Lectures on Modelling and Simulation; A selection from AMSE # 2017-N°2; pp 129-138
Selection at the AMSE Conference Calcutta/India, November 4-5, 2017

Study on the Chaotic Dynamics & its Control in a Time Delayed Optoelectronic Oscillator

* Dia Ghosh, ** Arindum Mukherjee ,*** Nikhil Ranjan Das, **** Baidya Nath Biswas

*Department of Electronics & Communication Engineering, Siliguri Institute of Technology, Siliguri-734009, West Bengal, India, (dia.slg42@gmail.com)

** Department of Electronics & Communication Engineering, Central Institute of Technology, Kokrajhar-783370, Assam, India,(arindum78@gmail.com)

*** Institute of Radio Physics and Electronics,
Calcutta University 92 A.P.C. Road, Kolkata 700009, West Bengal, India, (nrd@ieee.org)

****Education Division, SKF Group of Institutions,
Mankundu, Hooghly 712139, West Bengal, India, (baidyanathbiswas@gmail.com)

Abstract

The present literature reports the effect of an external synchronizing signal on the chaotic dynamics of a single Loop optoelectronic Oscillator (SLOEO). It has been observed that the OEO can produce chaotic oscillation with small change of feedback loop delay and application of an external periodic signal with suitably chosen amplitude and frequency can destroy the chaotic oscillation and produce single frequency oscillation. The proposed method can be used to suppress the chaotic oscillation in an OEO.

Key words

Delay dynamical system, optoelectronic oscillator (OEO), Chaos, synchronization.

1. Introduction

Over the last few years OEO has seen wide spread application in the field of RADAR, fiber optic communication system, long distance digital communication system, in view of the fact that it has the ability to produce high frequency signal with ultra high spectral purity. This oscillator was first introduced by Neyer & Voges[1]. Posterior to their pioneering work, Yao and Maleki introduced this oscillator as a high quality microwave oscillator [2]. The OEO contains a continuous wave laser source. The optical signal generated from the laser is fed to a Mach-Zehnder modulator (MZM), which is acting as intensity modulator .The intensity modulated optical signal is passed through an optical fiber delay line and applied to the photo detector. The detected RF signal is then filtered by a band pass filter (BPF). The output of the BPF is fed to the electrical port of the MZM. Generation of high spectrally pure signal is possible due to the long low loss optical fiber delay line in its feedback loop. The long delay line results in a high quality factor and spectral purity. The presence of optical fiber delay line facilitates OEO as a candidate of electro-optical system with delayed feedback. Therefore the study on the complex dynamics of OEO is an important aspect, both from academic and engineering application point of view. Considering the feedback gain as a control parameter Chembo et al described the generation of chaotic breathers in an OEO[3].Other schemes for chaotic signal generation and stability analysis in an OEO was also being contemplated [4-9]. By controlling both feedback delay and loop gain the complex dynamics and synchronization property of an OEO was reported [10-11] but the OEO in this report was implemented using discrete time DSP technology. The oscillator was designed with a laser, electro-optic modulator and a photo-detector but for delay and filtering purpose the DSP board was used. . In [12], it has been reported by the present authors that with the variation of loop delay the system loses its stability and following a period doubling route it produces chaotic oscillation.

In the present literature we report a study on the complex dynamics of an OEO under the influence of a synchronizing signal. In OEO in order to obtain high spectral purity of the signal a long feedback loop delay is required. But long feedback loop delay produces additional cavity modes. These adjacent cavity modes can even produce chaotic oscillation. It has been shown that with increment of feedback loop delay the system produces chaotic oscillation. The chaotic optoelectronic oscillator can have important application in chaos based secure communication. However the chaotic oscillation with the increment of feedback loop delay may be unwanted in many applications. To control the chaotic dynamics an external sync signal is applied in the OEO. It is observed that by controlling the amplitude of the external signal the chaotic oscillation at the output of the free running oscillator can be destroyed and period -1 oscillation can be produced. Although the method of chaos quenching is not new [13-14], as far as the knowledge of the authors is concerned, the effect of sync signal to control the chaotic dynamics of the OEO is addressed nowhere.

The paper is organized in the following way: Section 2 describes the basic configuration of the oscillator and derivation of the system equation. In section 3 the numerical study is presented. The simulation study is described in section 4. Finally the paper concludes in section 5.

2. Derivation of System Equation

Fig. 1 shows the basic configuration of an SLOEO. It consists of a continuous wave laser source which is fed in to a Mach-Zehnder modulator (MZM), the MZM acts as an intensity modulator of the optical signal. The optical output of the modulator is detected by a photo detector after passing through a long optical delay line. This signal is then passed through an electrical band pass filter (BPF). The output from the BPF is fed back to the electrical port of the MZM. The BPF implemented here using a single tuned circuit.

Let us consider the RF input to the MZM is $V_{in}(t) = V(t)e^{j(\omega_0 t + \theta(t))}$ where $V(t)$ is the amplitude of the signal with free-running frequency ω_0 and the initial phase of $\theta(t)$. The output power of the MZM can be expressed as [16].

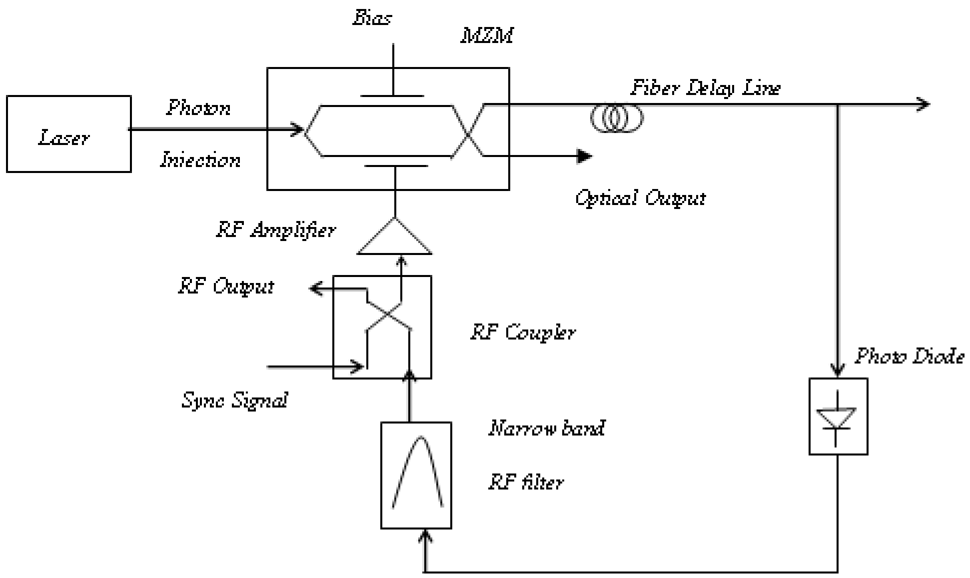


Fig.1. Basic configuration of a Single Loop Optoelectronic Oscillator

$$P(t) = \frac{1}{2}\alpha P_0\left[1 - \eta Sin\pi\left(\frac{V_{in}(t) + V_B}{V_\pi}\right)\right] \tag{1}$$

Where P_0 is the input optical power, α is the fraction of insertion loss of the modulator, η is the extinction ratio of the modulator, V_B is the bias voltage of the modulator , and V_π is the half wave voltage of the modulator. Therefore the photo-detector output can be expressed as $V_0(t) = \rho RP(t-\tau)$, where ρ is the sensitivity and R is the output impedance of the photo-detector and τ is the time delay resulting from the physical length of the optical fiber in the feed-back loop . Considering all these arguments it can be shown that [16, 17].

$$V_0(t) = -2\eta V_{ph}\cos\left(\frac{\pi V_B}{V_\pi}\right)J_1\left(\frac{\pi V(t-\tau)}{V_\pi}\right)\sin[\omega(t-\tau)] = \frac{N(V(t-\tau))}{V(t)}\exp(-s\tau)V_{in}(t)$$

where $N(V(t-\tau)) = -2\eta V_{ph}\cos\left(\frac{\pi V_B}{V_\pi}\right)J_1\left(\frac{\pi V(t-\tau)}{V_\pi}\right)$ and $V_{ph} = \frac{\alpha R\rho P_0}{2}$,

now for simplicity let us consider $\eta = 1; V_B = V_\pi; \pi V_{ph} = V_\pi; V_\pi = \pi$ and $N(V(t-\tau)) = 2J_1(V(t-\tau))$
Here J_1 is the Bessel's function of the first kind for order zero.
When the input signal $V_{in}(t)$ passes through the SLOEO the output voltage can be expressed as

$$V_0(t) = \beta(s).V_{in}(t) \tag{2}$$

$$\beta(s) = \left[\frac{N(V(t-\tau))}{V}G(s)e^{-s\tau}\right] \tag{3}$$

here $G(s)$ is the transfer function of the single tuned circuit and can be written as $G(s) = g_m Z(s)$, g_m is the gain of the tuned circuit.

Using (2) and (3) it can be shown that [16, 17, 18]

$$\left[\frac{V}{R} + C\frac{dV}{dt} + \frac{1}{L}\int Vdt\right] = 2J_1\left[V(t-\tau)e^{-s\tau}\right]g_m \tag{4}$$

To realize the transient behavior, we consider the operation of the system near resonance

$$\left(\frac{1}{R} + j\omega C + \frac{1}{j\omega L}\right) \cong G + 2C(j\omega - j\omega_0) \tag{5}$$

and

$$j\omega = \frac{1}{V(t)}.\frac{dV}{dt} + j\omega_0 + j\frac{d\theta}{dt} \tag{6}$$

using (4), 5) and (6) and equating the real and imaginary part the time varying amplitude and phase of an SLOEO can be written as.

$$\frac{dV}{dt} = \frac{\omega_0}{2Q}[G_1 2J_1[V(t-\tau)]Cos(\omega_0\tau) - V(t)]$$

$$\frac{d\theta}{dt} = -\frac{\omega_0}{2Q}\frac{G_1 2J_1[V(t-\tau)]}{V}Sin(\omega_0\tau) \tag{7}$$

Where $G_1 = g_m R$ is gain at resonance.
Multiplying equation (7) by 2 and considering the following quantities (7) can be rewritten as

$$t' = \frac{\omega_0 t}{Q}, \quad \tau' = \frac{\tau}{RC}, \quad b = 2G_1, \quad v = \frac{V(t)}{V_{\max}}, \quad v(t' - \tau') = \frac{V(t-\tau)}{V_{\max}}$$

$$\frac{dv}{dt'} = -v + bJ_1\left[v\left(t' - \tau'\right)\right] Cos(\omega_0 \tau') \qquad (8)$$

$$\frac{d\theta}{dt'} = -\frac{b\, J_1\left[v\left(t' - \tau'\right)\right]}{v} Sin(\omega_0 \tau')$$

Now let us consider a synchronizing signal having a form of $S(t) = E\, e^{j(\omega_1 t + \psi(t))}$ is injected in the free running oscillator, here E is the amplitude and $\psi(t)$ is the phase of the injected signal. The phase difference between the free running signal and the injected signal is $\phi(t) = \psi(t) - \theta(t)$. Thus it is not difficult to show that the closed loop amplitude and phase equation of the synchronized oscillator will take the following form.

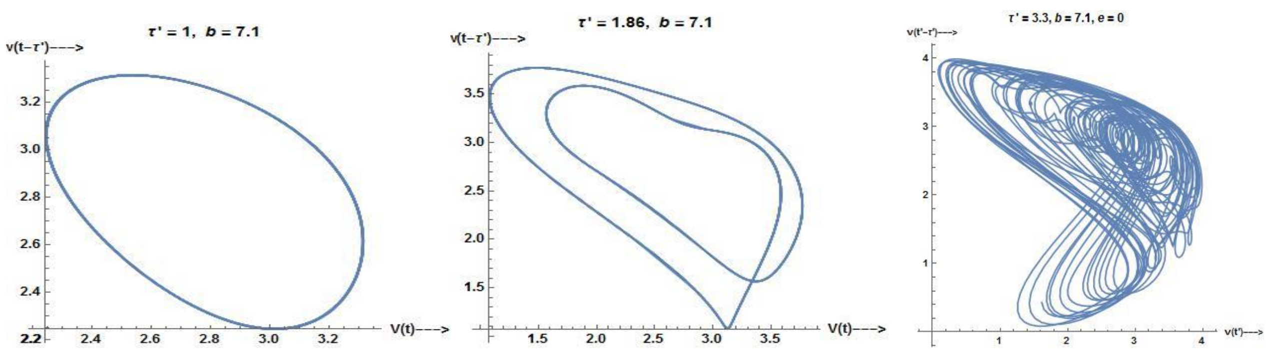
$$\frac{dv}{dt'} = -v + bJ_1\left[v\left(t' - \tau'\right)\right] Cos(\omega_0 \tau') + G_1 e\, Cos(\phi(t')) \qquad (9)$$

$$\frac{d\phi}{dt'} = \frac{2Q}{\omega_0}\Omega + \frac{b\, J_1\left[v\left(t' - \tau'\right)\right]}{v} Sin(\omega_0 \tau') - \frac{G_1 e}{v} Sin(\phi(t'))$$

here $\Omega = \omega_1 - \omega_0$ and e is the normalized amplitude of the sync signal.

3. Numerical Analysis

Equation (8) is the free running system equation of the oscillator. This equation is solved numerically using Mathematica version 10 considering $G_1 = 3.55, b = 2G_1 = 7.1$. In our previous work [12] it has been shown that with the variation of feedback loop delay τ' the system produces chaotic oscillation following a period doubling sequence. Fig. 2 depicts phase plane plot of the oscillator. In this figure it is shown that at τ'=1 period 1 oscillation is produced, period doubling oscillation is produced at τ'=1.86 and the hyper chaotic oscillation is obtained for τ'= 3.3.



(a) (b) (c)

Fig.2. Numerically obtained phase plane plot of free running oscillator (v , $v(t'-\tau')$ space)

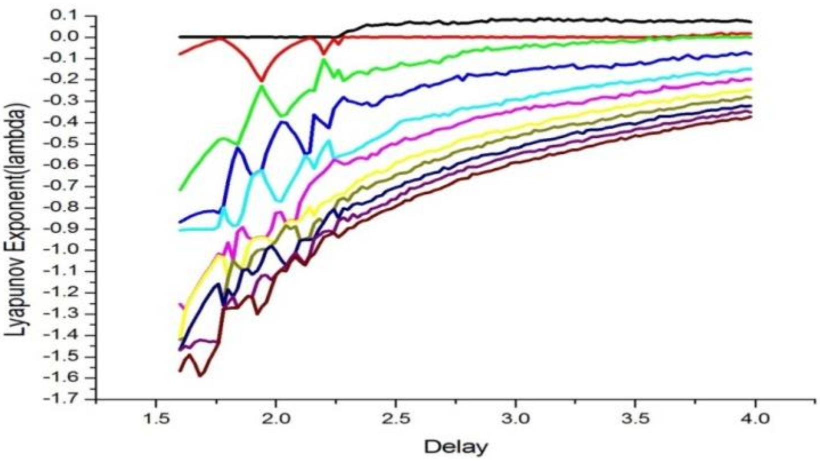
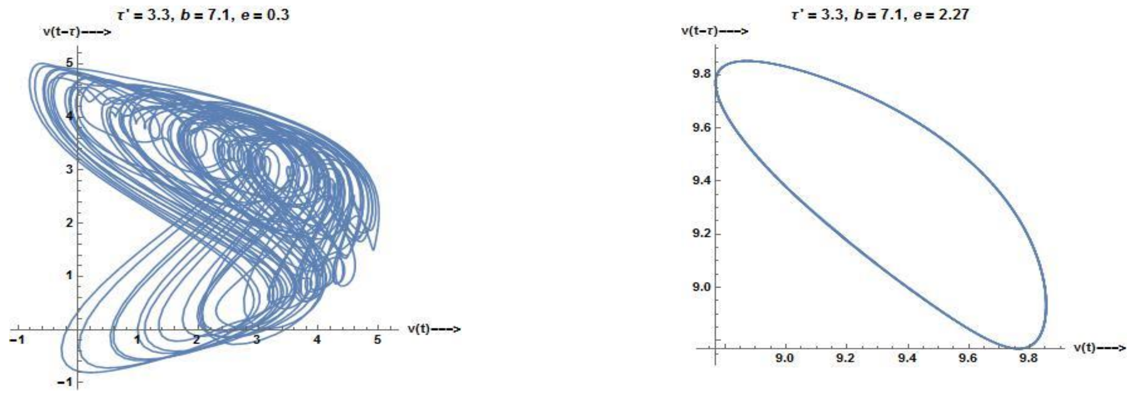


Fig.3. Lyapunov exponent (λ) with feedback delay

The chaotic dynamics is quantified using Lyapunov exponent spectrum, following the technique proposed by J.D Farmer [15]. The spectrum of Lyapunov exponent also ensures the existence of chaotic oscillation beyond τ' =2.3 (Fig.3). Now at τ' =3.3 keeping all other parameter values unchanged the external sync signal is injected into the oscillator. The injected signal frequency is same as the free running oscillation frequency. It has been observed using (9) that with suitable control of the sync signal amplitude the chaotic state of the free running oscillator disappears and period -1 oscillation is produced. Fig.4. shows the phase plane plot of the driven oscillator for $e = 0.3$ and $e = 2.27$ .It can be seen from the figure that at $e = 0.3$ the driven oscillator produces hyper chaotic oscillation but at $e = 2.27$ the chaotic oscillation completely disappears and single frequency oscillation is achieved at the output of the driven oscillator.



(a) (b)

Fig.4. Numerically obtained phase plane plot of the driven oscillator (v , $v(t'-\tau')$ space)

4. Simulation Study using MATLAB Simulink Software

The oscillator under study is realized using MATLAB™ 9.0 Simulink software. Fig.5 represents block diagram of the simulation set-up. In general OEO can generate high frequency signal in microwave and mm wave range. However it is difficult to carry out the simulation study in such a high frequency range. To overcome this difficulty the frequency of laser source is chosen as 500 M rad/s and the output signal amplitude of the laser is set at 1.4Volt. To design the BPF we have taken C=1nF, L=0.2μH, R=3kΩ, with these parameters the operating frequency becomes $f = 11.22MHz$, RF gain G_1 is set to 3.55. It can be shown that the fiber delay τ =10 μs ($\tau' = \frac{\tau}{RC} = 3.33$) produces the chaotic oscillation at the output of the oscillator in free running condition (Fig.6). Now an external RF signal is applied in to the oscillator. The operating frequency of the sync signal is kept same as the centre frequency of the RF spectrum of the free running chaotic oscillator and the amplitude E is varied. The output spectrum of the driven oscillator is shown in fig 7. It can be seen from the figure that at E=0.5 Volt chaotic oscillation is present at the output of the driven oscillator but as E is increased further, at $E = 2.55\,Volt$ single frequency oscillation is achieved at $11.25MHz$.

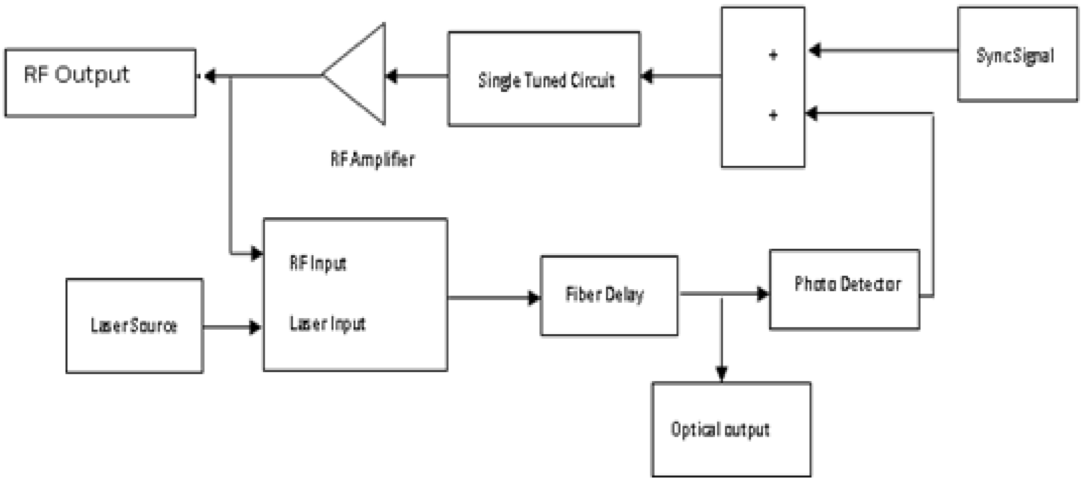


Fig.5. Schematic representation of simulation set-up

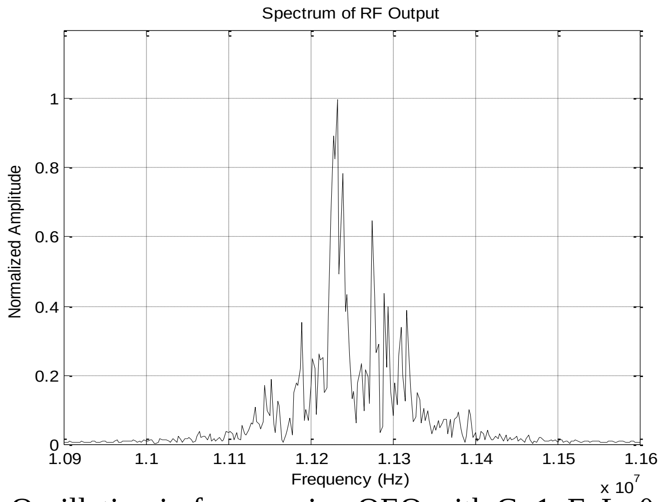
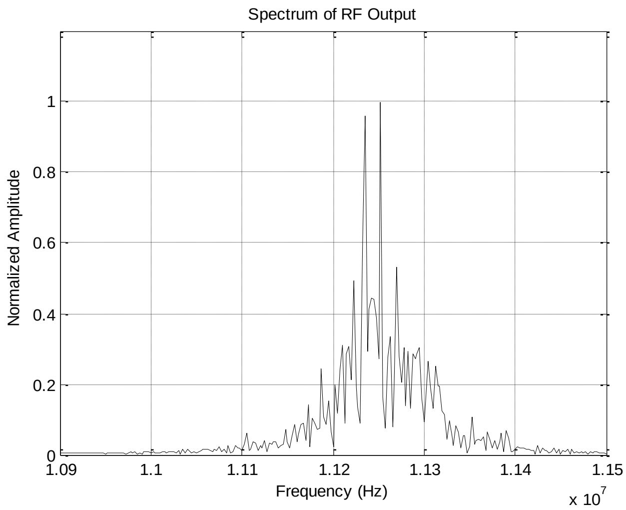
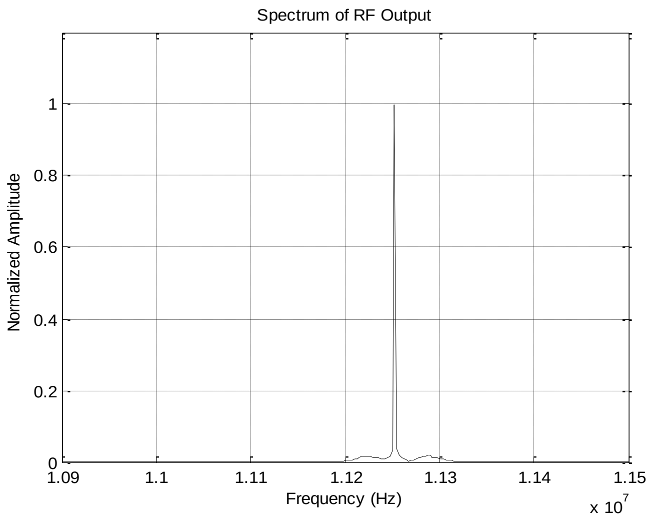


Fig.6. Chaotic Oscillation in free running OEO with C=1nF, L=0.2μH, R=3k Ω, τ =10μs



(a) E=0.5 Volt



(b) $E = 2.55\, Volt$

Fig.7. RF spectrum of the driven oscillator obtained from the simulation study with τ = 10μs and with different values of E, keeping all other parameters unchanged.

5. Conclusion

In this correspondence we have studied the complex dynamics of a time delayed SLOEO. Optoelectronic oscillator can efficiently produce high frequency signal with high spectral purity. Generation of high spectrally pure signal is possible due to the long low loss optical fiber delay line in its feedback loop. However long feedback loop delay may produce chaotic oscillation. This chaotic oscillation may find important application in chaotic RADAR, chaos based secure communication. However the chaotic oscillation may be unwanted in many applications It has been demonstrated through the numerical and simulation study that the application of the injected signal destroys the chaotic oscillation and suitable control of the injected signal amplitude can produce period -1 oscillation. The proposed technique can be efficiently used to remove chaotic oscillation and produce single frequency oscillation in an OEO.

Acknowledgement

The authors are thankful to the management of Siliguri Institute of technology, Siliguri, West Bengal, India, Central Institute of Technology, Assam, India for giving an opportunity to carry out this work. The authors also acknowledge the support from the management of Sir J C Bose School of Engineering to conduct the work at Sir J C Bose Creativity Centre of Supreme Knowledge Foundation Group of Institution, Mankundu, Hooghly,West Bengal, India.

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