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An Evaluation of the Strain-Based Critical Plane Parameters for Multiaxial Low-Cycle Fatigue Evaluation

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Abstract:

The paper provides a brief review of the exciting critical plane approaches for multiaxial fatigue evaluation. Three strain-based critical plane parameters i.e. Kandil-Brown-Miller (KBM), Modified KBM (MKBM) and Fatemi-Socie (FS) parameters are investigated by using the multiaxial fatigue test data of the metallic materials. A total of 117 multiaxial fatigue test data of six types of metallic materials from tubular specimens under axial-torsional straining using sinusoidal wave forms are used to verify the effectiveness of the strain-based critical plane parameters for three types of strain loading conditions. Results indicate that equivalent strain and the KBM parameters can give very well predicted fatigue lives for proportional loading conditions, but the two parameters give poor prediction lives evaluation for non-proportional loading conditions. However, the correlation of the life predictions based on the MKBM and FS parameters with the fatigue test results under the non-proportional loading can be greatly improved due to the consideration of the effect of the non-proportional additional cyclic hardening on the multiaxial fatigue damage.

Keywords: Multiaxial fatigue damage, critical plane parameter, non-proportional loading, experimental verification

1. Introduction

Many components in engineering applications are actually subjected to multiaxial cycle stresses [1], and multiaxial fatigue is the main failure mode under the dynamic vibration. According to the current research, established criteria for multiaxial fatigue evaluation can be subdivided into three categories [2-5], which are the effective strain approach, the energy-based approach and the critical plane approach. Reviews of exciting approaches for multiaxial fatigue life prediction have been presented by You [2], Karolczuk [3] and Wu [4].

Amount of experimental results have been proved that the equivalent strain parameter can give well predicted fatigue lives under proportional loading, but generate poor fatigue lives evaluation under multiaxial non-proportional loading compared with the observed ones. The energy-based approach firstly come from the static hypotheses of material resistance, and takes the strain energy density as the controlling parameter of the fatigue damage. However, for the efficient application of energy-based approach, an accurate constitutive equation of the investigated material is required. Furthermore, the quantity of strain energy density is a scalar, which cannot reflect the failure mechanism and the effect of loading mode on the multiaxial fatigue failure [6-7]. The concept of critical plane is presented based on a physical interpretation of the fatigue crack growth mechanisms, and the fatigue damage evaluation is fixed on a certain material plane, which can be applied to the multiaxial fatigue evaluation under the proportional and non-proportional loading. Among the various multiaxial fatigue criteria, the critical plane approach can be distinguished because of its effectiveness and extensive application range in multiaxial fatigue life prediction.

In the recent decades, researchers develop various critical plane parameters for multiaxial fatigue life prediction. However, almost fatigue criteria are limited to the investigated metallic materials and loading paths. Although a general applicable multiaxial fatigue criterion always needs more experimental measurement and further research, a widely recognition is that the fatigue parameters of critical plane can be defined as a combination of shear strain/stress parameters and normal strain/stress parameters on the well-defined critical plane. Socie [4] defined the material plane with maximum shear strain as the critical plane and take the maximum shear strain range as the fatigue parameter. Kandil-Brown-Miller (KBM) [5] takes the linear combination of shear and normal strain range in the critical plane as the fatigue parameter. Wang and Brown (WB) [8] proposed a similar linear criterion which modifies the definition of normal strain range in the KBM criterion to considering the variable-amplitude strains. Fatemi and Socie (FS) [9] proposed a widely accepted critical plane concept using the maximum normal stress to replace the normal strain on the maximum shear strain plane, because the stress parameter can reflect the effect of non-proportional cyclic additional hardening on the multiaxial fatigue damage. Based on the FS critical plane concept, Li and the co-authors [10] developed a stress-correlated factor to consider the effect of the maximum normal stress on the multiaxial fatigue strengthen, and then a modified KBM parameter (MKBM) is proposed for the fatigue life prediction under the multiaxial non-proportional loading conditions. A similar stress-correlated factor was proposed by Wang and Yao [11]. Li and Jiang [12] proposed a path-dependent factor to consider the influence of the change of the non-proportional loading paths on the fatigue damage and then proposed a multiaxial fatigue life prediction model for shear fatigue failure.

In the present paper, three strain-based critical plane parameters, i.e. KBM, MKBM and FS parameters are investigated by using the multiaxial fatigue test data of metallic materials in the exciting literature. A total of 117 multiaxial fatigue test data of six types of metallic materials from tubular specimens under axial-torsional straining using sinusoidal wave forms are used to verify the effectiveness of the strain-based critical plane parameters under three types of strain loading paths. Finally, the further studies of the critical plane parameter for predicting the multiaxial fatigue life are discussed.

2. Multiaxial strains analysis

A thin-walled tube specimen is widely used for the multiaxial fatigue test, the strain states of the tubular specimen under the combine tension-torsion loading conditions is shown Fig. 1.

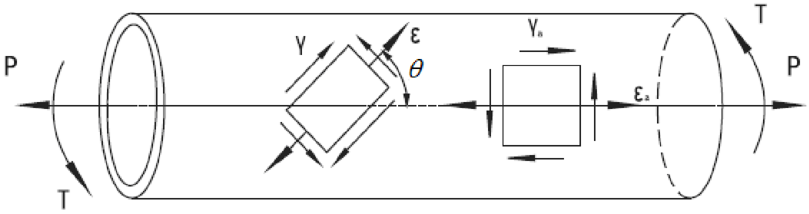


Fig 1 Strain state of the thin-walled tube specimen under the tension-torsion loading [10]

If the applied strains are sinusoidal, i.e.

$$\begin{cases} \varepsilon_{xx} = \varepsilon_a \sin wt \\ \gamma_{xy} = \gamma_a \sin(wt - \varphi) = \lambda \varepsilon_a \sin wt \end{cases} \tag{1}$$

where ε_a and γ_a are the applied axial strain range and shear strain range, respectively, φ is the phase angle between the tension and torsion cycle loading, λ is the applied strain ratio. If the material plane having the maximum shear strain range is defined as the critical plane, the orientation of the critical plane, i.e. θ_c, and the strain parameters acting on the critical plane are given by:

$$\Delta\gamma_{\max} = \varepsilon_a \sqrt{\left[\left(1+\upsilon_{eff}\right)\sin 2\theta_c - \lambda \cos 2\theta_c \cos\varphi\right]^2 + \left[\lambda \cos 2\theta_c \sin\varphi\right]^2} \tag{2}$$

$$\Delta\varepsilon_n = \frac{1}{2}\varepsilon_a \sqrt{\left[\left(1+\upsilon_{eff}\right)\left(1-\cos 2\theta_c\right) - \lambda \sin 2\theta_c \cos\varphi\right]^2 + \left[\lambda \sin 2\theta_c \sin\varphi\right]^2} \tag{3}$$

$$\tan 4\theta_c = \frac{2\lambda\left(1+\upsilon_{eff}\right)\cos\varphi}{\left(1+\upsilon_{eff}\right)^2 - \lambda^2} \tag{4}$$

where $\Delta\gamma_{\max}$ is the maximum shear strain range, $\Delta\varepsilon_n$ is normal strain range, θ_c is the orientation of the critical plane, υ_{eff} is the equivalent Poisson’s ratio, which can be determined by:

$$\upsilon_{eff} = \frac{\upsilon_e \varepsilon_e + \upsilon_p \varepsilon_p}{\varepsilon_e + \varepsilon_p} \tag{5}$$

where ε_e and ε_p is the elastic strain and the plastic strain, respectively, υ_e and υ_p is the elastic Poisson’s ratio and the plastic Poisson’s ratio, respectively. Consistency of volume requires the plastic Poisson’s ratio to be 0.5, and the elastic Poisson’s ratio typically equals 0.3 [5].

3. The strain-based critical plane approach

Under the uniaxial constant-amplitude loading, the relationship between the total strain and fatigue life, called as Manson-Coffin equation, is widely used for the fatigue life prediction [13], which is:

$$\frac{\Delta\varepsilon}{2} = \frac{\Delta\varepsilon_e}{2} + \frac{\Delta\varepsilon_p}{2} = \frac{\sigma'_f}{E}\left(2N_f\right)^b + \varepsilon'_f\left(2N_f\right)^c \tag{6}$$

where $\Delta\varepsilon/2$, $\Delta\varepsilon_e/2$ and $\Delta\varepsilon_p/2$ are the total strain amplitude, the elastic strain amplitude and the plastic strain amplitude, respectively. N_f is the number of cycles to fatigue failure. σ_f', ε_f', b and c are the fatigue strength coefficient, the fatigue ductility coefficient the fatigue properties, the fatigue strength exponent and the fatigue ductility exponent, respectively, which can be fitted by the uniaxial fatigue data of the investigated materials.

The strain-based critical parameters are widely adopted for predicting the multiaxial low-cycle fatigue life. According to the exciting study, the strain-based critical plane approach can be mainly divided into two categories. One considers that the fatigue life is a liner or nonlinear function of strain states of the critical plane, such as KBM parameter. The other proposal is to replace the normal strain in KBM parameter by the maximum normal stress acting on the critical plane, such as FS and MKBM parameters. The KBM model [5] can be written as:

$$\frac{\Delta\gamma_{\max}}{2}+S^*\Delta\varepsilon_n=\left[1+\upsilon_e+\left(1-\upsilon_e\right)S^*\right]\frac{\sigma_f'}{E}\left(2N_f\right)^b+\left[1+\upsilon_p+\left(1-\upsilon_p\right)S^*\right]\varepsilon_f'\left(2N_f\right)^c \tag{7}$$

where $\Delta\gamma_{\max}/2$ and $\Delta\varepsilon_n$ are the maximum shear strain amplitude and the normal strain range acting on critical plane, respectively, S^* is the normal strain influence coefficient.

The FS model [9] can be written as:

$$\frac{\Delta\gamma_{\max}}{2}\left(1+k\frac{\sigma_{n,\max}}{\sigma_y}\right)=\left(\left(1+\upsilon_e\right)\frac{\sigma_f'}{E}\left(2N_f\right)^b+\left(1+\upsilon_p\right)\varepsilon_f'\left(2N_f\right)^c\right)\left(1+k\frac{\sigma_f'}{2\sigma_y}\left(2N_f\right)^b\right) \tag{8}$$

where $\sigma_{n,\max}$ is the maximum normal stress acting on the critical plane, σ_y is the yield strength of the investigated material. *k* is an experimental coefficient. As an approximation, one may simply assume the coefficient *k* in FS model to be 1.0 [14].

The MKBM model proposed by Li et al [10] can be written as:

$$\frac{\Delta\gamma_{\max}}{2}+\left(1+\frac{\sigma_{n,\max}}{\sigma_y}\right)\frac{\Delta\varepsilon_n}{2}=\left(\frac{\sigma_f'}{E}\left(2N_f\right)^b+\varepsilon_f'\left(2N_f\right)^c\right)\left(1+\frac{\sigma_f'}{\sigma_y}\left(2N_f\right)^b\right) \tag{9}$$

The advantage of the MKBM model is that no empirical coefficient is needed in the Eq. (9), and the parameters in the MKBM model are well defined. The distinguishing feature of FS and MKBM parameters is that the maximum normal stress acting on the critical plane is considered on the strain-based critical plane parameter, which leads a non- linear combination of shear and normal strain ranges. The stress parameter can reflect the effect of the additional cyclic hardening due to the non-proportionality of the cycle loading on the multiaxial fatigue damage. It is worth to note here that for the three investigated critical plane parameters, the critical plane is defined as the plane having the maximum shear strain amplitude during the loading cycle.

4. Experimental Verifications

4.1. Fatigue Test Data

A total of 117 multiaxial fatigue test data of six types of metallic materials from tubular specimens under axial-torsional straining using sinusoidal wave forms exciting in the literature were used to verify the prediction accuracy of the three critical plane parameters. The investigated metallic materials have a widely range including the structural steel, stainless steel and alloys. The fatigue properties are summarized in Table 1.

Table 1. The investigated materials and fatigue properties

Metals	E(GPa)	σ_y (MPa)	σ'_f (MPa)	ε'_f	b	c
S460N	208	500	834	0.157	-0.079	-0.493
16MnR	212	544	966	0.842	-0.101	-0.618
304L SS	200	495	798	0.096	-0.055	-0.446
GH4169	182	1000	1565	0.162	-0.086	-0.580
7075-T651	71.7	501	540	0.222	-0.080	-0.542
Q235B	206	412	630	1.188	-0.080	-0.661

Fatigue data of S460N steel [15], Q235B steel [16] and GH4169 super alloy [17] include in-phase, 45°out-of-phase and 90°out-of-phase combined axial-torsional fatigue test data. Fatigue data of 16MnR steel [18], 304L SS stainless steel [19] and 7075-T651 aluminum alloy [20] include in-phase and 90°out-of-phase combined axial-torsional fatigue test data. The loading paths applied to the multiaxial fatigue tests are summarized in Fig. 2.

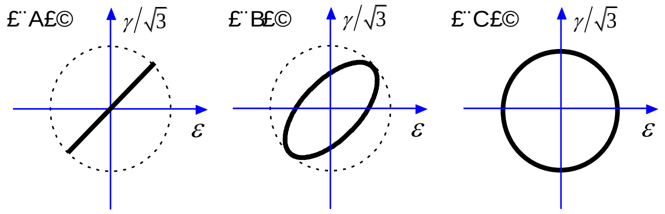
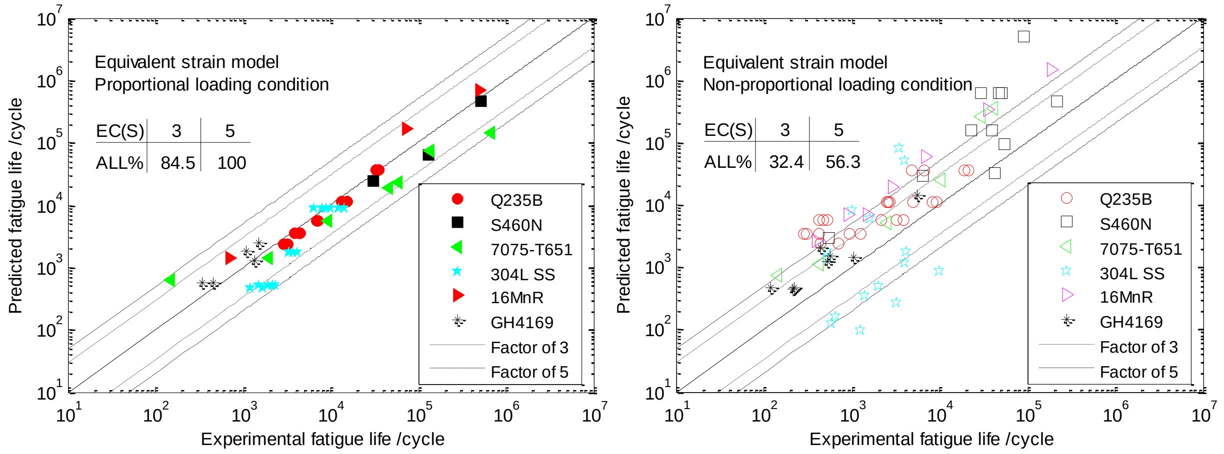


Fig.2 Strain paths: (A) In-phase, (B) 45°out-of-phase, (C) 90°out-of-phase

4.2. Fatigue life predictions

For the given multiaxial loading conditions, the orientation of the critical plane and the strain/stress parameters acting on the critical plane can be determined by using Eq.2~Eq.5, which can be used to calculate the critical plane parameters mentioned above, respectively. Then, the predicted fatigue lives can be generated combined with the Manson-Coffin equation and the critical plane parameters.

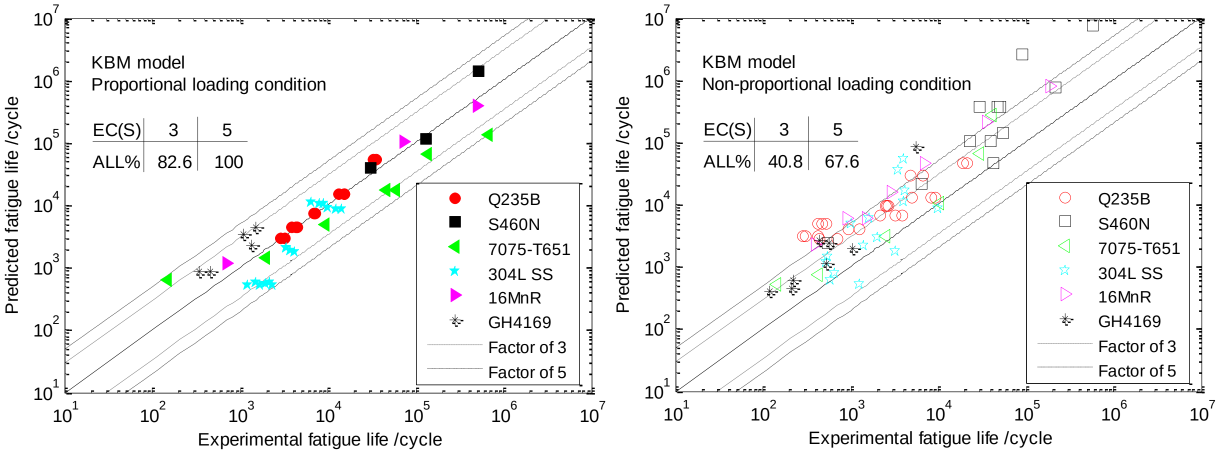
The predicted fatigue lives based on the von Mises equivalent strain parameter are shown in Fig. 3(a) for proportional loading conditions and Fig. 3(b) for non-proportional loading conditions. One can observe that the von Mises effective strain parameter and the fatigue test data can be correlated well for the multiaxial proportional loading conditions, and about 84.8% and 100% predicted fatigue lives are within a scatter band of 3 and 5, respectively. However, the von Mises effective strain parameter fails to correlate the fatigue lives under multiaxial non-proportional loading conditions, only about 32.4% and 56.3% predicted lives are within a scatter band of 3 and 5, respectively.



(a) Proportional loading (b) Non-proportional loading

Fig. 3 Fatigue life prediction based on von Mises equivalent strain parameter

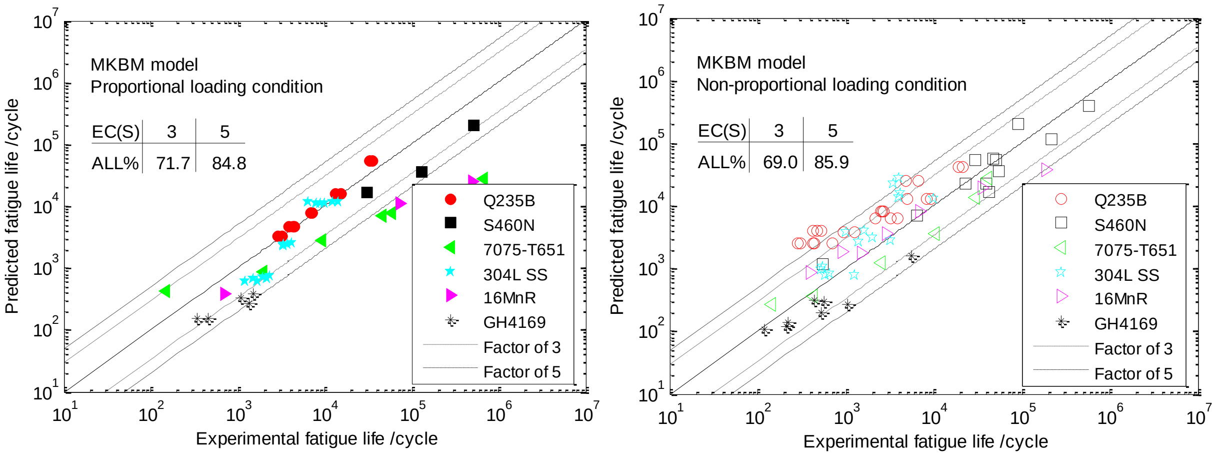
Fig. 4 shows the fatigue life prediction based on the KBM parameter. It can be seen that the KBM parameter can give very well predicted fatigue lives for proportional loading conditions, about 83% predicted lives are within a scatter band within scatter band of 3 and 100% predicted lives are within a scatter band of 5. However, the KBM parameter also give a poor fatigue lives prediction for the multiaxial non-proportional loading conditions, only 40.8% and 67.6% predicted fatigue lives are within a scatter band of 3 and 5, respectively.



(a) Proportional loading (b) Non-proportional loading

Fig.4 Fatigue life prediction based on KBM parameter

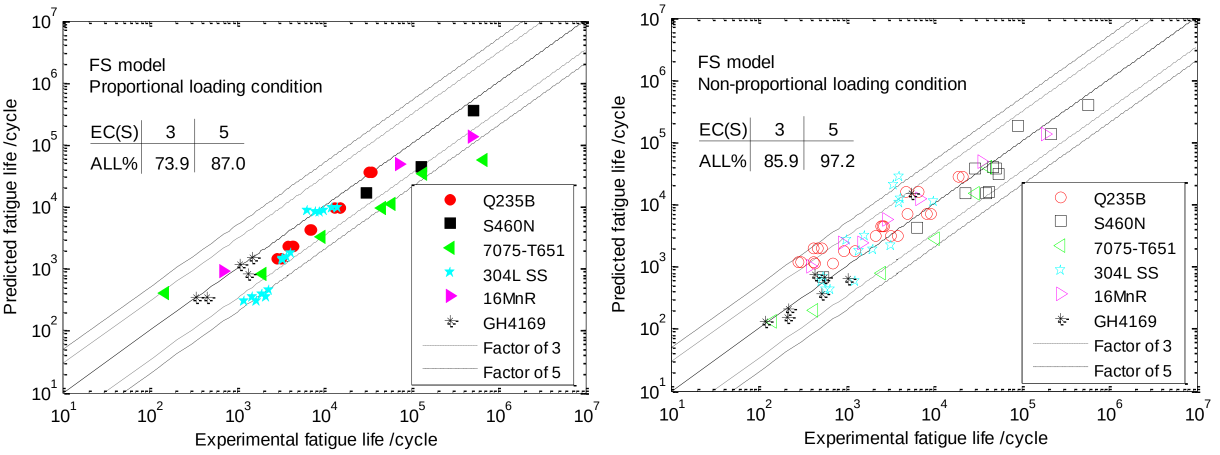
Fig. 4 shows the fatigue life prediction based on the MKBM parameter. It can be seen that the prediction accuracy of the MKBM parameter for proportional loading conditions is not as well as that of the KBM parameter and the equivalent strain parameter, and the predicted fatigue lives are generally tend to be conservative. However, the MKBM parameter can give a better predicted fatigue lives for non-proportional loading conditions, about 69% predicted lives are within a scatter band within scatter band of 3 and 85.9% predicted lives are within a scatter band of 5.



(a) Proportional loading (b) Non-proportional loading

Fig.5 Fatigue life prediction based on MKBM parameter

The fatigue life predictions based on the FS parameter are compared with the experimental fatigue lives in Fig. 6. The FS parameters give similar prediction results with MKBM parameters for proportional loading conditions. About 73.9% and 87.0% of the fatigue data for the proportional loading conditions falling within scatter bands of 2 and 5, respectively. However, the fatigue life predictions based on the FS parameter have very good correlations with the fatigue test data for non-proportional loading conditions. About 85.9% and 97.2% of the predicted fatigue lives for non-proportional loading conditions falling within scatter bands of 3 and 5, respectively. This indicates that the predictions accuracy of the FS parameter is better than that of the equivalent strain parameter, the KBM parameter and the MKBM parameter.



(a) Proportional loading (b) Non-proportional loading

Fig.6 Fatigue life prediction based on FS parameter

5. Discussion

For the most metallic materials, the cyclic stress-strain curves for the non-proportional loading conditions are higher than for uniaxial or proportional loading conditions, which indicate that the metallic materials exhibit additional cycle hardening. However, the strain parameters cannot reflect such additional hardening under the non-proportional loading. For the most metallic materials, the improving accuracy of the FS and MKBM parameters can be attributed to the introduced stress parameter, which can reflect the influence additional hardening due to the non-proportionality of the applied cycle loading on the multiaxial fatigue damage.

However, a types of metallic materials cannot be ignored, which do NOT exhibit non-proportional additional cycle hardening but also have shorter fatigue lives due to the non-proportionality of cycle loading, such as 1050 steel tested by Shamsael [21] and BT1-0 titanium alloy tested Gladskyi [22]. In other words, the cyclic stresses are independent of the loading paths, i.e. uniaxial, proportional or non-proportional loading conditions for these types of metals, which indicate that the FS parameters maybe have no obvious advantages for the multiaxial fatigue evaluation for the non-proportional loading conditions. Accordingly, a more general consideration of the effect of non-proportional loading paths on the multiaxial fatigue damage can be divided into two parts: material additional cyclic hardening and the rotation of principal stress/strain axes due to non-proportional loading paths. The combined effect of the two parts on the multiaxial fatigue damage still needs a detailed analysis and a large amount of experimental measurement, which is beyond the scope of the current study.

6. Conclusions

The existing critical plane models are empirical models and show different efficiency for the multiaxial fatigue life prediction of different metallic materials. Three strain-based critical plane criteria are investigated by using amount of multiaxial fatigue test data in the present study. The experimental results indicate that it is feasible to predict the multiaxial fatigue life for proportional loading conditions by using the equivalent strain and KBM parameters, but these two parameters give poor predicted fatigue lives for non-proportional loading conditions compared with the observed ones. In order to reflect the effect of the non-proportional additional cyclic hardening on multiaxial fatigue damage under the non-proportional loading, the FS and MKBM parameters take the maximum normal stress acting on the critical plane into account. As a consequence, the prediction accuracy of the FS and MKBM parameters are significantly improved for most types of materials exhibiting obvious additional cyclic hardening under non-proportional loading.

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