

A Numerical Study on Wire Coating Analysis in MHD Flow of an Oldroyd 8-Constant Fluid

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Abstract

Wire coating process is an industrial process to coat a wire for insulation, mechanical strength and environmental safety. The ever increasing applications in the industrial process are the cause of great interest in the study of viscoelastic flow in the wire coating process in a pressure type die. In the present study we provide a numerical solution of the Navier-Stokes equation for wire coating in a pressure type die using a bath of Oldroyd 8-constant fluid under constant pressure gradient in the axial direction. The novelty of this study is to analyze the effect of radial magnetic field on the wire coating. The governing differential equations characterizing the flow are solved numerically and the effects of emerging parameters such as dilatant parameter (α), pseudo-plastic parameter (β) and viscosity parameter (Ω) are discussed with the help of graphs. It is interesting to note that an increase in pseudo-plastic parameter accelerates the velocity in the absence of magnetic field. However, the presence of magnetic field slows down velocity of the coating fluid in the entire span of flow domain which is an important design requirement.

Key words

Wire coating, MHD, Oldroyd 8-constant fluid, Co-extrusion process.

1. Introduction

Boundary-layer behavior of a viscoelastic fluid over a continuously stretching surface leads to numerous applications in polymeric extrusion, drawing of plastic films and wires. The ever increasing applications in these industrial processes have generated deep interest in the study of viscoelastic fluid flow in the wire coating process. The wire-coating process is a continuous extrusion process for primary insulation of conducting wires using molten polymer for the purpose of mechanical strength and environmental protection. The plasticized polyvinyl chloride (PVC), low/high density polyethylene (LDPE/HDPE), nylon and polysulfone etc. are used as major plastic resins in coating of wires. The different processes usually implemented for coating of wire are

- Coaxial extrusion process/co- extrusion process
- Dipping process
- Electro-statical deposition process.

Nomenclature

R_w	radius of the wire	S	extra stress tensor
U_w	wire velocity	M	magnetic parameter
p	pressure	R_d	radius of die
B_0	strength of uniform transverse magnetic field	q	velocity of fluid
\vec{B}	magnetic field	η_0	zero stress viscosity
σ	electrical conductivity	μ	coefficient of viscosity
α	dilatant constant	β	pseudo-plastic constant
Ω	constant pressure gradient		

The co-extrusion process studied by Han and Rao [1] is an operation in which either the polymer is extruded on axial moving wire or the wire is dragged inside a die filled with molten polymer. The efficiency of co-extrusion process can be increased by implementing hydrodynamic model by Akter and Hashmi [2]. In this coating process, the velocity of continuum and the melt polymer generate high pressure in a specific region which results in strong bonding and provides fast coating. The experimental set-up of a typical wire coating process is shown in Fig.1.

In this set-up, the uncoated wire unwinds at the payoff reel passing through straightener, a preheater, a cross head die in turn wire meeting the melt polymer emerging from the extruder and gets coated. This coated wire then passes through a cooling through, a capstan and a tester finally ends on the rotating take-up reel. The co-extrusion process is simple to apply, time saving and economical in view of industrial applications.

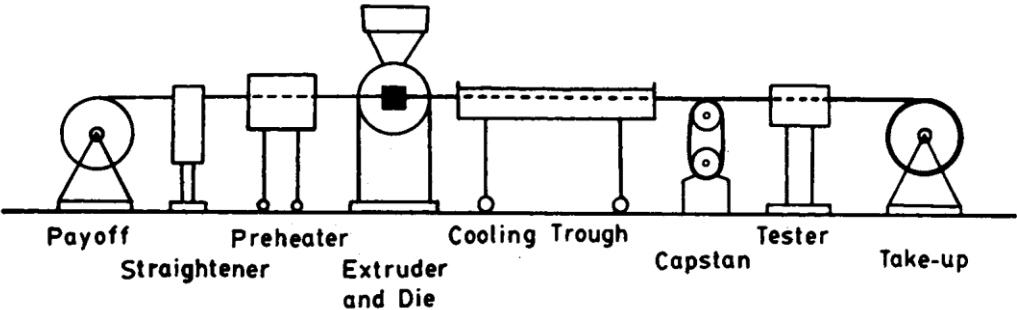


Fig.1 Typical wire coating process

Therefore, many researchers, Tadmor and Gagos [3], Nayak et al. [4], Shah et al. [5] contributed to this field of study. In the wire-coating process, a crosshead is fitted with a pressure type die where the molten plastic contacts with the wire. The molten polymer within the crosshead flows circumferentially through an annulus at an angle ($\leq 90^\circ$) to the extruder.

In the pressure type die (Fig. 2), the wire passes in the crosshead through a guider tip (or torpedo). Also in the pressure type die, the melt is still under pressure within the die when it comes into contact with the wire, which leaves the die already coated.

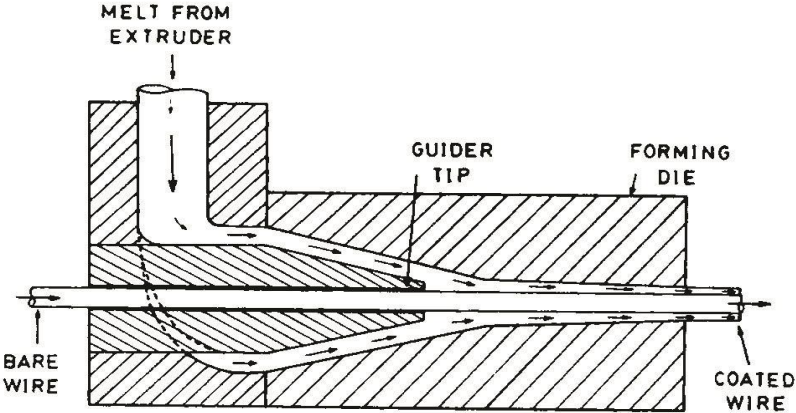


Fig. 2 Schematic representation of wire coating pressure type die.

The design of wire-coating dies is of primary importance as it greatly affects the quality of the final product. Because the melt meets the wire within the die, a complex flow field exists. Therefore, its understanding finds a compelling necessity for the design of better dies with optimum performance and avoiding excessive shear stresses so as to escape from elongation or frequent breakage, uneven and rough extrude coating etc. The effect of die design on the wire coating analysis was studied by Wagner and Mitsoulis [6]. The quality of the coated wire produced is determined by the flow phenomena taking place within the wire-coating dies.

Many fluids utilized in wire-coating exhibit the characteristics of third grade fluid. The third grade fluid is a viscoelastic fluid of industrial importance. Recently, Phan-Thien-Tanner (PTT) model [7], a viscoelastic fluid model is widely used for coating of wire. Siddiqui et al. [8] studied on wire-coating extrusion in a pressure type die with a flow of third grade fluid. Many other contributors to the above study are Aksoy and Pakdemirli [9], Siddiqui et al. [10], Nayak et al. [11].

Siddiqui et al. [12] used variational iteration approach in the study of thin film flow of an Oldroyd 8-constant fluid on a moving belt. Moreover, Saha et al. [13] analyzed effects of dilatant constant, pseudo-plastic constant and the constant pressure gradient in wire coating analysis with Oldroyd 8-constant fluid using optimal homotopy asymptotic method.

Motivated by the above studies, it is intended to investigate numerically the flow of an electrically conducting viscoelastic fluid in wire coating process where the coating material is Oldroyd 8-constant fluid.

The novelty of the present study is to undertake the wire coating process in the presence of a radial magnetic field assuming the coating material as conducting one which is a realistic assumption. The applied magnetic field may play an important role in controlling momentum in the boundary layer flow of melt polymer in wire coating process. Bearing this in mind authors endeavored to explore the effects of transverse magnetic field on the boundary layer flow for Newtonian and non-Newtonian fluids in wire coating process.

2. Formulation of the Problem

The internal geometry of the die considered here is shown in Fig. 3. The wire of radius R_w is extruded with velocity U_w in a pool of an incompressible Oldroyd 8-constant fluid in an annular die of radius R_d as shown in Fig. 3. The wire and die are concentric and the coordinate system is chosen at the centre of the wire in which z is taken in the direction of fluid flow and r is perpendicular to the direction of flow. Assuming that the die is uniform and the flow is steady, laminar and isothermal. The magnetic field is perpendicular to the direction of incompressible flow. The magnetic Reynolds number is taken to be small enough so that the induced magnetic field can be neglected.

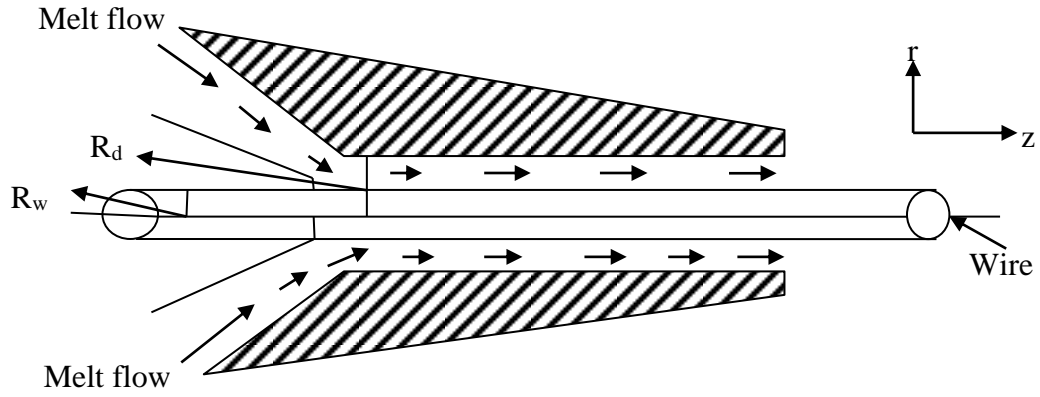


Fig. 3 Internal geometry of wire coating process in pressure type die.

With the above frame of reference and assumption the fluid velocity, extra stress tensor are considered as

$$q = [0, 0, w(r)], \quad S = S(r) \quad (1)$$

Boundary conditions are

$$\begin{aligned} w &= U_w \quad \text{at } r = R_w \\ w &= 0 \quad \text{at } r = R_d \end{aligned} \quad (2)$$

The constitutive equation of Oldroyd 8-constant fluids are defined as [14]

$$\begin{aligned} S + \lambda_1 \overset{\nabla}{S} + \frac{1}{2}(\lambda_1 - \mu_1)(A_1 S + S A_1) + \frac{1}{2}\mu_0 (tr S) A_1 + \frac{1}{2}\nu_1 (tr S A_1) I \\ = \eta_0 \left(A_1 + \lambda_2 \overset{\nabla}{A_1} + (\lambda_2 - \mu_2) A_1^2 + \frac{1}{2}\nu_2 (tr A_1^2) I \right) \end{aligned} \quad (3)$$

where the constants $\eta_0, \lambda_1, \lambda_2$ are respectively zero shear viscosity, relaxation and retardation time. The other five constants $\mu_0, \mu_1, \mu_2, \nu_1, \nu_2$ are associated with nonlinear terms.

The upper contra-variant convected derivative designed by ∇ over S and A_1 are defined as follows [15]

$$\overset{\nabla}{S} = \frac{DS}{Dt} - [(\nabla q)^T S + S(\nabla q)] \quad (4)$$

$$\overset{\nabla}{A_1} = \frac{DA_1}{Dt} - [(\nabla q)^T A_1 + A_1(\nabla q)] \quad (5)$$

where $A_1 = (\nabla q) + (\nabla q)^T$ and $\frac{DS}{Dt} = \left[\frac{\partial}{\partial t} + (q \cdot \nabla) \right] S$ (6)

Substituting the expression given in eqn. (1) into eqns. (3) – (6), the non zero components of extra stress S are obtained as

$$S_{rr} + (\nu_1 - \lambda_1 - \mu_1) \frac{dw}{dr} S_{rz} = \eta_0 (\nu_2 - \lambda_1 - \mu_1) \left(\frac{dw}{dr} \right)^2, \quad (7)$$

$$S_{rz} - \lambda_1 S_{rr} \frac{dw}{dr} + \frac{1}{2} (\lambda_1 - \mu_1 + \mu_0) (S_{rr} + S_{zz}) \frac{dw}{dr} + \frac{\mu_0}{2} S_{zz} \left(\frac{dw}{dr} \right) = \eta_0 \left(\frac{dw}{dr} \right), \quad (8)$$

$$S_{zz} + (\lambda_1 - \mu_1 + \nu_1) \frac{dw}{dr} S_{rz} = \eta_0 (\lambda_1 - \mu_2 + \nu_2) \left(\frac{dw}{dr} \right)^2, \quad (9)$$

$$S_{\theta\theta} + \nu_1 \frac{dw}{dr} S_{rz} = \eta_0 \nu_2 \left(\frac{dw}{dr} \right)^2, \quad (10)$$

Solving eqns. (7) – (10), the explicit expressions for the stress components are obtained as

$$S_{rr} = -(\nu_1 - \lambda_1 - \mu_1) \frac{dw}{dr} S_{rz} + \eta_0 (\nu_2 - \lambda_1 - \mu_1) \left(\frac{dw}{dr} \right)^2, \quad (11)$$

$$S_{\theta\theta} = -\nu_1 \frac{dw}{dr} S_{rz} + \eta_0 \nu_2 \left(\frac{dw}{dr} \right)^2, \quad (12)$$

$$S_{zz} = -(\lambda_1 - \mu_1 + \nu_1) \frac{dw}{dr} S_{rz} + \eta_0 (\lambda_2 - \mu_2 + \nu_2) \left(\frac{dw}{dr} \right)^2, \quad (13)$$

$$S_{rz} = \eta_0 \frac{\left[1 + \alpha \left(\frac{dw}{dr} \right)^2 \right] \frac{dw}{dr}}{1 + \beta \left(\frac{dw}{dr} \right)^2}, \quad (14)$$

where $\alpha = \lambda_1 \lambda_2 + \mu_0 \left(\mu_2 - \frac{3}{2} \nu_2 \right) - \mu_1 (\mu_2 - \nu_2)$,

$$\beta = \lambda_1^2 + \mu_0 \left(\mu_1 - \frac{3}{2} \nu_1 \right) - \mu_1 (\mu_1 - \nu_1)$$

The basic equations governing the flow of an incompressible fluid are:

$$\vec{\nabla} \cdot \vec{q} = 0 \quad (15)$$

$$\rho \frac{D\vec{q}}{Dt} = -\vec{\nabla} p + \vec{F} + \vec{J} \times \vec{B} \quad (16)$$

where \vec{q} is the velocity vector, $\frac{D}{Dt}$ is the material derivative.

Because of interaction of the current and the magnetic field, a body force $\vec{J} \times \vec{B}$ per unit volume of electromagnetic origin appears in the equation of motion given by eqn. (7). The electrostatic force due to charge density is considered to be negligible. A uniform magnetic field of strength B_0 is assumed to be applied in the positive radial direction normal to the wire i.e., along z-axis. Hence the retarding force per unit volume acting along z-axis is given by

$$\vec{J} \times \vec{B} = (0, 0, -\sigma B_0^2 w) \quad (17)$$

From eqn. (1), we observed that the velocity field q and the stress S are functions of r only, so the continuity eqn. (15) is satisfied identically and the dynamic eqn. (16) becomes

$$\frac{\partial p}{\partial r} = \frac{1}{r} \frac{d}{dr} (r S_{rr}) \quad (18)$$

$$\frac{\partial p}{\partial \theta} = 0 \quad (19)$$

$$\frac{\partial p}{\partial z} = \frac{1}{r} \frac{d}{dr} (r S_{rz}) \quad (20)$$

From eqn. (19), we have $p = p(r, z)$.

Substituting the nonzero shear stress given in eqn. (14) into eqn. (20), the differential equation of velocity field is obtained as

$$\begin{aligned}
& r \frac{d^2 w}{dr^2} + \frac{dw}{dr} - r \frac{\partial p}{\partial z} + (\alpha + \beta) \left(\frac{dw}{dr} \right)^3 - \beta r \left(\frac{dw}{dr} \right)^2 \frac{d^2 w}{dr^2} + \alpha \beta r \left(\frac{dw}{dr} \right)^4 \frac{d^2 w}{dr^2} \\
& + 3\alpha r \left(\frac{dw}{dr} \right)^2 \frac{d^2 w}{dr^2} + \alpha \beta \left(\frac{dw}{dr} \right)^5 - 2\beta r \frac{\partial p}{\partial z} \left(\frac{dw}{dr} \right)^2 - \beta^2 r \frac{\partial p}{\partial z} \left(\frac{dw}{dr} \right)^4 - \sigma B_0^2 w = 0
\end{aligned} \quad (21)$$

Let us scale the length with the radius of the uncoated wire, R_w , velocity with the mean velocity, U_w at the die exit, the pressure with a viscous scale, $\mu U_w / R_w$, the parameters α and β with a square of the ratio U_w and R_w , i.e., viscous scale, U_w^2 / R_w^2 . Thus, the dimensionless group that arise are as follows:

$$r^* = \frac{r}{R_w}, w^* = \frac{w}{U_w}, \alpha^* = \frac{\alpha U_w^2}{R_w^2}, \beta^* = \frac{\beta U_w^2}{R_w^2}, p^* = \frac{p}{\mu (U_w / R_w)}, M = \frac{\sigma B_0^2 R_w}{\mu} \quad (22)$$

Applying the above non-dimensional parameters and dropping the asterisks, and taking the assumption that the pressure gradient in the axial direction is constant, i.e., $\frac{\partial p}{\partial z} = \Omega$, eqn. (21) becomes

$$\begin{aligned}
& r \frac{d^2 w}{dr^2} + \frac{dw}{dr} - r \Omega + (\alpha + \beta) \left(\frac{dw}{dr} \right)^3 - \beta r \left(\frac{dw}{dr} \right)^2 \frac{d^2 w}{dr^2} + \alpha \beta r \left(\frac{dw}{dr} \right)^4 \frac{d^2 w}{dr^2} \\
& + 3\alpha r \left(\frac{dw}{dr} \right)^2 \frac{d^2 w}{dr^2} + \alpha \beta \left(\frac{dw}{dr} \right)^5 - r \beta^2 \Omega \left(\frac{dw}{dr} \right)^4 - 2r \beta \Omega \left(\frac{dw}{dr} \right)^2 - M w = 0
\end{aligned} \quad (23)$$

subject to the following physical conditions of no slip on boundaries

$$\begin{aligned}
w &= 1 \quad \text{at } r = 1 \\
w &= 0 \quad \text{at } r = \delta
\end{aligned} \quad (24)$$

where $\delta = \frac{R_d}{R_w} > 1$.

3. Numerical solution

The Runge-Kutta method has been used to solve equation (23) with boundary condition given in eqn. (24). This equation is reduced to a first order differential equation. This is because the value at $r=\delta$ (thickness of boundary layer) is not available for higher order equation. Therefore, the shooting method is used to solve the boundary value problem. The physical and computation domains are finite. For computational purpose, we have assigned $\delta=2$ [4].

Let $w = y_1$ and $\frac{dw}{dr} = y_2$ so that

$$y_2^1 = \frac{-\alpha\beta y_2^5 + r\beta^2\Omega y_2^4 - (\alpha + \beta)y_2^3 + 2r\beta\Omega y_2^2 - y_2 + My_1 + r\Omega}{\alpha\beta r y_2^4 + (3\alpha - \beta)r y_2^2 + r}$$

with $y_a(1)=1$ and $y_b(1)=0$.

4. Results and discussion

The following discussion delineates the results accomplished by Runge-Kutta method on the problem of wire-coating in an annular pressure type die with a bath of Oldroyd 8-constant fluid.

The effects of various pertinent parameters such as dilatant parameter, pseudo-plastic parameter, zero shear viscosity parameter, constant pressure gradient and magnetic parameter are discussed.

Fig. 4 depicts the effects of magnetic parameter (M) and dilatant parameter (α) on the velocity field. It is seen that the magnetic field has a decelerating effect on the velocity field due to resistive Lorentz's force that comes into play because of the interaction of magnetic field and the electrically conducting Oldroyd 8-constant fluid used as coating material.

It is also seen that an increase in dilatant parameter causes to decrease the velocity at all points of the flow domain in the presence as well as absence of magnetic field. However, the decelerating effect is more pronounced in the presence of magnetic field as was observed by Nayak et al. [4].

As the magnetic field contributes to slow down the velocity of coating fluid which is an important design requirement, magnetic field strength may be used as a controlling device for the desired quality.

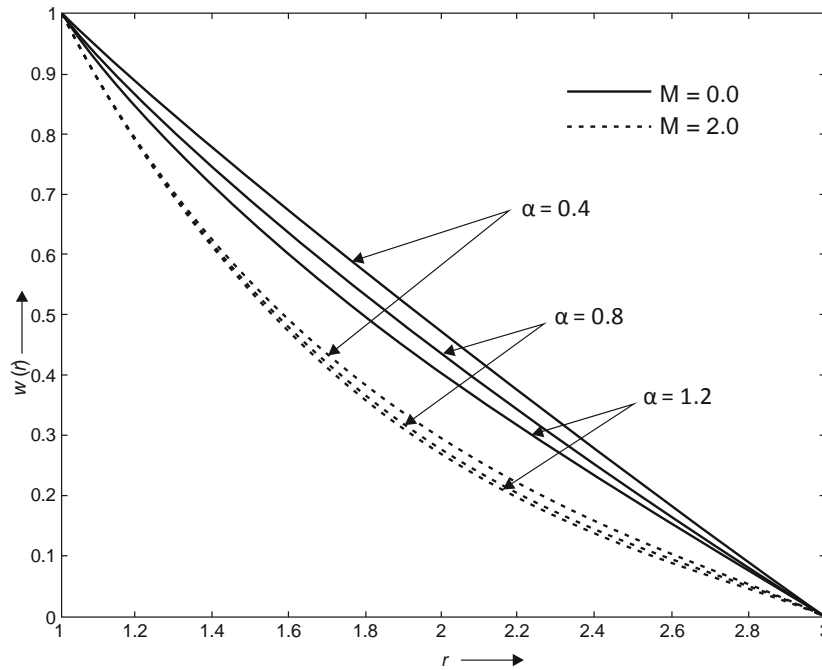


Fig. 4 Velocity distribution showing the effect of α and M for $\Omega = -0.5, \beta = 0.4$.

Fig. 5 shows that an increase in pseudo-plastic parameter (β) accelerates the velocity field in the absence of magnetic field as it was observed by Saha et al. [13]. However, magnetic field has decelerating effect on the velocity field at fixed values of dilatant parameter and constant pressure gradient, and pseudo-plastic parameter. The decelerating of the fluid velocity takes place only because of Lorentz force resulted from the interaction of magnetic field and the electrically conducting Oldroyd 8 constant fluid used as coating material. It is interesting to remark that an increase in pseudo-plastic parameter, keeping magnetic field strength fixed, leads to increase the velocity at all points in the annular region of flow domain. It is important to note that the presence of electromagnetic force due to higher magnetic field may contribute to non-linearity of velocity variation. Moreover, because of occurrence of non-

linearity in the constitutive equation, viscoelastic flows are full of instabilities such as in the flows of extrusion dies as claimed by Nhan-Phan-Thien [7].

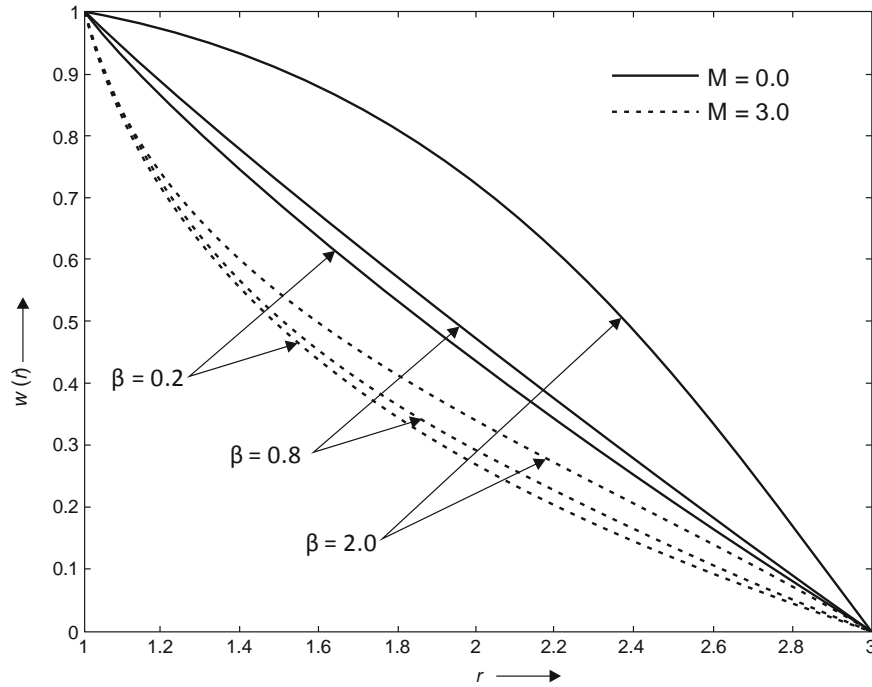


Fig. 5 Velocity distribution showing the effect of β and M for $\Omega = -0.5, \alpha = 0.5$.

Fig. 6 represents the shear stress profiles when the dilatant parameter, constant pressure gradient are set to fixed values. It is observed that in the absence of the magnetic field, an increase in β enhances the shear stress near the surface of the wire significantly but in the presence of magnetic field strength increase in β enhances the same more significantly near the surface of the wire. It is evident that this accelerating effect is more pronounced in the region $1 < r < 1.7$ for presence or absence of magnetic field. Another interesting aspect is the occurrence of point of inflexion vis-à-vis point of intersection in the middle of the annular region that proceeds slightly towards ($r = 1.7$). It is also evident that an increase in magnetic field strength decelerates the shear stress irrespective of lower or higher values of pseudo-plastic parameter within the layers $r < 1.7$ but thereafter, reverse effect is observed. It is physically accepted because excessive shear stress at the wire may lead to elongation or frequent breakage of the wire during coating operation and also uneven and rough extrudate coating.

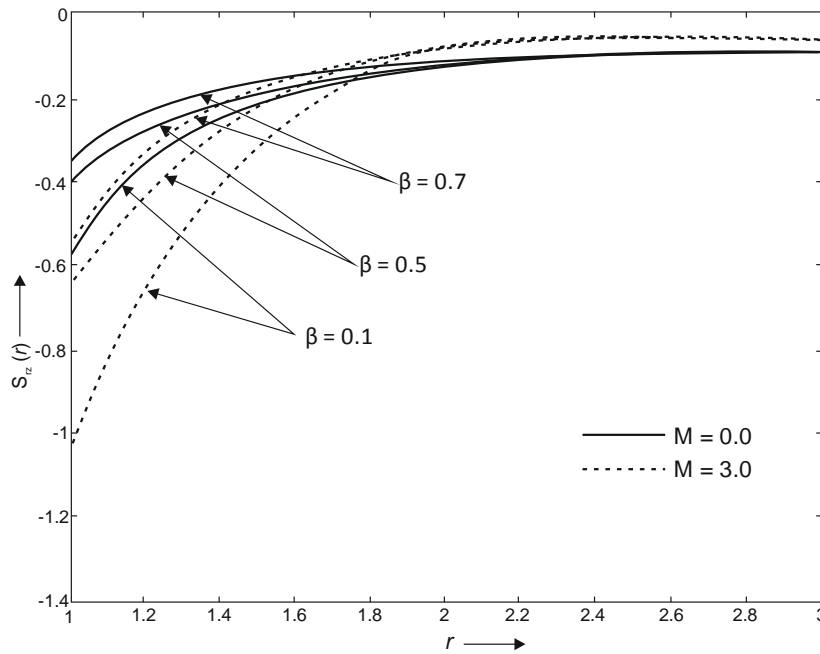


Fig. 6 Shear stress profiles showing the effect of β and M for $\Omega = -0.5, \alpha = 2.0$.

On very careful observation from Fig. 7 it is obvious that an increase in magnetic parameter (M) decreases the shear stress in the region $1 < r < 1.7$ and afterwards reverse effect occurs irrespective of lower, moderate and higher values of zero shear viscosity parameter (η_0). It is interesting to note that an increase in η_0 reduces the shear stress near the surface ($1 < r < 1.7$) of the wire but thereafter, reverse effect is observed for both presence as well as absence of magnetic field.

Fig. 8 delineates the effects of dilatant parameter and magnetic parameter on shear stress at fixed values of constant pressure gradient and pseudo-plastic parameter. First of all it is very very interesting to remark that in the absence of magnetic field at $\Omega = 0.5, \alpha = 0.2$ and $\beta = 0.2$ shear stress attains approximately a constant value at all points of the flow domain. However, the an increase in magnetic parameter leads to decrease the shear stress within the few layers of fluid ($1 < r < 1.7$) from the surface of the wire but thereafter reverse effect is observed. It is also evident that for an increase in α , there is a decrease in shear stress in the region $1 < r < 1.6$ in the absence of magnetic field and the presence of magnetic field is causing a significant decrease in shear stress in the same region. It is important to note that shear stress displays the same type behavior in the region far away from the surface of the wire irrespective

of low, moderate or higher values of α ($\alpha = 0.2, 0.4$ or 1.2) in presence or absence of magnetic field.

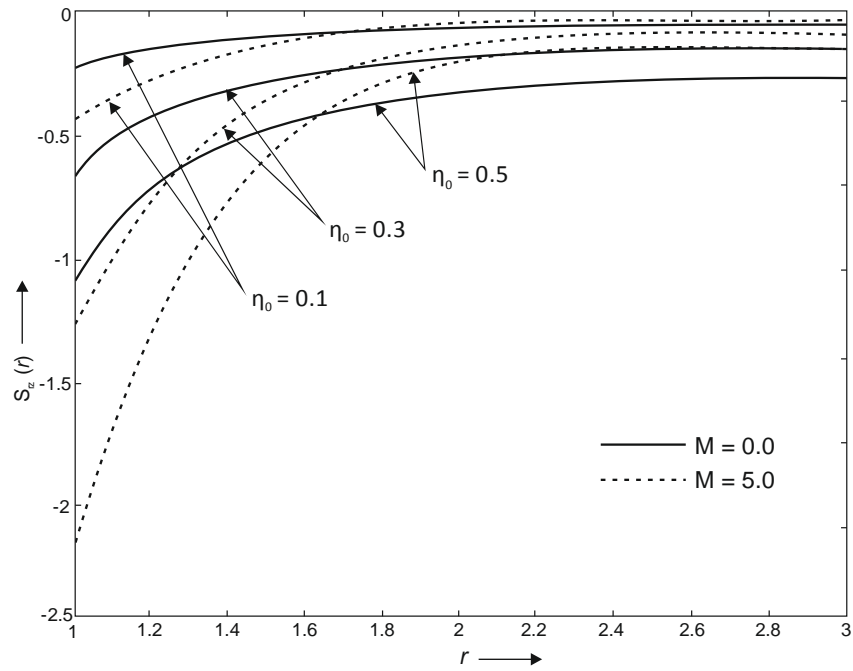


Fig. 7 Shear stress profiles showing the effect of M and η_0 for $\Omega = -0.5, \alpha = 2.0, \beta = 0.4$.

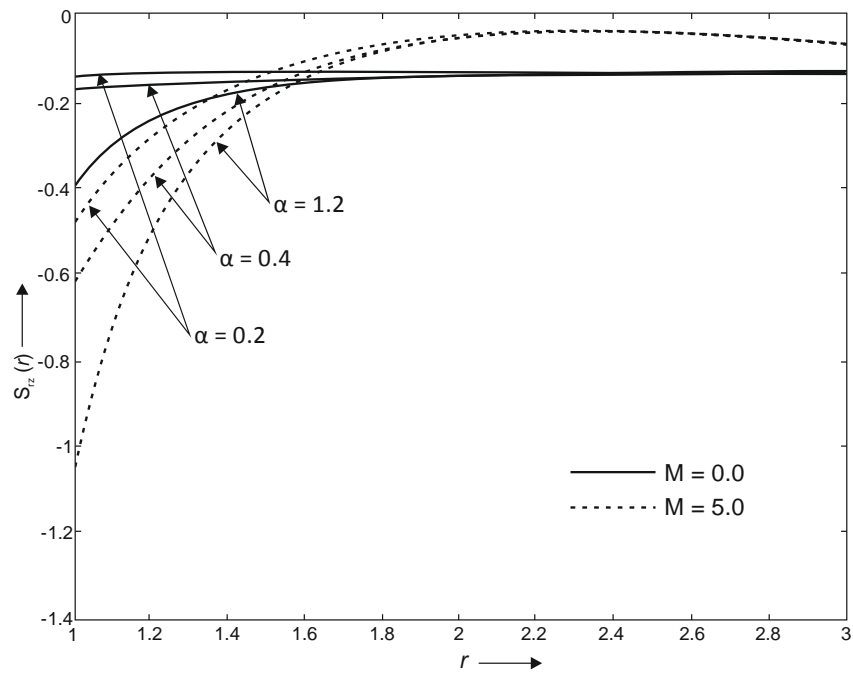


Fig. 8 Shear stress profiles showing the effect of α and M for $\Omega = -0.5, \beta = 0.2$.

5. Conclusion

Wire coating analysis on MHD flow of an Oldroyd 8-constant fluid in a pressure type die has been studied by solving the governing Navier-Stoke's equation numerically employing fourth order Runge-Kutta method along with shooting technique. It is worth mentioning here that non-Newtonian property of the fluid favours to enhance the velocity at all points of flow domain. Magnetic field has a decelerating effect at all points in the annular region of the die. An increase in pseudo-plastic parameter accelerates the velocity at all points of flow domain. It is interesting to note that the presence of magnetic field reduces the shear stress in the region $1 < r < 1.7$ and afterwards the reverse effect takes place irrespective of the values of η_0 . Also the shear stress gets reduced due to the enhancement of α . Moreover, the presence of electromagnetic force due to application of moderately large magnetic field may contribute to non-linearity of velocity variation.

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