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Flow and Mass Transfer Analysis of a Micropolar Fluid in a Vertical Channel with Heat Source and Chemical Reaction

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Abstract

This paper reports an analytical study of steady free convection and mass transfer flow of a micropolar fluid between two vertical walls in the presence of temperature dependent heat source/sink and chemical reaction. The analytical solutions of the governing coupled differential equations are presented for a wide range of emerging parameters. The intricacy of the solution arises due to the coupling of microrotation with velocity gradient giving rise to implicit boundary condition. A striking result is to note that microrotation is independent of the material property and vortex viscocity in the middle layers of the channel. It is observed that our result is well agreement with the result reported earlier in the absence of mass transfer associated with chemical reaction.

Keywords: Free convection; Micropolar fluid; Heat-source/sink; Chemical reaction

1. Introduction

Micropolar fluids proposed by Eringen [1] simulate accurately the flow characteristics of polymeric additives, geomorphological sediments, colloidal and haematological suspensions, liquid crystals, lubricants etc. Studies of the flows of heat convection in micropolar fluids focus mainly on flat surfaces by Rahman et al. [2-6]. Hassanien et al. [7] have considered natural convection flow of micropolar fluid along a vertical and a permeable semi-infinite plate embedded in a porous medium. The problem of fully developed natural convection heat and mass transfer of a micropolar fluid between porous vertical plates with asymmetric wall temperatures and concentrations is analyzed by Abdulaziz and Hashim [8]. Desseaux and Kelson [9] have studied the flow of a micropolar fluid bounded by a stretching sheet. Mitarai et al. [10] have presented the effect of collisional granular flow as a micropolar fluid. El-Arabawy [11] has shown the effect of suction/injection on the flow of a micropolar fluid past a continuously moving plate in the presence of radiation. Aydin and Pop [12-13] observed the natural convection in a heated enclosure filled with a micropolar fluid. The unsteady natural convection heat transfer of micropolar fluid over a vertical surface with constant heat flux was studied by Damseh et al. [14]. Chamkha et al. [15] analyzed numerical and analytical solutions of the developing laminar free convection of a micropolar fluid in vertical parallel plate channel with asymmetric heating.

Nomenclature

B	material parameter	C_p	specific heat at constant pressure
C	non-dimensional species parameter	C_0	reference species concentration
D	diffusion coefficient	E	total heat rate added to the fluid
g	acceleration due to gravity	j	micro-inertial density
k	kinematic rotational viscosity	K_c	chemical reaction parameter
L	distance between two vertical walls	m	temperature ratio parameter
m_1	concentration ratio parameter	Q	volume flow rate
R	vortex viscosity parameter	S	heat source/sink parameter
T	non-dimensional temperature	t	non-dimensional time
T_0	reference temperature	u	non-dimensional velocity
ω	micro-rotation velocity		

Greek letters

α	thermal diffusivity
γ	local buoyancy parameter
μ	kinematic viscosity
ρ	density of the fluid

Parida et al. [16] have studied MHD heat and mass transfer in a rotating system with periodic suction. Recently, Zucco et al. [17] discussed magneto-micropolar fluid over a stretching surface embedded in a Darcian porous medium by the numerical network method. Further, Eldabe et al. [18] reported their work on hydromagnetic peristaltic flow on micropolar biviscocity fluid. Another interesting work related to MHD visco-elastic flow with heat and mass transfer was reported very recently by Kar et al. [19]. Helmy et al. [20] studied MHD free convection flow of a micropolar fluid past a vertical porous plate.

Flow of fluids with internal heat sources/sinks are of great practical as well as theoretical interest. The fluid motion develops slowly following the development of non-uniformity in the temperature field. The volumetric heat generation / absorption term exerts strong influence on the flow and heat transfer when the temperature difference is appreciably large. The analysis of temperature field as modified by the heat source / sink in moving fluid is important in view of chemical reaction and problem concerned with dissociating fluids. Acharya et al. [21] studied the flow problems in the presence of heat source. Foraboschi et al. [22] have assumed two state volumetric heat generation as depending on temperature difference.

$$\theta = \begin{cases} \theta_0 (T - T_0), & T \geq T_0 \\ 0, & T < T_0 \end{cases}$$

In many chemical engineering processes chemical reaction takes place between a foreign mass and the working fluid which moves due to stretching otherwise of a surface. A chemical reaction is said to be first order and homogenous if its rate of reaction is directly proportional to the concentration and it occurs as a single phase volume reaction. Rath et al. [23] studied MHD flow with heat and mass transfer on a porous stretching wall embedded in porous medium with chemical reaction and Jha and Kaurangini [24] examined the unsteady/steady state pressure-driven fluid flow in composite microchannel.

The purpose of the present study is to develop a mathematical model of micropolar fluid flow, heat transfer and mass transfer in a vertical channel. The heat transfer associated with internal temperature dependent volumetric heat source/ sink and concentration distribution is accompanied by the chemical reaction of the reactive species obeying a first order reaction. The novelty of the present work is to study the effects of species concentration and chemical reaction over and above other physical parameters on the velocity, microrotation and the volumetric flow rate. The analytical solutions are obtained and analysed to bring out the effects of various emerging parameters.

The present investigation is likely to have bearing in geothermal areas where chemically treated ground water or industrial fluid exhibiting micropolar fluid properties.

2. Mathematical formulation

Consider steady laminar free convective flow of a micropolar fluid between two vertical walls in the presence of a temperature dependent heat source/sink and chemical reaction.

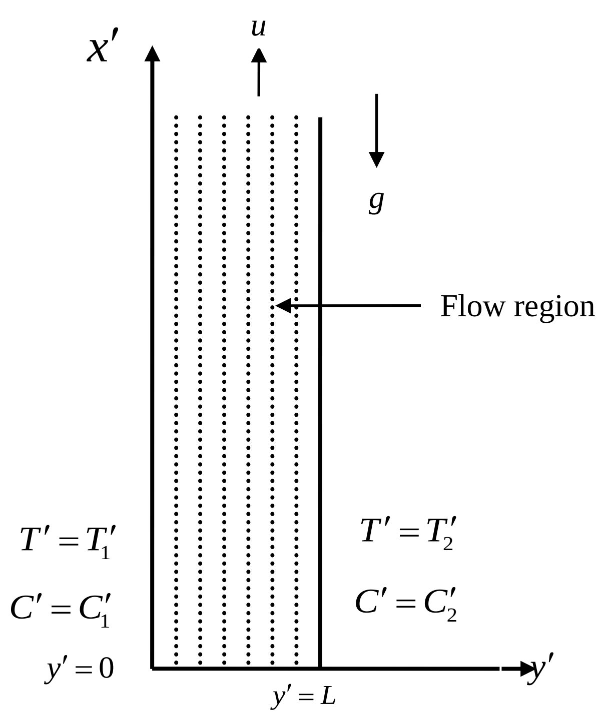


Fig. 1: Schematic diagram of the flow model.

The vertical walls are separated by a distance L and having temperatures T_1' and T_2' and concentrations C_1' and C_2'. The $x'-$ axis is taken along one of the vertical walls while $y'-$ axis is normal to it. The walls are assumed to be infinitely long so that the dependent variables are not dependent on the vertical co-ordinate. For free convection, let us consider the vertical walls maintained at different temperatures and concentrations in still micropolar fluid. The velocity of the fluid is induced by heat and mass transfer from the plates and it is small. Hence, the dissipation due to viscosity is neglected in this problem, but body forces due to gravity are added following Pai [25]. The micropolar fluid may be considered as an incompressible fluid if the temperature and concentration gradients are not large. Under these assumptions the governing equations corresponding to the considered model are derived as follows:

$$(\mu+k)\frac{d^2u'}{dy'^2}+k\frac{d\omega'}{dy'}+\rho g\frac{T'-T_0'}{T_0'}+\rho g\frac{C'-C_0'}{C_0'}=0 \tag{1}$$

$$\gamma\frac{d^2\omega'}{dy'^2}-k(2\omega'+\frac{du'}{dy'})=0 \tag{2}$$

$$\alpha\frac{d^2T'}{dy'^2}+S'(T'-T_0')=0 \tag{3}$$

$$D\frac{d^2C'}{dy'^2}+K_c{}'(C'-C_0')=0 \tag{4}$$

where $\gamma=(\mu+0.5k)\,j$, $\alpha=\frac{k}{\rho C_p}$

The boundary conditions are

$$\left.\begin{aligned}&u'=0,\omega'=-\frac{1}{2}\frac{du'}{dy'},T'=T_1',C'=C_1' \quad at \;\; y'=0\\ &u'=0,\omega'=-\frac{1}{2}\frac{du'}{dy'},T'=T_2',C'=C_2' \quad at \;\; y'=L\end{aligned}\right\} \tag{5}$$

In the energy equation the heat due to viscous dissipation is neglected for small velocities. Soret-Dufour (thermal diffusion and diffusion-thermo) effects are also ignored in the diffusion equations which is true when the species concentration level is very low.

Introducing the following non-dimensional quantities,

$$\left.\begin{aligned}&y=y'/L,\;\omega=\frac{\omega'\mu}{\rho g\,L},u=\frac{u'\mu}{\rho gL^2},\\ &T=\frac{T'-T_0'}{T_0'},C=\frac{C'-C_0'}{C_0'},m=\frac{T_2'-T_0'}{T_0'},m_1=\frac{C_2'-C_0'}{C_0'}\\ &B=L^2/j,S=S'L^2/\alpha,K_c=K'_c\,L^2\;/D,R=k/\mu\end{aligned}\right\} \tag{6}$$

the equations (1)-(4) reduce to

$$(1+R)\frac{d^2u}{dy^2}+R\frac{d\omega}{dy}=-T-C \tag{7}$$

$$\left(1+\frac{R}{2}\right)\frac{d^2\omega}{dy^2}-BR(2\omega+\frac{du}{dy})=0 \tag{8}$$

$$\frac{d^2T}{dy^2} + ST = 0 \tag{9}$$

$$\frac{d^2C}{dy^2} + K_c C = 0 \tag{10}$$

and the corresponding boundary conditions (5) become

$$\left.\begin{array}{l} u = 0, \omega = -\frac{1}{2}\frac{du}{dy}, T = 1, C = 1 \quad at \;\; y = 0 \\ u = 0, \omega = -\frac{1}{2}\frac{du}{dy}, T = m, C = m_1 \quad at \;\; y = 1 \end{array}\right\} \tag{11}$$

3. Analytical solution

The solutions are related to three different cases depicting different physical situations. Case-I deals with a physical model of fluid flow where the heat transfer process are not only affected by the temperature differences of the boundary surfaces but also by a heat source. Further, the mass diffusion process affecting the flow of fluids subject to chemically reactive species find various applications in polymer processing industries. There are other flow fields which arise from differences in concentration or material constitution alone and in conjunction with temperature effects.

Atmospheric flows, at all scales, are driven appreciably by both temperature and H_2O concentration difference. Flows in bodies of water are driven through the comparable effects upon density, temperature, concentration of dissolved materials and suspended particulate matters. Thus, the following cases are discussed taking all the cases or any particular case of interest.

Solutions of the equations (7) – (10) under the boundary conditions (11) for different cases are as follows:

Case – I Presence of temperature dependent source and exothermic reaction ($S > 0, K_c > 0$)

$$u = A_{32}A_{18}e^{\sqrt{A_4}y} + A_{33}A_{19}e^{-\sqrt{A_4}y} + A_{20}\cos\sqrt{S}y + A_{21}\sin\sqrt{S}y$$
$$+A_{22}\cos\sqrt{K_c}y + A_{23}\sin\sqrt{K_c}y + A_{24}A_{34}y + A_{35} \tag{12}$$

$$\omega = A_{32}.e^{\sqrt{A_4}y} + A_{33}.e^{-\sqrt{A_4}y} + A_7\sin\sqrt{S}y + A_8\cos\sqrt{S}y + A_9\sin\sqrt{K_c}y + A_{10}\cos\sqrt{K_c}y - \frac{A_{34}}{A_4} \tag{13}$$

$$T = \cos\sqrt{S}\,y + A_1 \sin\sqrt{S}\,y \tag{14}$$

$$C = \cos\sqrt{K_c}\,y + A_2 \sin\sqrt{K_c}\,y \tag{15}$$

The dimensionless volume flow rate is given by

$$Q = \int_0^1 u\,dy = \frac{A_{32}A_{18}}{\sqrt{A_4}}(e^{\sqrt{A_4}} - 1) - \frac{A_{33}A_{19}}{\sqrt{A_4}}(e^{-\sqrt{A_4}} - 1) + \frac{A_{20}}{\sqrt{S}}\sin\sqrt{S} - \frac{A_{21}}{\sqrt{S}}(\cos\sqrt{S} - 1)$$

$$+\frac{A_{22}}{\sqrt{K_c}}\sin\sqrt{K_c} - \frac{A_{23}}{\sqrt{K_c}}(\cos\sqrt{K_c} - 1) + \frac{1}{2}A_{24}A_{34} + A_{35} \tag{16}$$

The dimensionless total heat rate added to the fluid is given by

$$E = \int_0^1 uT\,dy = A_{32}A_{18}\left[\frac{\sqrt{A_4}}{S + A_4}(e^{\sqrt{A_4}}.\cos\sqrt{S} - 1) - \left(\frac{S}{S + A_4}\right)e^{\sqrt{A_4}}.\sin\sqrt{S}\right]$$

$$+A_{32}A_{18}A_1\left[\left(\frac{\sqrt{A_4}}{S + A_4}\right)e^{\sqrt{A_4}}.\sin\sqrt{S} - \frac{\sqrt{S}}{S + A_4}(e^{\sqrt{A_4}}.\cos\sqrt{S} - 1)\right]$$

$$+A_{33}A_{19}\left[\frac{\sqrt{A_4}}{S + A_4}(1 - e^{\sqrt{A_4}}.\cos\sqrt{S}) + \frac{\sqrt{S}}{S + A_4}.e^{-\sqrt{A_4}}.\sin\sqrt{S}\right]$$

$$+A_{33}A_{19}A_1\left[\frac{-\sqrt{A_4}}{S + A_4}e^{-\sqrt{A_4}}.\sin\sqrt{S} + \frac{\sqrt{S}}{S + A_4}(1 - e^{-\sqrt{A_4}}.\cos\sqrt{S})\right]$$

$$+\frac{A_{20}}{2}\left(1 + \frac{1}{2\sqrt{S}}.\sin 2\sqrt{S}\right) + \frac{A_{20}A_1}{4\sqrt{S}}(1 - \cos 2\sqrt{S}) + \frac{A_{21}}{4\sqrt{S}}(1 - \cos 2\sqrt{S}) + \frac{A_{21}A_1}{2}\left(1 - \frac{\sin 2\sqrt{S}}{2\sqrt{S}}\right)$$

$$+\frac{A_{22}}{2}\left[\frac{\sin(\sqrt{S} + \sqrt{K_c})}{\sqrt{S} + \sqrt{K_c}} + \frac{\sin(\sqrt{S} - \sqrt{K_c})}{\sqrt{S} - \sqrt{K_c}}\right] + \frac{A_{21}A_1}{2}\left[\frac{1 - \cos(\sqrt{S} + \sqrt{K_c})}{\sqrt{S} + \sqrt{K_c}} + \frac{1 - \cos(\sqrt{S} - \sqrt{K_c})}{\sqrt{S} - \sqrt{K_c}}\right]$$

$$+\frac{A_{23}}{2}\left[\frac{1 - \cos(\sqrt{S} + \sqrt{K_c})}{\sqrt{S} + \sqrt{K_c}} - \frac{1 - \cos(\sqrt{S} - \sqrt{K_c})}{\sqrt{S} - \sqrt{K_c}}\right]$$

$$+\frac{A_{23}A_1}{2}\left[\frac{\sin(\sqrt{S} - \sqrt{K_c})}{\sqrt{S} - \sqrt{K_c}} - \frac{\sin(\sqrt{S} + \sqrt{K_c})}{\sqrt{S} + \sqrt{K_c}}\right] + A_{34}A_{24}\left[\frac{1}{\sqrt{S}}.\sin\sqrt{S} + \frac{1}{S}(\cos\sqrt{S} - 1)\right]$$

$$+A_{34}A_{24}A_1\left[\frac{1}{\sqrt{S}}(1 - \cos\sqrt{S}) + \frac{1}{S}\sin\sqrt{S}\right] + A_{35}\left[\frac{\sin\sqrt{S}}{\sqrt{S}} - \frac{A_1}{\sqrt{S}}(\cos\sqrt{S} - 1)\right] \tag{17}$$

Case – II Absence of heat source and chemical reaction ($S=0, K_c=0$)

$$u = A_{54}A_{18}\,e^{\sqrt{A_4}\,y} + A_{55}A_{19}\,e^{-\sqrt{A_4}\,y} + A_{44}y^3 + A_{45}y^2 + A_{46}y + A_{24}A_{56}y + A_{57} \tag{18}$$

$$\omega = A_{54}\,e^{\sqrt{A_4}\,y} + A_{55}\,e^{-\sqrt{A_4}\,y} + A_{36}\left(y^2 - \frac{2}{A_4}\right) + A_{37}y + A_{38}\left(y^2 - \frac{2}{A_4}\right) + A_{39}y - \frac{A_{56}}{A_4} \tag{19}$$

$$T = (m-1)y + 1 \tag{20}$$

$$C = (m_1 - 1)y + 1 \tag{21}$$

The dimensionless volume flow rate is given by

$$Q = \int_0^1 u\,dy = \frac{A_{54}A_{18}}{\sqrt{A_4}}(e^{\sqrt{A_4}} - 1) - \frac{A_{55}A_{19}}{\sqrt{A_4}}(e^{-\sqrt{A_4}} - 1) + \frac{A_{44}}{4} + \frac{A_{45}}{3} + \frac{A_{46}}{2} + \frac{A_{24}A_{56}}{2} + A_{57} \tag{22}$$

The dimensionless total heat rate added to the fluid is given by

$$E = \int_0^1 uT\,dy = Q + (m-1)\left[A_{54}A_{18}\left\{\frac{e^{\sqrt{A_4}}}{\sqrt{A_4}} - \frac{1}{A_4}(e^{\sqrt{A_4}} - 1)\right\} - A_{55}A_{19}\left\{\frac{e^{-\sqrt{A_4}}}{\sqrt{A_4}} + \frac{1}{A_{14}}(e^{-\sqrt{A_4}} - 1)\right\} + \frac{A_{44}}{4} + \frac{A_{45}}{3} + \frac{A_{46}}{2} + \frac{A_{24}A_{56}}{2} + A_{57}\right] \tag{23}$$

Case – III Presence of sink and endothermic reaction ($S<0, K_c<0$)

$$u = A_{72}A_{31}\,e^{\sqrt{A_4}y} + A_{73}A_{32}\,e^{-\sqrt{A_4}y} + A_{74}\,e^{\sqrt{S}y} + A_{75}\,e^{-\sqrt{S}y} + A_{76}\,e^{\sqrt{K_c}y} + A_{77}\,e^{-\sqrt{K_c}y} + A_{78}A_{90}\,y + A_{91} \tag{24}$$

$$\omega = A_{87}\,e^{\sqrt{A_4}y} + A_{88}\,e^{-\sqrt{A_4}y} + A_{61}\,e^{\sqrt{S}y} + A_{62}\,e^{-\sqrt{S}y} + A_{63}\,e^{\sqrt{K_c}y} + A_{64}\,e^{-\sqrt{K_c}y} - \frac{A_{89}}{A_4} \tag{25}$$

$$T = A_{58}\,e^{\sqrt{S}y} + A_{59}\,e^{-\sqrt{S}y} \tag{26}$$

$$C = A_{60}\,e^{\sqrt{K_c}y} + A_{61}\,e^{-\sqrt{K_c}y} \tag{27}$$

The dimensionless volume flow rate is given by

$$Q = \int_0^1 u\,dy = \frac{A_{72}A_{31}}{\sqrt{A_4}}(e^{\sqrt{A_4}} - 1) - \frac{A_{73}A_{32}}{\sqrt{A_4}}(e^{-\sqrt{A_4}} - 1) + \frac{A_{74}}{\sqrt{S}}(e^{\sqrt{S}} - 1) - \frac{A_{75}}{\sqrt{S}}(e^{-\sqrt{S}} - 1) + \frac{A_{76}}{\sqrt{K_c}}(e^{\sqrt{K_c}} - 1) - \frac{A_{77}}{\sqrt{K_c}}(e^{-\sqrt{K_c}} - 1) + \frac{1}{2}A_{78}A_{90} + A_{91} \tag{28}$$

The dimensionless total heat rate added to the fluid is given by

$$E=\int_0^1 uT\,dy=\frac{A_{72}A_{31}A_{58}}{\sqrt{A_4}+\sqrt{S}}(e^{\sqrt{A_4}+\sqrt{S}}-1)+\frac{A_{72}A_{31}A_{59}}{\sqrt{A_4}-\sqrt{S}}(e^{\sqrt{A_4}-\sqrt{S}}-1)+\frac{A_{73}A_{32}A_{58}}{\sqrt{S}-\sqrt{A_4}}(e^{\sqrt{S}-\sqrt{A_4}}-1)$$

$$-\frac{A_{73}A_{32}A_{59}}{\sqrt{A_4}+\sqrt{S}}\left[e^{-(\sqrt{A_4}+\sqrt{S})}-1\right]+\frac{A_{58}A_{74}}{2\sqrt{S}}(e^{2\sqrt{S}}-1)$$

$$+A_{59}A_{74}+A_{58}A_{75}-\frac{A_{59}A_{75}}{2\sqrt{S}}(e^{-2\sqrt{S}}-1)+\frac{A_{58}A_{76}}{\sqrt{S}+\sqrt{K_c}}(e^{\sqrt{S}+\sqrt{K_c}}-1)+\frac{A_{59}A_{76}}{\sqrt{K_c}-\sqrt{S}}(e^{\sqrt{K_c}-\sqrt{S}}-1)$$

$$+\frac{A_{58}A_{77}}{\sqrt{S}-\sqrt{K_c}}(e^{\sqrt{S}-\sqrt{K_c}}-1)-\frac{A_{59}A_{77}}{\sqrt{S}+\sqrt{K_c}}\left[e^{-(\sqrt{S}+\sqrt{K_c})}-1\right]$$

$$+A_{58}A_{78}A_{90}\left[\frac{e^{\sqrt{S}}}{\sqrt{S}}-\frac{1}{S}(e^{\sqrt{S}}-1)\right]-A_{59}A_{78}A_{90}\left[\frac{e^{-\sqrt{S}}}{\sqrt{S}}+\frac{1}{S}(e^{-\sqrt{S}}-1)\right]+\frac{A_{58}A_{91}}{\sqrt{S}}(e^{\sqrt{S}}-1)-\frac{A_{59}A_{91}}{\sqrt{S}}(e^{-\sqrt{S}}-1) \qquad (29)$$

4. Results and discussion

An analytical solution for the problem of steady free convection and mass transfer flow of a micropolar fluid between two vertical walls in the presence of heat source/sink and chemical reaction is analyzed for different values of governing parameters and the results are manifested graphically.

From the practical point of view, it is important to know the effects of mass transfer, heat transfer and chemical reaction parameter of the reacting species on the flow field. For the purpose of calculation we have considered two cases (i) under symmetric distribution of temperature and concentration ($m=m_1=1$) i.e., $T_2'=2T_0'$ and $C_2'=2C_0'$ which correspond to the temperature and concentration at the plate $y=1$ are twice the reference temperature and concentration. Moreover, $T=1$ and $C=1$, boundary conditions, correspond to $T_1'=2T_0'$ and $C_1'=2C_0'$. Hence, the results are pertaining to the cases of equal temperature and concentration (ii) asymmetric distribution ($m=m_1=0$) corresponds to unequal temperature and concentration at the plates.

The parabolic velocity distribution profiles bear a common characteristic i.e. presence of heat source and exothermic reaction**(** $S>0, K_c>0$ **)** escalates the velocity at all points in comparison with the case of sink and endothermic reaction ($S<0, K_c<0$) overriding the effects of other parameters on the flow phenomena for both symmetric and asymmetric temperature and concentration distributions.

The velocity distribution is almost symmetrical about the middle of the channel with a crest at $y = 0.5$ (approx.). This is a common phenomena encountered in channel flow reported in literature earlier and in particular by Ravi et al. [26] recently. They have considered the micropolar fluid flow in a vertical channel in the absence of mass transfer with chemical reaction.

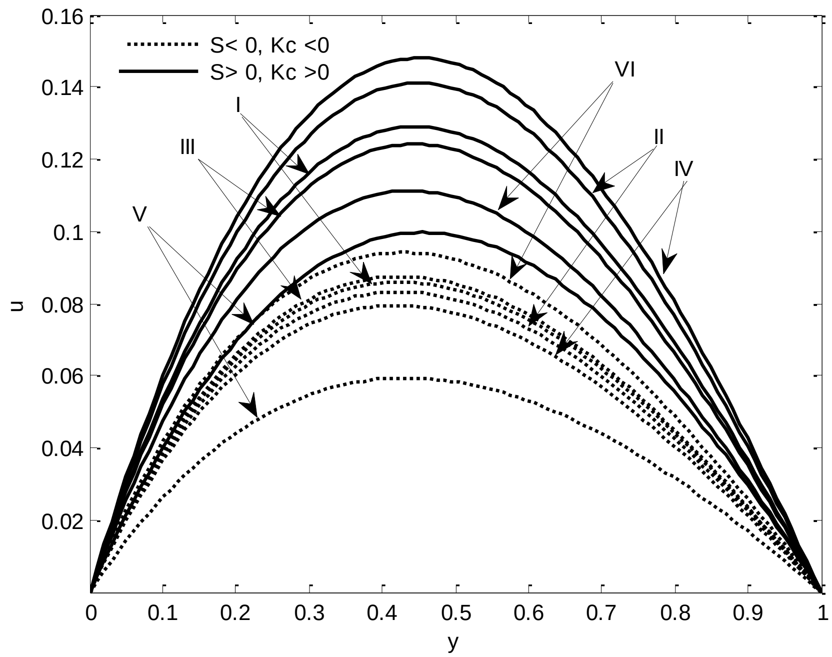
Moreover, from equations (7) and (8), it is evident that vortex viscosity parameter R attributes micropolar property. If R is set to zero both the governing equations reduce to

$$\frac{d^2u}{dy^2} = -T - C \quad \text{and} \quad \frac{d^2\omega}{dy^2} = 0$$

which indicate that effect of material parameter automatically becomes ineffective.

On the other hand, in the absence of material parameter, $B = 0$, the equations still contain R exhibiting the effect of vortex viscosity. Hence, the role of vortex viscosity persists in the absence of material parameter. It is also evident that the velocity attains maximum in case of source and exothermic reaction.

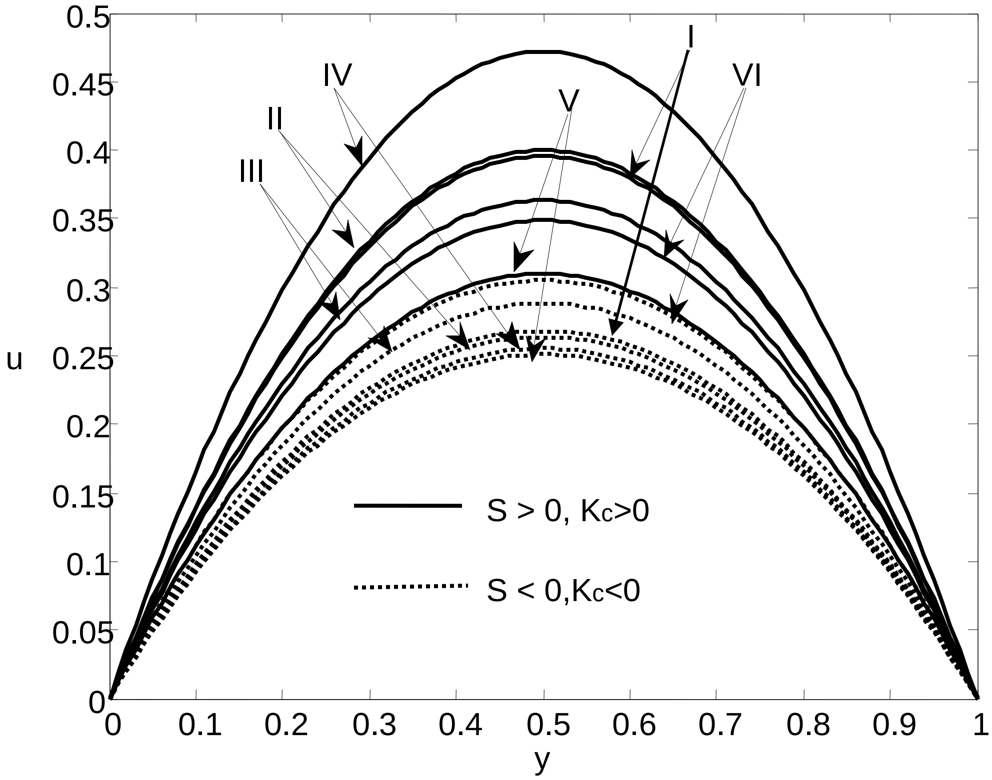
Fig. 2 (asymmetric case: $m = m_1 = 0$) exhibits the velocity profiles for different values of vortex viscosity parameter *R*, heat source/sink parameter *S*, material parameter *B* and chemical reaction parameter K_c for both source and exothermic reaction $(S > 0, K_c > 0)$ as well as sink and endothermic reaction $(S < 0, K_c < 0)$. It is observed that an increase in *S* and K_c, increases the velocity whereas increase in *R*, decreases it in the presence of source and exothermic reaction as well as sink and endothermic reaction. But increase in *B* causes a decrease in velocity in the presence of source and exothermic reaction where as an opposite effect is observed in the presence of sink and endothermic reaction.



Curve	R	S	B	Kc	Curve	R	S	B	Kc
I	0.5	1.0	0.5	3.0	I	0.5	-1.0	0.5	-3.0
II	0.5	1.0	0.5	4.0	II	0.5	-1.0	0.5	-4.0
III	0.5	1.0	1.0	4.0	III	0.5	-1.0	1.0	-4.0
IV	0.5	2.0	0.5	4.0	IV	0.5	-2.0	0.5	-4.0
V	1.5	1.0	0.5	4.0	V	1.5	-1.0	0.5	-4.0
VI	0.5	1.0	0.5	0.7	VI	0.5	-1.0	0.5	-0.7

Fig 2: Velocity profiles showing effect of R, S, B , K_c for $m = m_1 = 0$ (asymmetric).

Fig. 3 (symmetric case : $m = m_1 = 1$) presents the velocity profiles for different values of *R*, *S*, *B* and K_c for both $S > 0, K_c > 0$ and $S < 0, K_c < 0$. The effects of all the parameters qualitatively remain same as that of asymmetric case besides symmetricity of the profiles.



Curve	R	S	B	Kc	Curve	R	S	B	Kc
I	0.5	1.0	0.5	3.0	I	0.5	-1.0	0.5	-3.0
II	0.5	1.0	0.5	4.0	II	0.5	-1.0	0.5	-4.0
III	0.5	1.0	1.0	4.0	III	0.5	-1.0	1.0	-4.0
IV	0.5	2.0	0.5	4.0	IV	0.5	-2.0	0.5	-4.0
V	1.5	1.0	0.5	4.0	V	1.5	-1.0	0.5	-4.0
VI	0.5	1.0	0.5	0.7	VI	0.5	-1.0	0.5	-0.7

Fig 3: Velocity profiles showing effect of R, S, B , K_c for $m = m_1 = 1$(symmetric).

Figs.4 and 5 display the velocity profiles for asymmetric and symmetric distribution of temperature and concentration in the absence of source and chemical reaction. It is interesting to note that change in material parameter does not affect the profiles which are symmetric about the middle of the channel in the absence of heat source and chemical reaction but the vortex viscosity $R\left(=\frac{k}{\mu}\right)$ reduces the velocity substantially. This observation is in good agreement with Ravi et al. [26]. Thus, it is inferred that kinematic rotational viscosity resists the flow producing a thinner hydrodynamic boundary layer.

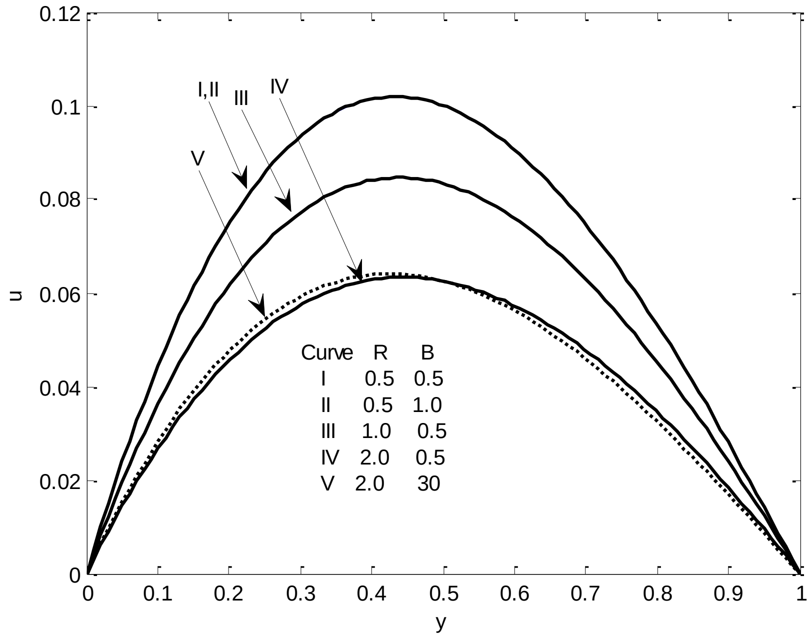


Fig 4: Velocity profiles showing effect of *R*, *B* $(S = 0, K_c = 0)$ **for** $m = m_1 = 0$.

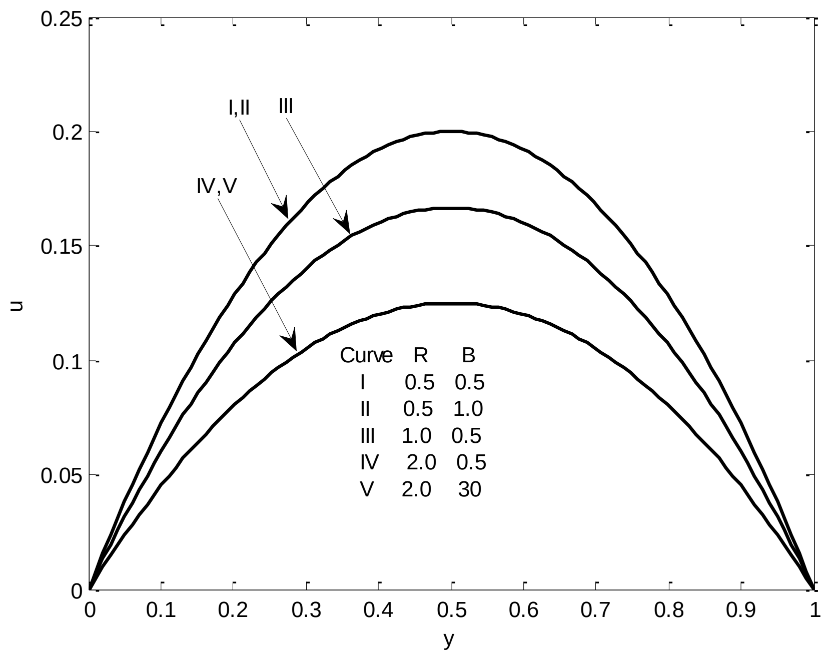


Fig 5: Velocity profiles showing effect of *R*, *B* $(S = 0, K_c = 0)$ **for** $m = m_1 = 1$.

Fig.6 presents the micro rotation profiles for asymmetric distribution of temperature and concentration in the presence of source/sink for both types of reactions. One of the striking features of the variation is that middle layers of the profiles remain unaffected due to variation of pertinent parameters governing the flow. This phenomena is an outcome of the reverse rotation sets in the layers beyond the middle of the channel. Another feature is to note that presence of sink under the influence of endothermic reaction attains higher values of $|\omega|$ in the lower half of the channel than the presence of source with exothermic reaction. It is also evident that micro-rotation assumes negative values in the lower half whereas positive values in the upper half. As regard to the effects of individual parameters, it is clear that an increase in absolute values of heat sink and endothermic reaction, $|\omega|$ increases whereas an opposite effect is observed in case of material parameter *B*. The decrease of micro-rotation for higher value of *B* was also observed by Ravi et al. [26]. Further, it is concluded that opposite flow behavior is exhibited in the upper half of the channel with respect to material property when $S > 0, K_c > 0$.

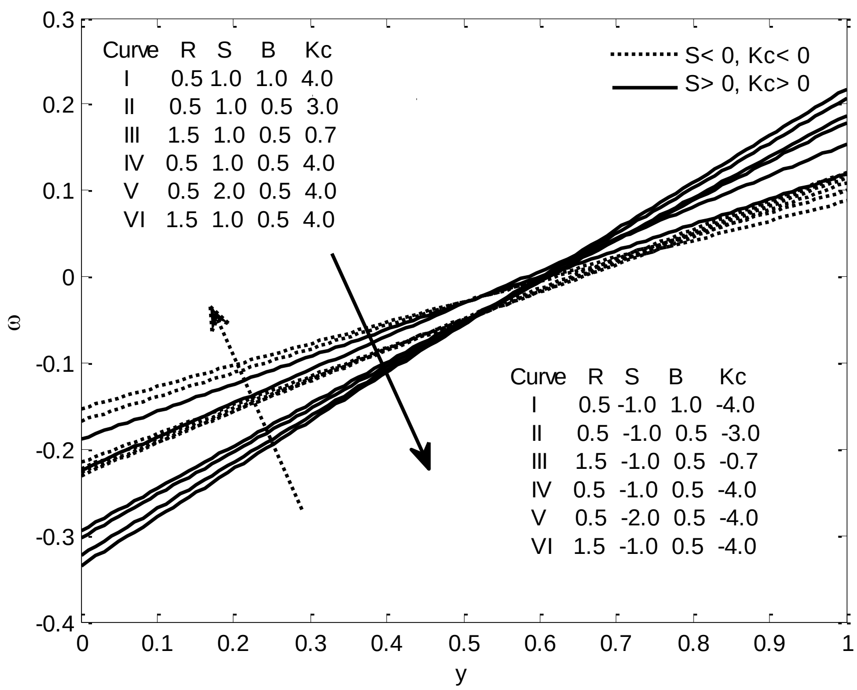


Fig 6: Micro rotation profiles for $m = m_1 = 0$.

Fig.7 exhibits the case of symmetric distribution. One interesting feature of micro-rotation is that both the categories of profiles ($S > 0, K_c > 0 \,\&\, S < 0, K_c < 0$) have common point of intersection at middle of the channel where as in case of asymmetric case it is not the case. Other features of the profile remain same as that of asymmetric case.

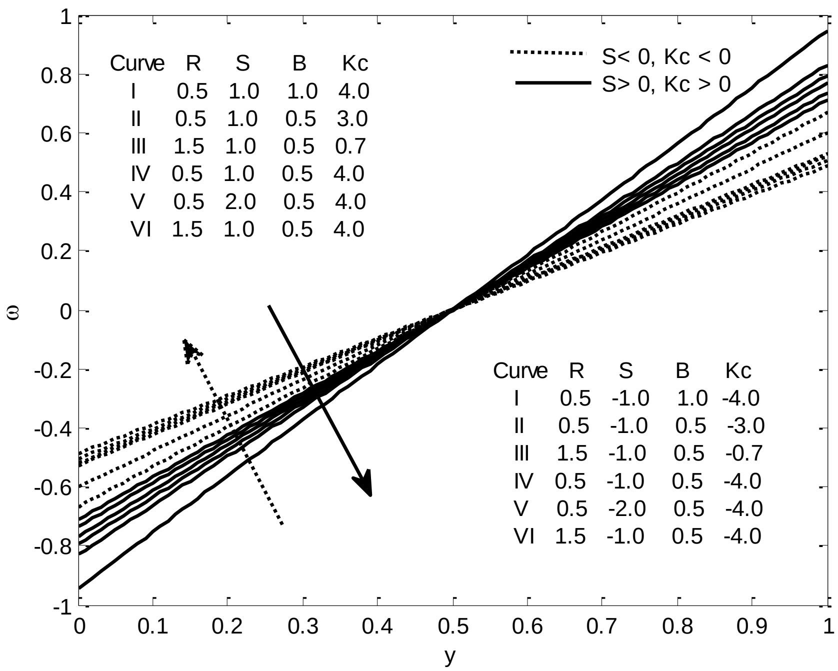


Fig 7: Micro rotation profiles for $m = m_1 = 1$.

Now, the special case in the absence of source and chemical reaction is discussed with the help of Figs. 8 and 9. One important observation is that the effect of material property of micropolar fluid on micro-rotation is not significant in the absence of source and chemical reaction which is evident from the coincident curves IV and V. It is further observed that an increase in vortex viscosity parameter decelerates micro-rotation $|\omega|$ in the lower half while other features of the profiles remaining the same. This observation is well supported by Ravi et al. [26]. Thus, the special cases have been discussed in respect of velocity and the micro-rotation.

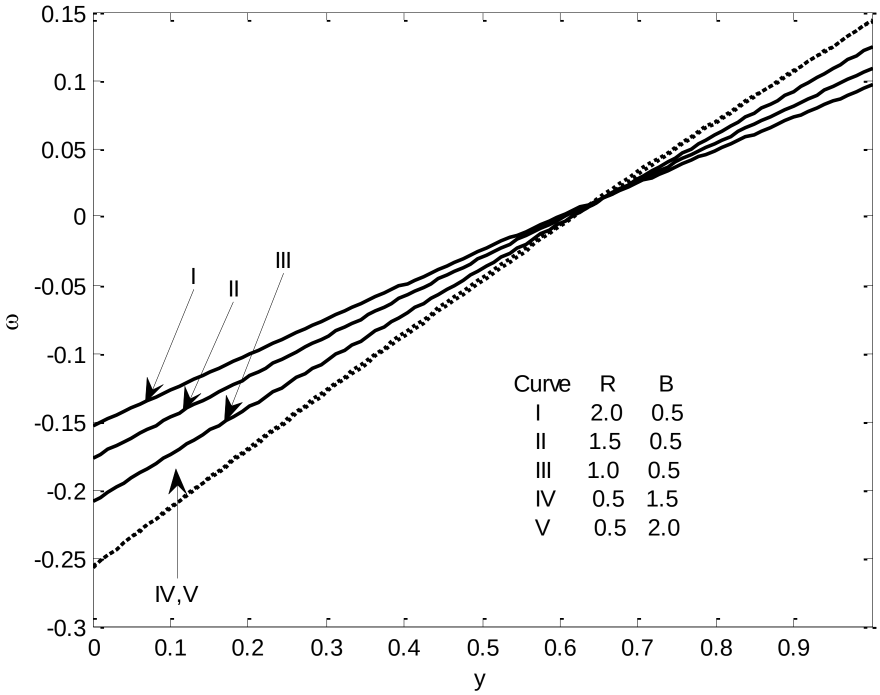


Fig 8: Micro rotation profiles for $S = K_c = m = m_1 = 0$.

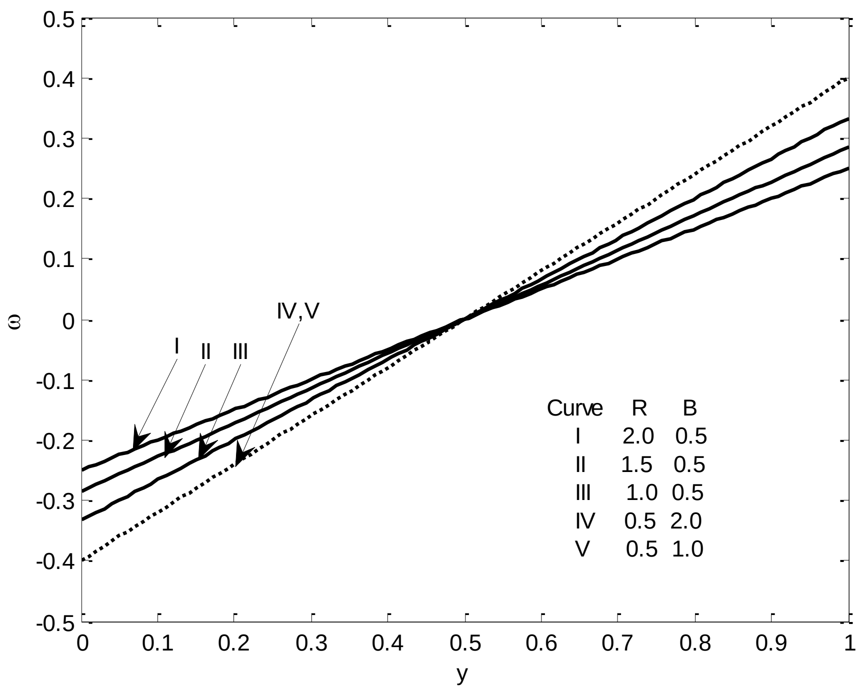


Fig 9: Micro rotation profiles for $S = K_c = 0;\ m = m_1 = 1$.

Fig.10 provides a graphical representation of temperature variation for source or sink and two values of m ($m=0$ and 1). Equation (9) is a linear equation and the corresponding boundary conditions indicate that temperature variation is controlled by source and temperature parameter. Fig.10 also shows that temperature increases at all points when S varies from -1 to 3. It is to note that the temperature ratio parameter m characterizes thermal boundary layer. In the present case when $m=1$, the distribution is almost symmetrical and parabolic but when $m=0$, the sharp fall of temperature is marked.

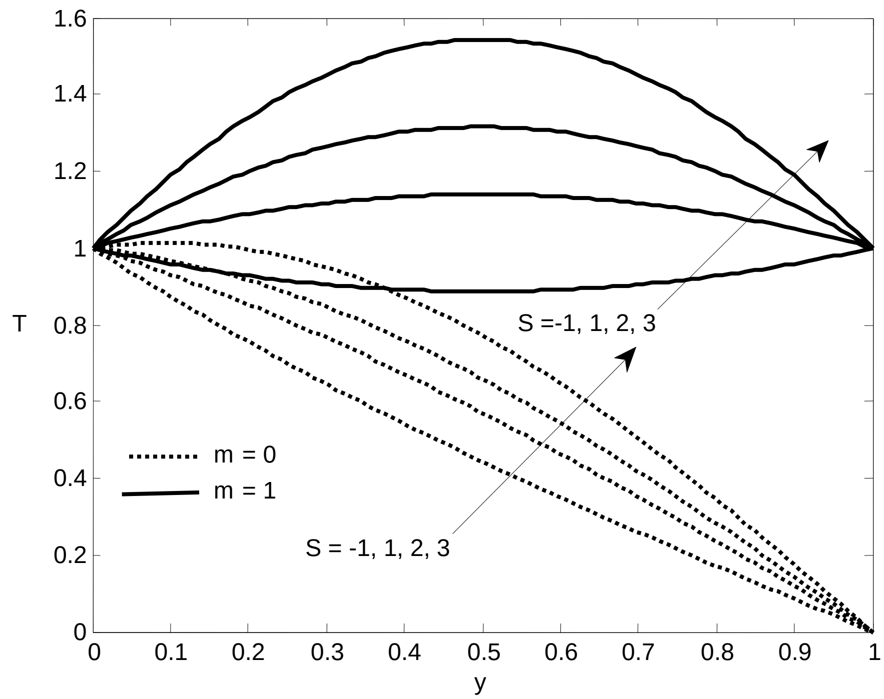


Fig 10: Temperature profiles for different values of *S* with $m=0$ **and** $m=1$.

The variation of concentration is similar to that of temperature as the source/ sink is replaced by exothermic/endothermic reaction of first order.

Table 1 shows the effect of pertinent parameters on volumetric flow rate and heat flow rate for symmetric and asymmetric cases. It is observed that an increase in exothermic reaction rate K_c increases the volume flow rate (Q) and total heat rate (E) added to the fluid in both symmetric and asymmetric distribution of plate temperature and concentration. Further, it is observed that presence of sink also increases the values of Q and E in all the cases but on careful observation it is noticed that when the magnitude of sink strength increases (i.e., from $|-1|$ to $|-2|$)

both Q and E decrease. Thus, it is concluded that for moderately high value of sink parameter both Q and E decrease. Moreover, increasing vortex viscosity parameter, R and material parameter, B enhance Q and E also. From the above discussion it is concluded that all the parameters except moderately high value of sink $(S=-2)$, enhance both volume flow rate and rate of heat flow. Thus, presence of sink of high strength is not desirable if higher rate of volumetric flow and heat rate is required.

Table 1: Variation of volume flow rate (Q) and total heat rate (E).

R	B	S	K_c	Q		E	
				$m=0, m_1=1$	$m=1, m_1=1$	$m=0, m_1=1$	$m=1, m_1=1$
0.5	1	1	1	0.16023554	0.20971917	0.0802643	0.2066555
0.5	1	1	2	0.17378868	0.22327231	0.0869722	0.2200713
0.5	1	1	3	0.19127878	0.24076242	0.0956287	0.2373842
0.5	1	3	3	0.18771961	0.23364407	0.0853349	0.2067268
0.5	1	-1	3	0.19127878	0.24076242	0.0956287	0.2373842
0.5	1	-2	3	0.18987831	0.23796147	0.0915155	0.2250687
0.5	2	1	3	0.19118178	0.24063605	0.0955829	0.2372594
0.5	2	3	3	0.18762329	0.23351907	0.0853349	0.2066147
0.5	2	-1	3	0.19118178	0.24063605	0.0955829	0.2372594
1	1	1	1	0.17762534	0.23253658	0.0889824	0.2291551
1	1	3	2	0.18855934	0.23957977	0.0856875	0.2114968
1	2	3	3	0.20731438	0.25823201	0.0942812	0.2285904
1	2	-1	3	0.21120067	0.2660046	0.1056033	0.262286
1	2	-2	3	0.20967149	0.26294623	0.1010798	0.2487522
0.5	2	1	2	0.17369526	0.22314954	0.0869282	0.2199501
0.5	1	1	-1	0.21984949	0.19933004	0.1230469	0.2215092
0.5	1	1	-2	0.2133675	0.19284805	0.1196712	0.2142978

Conclusion

The problem of steady free convective flow and mass transfer of a micropolar fluid in a vertical channel with heat source and chemical reaction has been investigated. The results are delineated for the major physical parameters and a systematic study on the effects of the various parameters on flow, heat and mass transfer characteristics is carried out.

Presence of heat source and exothermic reaction enhance the velocity at all points overriding the effects of vortex viscosity and material property in both symmetric and asymmetric cases. Vortex viscosity has a retarding effect on fluid velocity. Micro-rotation remains unaffected by the pertinent parameters of the flow phenomena in the middle layers of the channel as the middle layers experience two mutual opposite rotations. Exothermic reaction, vortex viscosity and material parameter enhance the volume flow rate and total heat rate added to the fluid, whereas sink of moderately high strength decreases them.

Appendix

$$A_1 = \frac{m - \cos\sqrt{S}}{\sin\sqrt{S}},\ A_2 = \frac{m_1 - \cos\sqrt{k_c}}{\sin\sqrt{k_c}},\ A_3 = \frac{BR}{1+R/2},\ A_4 = 2A_3 - \frac{RA_3}{1+R},\ A_5 = -\frac{A_3}{1+R},\ A_6 = \frac{-A_3}{1+R},$$

$$A_7 = \frac{-A_5}{\sqrt{S}(S+A_4)},\ A_8 = \frac{A_5 A_1}{\sqrt{S}(S+A_4)},\ A_9 = \frac{-A_6}{\sqrt{k_c}(k_c+A_4)},\ A_{10} = \frac{A_6 A_2}{\sqrt{k_c}(k_c+A_4)},\ A_{11} = A_8 + A_{10},$$

$$A_{12} = A_7 \sin\sqrt{S} + A_8\cos\sqrt{S} + A_9 \sin\sqrt{k_c} + A_{10}\cos\sqrt{k_c},\ A_{13} = \frac{R}{1+R},\ A_{14} = \frac{A_1}{\sqrt{S}} + \frac{A_2}{\sqrt{k_c}},$$

$$A_{15} = A_{14} - (A_{13}-2)A_{11},\ A_{16} = -\frac{1}{\sqrt{S}}\sin\sqrt{S} + \frac{A_1}{\sqrt{S}}\cos\sqrt{S} - \frac{1}{\sqrt{k_c}}\sin\sqrt{k_c} + \frac{A_2}{\sqrt{k_c}}\cos\sqrt{k_c},$$

$$A_{17} = A_{16} - (A_{13}-2)A_{12},\ A_{18} = \frac{-A_{13}}{\sqrt{A_4}}e^{\sqrt{A_4}\,y},\ A_{19} = \frac{A_{13}}{\sqrt{A_4}}e^{-\sqrt{A_4}\,y},\ A_{20} = \frac{A_{13}A_7 + \frac{1}{\sqrt{S}}}{\sqrt{S}},\ A_{21} = \frac{A_{13}A_8 - \frac{A_1}{\sqrt{S}}}{\sqrt{S}},$$

$$A_{22} = \frac{A_{13}A_9 + \frac{1}{\sqrt{k_c}}}{\sqrt{k_c}},\ A_{23} = \frac{-\left(A_{13}A_{10} - \frac{A_2}{\sqrt{k_c}}\right)}{\sqrt{k_c}},\ A_{24} = \frac{A_{13}}{A_4},$$

$$A_{25} = A_{20}\cos\sqrt{S} + A_{21}\sin\sqrt{S} + A_{22}\cos\sqrt{k_c} + A_{23}\sin\sqrt{k_c},\ A_{26} = (A_{13}-2)\left[\frac{(e^{\sqrt{A_4}}-1)A_{18}}{A_4} + A_{24}e^{\sqrt{A_4}}\right],$$

$$A_{27} = (A_{13}-2)\left[\frac{(e^{-\sqrt{A_4}}-1)A_{19}}{A_4} + A_{24}\,e^{-\sqrt{A_4}}\right],\ A_{28} = \frac{(A_{20}+A_{22}-A_{25})(A_{13}-2)}{A_4} + A_{24}\,A_{17},$$

$$A_{29} = (A_{13}-2)\left[\frac{A_{18}(e^{\sqrt{A_4}}-1)}{A_4} + A_{24}\right],\ A_{30} = (A_{13}-2)\left[\frac{(e^{-\sqrt{A_4}}-1)}{A_4} + A_{24}\right],$$

$$A_{31} = \frac{(A_{20}+A_{22}-A_{25})(A_{13}-2)}{A_4} + A_{15}A_{24},\ A_{32} = \frac{A_{31}A_{27} - A_{28}A_{30}}{A_{29}A_{27} - A_{26}A_{30}},\ A_{33} = \frac{A_{31}A_{26} - A_{28}A_{29}}{A_{30}A_{26} - A_{27}A_{29}},$$

$$A_{34} = \frac{[(A_{13}-2)(A_{32}+A_{33}) - A_{15}]A_4}{A_{13}-2},\ A_{35} = -A_{20} - A_{22} - A_{18}A_{32} - A_{19} - A_{33},\ A_{36} = \frac{(m-1)A_5}{2A_4},$$

$$A_{37} = \frac{-A_5}{A_4},\ A_{38} = \frac{-(m_1-1)A_6}{2A_4},\ A_{39} = \frac{-A_6}{A_4},\ A_{40} = -\frac{2A_{36}}{A_4} - \frac{2A_{38}}{A_4},$$

$$A_{41}=A_{36}\left(1-\frac{2}{A_4}\right)+A_{37}+A_{38}\left(1-\frac{2}{A_4}\right)+A_{39}\,,\,A_{42}=-\frac{1}{(1+R)}\left[\frac{m-1}{2}+1\right]-\frac{1}{(1+R)}\left[\frac{m_1-1}{2}+1\right],$$

$$A_{43}=A_{42}-(A_{13}-2)A_{41}\,,\,A_{44}=\frac{1}{3}\left[A_{13}A_{36}+\frac{m-1}{2(1+R)}+\frac{m_1-1}{2(1+R)}+A_{38}\right],$$

$$A_{45}=-\frac{1}{2}\left[A_{13}A_{37}+A_{13}A_{39}+\frac{2}{1+R}\right],A_{46}=\frac{2A_{13}A_{36}}{A_4}+\frac{2A_{13}A_{38}}{A_4}\,,\,A_{47}=A_{44}+A_{45}+A_{46}\,,$$

$$A_{48}=\frac{A_{18}(e^{\sqrt{A_4}}-1)(A_{13}-2)}{A_4}+(A_{13}-2)A_{24}\,e^{\sqrt{A_4}}\,,\,A_{49}=\frac{A_{19}(e^{-\sqrt{A_4}}-1)(A_{13}-2)}{A_4}+(A_{13}-2)\,e^{-\sqrt{A_4}}A_{24}\,,$$

$$A_{50}=\frac{-A_{47}(A_{13}-2)}{A_4}+A_{24}A_{43}\,,\,A_{51}=\frac{A_{18}(e^{\sqrt{A_4}}-1)}{A_4}+A_{24}\,,\,A_{52}=\frac{A_{19}(e^{-\sqrt{A_4}}-1)}{A_4}+A_{24}\,,$$

$$A_{53}=\frac{-A_{47}}{A_4}-A_{24}A_{40}\,,\,A_{54}=\frac{A_{53}A_{49}-A_{50}A_{52}}{A_{51}A_{49}-A_{48}A_{52}}\,,\,A_{55}=\frac{A_{53}A_{48}-A_{50}A_{51}}{A_{52}A_{48}-A_{49}A_{51}}\,,\,A_{56}=(A_{54}+A_{55}+A_{40})A_4\,,$$

$$A_{57}=-A_{18}A_{54}-A_{19}A_{55}\,,\;A_{58}=\frac{1}{2}\times\frac{m-e^{-\sqrt{S}}}{\sinh\sqrt{S}}\,,\;A_{59}=-\frac{1}{2}\times\frac{m-e^{\sqrt{S}}}{\sinh\sqrt{S}}\,,\;A_{60}=\frac{1}{2}\times\frac{m_1-e^{-\sqrt{k_c}}}{\sinh\sqrt{k_c}}\,,$$

$$A_{61}=-\frac{1}{2}\times\frac{m_1-e^{-\sqrt{k_c}}}{\sinh\sqrt{k_c}}\,,\,A_{62}=\frac{A_5A_{58}}{\sqrt{S}\,(S-A_4)}\,,\,A_{63}=-\frac{A_5A_{59}}{\sqrt{S}\,(S-A_4)}\,,\,A_{64}=\frac{A_6A_{60}}{\sqrt{k_c}\,(k_c-A_4)}\,,$$

$$A_{65}=-\frac{A_6A_{61}}{\sqrt{k_c}\,(k_c-A_4)}\,,\,A_{66}=A_{62}+A_{63}+A_{64}+A_{65}\,,\,A_{67}=A_{62}e^{\sqrt{S}}+A_{63}e^{-\sqrt{S}}+A_{64}e^{\sqrt{k_c}}+A_{65}e^{-\sqrt{k_c}}\,,$$

$$A_{68}=-\frac{1}{(1+R)}\left[\frac{A_{58}}{\sqrt{S}}-\frac{A_{59}}{\sqrt{S}}+\frac{A_{60}}{\sqrt{k_c}}-\frac{A_{61}}{\sqrt{k_c}}\right],A_{69}=A_{68}-(A_{13}-2)\,A_{66}\,,$$

$$A_{70}=-\frac{1}{(1+R)}\left(\frac{A_{58}e^{\sqrt{S}}}{\sqrt{S}}-\frac{A_{59}e^{-\sqrt{S}}}{\sqrt{S}}+\frac{A_{60}e^{\sqrt{k_c}}}{\sqrt{k_c}}-\frac{A_{61}e^{-\sqrt{k_c}}}{\sqrt{k_c}}\right),A_{71}=A_{69}-(A_{13}-2)\,A_{67}\,,\,A_{72}=\frac{-A_{13}}{\sqrt{A_4}}\,,$$

$$A_{73}=\frac{A_{13}}{\sqrt{A_4}}\,,\,A_{74}=-\left[\frac{A_{13}A_{62}+\dfrac{A_{58}}{\sqrt{S}\,(1+R)}}{\sqrt{S}}\right],\,A_{75}=\frac{A_{13}A_{63}-\dfrac{A_{59}}{\sqrt{S}\,(1+R)}}{\sqrt{S}}\,,\,A_{76}=-\left[\frac{A_{13}A_{64}+\dfrac{A_{60}}{\sqrt{k_c}\,(1+R)}}{\sqrt{k_c}}\right],$$

$$A_{77}=\frac{A_{13}A_{65}-\dfrac{A_{61}}{\sqrt{k_c}\,(1+R)}}{\sqrt{k_c}}\,,\,A_{78}=\frac{A_{13}}{A_4}\,,\,A_{79}=A_{74}+A_{75}+A_{76}+A_{77}\,,$$

$$A_{80}=A_{74}\,e^{\sqrt{S}}+A_{75}\,e^{-\sqrt{S}}+A_{76}\,e^{\sqrt{k_c}}+A_{77}\,e^{-\sqrt{k_c}}\,,\,A_{81}=A_{79}-A_{80}\,,\,A_{82}=A_{71}-A_{69}\,,$$

$$A_{83}=(A_{13}-2)(e^{\sqrt{A_4}}-1)\,,\,A_{84}=(A_{13}-2)(e^{-\sqrt{A_4}}-1)\,,\,A_{85}=(A_{13}-2)\left[\frac{e^{\sqrt{A_4}}-1}{A_4}A_{72}+A_{78}\right],$$

$$A_{86}=(A_{13}-2)\left[\frac{e^{-\sqrt{A_4}}-1}{A_{14}}A_{73}+A_{78}\right], A_{87}=\frac{A_{81}(A_{13}-2)}{A_4}+A_{71}, A_{88}=\frac{A_{82}A_{86}-A_{84}A_{87}}{A_{83}A_{86}-A_{84}A_{85}},$$

$$A_{89}=\frac{A_{82}A_{85}-A_{87}A_{83}}{A_{83}A_{86}-A_{84}A_{85}}, A_{90}=\left[\frac{(A_{13}-2)(A_{88}+A_{89})-A_{69}}{(A_{13}-2)}\right]A_4, A_{91}=-A_{79}-A_{72}A_{88}-A_{73}A_{89},$$

$$A_{92}=-\frac{2A_{36}}{A_4},\ A_{93}=A_{36}\left(1-\frac{2}{A_4}\right)+A_{37},\ A_{94}=-\frac{1}{(1+R)}\left(\frac{m-1}{2}+1\right),\ A_{95}=A_{94}-(A_{13}-2)A_{93},$$

$$A_{96}=\frac{1}{3}\left[A_{13}A_{36}+\frac{m-1}{2(1+R)}\right],\ A_{97}=-\frac{1}{2}\left(A_{13}A_{37}+\frac{1}{1+R}\right),\ A_{98}=\frac{2A_{13}A_{36}}{A_4},\ A_{99}=A_{96}+A_{97}+A_{98},$$

$$A_{100}=\frac{\left(A_{18}.e^{\sqrt{A_4}}-1\right)(A_{13}-2)}{A_4}+(A_{13}-2)A_{24}.e^{\sqrt{A_4}},\ A_{101}=\frac{A_{19}(e^{-\sqrt{A_4}}-1)(A_{13}-2)}{A_4}+(A_{13}-2)A_{24}.e^{-\sqrt{A_4}},$$

$$A_{102}=A_{24}.A_{95}-\frac{A_{99}(A_{13}-2)}{A_4},\ A_{103}=\frac{A_{18}(e^{\sqrt{A_4}}-1)}{A_4}+A_{24},\ A_{104}=\frac{A_{19}(e^{-\sqrt{A_4}}-1)}{A_4}+A_{24},$$

$$A_{105}=-\frac{A_{99}}{A_4}-A_{92}.A_{24},\ A_{106}=\frac{A_{102}A_{104}-A_{101}A_{105}}{A_{100}A_{104}-A_{101}A_{103}},\ A_{107}=\frac{A_{102}A_{103}-A_{100}A_{105}}{A_{101}A_{103}-A_{100}A_{104}},$$

$$A_{108}=(A_{92}+A_{106}+A_{107})A_4,\ A_{109}=-A_{18}A_{106}-A_{19}A_{107},\ A_{110}=\frac{A_{107}}{A_4}.$$

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