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Studying the Model that Predict and Monitor Constant Area Variable Pressure Drop in Orifice Plate

*C.P. Ukpaka, and ** C. Orji

*Department of Chemical/Petrochemical Engineering
**Department of Marine Engineering
Rivers State University of Science and Technology,
Nkpolu, P.M.B. 5080, Port Harcourt, Nigeria.
(chukwuemeka24@yahoo.com)

Abstract

Mathematical models for the design of pneumatic proportional control to predict and monitor constant area variable pressure drop using different fluids were developed and tested for an orifice plate. The different fluids used are water, Benzene, ethyl ethanol and butane. Results obtained from the investigation revealed that density, viscosity and flow rate are functional parameters that influence the pressure drop in the system. The proportional gain is also other functional parameters that govern and improve effectiveness of control process by reducing the error value in the system. The developed mathematical model of constant area, variable pressure drop was correlated with pneumatic proportional control. Computer simulation of the different fluids characteristics were carried out using C- programming language software developed for this purpose. The model developed in this research work is found useful in monitoring and predicting the effect of functional parameters on the characteristics of different fluids flowing through an orifice plate.

Keywords: Pneumatic, fluids, predict, monitor, orifice plate,

1. Introduction

Orifice plates are devices widely used in industry for continuous measurement of the rate of fluid flow in pipes. For the purpose of design and development of mathematical model, they use the same principle as a venture nozzle, namely Bemoulli's principle which states that there is a relationship between the pressure of the fluid and the velocity of the fluid, when the velocity increases, the pressure decreases and vice versa. In this study, the orifice plate was be used as the key device to propagate the mathematical model that will predict and monitor constant area, variable pressure drop using different fluids. It is a thin plate with a hole at the middle usually placed in a pipe in which fluid flows. When the fluid reaches the orifice plate with the hole in the middle, the fluid is forced to converge and pass through the small hole the point of maximum convergence actually occurs shortly downstream of the physical orifice, at the so called vena contracta point, as shown in the Figure 1 below. (Cunningham, 1951; Ogoni and Ukapaka, 2004; Creankoplis, 1993; Perry and Perry, 1984; Yuxi, Zhen, Yamin, Riyao and Xi, 2008 and Zhn, Ndegwa and Luo, 2001). As it does so, the velocity and the pressure changes. Beyond the vena contracta, the fluid expands and the velocity pressure change once again. By measuring the difference in pressure between the normal pipe section and at the vena the volumetric and mass flow rates can be obtained from Bernoulli's equation. (Jiang, Graham, Andre, Kelsalt and Brandon 2002; Bradford, Tadassa, and Jin, 2006; JICA, 2000; Uyigue and Umoh 2007; Fried, 2000; Winkler, 1997; and LENTECH, 2005)

The importance of this study of using an orifice plate device rather than other instrument is that the orifice affords a larger surface area per unit volume for mass transfer through the pipes. They are readily used extensively for continuous measurement of fluid in pipes and in some small river system to measure flow at locations where the river passes through a culvert or drain. Only a small number of rivers are appropriate for the use of the technology since the plate must remain completely immersed i.e the approach pipe must be full and the river must be substantially free of debris. In the natural environment large orifice plates are used to control onward flow in flood relief dams, in these structures a low dam is placed across a river and in normal operation the water flows through the orifice plate unimpeded as the orifice is substantially larger than the normal flow cross section. However, in floods the flow rate rises and floods out the orifice plate which can then only pass a flow determined by the physical dimensions of the orifice. Flow is then held back behind the low dam in a temporary reservoir which is slowly discharged through the orifice when the flood subsides. (Chrysikopoulos, Hsuan, Fyrillas and Lee, 2003; Glaso, 1980; Al-Marhoun, 2003; Bergen, Kiko, and Weisenbor 1999, Labede, 1990; Halliburton services, 1978 and Gerard, 1998).

The purpose of this study is to predict and monitor the behaviour of physical and chemical processes of different fluids as they flow in the pipe of the orifice with respect to a constant area and variable pressure drop. In this study, mathematical models were developed and tested to monitor and predict the characteristics of the process as well as the functional parameters. To understand the concept of variable pressure drop and constant area over the orifice under different fluid flow conditions and interpret the values obtained by simulation and theoretically in the concepts to industrial applications was conducted by various research groups. (Donald, 1950; William, 1989; Consider, 1957; Byron, Warren and Edwin 1976 and Eckman 1958).

This study had contributed extensively to knowledge as it provides a better way of controlling onward flow in flood relief dams in some of our natural environment. It also gives the design engineer the opportunity of using mathematical model to measure flow at location where rivers passes through culverts or drains. The scope of the work involves the flow of different fluids through an orifice and designing and developing a mathematical model that predict and monitor the constant area and variable pressure drop, and this developed models were related to the pneumatic proportional controller. The principle involves placing a suitable fixed area flow restriction in the pipe. This restriction causes a pressure drop which varies with the flow-rate. Thus, measurement of the pressure drop by means of a suitable differential pressure pickup allows flow rate measurement. In the case of orifice is mainly used for the determination of the above parameters with different fluids flowing through the pipe.

2. Materials and Methods

2.1 Density Measurement/Sampling

The density of the various substance used for the investigation was determined using hydrometer. The various fluid used for the laboratory experiment was obtained in the Department of chemical Petrochemical Engineering, Rivers State University of Science and Technology Nkpolu, Port Harcourt in Nigeria.

2.2 Computational Procedure

The developed model was simulated using the experimentally determine values inconjection with theoretically assumed values of flow rate 8m^3/s, 10m^3/s, 17m^3/s and 14m^3/s, cross sectional area of orifice = 2m^2 and cross sectional area of pipe = 5m^2. The experimental values of densities of various fluid is given as water ρ_{H2o}= 1000kg/m^3 benzene $\rho_{c_6H_6}$= 873.8kg/m^3, ethyl ethanol $\rho_{c_2H_5 - C_2H_5O}$ = 785.1kg/m^3 and Butane $\rho_{C_4 H_{10}}$ = 599kg/m^3 all samples analyzed at 25^{o}C. The experimental and the theoretical data were fed into the developed equations (25) and (27) to determine the functional parameters.

2.3 Experimental Procedures

The method involves placing a suitable fixed area flow restriction in the pipe of an orifice the restriction causes a pressure drop which varies with flow-rate as the different fluids flow through the pipe. Thus measurement of the pressure drop by means of a suitable differential pressure drop pick up allows flow rate measurement. Several empirical correlations exist in this methods to predict how the orifice will operate relating pressure drop and assuming steady-state, incompressible (constant fluid density), in viscid, laminar flow in a horizontal pipe with negligible friction losses. One of those most useful equations is the Bernoulli's equation which reduces to an equation relating the conservation of energy between two points on the same streamline as will be shown in the mathematical model.

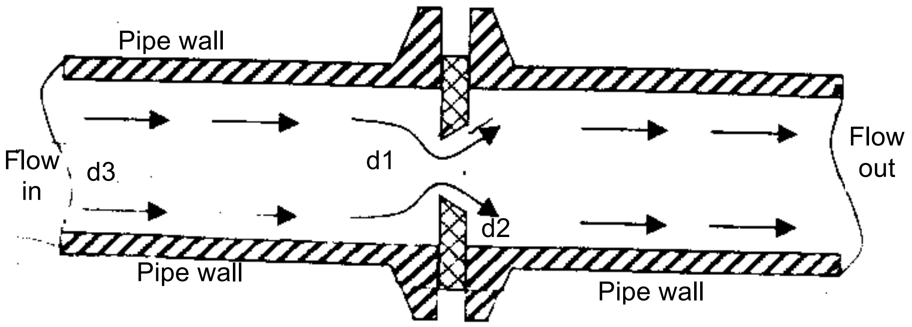

Figure 1: Systematic diagram of fluid flow in pipe system

2.4 The Model

Mathematical concept was used in developing the model for this paper based on flow characteristics such as

$$\frac{V_1^2}{2} + \frac{P_1}{P} + gZ_1 = \frac{V_2^2}{2} + \frac{P_2}{P} + gZ_2$$

$$\frac{V^2}{2} + \cancelto{0}{gZ} + \frac{P}{\rho} = \text{constant} \qquad (1)$$

If the elevation effects are negligible, if the potential energy term 'gZ' becomes zero (gZ = 0), then equation (1) reduces to,

$$\frac{V^2}{2} + \frac{P}{\rho} = \text{constant} \qquad (2)$$

Multiply through equation (1) by ρ, we have

$$\frac{\rho V^2}{2} + P = \text{constant} \qquad (3)$$

$$\frac{\rho V_1^2}{2} + P_1 \;=\; + \frac{\rho V_2^2}{2} + P_2 \tag{4}$$

From continuity equation, we have that

$$Q = A_1 V_1 = A_2 V_2 \tag{5}$$

$$Q = ApV_1 = A_{orif} V_2 \tag{6}$$

From equation (6) we have

$$V_1 = \frac{Q}{A_p} \tag{7}$$

$$V_2 = \frac{Q}{A_{orif}} \tag{8}$$

Substituting equations (7) and (8) into equation (4), we have

$$\rho_2 \left(\frac{Q}{Ap}\right)^2 + P_1 = \frac{\rho}{2}\left(\frac{Q}{A_{orif}}\right)^2 + P_2 \tag{9}$$

Simplifying equation (9), we have

$$P_1 - P_2 = \frac{\rho}{2}\left(\frac{Q}{A_{orif}}\right)^2 - \frac{\rho}{2}\left(\frac{Q}{A_p}\right)^2 \tag{10}$$

$$P_1 - P_2 = \frac{Q^2\rho}{2}\left(\frac{1}{A^2{}_{orif}} - \frac{1}{A^2{}_p}\right) \tag{11}$$

$$P_1 - P_2 = \frac{Q^2\rho}{2}\left(\frac{A^2{}_p - A^2{}_{orif}}{A^2{}_{orif} A^2{}_p}\right) \tag{12}$$

$$Q^2 = \frac{2\left(P_1 - P_2\right)\left(A^2{}_{orif} A^2{}_p\right)}{\rho\left(A^2 p - A^2{}_{orif}\right)} \tag{13}$$

Multipling through equation (13) by $1/A^2P$, we have

$$Q^2 = \frac{2\left(P_1 - P_2\right) A^2 orif}{\rho\left(1 - \frac{A^2{}_{orif}}{A^2{}_p}\right)} \tag{14}$$

$$= \frac{2\left(P_1 - P_2\right) A^2 orif}{\rho\left(1 - \frac{A_{orif}}{A_p}\right)^2} \tag{15}$$

$$Q = \sqrt{\frac{2(P_1 - P_2)\,A^2 orif}{\rho\left[1 - \left(\frac{A^2{}_{orif}}{A^2{}_p}\right)^2\right]}} \tag{16}$$

$$Q = \sqrt{\frac{A_{orif}}{1\left(\frac{A_{orif}}{A_p}\right)^2}}\sqrt{\frac{2(p - P_{21})}{\rho}} \tag{17}$$

$$Q = \frac{A_{orif}}{\sqrt{1 - \left(\frac{A_{orif}}{Ap}\right)^2}}\sqrt{\frac{2\Delta P}{\rho}} \tag{18}$$

Where, Δp = Pressure drop. All other parameters remain the same equation (17) is the therefore the mathematical model that is related to the pneumatic proportional controller, which can be represented in form of flow diagram as shown in Figure 2.

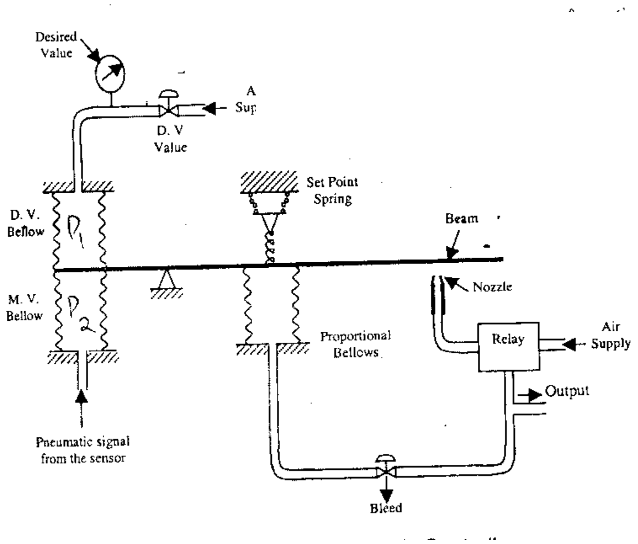

Figure 2: Proportional Pneumatic Controller

The output pressure can be expressed as;

$$P = P_o + K_c E \tag{19}$$

Recalling equation (17) and making P_2 = output pressure the subject of formula, we have

$$Q = \frac{A_{orif}}{\sqrt{1 - \left(\frac{A_{orif}}{Ap}\right)^2}}\sqrt{\frac{2(P_1 - P2)}{\rho}} \tag{20}$$

Equation (20) can be written as

$$Q^2 = \frac{2(P_1 - P_2)A_{orif}^2}{\rho\left[1 - \left(\frac{A_{orif}}{Ap}\right)^2\right]}$$

(21)

$$\frac{Q^2\rho\left[1 - \frac{A_{orif}^2}{A^2 p}\right]}{A_{orif}^2} = 2P_1 - 2P_2$$

(22)

$$\Rightarrow P_1 - P_2 = \frac{Q^2\rho\left[1 - \left(\frac{A_{orif}^2}{A^2 p}\right)\right]}{2A_{orif}^2}$$

(23)

$$\left[\Delta P = \frac{(Q^2)(\rho)\left[1 - \left(\frac{A_{orif}}{Ap}\right)^2\right]}{2\ A_{orif}^2}\right]$$

(24)

$$P_2 = P_1 - \frac{Q^2\rho\left[1 - \left(\frac{A_{orif}^2}{Ap}\right)^2\right]}{2A_{orif}^2}$$

(25)

Equating equation (19) and (25), that is $P = P_2$ therefore we have

$$P_o + K_C E = P_1 - \frac{Q^2\rho\left(1 - \left(\frac{A_{orif}}{Ap}\right)^2\right)}{2A_{orif}^2}$$

(26)

$$P_1 - P_o = K_C E + \frac{Q^2\rho\left(1 - \left(\frac{A_{orif}}{Ap}\right)^2\right)}{2A_{orif}^2}$$

(27)

Thus $P_1 - P_o = \Delta_p^{1}$ therefore equation (27) can be written as

$$\Delta p^{I} = K_c E + \frac{Q^2 \rho \left(1 - \left(\frac{A_{orif}}{Ap}\right)^2\right)}{2A_{orif}^2}$$

(28)

3. Results and Discussion

The results obtained from the investigation are presented in Tables and Figures as shown below. The various functional parameters were evaluated and results obtained presented in this paper. The result presented in Figure 3 illustrates the variation in pressure drop of the system in present of pneumatic proportional control as well as absent of pneumatic proportional control with variation in flow rate. Increase in pressure drop was observed for both systems with increase in flow rate. The pressure drop was higher un system with the absent of pneumatic proportional control than the one with the present of pneumatic proportional control for water.

From the results, shown it can be deduced that there is variation in the pressure drop of the different fluids as the flow rate varies. An increase in the flow rate shows an increase in the pressure drop if the different fluid flowing through the pipe. Also, the deviation in densities of the different fluids shows a variation in the pressure drop. Fluids with reduced densities had a lower pressure drop. Hence the lower the density the lower the pressure drop and the higher density the higher he pressure drop. There the density of the fluids is directly proportional to the pressure drop at constant area.

Table 1. Experimentally and theoretically values of functional parameters of different fluids.

S/N	**Fluids**	**Density ρ at**	**Flow rate**	**Pressure**	**Pressure**	**Q^2**

		25^{o}C (kg/m^3)	**Q (m^3/S)**	**drop Δp (kpa)**	**drop Δp' (kpa)**	**(m^6/s^2)**
1	Water	1000	8	6.72	11.72	64
			10	10.5	15.5	100
			12	15.12	20.12	144
			14	20.58	25.58	196
2	Benzene	873.8	8	5.87	10.87	64
			10	9.17	14.17	100
			12	13.21	18.21	144
			14	17.98	22.98	196
3	Ethyl ethanol	785.1	8	5.28	10.28	64
			10	8.24	13.24	100
			12	11.87	16.87	144
			14	16.16	21.16	196
4	Butane	599	8	4.025	9.025	64
			10	6.29	11.29	100
			12	9.66	14.06	144
			14	12.33	17.33	196

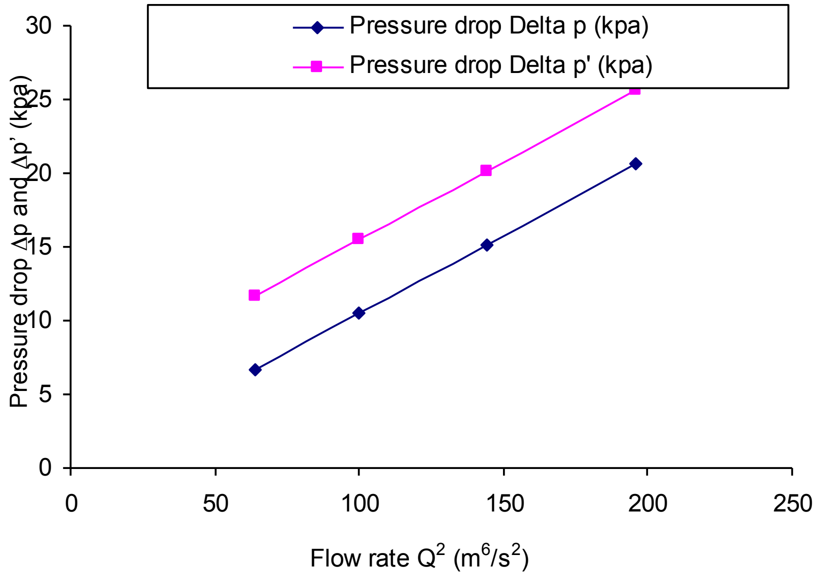

Figure 3 Graph of Pressure drop Δp and Δp' (kpa) versus flow rate Q^2 (m^6/s^2) for water

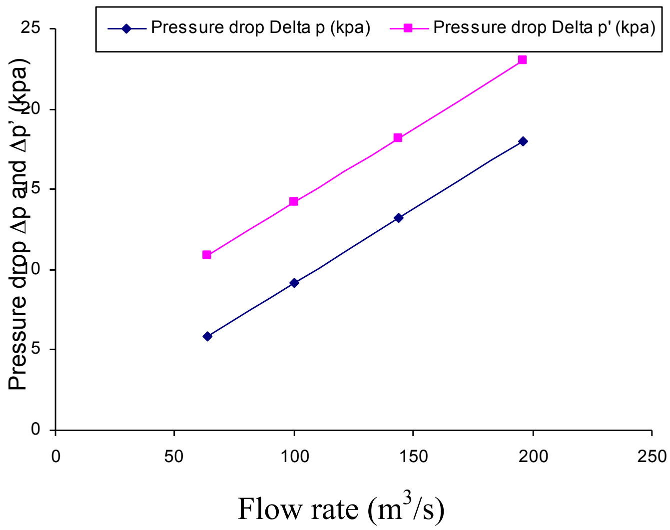

Figure 4: Graph of Pressure drop Δp and Δp' (kpa) versus flow rate Q^2 (m^3/s) for Benzene

In Figure 4, it is seen that both pressure drop increase with increase in flow rate. The variation in concentration of the pressure drop can be attributed to the variation in the flow rate. The pressure drop is higher for the normal operation of fluid flow characteristics where the actual flow rate was evaluated in the absent of proportional pneumatic control involved. The aim of the pneumatic proportional control is to help reduce the pressure drop in the system as well as the error range value for benzene.

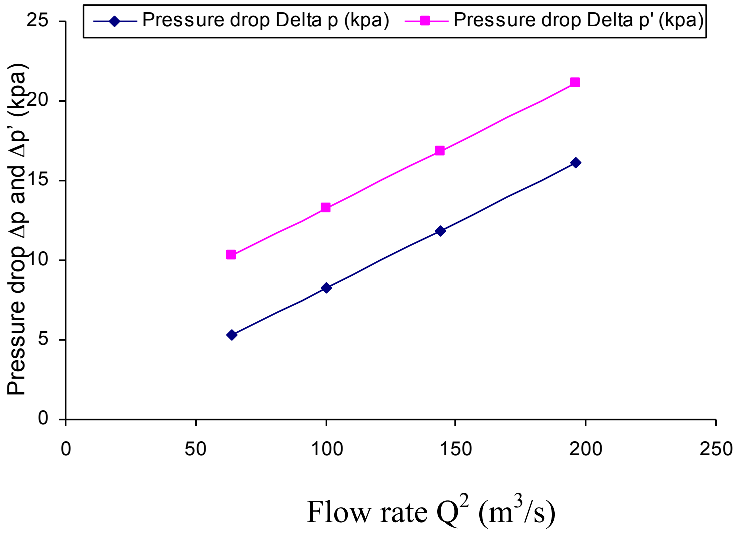

Figure 5: Graph of Pressure drop Δp and Δp' (kpa) versus flow rate Q^2 (m^6/s^2) for Ethyl ethanol

Figure 5 illustrates the relationship between the pressure drop in a fluid flow system of normal characteristics that is in the absent of proportional pneumatic control as well as the process that involved proportional pneumatic control system. The results obtained sowed that minimum error is observed in the pressure drop with the system in present of proportional pneumatic control than the system in the absent of proportional pneumatic control in process fluid flow of ethyl ethanol. The variation in the concentration of pressure drop for both systems can be attributed to variation in the flow rate of fluid in the pipe.

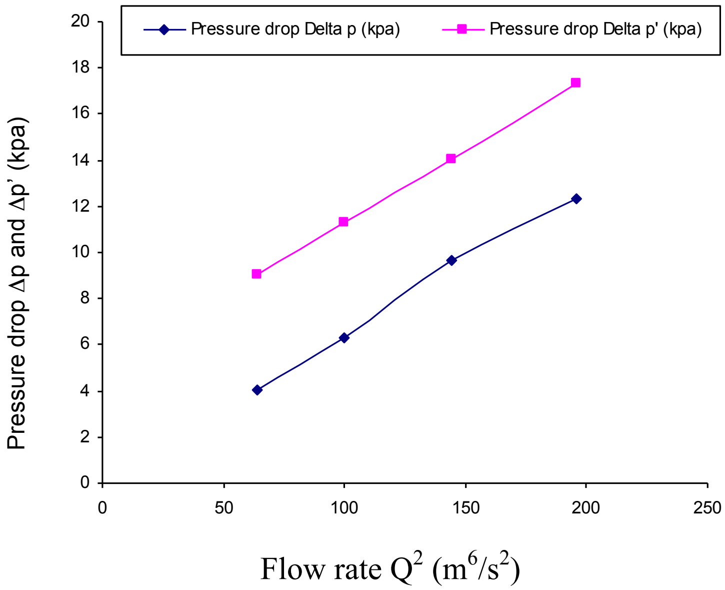

Figure 6: Graph of Pressure drop Δp and Δp' (kpa) versus flow rate Q^2 (m^6/s^2) for Butane

Figure 6 illustrate the relationship between the pressure drops of the both process with flow rate characteristic of butane. Increase in pressure drop for both systems was observed with increase in flow rate of butane. The variation in the pressure drop for both systems can be attributed to the variation in the flow rate characteristics of fluid (butane).

Conclusion

In conclusion, the flow of fluids through thin-plate orifices promotes increase in the flow rate as the down stream pressure is lowered, and pressure drop increases. Hence the mathematical model developed is of great relevance as it is used to determine the gradual discharge of fluids through an orifice. For example, in the natural environment, where large orifice plates are used to control onward flow in flood relief dams, the efficiency is dependent of the pneumatic proportional control of the system. The flow characteristics of water, ethyl ethanol, benzene and butane are lowered as the different fluids passes through the orifice plate, which resulted to decrease in pressure.

Nomenclature

d3 = pipe diameter (mm)

d2 = vena contracta diameter (mm)

d1 = orifice diameter (mm)

V_1 = upstream fluid velocity (m/s)

V_2 = fluid velocity through the orifice hole (m/s)

P_1 = Fluid upstream pressure, Pa (kg/m.s^2)

P_2 = fluid down stream pressure Pa (kg/ms^2)

ρ = Fluid density, kg/m^3

$A_1 = A_p$ = Cross-sectional area of the pipe (m^2)

$A_2 = A_{orif}$ = Cross-sectional area of the orifice plate (m^2)

Q = Volumetric flow rate (m^3/s)

Q = $ApV_1 - A_{orif}\,V_2$
P = Output signal pressure (kg/m.s^2)

K_c = proportional gain

P_o = output signal when there is no error

E = Error

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