamed asphalt with high expansion ratio, the foaming cavity as the key component of the asphalt foaming device is studied. By utilizing the computational fluid dynamics method, the characteristics of the internal flow field in the cavity of asphalt foaming process is studied to determinethe main structural parameters affecting the expansion rate. Then the optimization of structural parameters for asphalt foaming cavity is performed by using and the method of response surface. To construct the second-order response surface model used in the optimization of the cavity parameters, the influencing factors like the cavity volume, the inlet size and the outlet size of the cavity are taken as the argumentsand the expansion ratio is served as theresponse value, in which the method of Box-Behnken Design was adopted to select test points. The results show thatthe cavity volume affects the expansion rate most significantly, the asphaltinletsize secondly, and the interaction between the asphalt inlet size and volume of the foaming cavity on the expansion ratio is remarkable. By parameters optimizing, a great expansion ratecan be achieved when the volume of the asphalt foaming cavity is 87.5ml, the inlet size is 2.79mm, and the outlet size is 7.68mm.

Key words

Foamed Asphalt, Cavity Structure Parameters, Response Surface, Optimization Design

1. Introduction

As the national environmental protection and resource conservation policies are considered more and more significant, the foamed asphalt recycling technology has become the main technology in the large-scale road maintenance because of its great advantages like energy saving, environmental protecting and cost reduction. It can reduce the fossil oil resources and soil cost by reusing the asphalt pavement material, and also save the water at the same time. Furthermore, because of its fast proceeding, the construction is not influenced by weather and the traffic is rarely affected. Moreover, the foamed asphalt apply can not only save asphalt cost, but also greatly enhance the coating performance [1]. Compared with ordinary asphalt mixture, the foamed asphalt mixture can improve production efficiency and reduce energy consumption. It can reduce 5% to 10% asphalt amount and save 15% to 20%mixing time [2].

The asphalt foaming cavity is a certain volume container in which asphalt foaming is proceeded. The structural design of asphalt foaming cavity is so important that it directly affects the effect of asphalt foaming. Expansion rate (R) and half-life of foamed asphalt (T1/2), two indicators come from the Technology Specification for Highway Asphalt Pavement Recycling (F41-2008 JTG), comprehensively evaluate asphalt foaming quality. Expansion ratio is the ratio of the maximum volume of the asphalt after foaming and the volume before foaming. The half-life of foamed asphalt is the time when the maximum volume of the asphalt is reduced to half of the volume [3-4].

Engineering appliance expects foamed Asphalt with the large expansion rate and the long half-life. Performance of foamed asphalt depends on foaming technology and equipment [2]. Although many domestic and foreign scholars have studied the control factors of asphalt foaming process, such as asphalt type, asphalt temperature, foaming water content. But the analysis of foaming cavity structure is very few [5-8]. In order to dynamically simulate the asphalt foaming process, the analysis method of computational fluid dynamics is utilized. Then the expansion rate influenced by the structure parameters is studied respectively, which are cavity volume, inlet size, and the outlet size. In order to analyze the asphalt foaming behavior and the expansion rate clearly, the foamed asphalt at the cavity exit is considered as the observing point. Finally, the response surface analysis method was utilized to optimize the structural parameters of asphalt foaming cavity.

2. Computational fluid dynamics analysis of asphalt foaming behavior

2.1Control equation of asphalt foaming

Asphalt foaming is a complex multiphase flow coupling process in a specific vessel, which contains heat asphalt, air, water and water vapor transformed from water. The internal flow field of asphalt foaming is a strong fuzzy and coupled process [9]. In this paper, the Mixture model is used to simulate the flow field inside the cavity, and the governing equations are established as follows:

Continuity equation：

$$\frac{\partial}{\partial t}(\rho_m)+\nabla\cdot(\rho_m\vec{v}_m)=\dot{m} \tag{1}$$

Momentum equation：

$$\frac{\partial}{\partial t}(\rho_m\vec{v}_m)+\nabla\cdot(\rho_m\vec{v}_m\vec{v}_m)=-\nabla p+\nabla\cdot[\mu_m(\vec{v}_m+\vec{v}_m^T)]+\rho_m\vec{g}+\vec{F}+\nabla\cdot(\sum_{k=1}^{n}\alpha_k\rho_k\vec{v}_{dr,k}\vec{v}_{dr,k}) \tag{2}$$

Energy equation：

$$\frac{\partial}{\partial t}\sum_{k=1}^{n}(\alpha_k\rho_k E_k)+\nabla\cdot\sum_{k=1}^{n}[\alpha_k\vec{v}_k(\rho_k E_k+p)]=\nabla\cdot(k_{eff}\nabla T)+S_E \tag{3}$$

Drift velocity equation：

$$\vec{v}_{dr.p}=\vec{v}_{qp}-\sum_{k=1}^{n}\frac{\alpha_k\rho_k}{\rho_m}\vec{v}_{qk} \tag{4}$$

The standard k-ε turbulent model equations

$$\rho\frac{\mathrm{d}k}{\mathrm{d}t}=\frac{\partial}{\partial x_i}\left[\left(\mu+\frac{\mu_t}{\sigma_k}\right)\frac{\partial k}{\partial x_i}\right]+G_k+G_b-\rho\varepsilon-Y_M \tag{5}$$

$$\rho\frac{\mathrm{d}\varepsilon}{\mathrm{d}t}=\frac{\partial}{\partial x_i}\left[\left(\mu+\frac{\mu_t}{\sigma_\varepsilon}\right)\frac{\partial \varepsilon}{\partial x_i}\right]+C_{1\varepsilon}\frac{\varepsilon}{k}(G_k+C_{3\varepsilon}G_b)-C_{2\varepsilon}\rho\frac{\varepsilon^2}{k} \tag{6}$$

The volume fraction equation of the second phase：

$$\frac{\partial}{\partial t}(\alpha_p\rho_p)+\nabla\cdot(\alpha_p\rho_p\vec{v}_m)=-\nabla\cdot(\alpha_p\rho_p\vec{v}_{dr,p}) \tag{7}$$

The variables in the formulas are:

$\vec{v}_m$-Quality average speed; ρ_m-Mixed density; α_k-The k phase of the volume fraction; $\dot{m}$-Mass transfer; $\vec{F}$-Body force; μ_m-Mixed sticky; $\vec{v}_{dr,k}$-The second phase k shiftspeed; k_{eff}-Effective thermal conductivity; S_E-Volumetric heat source; $\vec{\alpha}$-The second phase particle acceleration; τ_{qp}-The particles of relaxation time; G_k-Turbulent kinetic energy;

G_b-buoyancy;ε-Rate of dissipation of unit mass; Y_M-Correction term of compressibility;
σ_ε-Turbulent Prandtl number;

2.2Establishment of asphalt foaming cavity model and mesh generation

As Fig. 2 shows, the model of asphalt foaming cavity is established, which contains asphalt foaming cavity, asphalt inlet, water vent, air vent and foamed asphalt outlet[10-11]. The volume change of the foamed asphalt can be easily observed at the exit of the foamed asphalt, at which the cylindrical calculation field is arranged. As Fig. 3 shows, the symmetry of the model and the numerical solution make it possible to reduce the computation without quality losing. Finally, a half cavity model isestablished to numerically simulate the asphalt foaming behavior by using a symmetric boundary.

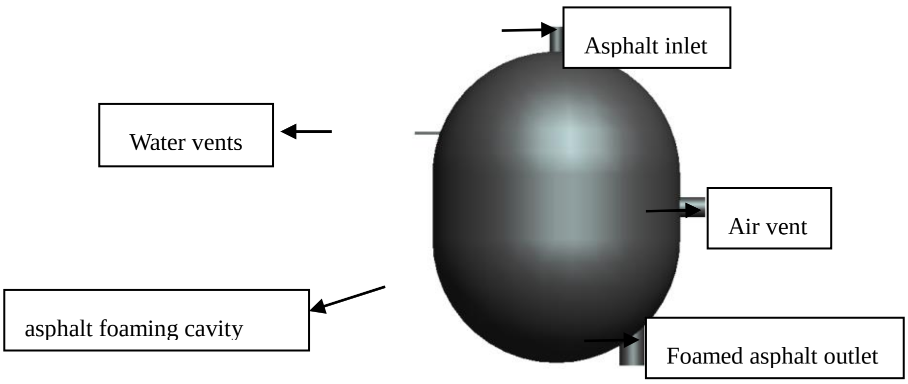


Fig.2 Geometric model of foaming cavity

The volumesof cavity partsare quite different. In order to obtain a high quality grid, the computational domain is divided into six parts and GAMBITsoftware is used to mesh eachpart of the model. Fig. 3 shows the properties of the mesh.The mesh number is 1169122, equiangle skewis 0.835 and aspect ratiois less than 5.

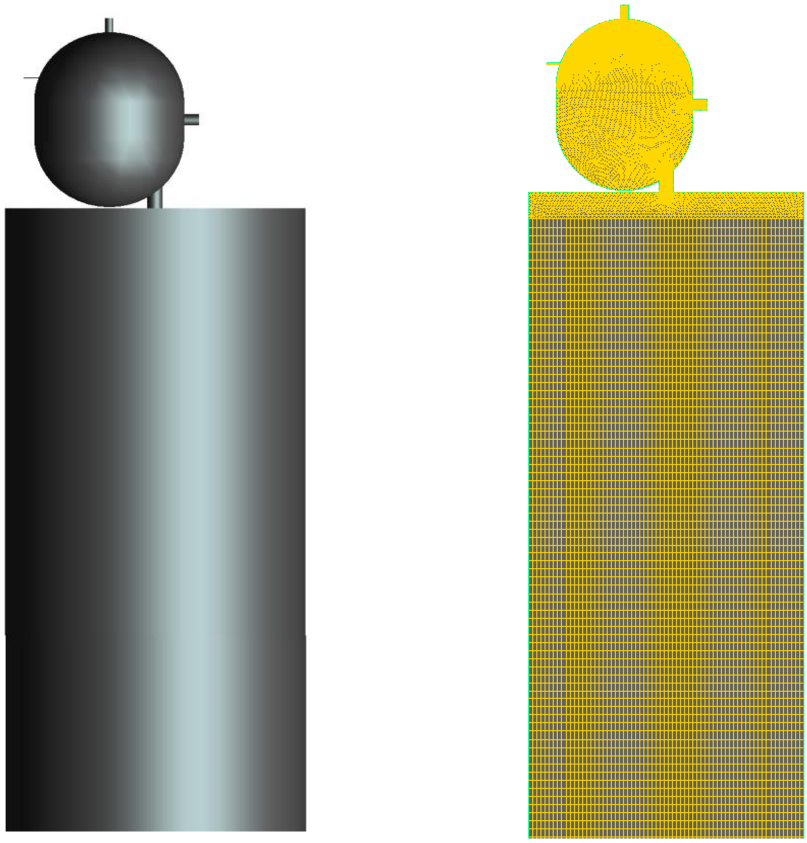


Fig.3 Model and meshing of asphalt foaming process

2.3 Solver settings

A non-steady state implicit solver based on pressure solution is established in fluent software. In the process of asphalt foaming, foaming water at room temperature can be heated to 100 ℃by contacting with hot asphalt, then vaporizing and forming a large amount of water vapor. Therefore, the process of foaming water phase transformation will be compiled in UDF, in which asphalt is set to the main phase and the air, water vapor and waterare set to the second phase. Physical parameters of various materials are set as follows: Firstly, the data about water, water vapor and air can be exported from physical property database. Secondly, air duringthe process of asphalt foaming can be seen as incompressible gas, because its Mach number is less than 0.3. Finally, the physical properties of asphalt are set as Table 1 shows. The viscosity temperature curve of asphalt can be obtained by Block Field viscosity method[12]. As Fig. 4shows, the viscosity of asphalt will be decreased with the increase of temperature.

The regression equation of viscosity and temperature of asphalt was established according to Saal formula come from Standard Viscosity-Temperature Chart for Asphalts(ASTM D 2493-2001)[13].

$$lglg(\eta\times1000)=n-mlg(T+273.13) \quad (8)$$

The variables in the formulas individually are:

η- Viscosity; T-The Celsius temperature; n、 m- Regression coefficient.

Regression analysis give the result as follows: n=9.3143,m=3.4154. 165 ℃,the temperature used in the asphalt foaming testis substituted into formula (8) to give the result as follows: η=0.0905 Pa·s.

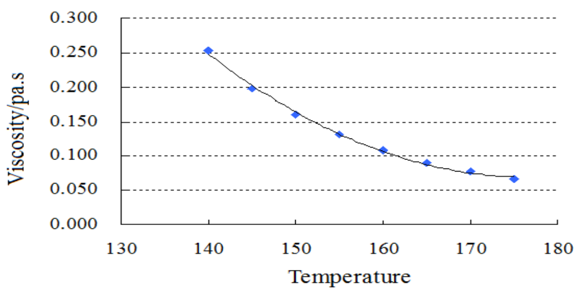


Fig.4 Viscosity-temperature curve

Tab.1 Physical parameters of asphalt

density(kg/m³)	Specific heat(j/kg-k)	Thermal conductivity(w/m-k)	viscosity(pa·s)
1011	1628	0.628	0.0905 Pa·s

Asphalt foaming conditions are set as follows:

asphalt temperature	flow	oil water mass ratio	air pressure
165 ℃	100 g/s	2.5%	0.3 MPa

The boundary conditions are set as follows:

Velocity-inlet	symmetry	outflow	wall
asphalt inlet water inlet air inlet	symmetric boundary	semi cylindrical outlets	the foaming cavity the nozzle wall boundary

The fluid velocity, turbulence intensity and the hydraulic diameter of each inlet can be calculated according to the asphalt foaming conditionsand structure parameters of the model. Meanwhile, the energy equation is considered because of the transfer and exchange of heat in the field. In addition, the fully implicit solution and the SIMPLEC algorithm are performed in the initialization of computational domains. In the iterative calculation process, the time step is set to

0.01s and the step size is set to 1500, the residual plotmonitor is set up to show convergence results and to analyze convergence judge.

2.4 Evaluation method of expansion rate in numerical calculation

The foamed asphalt volume flow can be measured on the certain distance section at the foaming cavity exit after numerical calculation of asphalt foaming behavior. Volume change during the process of transition from asphalt to the foamed asphalt can be analyzed after compared volume flow of asphalt inlet and the foaming cavity exit. The maximum ratio of the volume flow of asphalt inlet and the foaming cavity exit is defined as the expansion rate of the foamed asphalt[14]

Fig.5 Pressure nephogram of foamed asphalt outlet

As Fig. 5 shows, the pressure of foamed asphalt varied greatly at the vicinity outlet. In addition, a vortex was formed at the exit corner where the pressure distributed. The relation curve between the expansion rate and the distance to theexit was shown in Figure 6. The expansion ratio increased with the increase of the distance along the vertical direction of the exit and then began to decrease slowly after it reached the maximum value of 10.9 at a certain distance. Furthermore, as shown in Figure 7,under the same conditions, the asphalt foaming test have been carried out and the expansion rate of the foamed asphalt obtained from the experimentwas about 10.8.The results of numerical simulation and test results showed that the error was about 0.92%, which indicated that the numerical simulation results were reliable.

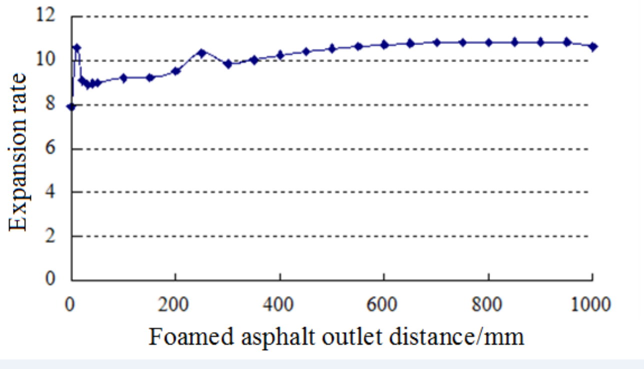


Fig.6The curve of expansion ratio of foamed asphalt outlet

Fig.7 The test of asphalt foaming

3. Optimization for Asphalt Foaming Cavity by Response Surface Analysis

The response surface method is an optimization method of the asymptotic approximation, which can solve the nonlinear and complex problem. The basic idea of the response surface method is to construct an approximate polynomial function to explicitly express the implicit function. Its essence is a statistical algorithm, which finds the best response value in the different values of the variable factors through some test points. The response surface model of quadratic polynomial is adopted to describe the optimization [15-18]. The formula for response surface is:

$$\hat{y} = \beta_0 + \sum_{i=1}^{k} \beta_i x_i + \sum_{i=1}^{k} \beta_{ii} x_i^2 + \sum_{i<j}^{k} \beta_{ij} x_i x_j + \varepsilon \tag{9}$$

The variables in the formulas individually are: $\hat{y}$ -Response variable; x_i, x_j -Variable; ε -Normal random error; β_0 -Regression intercept; $\beta_i, \beta_{ii}, \beta_{ij}$ -Regression coefficient; k -Number of independent variables.

The most commonly used test design method about the response surface methodology includes central composite design and Box-Behnken test design. Firstly, the Box-Behnken test method can use less test design points to test. Secondly, Box Behnken design can provide optimization method to solve the problem with multi variables by adopting non-linear mathematical model to fit. Thirdly, the Box-Behnken test method can find the best combination among the various factors and examine the interactions with such advantages as less number of experiments [19], high precision of fitting and high reliability about predicted results. Finally, this paper uses the Box-Behnken test design method for high efficiency of test design and the reliability of the calculation results [20].

In the experiment, in order to obtain the maximum expansion rate, the cavity volume, the entrance size of the asphalt and the exit size of foamed asphalt are set as the three elements to optimize the foaming. The design variable values of the sample points are selected within a certain range according to the Box-Behnken design method. Then the foaming process is numerically simulated with FLUENT software and the response values of the numerical calculation can be optimized and analyzed.

3.1 Response surface test design

The influence rules of main effect and interaction effect on expansion ratio of foamed asphalt are analyzed by using the Box-Behnken method, in which three structural parameters of the foaming cavity, namelythe cavityvolume, the size of the asphalt inlet and the foamed asphaltoutlet are defined as independent variables. Based on 3×3 factorial designed experiment, the factors and codes of the test pointsare shown in Table 2: where A, B, C represents the volume of the cavity, the asphalt inlet size and the size of the foam asphalt outlet respectively; three levels about low, middle and high of independent variables were expressed by -1, 0 and +1 respectively.Seventeensets of test points produced from Expert Design 8 software based on Box-Behnken test design method are shown in Table 3: where number 1~12 is factorial experiment, 13~17 is the center of the test. Twelve sets of data from 1 to 12 are used to perform factorial experiments and the corresponding points are referred to factorial points, which represent the center point of 12 edgesin hexahedron. Five sets of data from 13 to 17 are defined as zero points, namely, the central point of the regional design, which can represent 5 times repeated teststo estimate the test errors.

Tab.2 The factor and level of Box-Behnken Design

Code	Factor	-1	Level(0)	1
A	Volume of the cavity(L)	0.042	0.091	0.14
B	The entrance to the asphalt(mm)	2.5	2.8	3.1
C	Foamed asphalt exports(mm)	4	5	6

Tab.3 Box-Behnken Design and analysis results

Code	Volume of the cavity	The entrance to the asphalt(mm)	Foamed asphalt exports(mm)	expansion rate
1	0.042	2.50	5.00	9.3940
2	0.140	2.50	5.00	9.7879
3	0.042	3.10	5.00	9.8798
4	0.140	3.10	5.00	9.7588
5	0.042	2.80	4.00	9.7329
6	0.140	2.80	4.00	10.1269
7	0.042	2.80	6.00	10.3901
8	0.140	2.80	6.00	10.4592
9	0.091	2.50	4.00	10.3687
10	0.091	3.10	4.00	10.5649
11	0.091	2.50	6.00	10.5749
12	0.091	3.10	6.00	10.5783
13	0.091	2.80	5.00	10.8676
14	0.091	2.80	5.00	10.9199
15	0.091	2.80	5.00	10.8927
16	0.091	2.80	5.00	11.3726
17	0.091	2.80	5.00	11.2131

3.2 Results and Analysis

3.2.1 Model establishment and Significance tests

Tab.4 The variance analysis

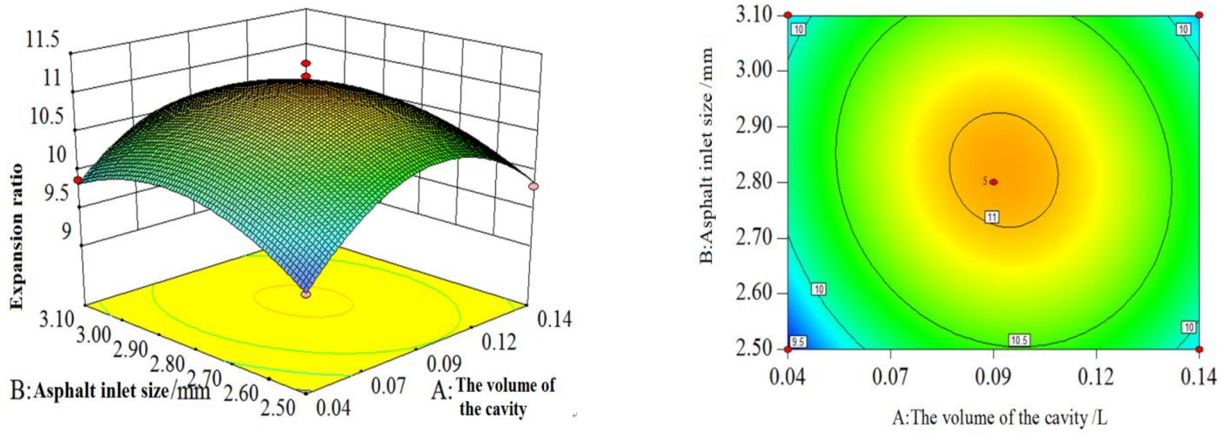
Source	Sum of Squares	Freedom	Mean Square	F	P(Prob>F)
Model	4.73	9	0.53	12.58	0.0015
A	0.068	1	0.068	1.62	0.2436
B	0.054	1	0.054	1.29	0.2936
C	0.18	1	0.18	4.37	0.0748
AB	0.066	1	0.066	1.59	0.2482
AC	0.026	1	0.026	0.63	0.4528
BC	9.293E-003	1	9,293E-003	0.22	0.6515
A^2	3.02	1	3.02	72.17	<0.0001
B^2	1.06	1	1.06	25.38	0.0015
C^2	3.705E-003	1	3.705E-003	0.089	0.7745
residual	0.29	7	0.042	—	—
Loss of fitting	0.087	3	0.029	0.56	0.6673
absolute error	0.21	4	0.051	—	—
Total deviation	5.02	16	—	—	—

Based on the ANOVA analysis method of Design expert 8.0software,the multiple regression analysis of A, B, C and Y are takenaccording tothe table 3.And the obtainedresults are shown in Table 4. Obviously, the three factors can influencethe foaming expansion ratesignificantly.According to the degree of influence,they are the asphalt outlet size, the cavityvolume, and theasphalt inlet size. Because the P value of the interaction term is greater than 0.2, the impact of the interaction terms onthe expansion rate is not significant. According to the test results in the table, the quadratic multidimensional regression model of response surface can be established.

$$Y=11.05+0.092A+0.082B+0.15C-0.13AB-0.081AC-0.048BC-0.85A^2-0.50B^2-0.030C^2 \quad (10)$$

Inorder to analyze the regression parameters of the response surface, F test is used to determine the significant level of the influence of each variable in the regression equation on the responsevalue. From table 4 it can be concluded that: Firstly, the P valueis 0.0015by F testing the model, less than 0.01, which showed that theexpansion rate affectedby the cavityvolume is extremely remarkable; secondly, P value of the lack of fit by F test is 0.6673, greater than 0.05, which indicates that the influence of lack of fit was not significant. The favorable factors for the model of the regression equation, namely, R^2=0.9418, indicates that the fitting of the model is good and regression equation has good representative trait. It can be concluded that the model not only explained 86.7 percent of the change for effects value, but also had little test error, when the correction factor is 0.8669. Furthermore, the regression equation can describe the relationship between the variablesandthe response valuevery well.

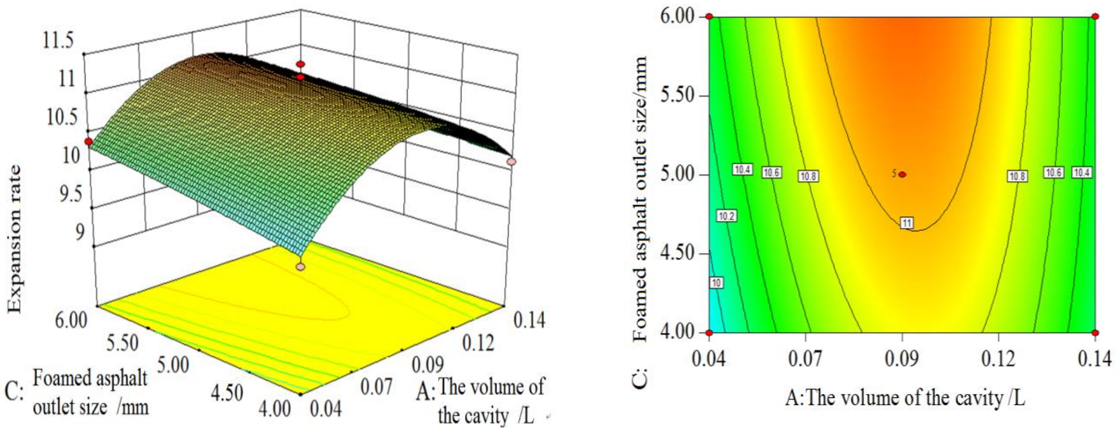
3.2.2Response surface analysis of the interaction effect of factors



a) Response surface

b) Contour

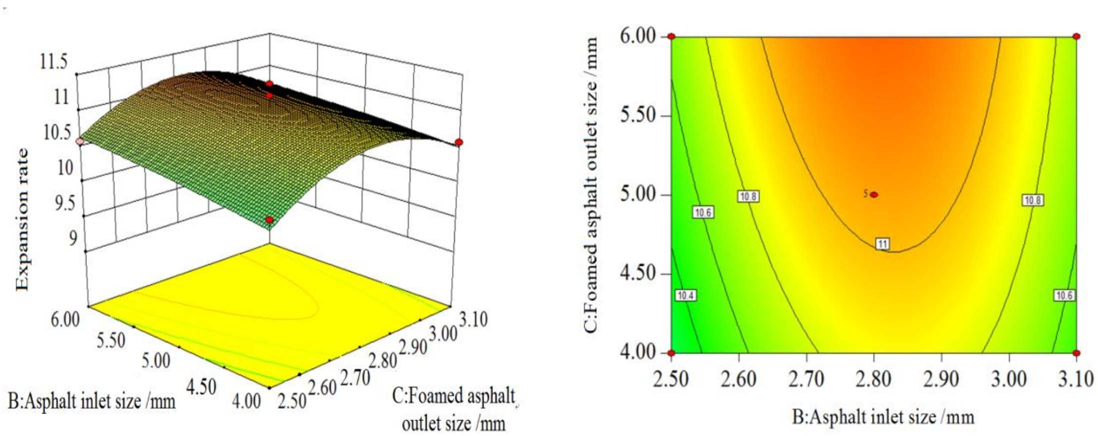
Fig.8 The expansion ratio influenced by cavity volume and asphalt entrance size



a) Response surface

b) Contour

Fig.9 The expansion ratioinfluenced by cavity volume and foamed asphalt outlet size



a) Response surface

b) Contour

Fig.10 The expansion ratioinfluenced byasphalt inlet size and foamed asphalt outlet size

The three dimensional response surface and contour map are drawn according to the effects of different factors on the responsevalues. Figure 8, figure 9, Figure 10 respectively indicate the impact of the interaction of the two factors on the expansion rate. As Figure 8 shows, the responsesurface is aconvex surface,which indicates that the interaction between the asphalt inlet size and volume of the cavity is remarkable. On the other hand,as Figure 9 and Figure 10 shows, the response surface is a cylindrical surface, which indicates that the interaction of two factors is not remarkable.

3.2.3 Response surface optimization prediction and verification

The optimal conditions are calculated by utilizing Design Expert 8.0software and the calculation resultscan be forecasted by regression simulation are as follows:

the volume of the cavity	the asphalt inlet size	the outlet size	Predicted Expansion rate	Desirability
0.0875L	2.79mm	7.68mm	11.25	0.974

In order to verify the optimization results, the structure parameters of the asphalt foaming cavity obtained by optimizationwere numerically calculated in Fluent and the values are: the expansion rate calculated is 11.31, and the contrast error between the predicted value and the experimental results is 0.53%.

4. Conclusions

1) Among the three optimized parameters of the asphalt foaming cavity, the influence of the foamed asphalt output size on the expansion rate during the asphalt foaming process is the most significant; the cavity volume takes the second.

2) Among the three optimized parameters of the asphalt foaming cavity, the interaction between the asphalt inlet size and the foaming asphalt cavity volume is remarkable, nevertheless, the interaction effect on the expansion rate of the foamed asphalt are quite small between both volume of the foamed asphalt cavity and foam asphalt outlet size or foamed asphalt outlet size and asphalt inlet size.

3) Through the response surface analysis and optimization, the design parameters of asphalt foaming cavity can be obtained as: the cavity volume is 87.52mL, the asphalt inlet size is 2.79mm, the outlet size of the foamed asphalt is 7.68mm and the prediction value of the expansion ratio of foamed asphalt can reach 11.25.

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