

## The ordered weighted average and its extension in volatility

Ezequiel Avilés-Ochoa<sup>1</sup>, Luis F. Espinoza-Audelo<sup>1</sup>, Ernesto León-Castro<sup>1\*</sup>, Jose M. Merigó<sup>2</sup>

<sup>1</sup> University of Occidente, Blvd. Lola Beltrán s/n esq. Circuito Vial, Culiacán 80200, México

<sup>2</sup> Department of Management Control and Information Systems, School of Economics and Business, University of Chile, Av. Diagonal Paraguay 257, 8330015 Santiago, Chile

Corresponding Author Email: ernesto.leon@udo.mx

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### ABSTRACT

The agricultural products have had sudden changes of prices in the last years, this derived from the volumes of production and demand that they have at the international level. In this way, it is important to be able to have knowledge of their future prices to implement a hedging strategy. A key element to know the difficulties that the market may have is volatility. Among the problems that this has is that its traditional formula only considers the historical information, without considering the knowledge and skills that the decision making has. In order to solve this situation, the use of the ordered weighted average (OWA) operator.

## 1. INTRODUCTION

They comment that one of the biggest concerns of agricultural producers is the risk in price. Per manufacturers Jankelova, such uncertainty has increased by the release of markets and high specialization in production systems [6, 9].

The fluctuation in the prices of raw materials generates a great impact both in the organizations, as in the economies, that sometimes discourages the realization of projects of investment [8], it is this way, to understand the Price behavior is important for generating short-and long-term strategies [1].

## 2. VOLATILITY

To capture the dynamics of prices, it is important to measure the volatility that they present during the analysis period, this is usually calculated as a coefficient of variation [4, 10], this can be defined as follows [3]

**Definition 1.** Volatility is the formula:

$$v = \frac{\sigma}{\mu} \quad (1)$$

Dónde  $v$  está la volatilidad  $\sigma$  es la desviación estándar y  $\mu$  la media.

One of the weaknesses of the traditional formulation of volatility is that only consider the historical data, leaving aside another macroeconomics variables such as interest rate, GDP and others [5, 11]. Also, it is important to include the knowledge, expectations and aptitude of the decision maker, that in many cases is very important to aggregate them in order to obtain a relevant result of the future expectations of the market, that usually is characterized by the uncertainty in them [16-17].

## 3. OWA

Among the ways that exists to add information of the decision maker in the mathematical operation is the Ordered Weighted Average (OWA) operator [15]. The main advantage of this operator is that it is possible to rearrange the average in different criteria and between a maximum and minimum results. The definition is as follows.

**Definition 2.** An OWA operator of dimension  $n$  is a mapping OWA:  $R^n \rightarrow R$  with an associated weight vector  $W$  of dimension  $n$  such that  $\sum_{j=1}^n w_j = 1$  and  $w_j \in [0,1]$ , according to the following formula:

$$OWA(a_1, a_2, \dots, a_n) = \sum_{j=1}^n w_j b_j, \quad (2)$$

where  $b_j$  is the  $j$ th largest element of the collection  $a_i$ .

## 4. OWA OPERATOR AND ITS EXTENSION IN VOLATILITY

As can be seen in the classical formulation for volatility, it only takes into account the historical data in order to get the final results, being this one of its main disadvantages. In order to obtain results according to the reality of the topic being studied it is necessary to add information of the future scenarios, not only the past one. In this sense, with the use of the OWA operator and its extension it is possible to aggregate the knowledge, expectations and attitude of the decision maker in the final results. Taking these into account, the OWA-volatility can be defined as follows.

**Definition 3.** An OWA-Volatility operator of dimension  $n$  is a mapping OWA-Volatility:  $R^n \rightarrow R$  with an associated weight vector  $W$  of dimension  $n$  such that  $\sum_{j=1}^n w_j = 1$  and  $w_j \in [0,1]$  such that

$$OWA - Volatility (a_1, a_2, \dots, a_m) = \frac{\sigma - OWMA}{\mu - OWMA} \quad (3)$$

where  $\sigma - OWA$  is the OMA standard deviation,  $\mu - OWA$  is the OMA average.

## 5. OWA-VOLATILITY IN ITS APPLICATION IN CORN PRICE

### 5.1 Theoretical approach

The volatility of market conditions and the prices of products traded between them have become vital information for market participants, be they producers, consumers, investors, among others. [7]. Thus, considering the volatility within the forecasts of different financial market variables has allowed to visualize in a clearer way the future of the same, hence the importance that is calculated efficiently [13, 2].

Within the markets that have had greater volatility in recent years is the agricultural products [14]. In the specific case of Sinaloa, Mexico, corn is one of the main produced agricultural product, generating 15.8% of the total national production [12], that is why is very important for the state to calculate in an efficient way the volatility, because the profit of many enterprises is in risk.

In this paper, new formulations to calculate volatility are presented, among there is OWA-Volatility.

The step to use this new formulation is as follows.

Step 1. It is necessary to identify the number of months that will be used to calculate volatility. Can be a quarter, half year or the whole years. This step depends on the decision maker.

Step 2. The information of the prices that volatility wanted to calculate.

Step 3. The weighting vector that will be applied to the arguments based in the importance of the information and the decision maker knowledge.

Step 4. With the information provided by Step 1 to 3, it is possible to generate different formulations considering OWA-Volatility.

### 3.2 Numerical example

**Table 1.** Monthly average corn price for 2016-2017.

Date	2017
December	350.50
November	344.55
October	343.18
September	349.39
August	347.95
July	353.12
June	377.60
May	372.28
April	366.61
March	363.01
February	362.78
January	368.74

Step 1. The number of months that will be taken into account for the volatility will be six.

Step 2. The information of the monthly average price of corn for 2016 and 2017 are presented in table 1.

Step 3. A weighted vector  $W=(0.10,0.10,0.15,0.15,0.25,0.25)$

Step 4. Con la información proporcionada en el paso 1-3, la volatilidad de 2017 se calculará utilizando OWA volatilidad. formula. Results are presented in Table 2.

**Table 2.** Volatility for corn price 2017 using the general formulation

Date	Volatility	OWA-Vol
12-17	0.0330	0.0253
11-17	0.0918	0.0400
10-17	0.0995	0.0346
09-17	0.0989	0.0321
08-17	0.0872	0.0248
07-17	0.0644	0.0109
06-17	0.0330	0.0182
05-17	0.0918	0.0200
04-17	0.0995	0.0262
03-17	0.0989	0.0266
02-17	0.0872	0.0421
01-17	0.0644	0.0294

With the results presented in table 2 it is possible to visualize that the results obtained using the classical volatility formula only one result is possible, because the historical data doesn't change, but taking into account different information provided by the decision maker. Los rangos en las volatilidades obtenidos utilizando el operador OWA, que utilizando la formula tradicional de volatilidad, ampliando de manera la perspectiva del tomador de decisión sobre el nivel de precios en los periodos de cosecha.

Date	Volatility	OWA-Vol
Máxima	0.0995	0.0421
Mínimo	0.0330	0.0109
Rango	0.0665	0.0312

## 6. CONCLUSIONS

The aim of the paper is to introduce new formulations to the traditional volatility formulation using as a base to aggregate the ordered weighted average (OWA) operator. An application in corn price volatility for 2017 have been presented. Corn is one of the main products in Sinaloa, Mexico and that is why forecast the future price of the same is a crucial factor for the profits of the enterprises. With the results, it can be seen that with the use of more complex situations and adding new vector in the aggregation process is possible to visualize new scenarios of volatility that in the classical volatility formula cannot be seen, this will help the decision maker to understand better the market complexity and take alternatives that provide less risk to the company.

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