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Elastoplastic Analysis of Circular Tunnel using Nonlinear Improvement of Unified Strength Theory

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Abstract

The original two-parameter twin shear unified strength theory proposed by Yu which can consider the influence of intermediate principle stress has the linear failure envelope. The improvement of unified strength theory with parabolic failure envelope can describe rock failure properties, especially the rock tensile property better than the original one. Based on the improvement of unified strength theory, solutions are presented for plastic zone radius and stress with considering the intermediate principle stress and seepage. The results show that the parameter *b* in the unified strength theory plays a significant effect on the radius of the plastic zone. The radius of plastic zone is decreased due to the intermediate principal stress. Compared with and without consider seepage, the radius of the plastic zone with considering influence of seepage is larger than that without considering the influence. The results also indicate that the parameter *b* and seepage have considerable influences on tangential stress and radial stress. In tunnel design, the unified strength theory parameter should be reasonably adopted according to surrounding rock material properties. When the failure envelopes of rock take the form of parabolic, it can provide theoretical supports for tunnel design.

Key words

Tunnel, unified strength theory, parabolic failure envelope, intermediate principle stress, seepage

1. Introduction

Accurate predictions for plastic zone and stress change surrounding rocks of tunnel play a crucial role in designing and constructing the tunnel, which are based on selecting reasonable strength criterion of rock. As reported in the literatures, the rock mass is conventionally assumed to be governed by the Mohr-Coulomb failure criterion in the elasto-plastic analysis [1][2][3]. As is well known, the disadvantage of Mohr-Coulomb failure criteria cannot consider the intermediate principal stress. Through numerous tests on rock, the existence of the intermediate principal stress effect has now been well recognized as characteristics of the materials [4][5], the rock strength is generally related to intermediate principal stress[6][7]. The unified strength theory proposed by YU takes the effect of all the stresses into consideration in the characteristics line field [8], which can reflect the effects of the intermediate principle under various stress states[9][10].

However for failure envelope, the linear type is inferior to the hyperbolic type in describing the rock tensile properties. The failure envelope of the some rock, for example, the marlstone, the sandstone and the shale et al., is approximated by a parabolic failure envelope [11]. The parabolic function of Mohr strength criterion can describe the rock strength much better than the linear Mohr-Coulomb criterion, especially when the embedded depth of rock is larger[12].The Mohr circle comparisons show that the two-parameter twin shear unified strength theory used now has the linear the failure envelope same as the Mohr-Coulomb strength theory. Improvement of two-parameter twin shear unified strength theory with parabolic failure envelope is given out, which can explain the rock failure properties, especially the rock tensile property better than the original one. It was used in elastoplastic analysis for the wall rock around the tunnel with circular cross-section in static hydraulic pressure condition [13][14], but it did not consider the effect of the seepage, and the tunnels are assumed in dry ground conditions. When a tunnel is excavated below groundwater table, the groundwater may flow into the tunnel, and consequently seepage forces will occur through the ground. This may seriously influence on the behavior of the tunnel [15][16][17]. Thus, the hydro-mechanical aspects should be taken into account in designing the tunnel.

In this paper, based on the improvement of unified strength theory which is the nonlinear improved expression of the two-parameter twin shear unified strength theory, the elastoplastic analysis of tunnels is presented by considering the seepage. The solutions of plastic zone radius and stress field around circular tunnel are derived. The proposed method considers the effects of intermediate principle stress and seepage induced in the plastic zone and stress field, assuming that initial stress field is hydrostatic.

2. Improvement of unified strength theory

The well-known strength theory was established by Mohr in 1900. The Mohr-Coulomb yield criterion can be expressed a

$$|\tau| = f(\sigma)$$

(1)

However, numerous experiments have shown that the strength envelopes of almost all geomaterials are nonlinear in the σ-τ stress space, where σ and τ are the normal and shear stresses respectively. Assuming the failure envelope of the surrounding rocks satisfies parabolic .Two-parameter parabolic failure envelope is shown in Fig.1.

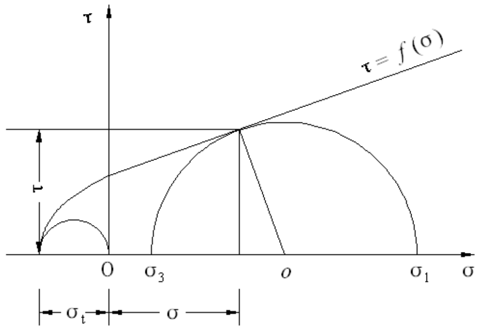

Fig.1 Parabolic failure envelope

The parabolic failure envelope can be written as[18]:

$$|\tau|^2 = \lambda(\sigma + \sigma_t)$$

(2)

where τ and σ are the shear stress and normal stress, σ_t is the material's uniaxial tensile-strength, λ is a constant which can be determined as

$$\lambda \approx \frac{\sigma_c^2}{2(\sigma_c + 2\sigma_t)}$$

(3)

where σ_c is the material's uniaxial compressive strength.

It can be expressed in terms of the major and minor principal stress σ_1 and σ_3 as follows

$$\left(\frac{\sigma_1-\sigma_3}{2}\right)^2=\lambda\left(\frac{\sigma_1+\sigma_3}{2}\right)+B \tag{4}$$

where $B=\lambda\sigma_t-\dfrac{\lambda^2}{4}$.

The unified strength theory considers different contributions from all of the components acting on an orthogonal octahedron, proposed by YU with the linear failure envelope. According the strength behaviors and method by Yu, the linear failure envelope is replaced by parabolic failure envelope, the idea and expressions of improvement of unified strength theory with parabolic failure envelope are expressed in principle stress space as follows [15]

when $\sigma_2\le\dfrac{\alpha\sigma_1+\sigma_3}{1+\alpha}$

$$\left[\frac{(1+b)\sigma_1-b\sigma_2-\sigma_3}{2(1+b)}\right]^2=\lambda\left[\frac{(1+b)\sigma_1+b\sigma_2+\sigma_3}{2(1+b)}\right]+B \tag{5a}$$

when $\sigma_2\ge\dfrac{\alpha\sigma_1+\sigma_3}{1+\alpha}$

$$\left[\frac{\sigma_1+b\sigma_2-(1+b)\sigma_3}{2(1+b)}\right]^2=\lambda\left[\frac{\sigma_1+b\sigma_2+(1+b)\sigma_3}{2(1+b)}\right]+B \tag{5b}$$

where $\alpha=\sigma_t/\sigma_c$ is the material's tensile-compressive strength ratio, and the b $(0\le b\le 1)$ is the intermediate stress parameter that reflects the influence of intermediate principle stress. The value of b can be determined by material tests. The theory is especially versatile in reflecting σ_2 effect to different extents for different materials. The Eq.(4) can be derived from Eq.(5) when the parameter $b=0$. This is one of main advantages of improvement of unified strength theory.

3. Theoretical analysis under axial-symmetry condition

Let us consider the case of a circular tunnel of radius r_0 excavated in an elastoplastic rock, which satisfies the improvement of unified strength theory with parabolic failure envelope. This problem is considered with axial-symmetry condition, and the initial stress state q is assumed to be hydrostatic. The tunnel is shown in Fig.2. Based on the presumption that there is a support resistance p distributed uniformly on the rock wall ($r = r_0$), the radius of the plastic zone R. The radial stresses σ_r and tangential stresses σ_θ in the elastic zone can be expressed as follows.

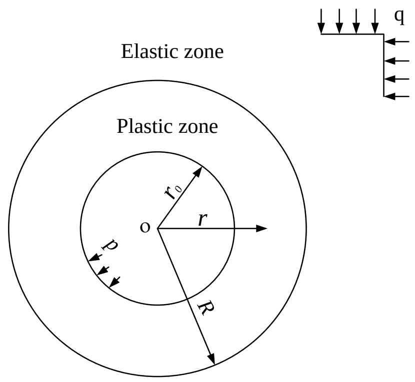

Fig.2 Simplified theoretical calculation model

$$\sigma_r = \left(1 - \frac{R^2}{r^2}\right) q + \frac{a^2}{r^2}\sigma_p + \eta p_w$$

(6)

$$\sigma_\theta = \left(1 + \frac{R^2}{r^2}\right) q - \frac{a^2}{r^2}\sigma_p + \eta p_w$$

(7)

where σ_p is the stresses at interface between the elastic and plastic zones, p_w is the seepage forces, η $(0 < \eta < 1)$ respects the area coefficient that depend on the void ratio of the material.

For an axial-symmetry problem, the differential equilibrium equation of the tunnel is given by

$$\frac{d\sigma_r}{dr} + \frac{\sigma_r - \sigma_\theta}{r} - \eta \frac{dp_w}{dr} = 0$$

(8)

Assuming the water flow is stable, and its movement obeys Darcy's law. Based on the effect of the initial seepage forces p_0, the boundary condition is $p_w\big|_{r=r_0} = 0$, $p_w\big|_{r=R_0} = p_0$, the seepage forces can be obtained as follows

$$p_w = p_0 \frac{\ln(r/r_0)}{\ln(R_0/r_0)}$$

(9)

where R_0 is the radius of the initial seepage force that obtained by pumping test.

Under the axisymmetrical plane strain condition, the radial stresses σ_r and tangential stresses σ_θ in the plastic zone will be major and minor principal stresses σ_1 and σ_3, respectively. In the plane strain state, the intermediate principle stress $\sigma_z = \sigma_2 = k(\sigma_r + \sigma_\theta)/2$ [19][20]. For simplicity, it is assumed that $k = 1$ in plastic zone. Therefore, $\sigma_2 \geq \frac{\alpha\sigma_1 + \sigma_3}{1+\alpha}$, Eq.(5b) should be adopted in plastic zone and can be expressed as

$$\left[\frac{(2+b)(\sigma_\theta - \sigma_r)}{4(1+b)}\right]^2 = \lambda\left[\frac{(2+b)\sigma_\theta + (2+3b)\sigma_r}{4(1+b)}\right] + B$$

(10)

Eq.(10) can be rewritten as follow

$$\frac{(2+b)(\sigma_\theta - \sigma_r)}{4(1+b)} = (\lambda m + B)^{1/2}$$

(11)

where $m = \frac{(2+b)\sigma_\theta + (2+3b)\sigma_r}{4(1+b)}$

By Eq.(11), the radial and tangential stresses in the plastic zone can be given as

$$\sigma_r = m - (\lambda m + B)^{1/2}$$

(12)

$$\sigma_\theta = m + \frac{2+3b}{2+b}(\lambda m + B)^{1/2}$$

(13)

Substitution of Eqs.(12), (13) and (9) into the equations of Eq.(8), provides

$$\left[1 - \frac{\lambda}{2}(\lambda m + B)^{-1/2}\right]\frac{dm}{dr} - \frac{4(1+b)(\lambda m + B)^{1/2}}{(2+b)r} - \frac{A}{r} = 0$$

(14)

where $A = \frac{\eta p_0}{\ln(R_0/r_0)}$.

The equation for r can be obtained as following

$$r = (4\sqrt{\lambda m + B} + 4b\sqrt{\lambda m + B} + 2A + Ab)^{-\frac{2\lambda + 2b\lambda + 2A + Ab}{8\lambda(1+b)^2}} \exp\left\{\frac{(2+b)\sqrt{\lambda m + B}}{2\lambda(1+b)} - C\right\}$$

(15)

where C is an integral constant.

The boundary conditions of this problem is

$$r = r_0,\ \sigma_r = p$$

(16)

By using Eqs.(12) and (16), the following formula can be obtained

$$m_0 = p + \frac{\lambda}{2} + \sqrt{\lambda(p + \sigma_t)}$$

(17)

Recalling the boundary conditions, the integral constant C can be calculated, and provides

$$r = r_0\left(\frac{4\sqrt{\lambda m_0 + B} + 4b\sqrt{\lambda m_0 + B} + 2A + Ab}{4\sqrt{\lambda m + B} + 4b\sqrt{\lambda m + B} + 2A + Ab}\right)^{D_1} \exp\left\{\frac{(2+b)\left(\sqrt{\lambda m + B} - \sqrt{\lambda m_0 + B}\right)}{2\lambda(1+b)}\right\} \tag{18}$$

where $D_1 = \dfrac{2\lambda + 2b\lambda + 2A + Ab}{8\lambda(1+b)^2}$.

Substituting m in Eq. (18), the r can be determined. Also utilizing Eq.(12) and (13), the relationship of the σ_r, σ_θ and r can be obtained.

The elastic and plastic zones interact with each other at the plastic radius R, Applying the continuous stress condition on interface between the elastic and plastic zones ($r = R$), the equation can obtain as

$$(\sigma_r + \sigma_\theta)^e = (\sigma_r + \sigma_\theta)^p$$

(19)

The equation can be deduced by superposition of the Eq. (6) and (7) and Eq. (12)and(13) as follows

$$m + \frac{b}{2+b}(\lambda m + B)^{1/2} = q + \eta p_w$$

(20)

By Eq.(9), (18) and (20), the m can be obtained. Substituting m into Eq. (18), the radius of plastic R can be given. The stresses acting on interface between elastic and plastic zones also can obtain.

4. Discussion

In order to investigate the integrated influences of the intermediate principle stress and seepage forces, an example of rock mass is analyzed. The parameters are internal radius $r_0 = 8.0$ m, support resistance $p = 1.2$ MPa, the material's uniaxial compressive strength

$\sigma_c = 4.2$ MPa, the material's uniaxial tensile- strength $\sigma_t = 1.05$ MPa, the area coefficient $\eta = 1$, the initial field stress $q = 5.0$ MPa. The radius of the initial seepage force $R_0 = 12r_0 = 96$ m that obtained by pumping test [7]. Five values for the unified strength parameter b (0, 0.25, 0.5, 0.75, 1) are considered. When the parameter b equals to zero, the analytical solution degenerates to that of parabolic failure criterion which does not consider the intermediate stress effect.

To investigate the effects of parameter b on plastic zone, the parameter b varies from 0 to 1, different b values correspond to different intermediate principle stress effects. The variation of plastic zone radius R versus parameter b is plotted in Fig.3. It is show that the parameter b in unified strength theory has significant effects on the radius of the plastic zone. The plastic zone radius R reduces greatly with increase of b value for a given condition. The R value is decreased by 19.4%, from 10.45 to 8.75 for considering the seepage, 17.4%, from 10.18 to 8.67 for without considering the seepage, respectively, when $b = 1$ compared with that of $b = 0$. Therefore, the plastic zone is overestimated when the intermediate principle stress effect is neglected. In this study, the effect of seepage force on tunnels is investigated. In comparison with without considering seepage force, the radius of the plastic zone is large because of taking into account the seepage force ($p_0 = 1.0$ MPa).

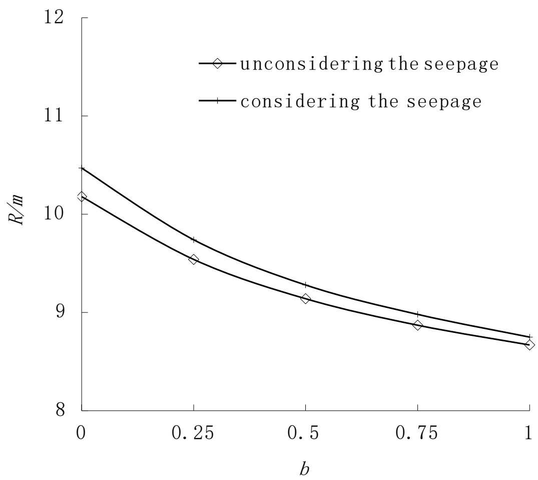

Fig. 3 Influences of b on the radius of plastic zone

The response of tunnels under different hydro-mechanical conditions is analyzed. Fig.4 is the curves of the radius R of the plastic zone and porous hydraulic pressure p_0 for different b. The values of porous hydraulic pressure vary zero to 10MPa. The case of $b = 0$ gives the smallest value of R for a giving hydraulic pressure. It is shown that the flow of groundwater into tunnels results in significant increase in the radius of the plastic zone of the tunnel wall due to seepage. There are significant differences in the results with different b values.

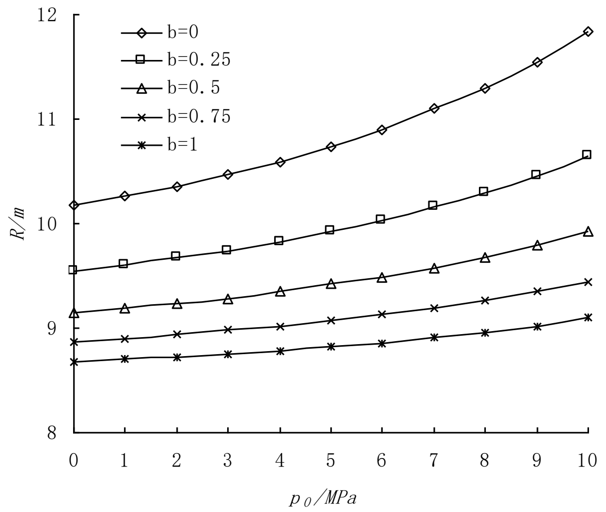

Fig.4 Influences of hydraulic pressure p_0 on Radius R of plastic zone for different b

Fig.5 indicates influences of b on the stresses acting on interface between elastic and plastic zones. It is clear that the radial stresses are increased and tangential stresses are decreased because of the effect of intermediate principal stress. In comparison with dry conditions, the existence of flow into a tunnel causes seepage force and consequently has increase radial stresses and decrease tangential stresses acting on the interface between elastic and plastic zones.

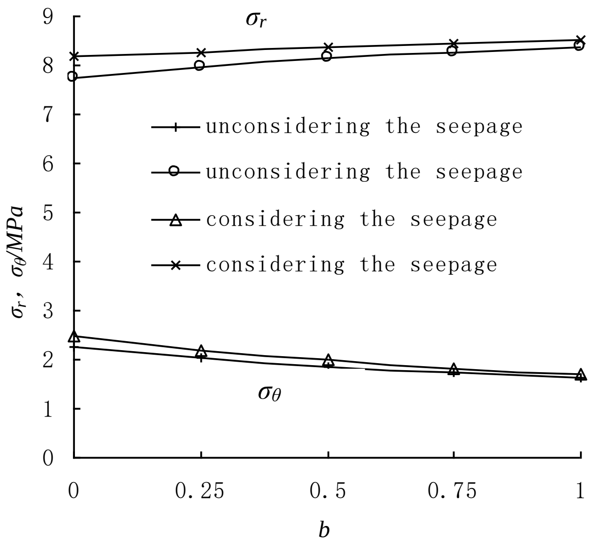

Fig .5 Influences of b on stresses acting on interface between elastic and plastic zones

5. Conclusion

In this paper, the elastoplastic analysis of surrounding rocks of circular tunnel is presented using the improvement of unified strength theory with parabolic failure envelope, assuming the axial-symmetric condition. The solution for plastic zone radius and stresses acting on the interface between elastic and plastic zones are obtained considering the seepage. The plastic zone radius can be predicted by the proposed solution, and the influence of intermediate principle stress and porous hydraulic pressure are discussed. The effects of the intermediate principal stress on the stresses and the radius of the plastic zone are significant. The radius of the plastic zone and tangential stresses acting on interface between elastic and plastic zones are reduced with increasing the parameter b, and more strength potentials of rock mass can be achieved when the effect of intermediate principle stress is considered. The increase in parameter b value leads to increase in the level of radial stresses. The influence of seepage force on the plastic zone radius is considerable. The radius of the plastic zone increases as the seepage force increases. The proposed solution can be used for the actual design of tunnel below the groundwater. For a reasonable and safe design of underwater tunnels, the effect of seepage force should be taken into consideration.

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