

## **Design of Concentric Circular Antenna Arrays for Sidelobe Reduction using Differential Evolution Algorithm**

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### **Abstract**

Difference patterns are often used in tracking radar applications. It is essential to generate these patterns with low side lobes to improve target tracking due to interference, clutter, jammers etc. Although reduced side lobes can be obtained by amplitude only synthesis method, thinning is another technique for the same purpose. It brings down the number of active elements in the array without degrading the system performance. The objective of the present work is to generate low side lobe difference patterns from concentric ring arrays by amplitude only synthesis method along with thinning the array at the same time. Thinning not only reduces side lobes but also reduces cost and weight. A Differential Evaluation algorithm is employed for obtaining optimum array configurations. Results are presented for different arrays of concentric rings.

### **Keywords**

Concentric Circular Arrays, Side lobe Reduction, Thinning, Difference patterns, Differential Evolution algorithm.

### **1. Introduction**

In target tracking radar systems, difference patterns are employed to locate the target with high accuracy [1-2]. Difference patterns have a sharp, deep null in the bore sight direction with twin lobes on either side. This deep null is utilized in target detection. When the target is placed exactly in the null between the two principal lobes, its angular location can be determined with high accuracy. This is more precise than using a sum pattern with broad main beam peak as the null is having a very narrow

angular width. Array designs with desired radiation pattern characteristics can be generated by careful design [3-4], [5].

Concentric ring arrays find many applications in air & space navigation, radio direction finding [6]. They are given more concentration as they offer several advantages like scan capability in entire  $360^{\circ}$  azimuthal plane and a compact antenna structure. Moreover owing to the fact that there are no edge elements, these arrays are less sensitive to mutual coupling effects. Most of the earlier works, contributed in the field of difference pattern generation from circular arrays, used conventional methods. Bayliss [7] developed a two parameter difference pattern with nearly equal side lobes similar to those of Taylors sum pattern for a circular aperture antenna. Elliot [8] proposed a perturbation technique which modifies the Taylor-type linear and circular aperture distributions to generate sum and difference patterns with arbitrary side lobes. Hansen [9] presented a synthesis method for linear and circular planar arrays that provide pencil beams and difference patterns with variable side lobe level based on placement of zeros of the array polynomial. Ares et.al [10] presented a design technique to get an aperture distribution for both linear and circular apertures, which yields difference patterns with arbitrary side lobe topography. Elliot [11] reported various synthesis techniques for finding excitations that result in desired radiation patterns. The techniques are useful for linear and circular array geometries to generate sum and difference patterns with lobes characterized by scattered deep nulls. Keizer [12] presented a pattern synthesis method which yields low side lobe sum and difference patterns from circular and elliptical apertures with periodic arrangement of elements. The proposed method is specifically suitable for large planar antenna arrays with ultra low side lobe requirements.

Thinning is a technique that selectively turns off certain array elements without disturbing the system performance. It results in optimum design of arrays with a reduction in cost and weight. Moreover, all the elements are excited uniformly which requires a simple feed network. Very limited literature is available on generating low side lobe difference patterns from thinned concentric circular arrays. Keizer [13] carried out synthesis of continuous amplitude tapers for illuminating turned ON elements of large thinned circular arrays for generating both sum and difference patterns. Iterative Fourier Transform method is employed to synthesize the low side lobe patterns. The work done mainly refers to highly thinned circular arrays with large diameters ranging from 25 to 133.3 wavelengths.

The current work is aimed at generating low sidelobe difference patterns from concentric ring arrays using amplitude only synthesis along with array thinning. To the author's knowledge, no such work has been reported earlier. The fill factor is not allowed to exceed 55%. Both optimum values satisfying the set criteria are obtained using a Differential Evolution Algorithm. Results are presented for 8 and 10 concentric ring arrays. All results are simulated using Matlab software.

The present work is categorized as follows: Section II describes the working principles of Differential Evolution algorithm. Section III gives the problem formulation. Results are discussed in section IV. Conclusions are discussed in section V.

## **2. Differential Evolution**

DE is a simple population based stochastic search algorithm. It was first proposed by [14-15]. It is another evolutionary algorithm which paved way for solving complex optimization problems. It is a powerful search technique which is successfully applied in many fields like communications, pattern recognition etc. The algorithm offers following advantages:

- It can easily handle non-linear, non-differentiable complex cost functions
- It has few easy to choose control parameters which influence the convergence of the algorithm
- Good convergence speed in finding optimum value

The main steps involved in the algorithm are depicted in the following flowchart:

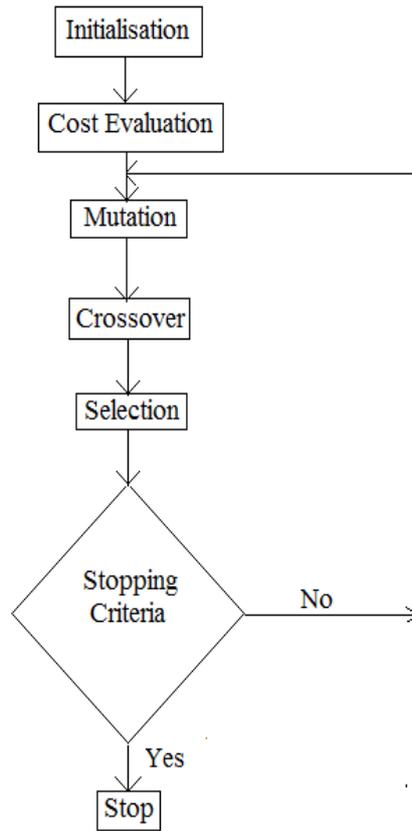


Fig. 1. Flowchart for DE

Step 1: Initialization: The algorithm starts with ‘N’ (at least equal to 4) population vectors. The individuals are called target vectors. The total number of these parameter vectors remains same throughout the algorithm. Let ‘ $x_{i,G}$ ’ represent the  $i^{\text{th}}$  parameter vector where  $i=1, 2, \dots, N$ . ‘G’ is the generation number. The parameter vectors are randomly initialized in step 1.

Step 2: Cost Evaluation: The initial  $x_{i,G}$  parameter vectors are evaluated for their cost using the objective function.

Step 3: Mutation: In this step, new parameter vectors are generated by adding weighted difference between two target vectors to a third target vector, i.e. for a given target vector ‘ $x_{i,G}$ ’, select three target vectors  $x_{r1,G}$ ,  $x_{r2,G}$ ,  $x_{r3,G}$  such that  $i, r1, r2, r3$  are distinct to form mutant vectors called ‘donor vectors’.

$$v_{ji,G+1} = x_{r1,G} + F * (x_{r2,G} - x_{r3,G})$$

here  $r1, r2, r3 \in \{1, 2, \dots, N\}$

Mutation expands the solution space. The factor ‘F’ is called mutation factor. Usually it is a real constant chosen in the range 0 to 2.

Step 4: Crossover: It increases the diversity of parameter vectors by including good solutions or vectors from previous generations. It forms the new so called ‘trial vectors ( $u_{i,G+1}$ )’ by mixing elements of target vector ‘ $x_{i,G}$ ’, and donor vector ‘ $v_{i,G+1}$ ’.

$$u_{ji,G+1} = v_{ji,G+1} \text{ if } rand \leq CR \text{ or } j = I_{rand}$$

$$= x_{ji,G+1} \text{ if } rand > CR \text{ and } j \neq I_{rand}$$

Here  $i=1,2,\dots,N$ ;  $j=1,2,\dots,D$ .  $D$  is the number of parameters in one vector.  $I_{rand}$  is a randomly number chosen in the range 1 to  $D$  which ensures that  $u_{i,G+1}$  gets at least one parameter from  $v_{i,G+1}$ .  $CR$  is the crossover constant to be taken in the range  $(0,1)$ .

Step 5: Selection: It imitates survival-of-the-fittest. It follows greedy scheme and selects vectors for next generation. The process is as follows:

$$x_{i,G+1} = u_{i,G+1} \text{ if } cost(u_{i,G+1}) \leq cost(x_{i,G})$$

$$= x_{i,G} \text{ otherwise}$$

That means, the newly generated trial vectors replace parent target vectors if they yield lower cost otherwise the parent target vectors are passed on to next generation.

Step 6: Stopping criteria: Steps 2 to 5 are repeated until some stopping criteria is met. Stopping criteria in general may be fixed number of generations or a predetermined cost.

There are different variants of DE as suggested by Storn and Price [14]. In the present work, a DE/rand/1/binary scheme is used.

### 3. Formulation

The geometry of a ‘ $m$ ’ ring concentric circular array is as shown in figure 2. Assume all elements in all rings are isotropic elements. Let  $r_m$  represent the radius of  $m^{\text{th}}$  ring and let the number of elements present in  $m^{\text{th}}$  ring be  $N_m$  where  $m=1,2,\dots,M$ . Let  $d_m$  be the inter element spacing.

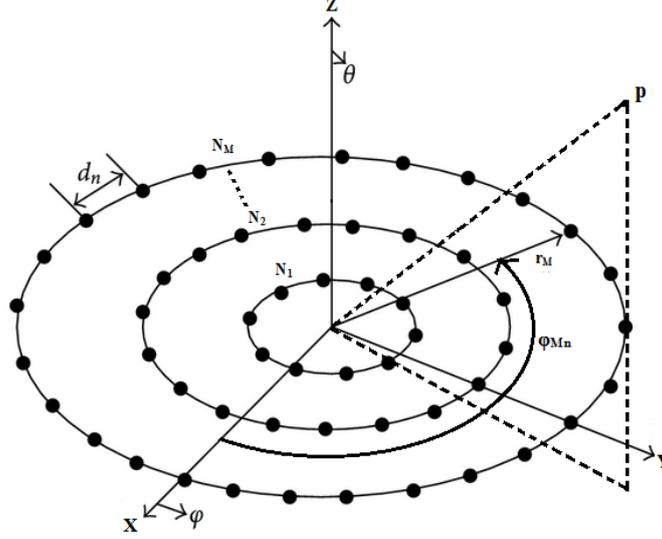


Fig. 2. Geometry of concentric circular array

The generalized array factor for the array [16] is given by

$$E(\theta, \varphi) = \sum_{m=1}^M \sum_{n=1}^{N_m} I_{mn} A_{mn} \exp(jkr_m \sin \theta \cos(\varphi - \varphi_{mn})) \quad (1)$$

Here

$M$  = number of rings

$N_m$  = number of elements in ring  $m$

$A_{mn}$  = Amplitude excitation of  $n^{\text{th}}$  element of  $m^{\text{th}}$  ring

$I_{mn}$  = excitation of  $n^{\text{th}}$  element of  $m^{\text{th}}$  ring =

$r_m$  = radius of ring  $m$

$\varphi_{mn}$  = angular position of  $n^{\text{th}}$  element of  $m^{\text{th}}$  ring

$$= \frac{2\pi(m-1)}{N_m} \quad (2)$$

$$k = 2\pi/\lambda$$

$\theta$  = elevation angle

$\varphi$  = Azimuthal angle

In 'u' domain

$$E(u)_{\varphi = \cos \theta \cos \varphi} = \sum_{m=1}^M \sum_{n=1}^{N_m} I_{mn} A_{mn} \exp(jkr_m u \cos(\varphi - \varphi_{mn}))$$

where  $u = \sin(\theta)$

The radius of  $m^{\text{th}}$  ring is given by

$$r_m = m\lambda/2 \quad (3)$$

The inter element spacing is assumed to be approximately  $\lambda/2$  i.e.  $d_m = \lambda/2$  .

The number of equally spaced elements present in ring ‘m’ is given by

$$N_m = 8 * m \quad (4)$$

All the elements have uniform excitation phase of zero degrees. For attaining the difference patterns, half the array must be excited in out of phase. Hence the resultant expression for the array factor is given by

$$AF(u) = |E_1(u) - E_2(u)| \quad (5)$$

where

$$E_1(u) = \sum_{m=1}^M \sum_{n=1}^{N_m/2} I_{mn} A_{mn} \exp(jkr_m u \cos(\varphi - \varphi_{mn}))$$

$$E_2(u) = \sum_{m=1}^M \sum_{n=\left(\frac{N_m+1}{2}\right)}^{N_m} I_{mn} A_{mn} \exp(jkr_m u \cos(\varphi - \varphi_{mn}))$$

In the above equations,  $\varphi$  is assumed to be constant.

#### 4. Method of Sidelobe Reduction

Low sidelobe difference patterns can be generated by nonuniformly exciting the elements in each ring. Thinning of the array also reduces the sidelobe levels. In this paper, both amplitude tapering and thinning techniques are used. The DE algorithm is used to find the optimum amplitude excitations as well as thinning coefficients.

The objective function which is to be minimized for finding the optimum solution is as follows:

$$Fit = w1 * (PSLL_o - SLL_d) + w2 * (FF_o - FF_d) \quad (6)$$

where

$$PSLL_o = \max \left\{ 20 \log \left| \frac{AF(u)}{AF_{\max}(u)} \right| \right\} = \text{Obtained Peak Sidelobe level}$$

$u \in$  side lobe region.

$SLL_d$  = Desired Sidelobe level

$AF_{\max}(u)$  = Main beam peak value

$FF_o$  is the obtained Fill factor,  $FF_d$  is the desired Fill factor. Fill factor is defined as the number of turned on elements divided by total number of elements present.  $w1$  and  $w2$  are weighing factors for controlling the amount of significance given to each term in eq.(6).

#### 5. Results

This section provides various computational results of concentric circular array designs obtained using DE algorithm. In the present work, eight and ten ring concentric circular arrays are considered. The corresponding optimum amplitude excitations and thinning coefficients for generating difference pattern are obtained using eq. (5). Since the optimum results depend on the parameters of the algorithm, the control parameters must be carefully chosen. The algorithm started with a population size of 12, Crossover rate of 0.85, and Mutation factor of 0.7. The algorithm is run for a maximum of 350 generations.

An 8 ring concentric circular array is considered initially and the DE optimized values are introduced in eq. (5) and the resulting far field radiation pattern is presented in fig.3. The resultant pattern has a peak sidelobe level of -22.08dB with 54.166% fill factor. The pattern for a fully populated array is also shown for comparison. Uniform excitation of the elements gives a peak sidelobe level of -10.92dB for a fully filled array. An improvement of 11.16dB can be observed.

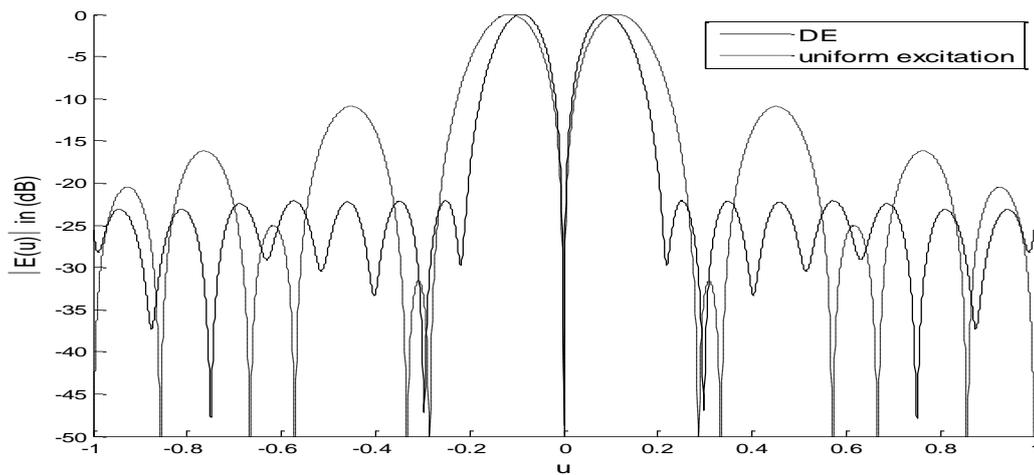


Fig. 3. Patterns for DE optimized and uniformly excited arrays

The nonuniform aperture distribution across the elements is presented in fig.4. To get the difference pattern, one half of the array is excited out of phase. This is presented clearly in fig.5. The '+' sign indicates element excited with zero phase whereas a '.' sign indicates element excited out of phase.

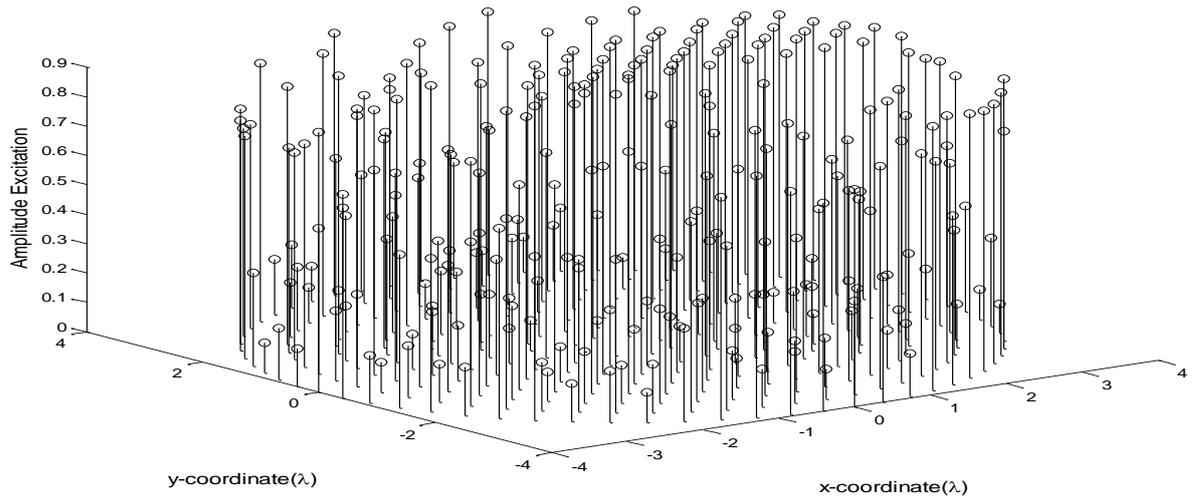


Fig. 4. Aperture distribution across the array

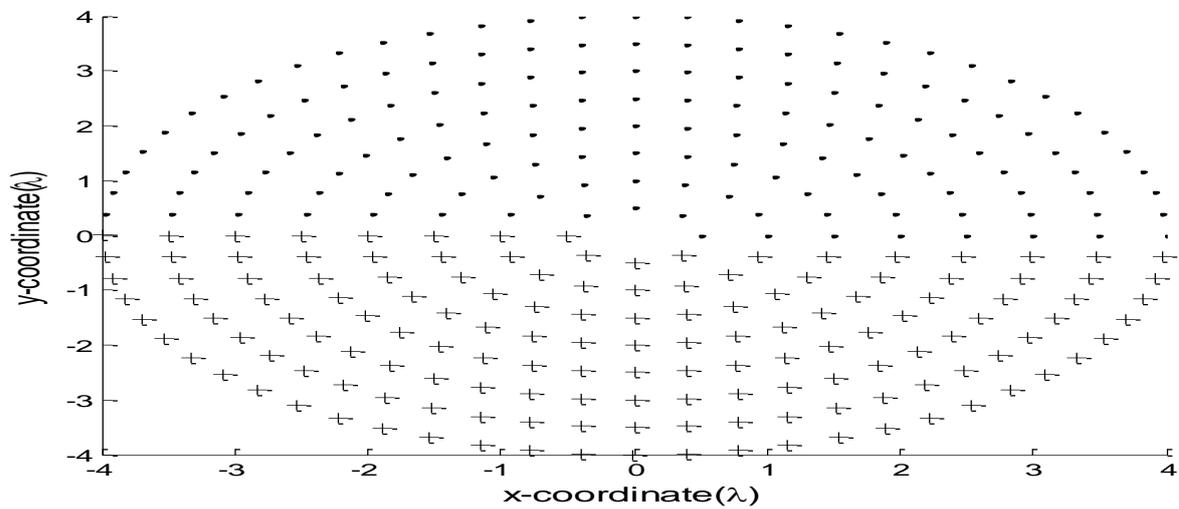


Fig. 5. Half of the array excited out of phase

The thinned aperture is shown in fig. 6. Turning off 132 elements out of a total number of 288 elements, brought down the sidelobe level to -22.08dB. As pointed earlier, this lowers the antenna weight and design cost.

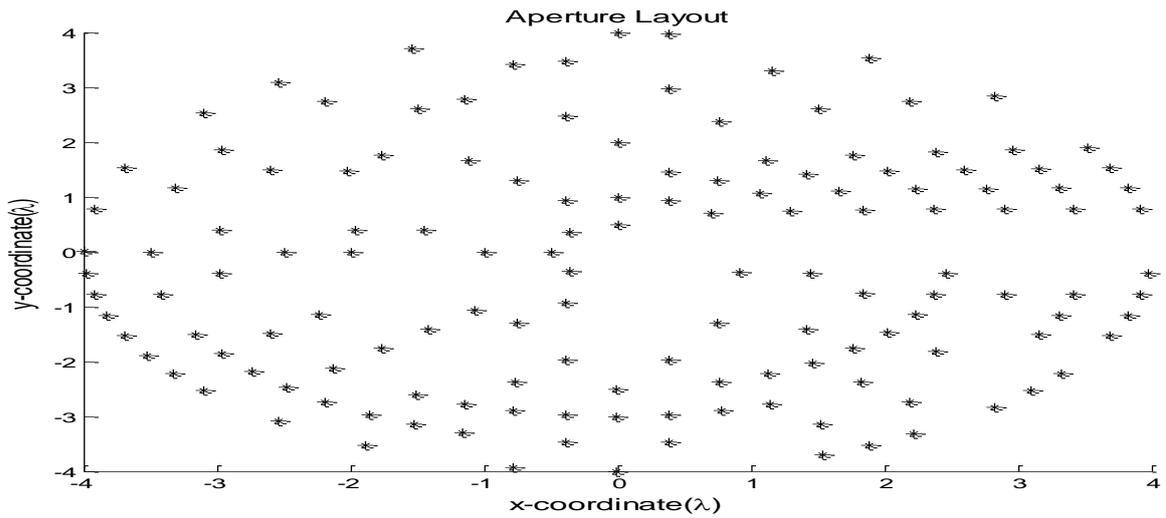


Fig. 6. Aperture layout of ON elements

Figures 7, 8, 9 show the optimized far field difference pattern, Amplitude distribution and Thinned aperture of a 10 ring concentric circular array. Out of 440 elements, only 230 elements are excited and remaining 210 elements are turned ‘OFF’.

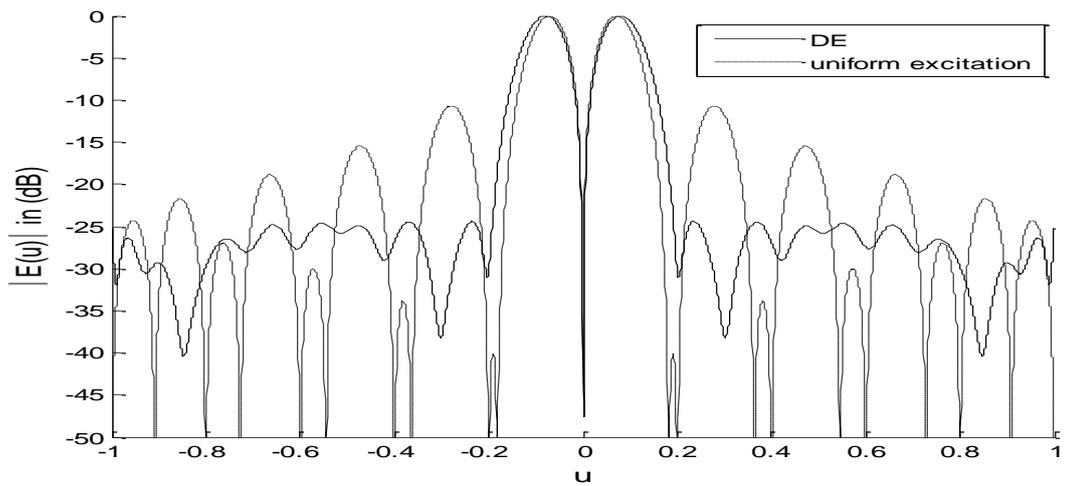


Fig. 7. Patterns for DE optimized and uniformly excited arrays

A peak SLL of -24.399 dB is obtained with a fill factor of 52.2727%. Fig.7 also gives a comparison of radiation patterns from a uniformly excited array with DE optimized array. The optimization gives an improvement of 13.7dB over the -10.699dB SLL achieved from a fully populated uniformly excited array.

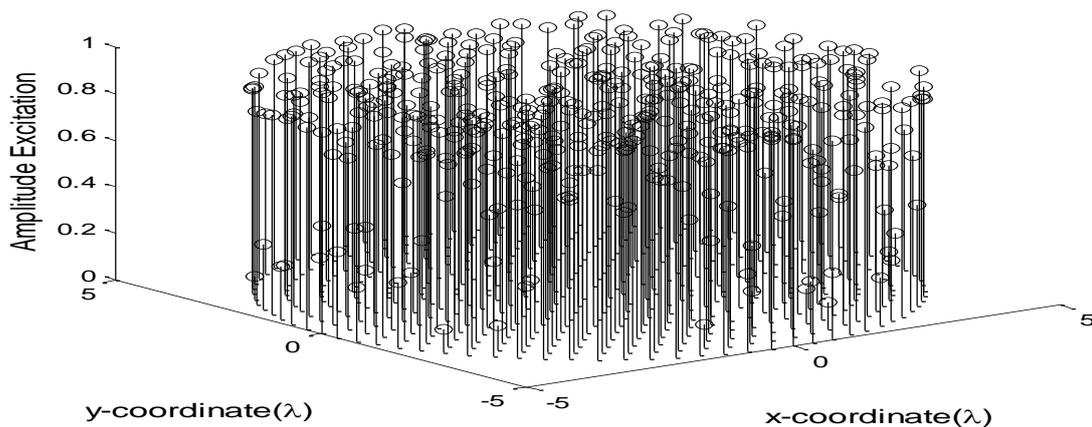


Fig. 8. Aperture distribution across the array

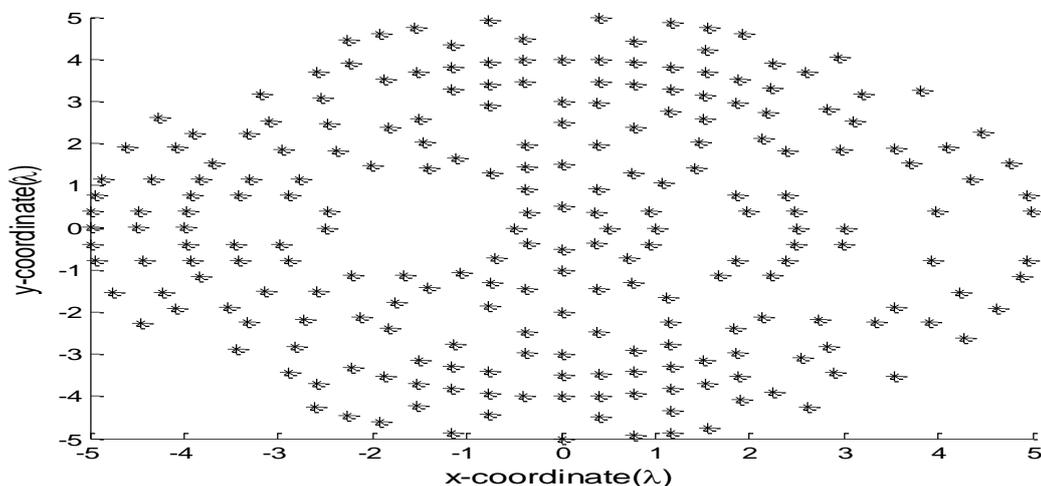


Fig. 9. Aperture layout of ON elements

## 6. Conclusion

This paper presents useful array designs for generation of low sidelobe tapered difference patterns from concentric circular arrays. It is worth to mention that no work has been reported earlier in this regard. Such patterns with low side lobes find wide applications in radar target tracking. Amplitude only synthesis and thinning techniques are employed for reducing the sidelobe levels. The optimum array configurations are derived using Differential Evolution algorithm. The designed configurations have a fill factor restricted to a maximum of 55%. Results are presented for 8 and 10 number of concentric circular arrays. PSLR of -22.08dB and -24.399dB are attained for 8 ring and 10 ring concentric circular arrays respectively. The results show good improvement in reduction of peak SLL, as the obtained sidelobe levels are atleast 11dB better than those attained from uniformly fed fully filled arrays. The work can be extended for thinning

arrays of practical elements and arrays of different geometries. G. S. N. Raju (2008), "Radar Engineering and Fundamentals of Navigational Aids," I.K. International Publishing House Pvt. Ltd., India.

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