

$$I(t) = \sin 4t \left(\frac{3}{2} \cos 9t - \frac{1}{2} \sin 9t + \frac{3}{2} \cos t - \frac{9}{2} \sin t \right) - \cos 4t \left(\frac{1}{2} \cos 9t + \frac{3}{2} \sin 9t - \frac{9}{2} \cos t - \frac{3}{2} \sin t \right) - 4e^{-3t} \cos 4t - 3e^{-3t} \sin 4t$$

Applying DTM, we have $Y(0) = 0, Y(1) = 0$ and the recursive relation

$$Y(k+2) = \frac{1}{(k+1)(k+2)} (-6(k+1)Y(k+1) - 25Y(k) - 120 \frac{5^k}{k!} \sin \frac{k\pi}{2})$$

Application of inverse DTM gives the polynomial solution as

$$I(t) = -100t^3 + 150t^4 + 70t^5 - 195t^6 + \frac{715}{14}t^7 + \frac{195}{4}t^8 - \frac{2635}{108}t^9 + \frac{79}{72}t^{10} \dots \dots \dots$$

Example 6:
Solution for the current $I(t)$ in an R-L-C circuit with $R = 80 \text{ ohms}, L = 10 \text{ henry}, C = .004 \text{ farad}$

and $E = 240.5 \sin t$ assuming zero current and charge at initial time:

The required IVP is

$$I'' + 8I' + 25I = 24.05 \cos t, I(0) = 0, I'(0) = 0$$

Exact solution:

Applying DTM we have $Y(0) = 0, Y(1) = 0$ and the recursive relation

$$Y(k+2) = \frac{1}{(k+1)(k+2)} (-8(k+1)Y(k+1) - 25Y(k) + 24.05 \cos \frac{k\pi}{2})$$

Application of inverse DTM yields the polynomial solution

$$I(t) = \frac{481}{40}t^2 - \frac{481}{15}t^3 + \frac{9139}{240}t^4 - \frac{6253}{300}t^5 - \frac{6253}{1600}t^6 + \dots \dots \dots$$

The following tables show the exact value, approximate value and absolute error in the solution of given IVPs by executing DTM.

Table 1. Example 1

x	Exact Value	Approximate Value	Absolute Error
0.1	2.099824744343730	2.099824744343730	0.000000000000000
0.2	2.198524964688300	2.198524964688310	4.884981308350689e-15
0.3	2.294759972879200	2.294759972879610	4.107825191113079e-13
0.4	2.386919516226030	2.386919516235850	9.819256518994735e-12
0.5	2.473081906050190	2.473081906165600	1.154130124803032e-10
0.6	2.550966320546710	2.550966321412570	8.658598282806906e-10
0.7	2.617878519711620	2.617878524476930	4.765317918042911e-09
0.8	2.670649114190960	2.670649135096770	2.090581485703069e-08
0.9	2.705563422272640	2.705563499406090	7.713345295812246e-08
1	2.718281828459040	2.718282076719570	2.482605299114482e-07

Table 2. Example 2

x	Exact Value	Approximate Value	Absolute Error
0.1	-0.900313230945780	-0.900313230945790	9.992007221626400E-15
0.2	-0.802356138016422	-0.802356138026808	1.038602537306580E-11
0.3	-0.707481400751040	-0.707481401349163	5.981229955764880E-10
0.4	-0.616694442352788	-0.616694452960846	1.060805798758220E-08
0.5	-0.530714327248554	-0.530714425922912	9.867435790678050E-08
0.6	-0.450025541597102	-0.450026151828571	6.102314689671040E-07
0.7	-0.374921904362514	-0.374924751770211	2.847407697015300E-06
0.8	-0.305543695599711	-0.30554506587654	1.081098794297740E-05
0.9	-0.241908947545656	-0.241944014314341	3.506676868500480E-05
1	-0.183939720585721	-0.184040178571429	1.004579857079850E-04

Table 3. Example 3

x	Exact Value	Approximate Value	Absolute Error
0.1	0.001356464598500	0.001356464598485	1.499993705633695e-14
0.2	0.009798598745439	0.009798598715429	3.001000142832666e-11
0.3	0.029801158441399	0.029801155907170	2.534228996792232e-09
0.4	0.063497061223301	0.063497002617905	5.860539599444881e-08
0.5	0.111140193030166	0.111139526367187	6.666629790014200e-07
0.6	0.171495395053878	0.171490552827429	4.842226449003340e-06
0.7	0.242178918565363	0.242153108741396	2.580982396699350e-05
0.8	0.319964492938543	0.319854795629714	1.096973088290376e-04
0.9	0.401064032741230	0.400671801838366	3.922309028639992e-04
1.0	0.481387476388215	0.480163690476191	0.001223785912024

Table 4. Example 4

x	Exact Value	Approximate Value	Absolute Error
0.1	0.085963177796689	0.085963177796690	9.992007221626400E-16
0.2	0.147421965065099	0.147421965066131	1.032007812540310E-12
0.3	0.189130851998201	0.189130852086716	8.851500088447040E-11
0.4	0.215113997716010	0.215113999795245	2.079235006391400E-09
0.5	0.228750064528217	0.228750088538653	2.401043600053220E-08
0.6	0.232853257577284	0.232853434512000	1.769347159896730E-07
0.7	0.229748790415279	0.229749746699376	9.562840970200880E-07
0.8	0.221341795521528	0.221345914634384	4.119112855999190E-06
0.9	0.209179272281065	0.209194191246877	1.491896581198850E-05
1	0.194505064836663	0.194505064836663	4.712872594700430E-05

Table 5. Example 5

x	Exact Value	Approximate Value	Absolute Error
0.1	-0.084489429654472	-0.084489429645569	8.902989456771593e-12
0.2	-0.549313882391200	-0.549313865210582	1.718061803934745e-08
0.3	-1.443162311236885	-1.443160940833929	1.370402956002437e-06
0.4	-2.532605788606551	-2.532576547487832	2.924111871882573e-05
0.5	-3.441829862462138	-3.441530903811178	2.989586509598574e-04
0.6	-3.807368479298738	-3.805476693942859	1.891785355879e-03
0.7	-3.407366376454890	-3.398922232355096	8.444144099794e-03
0.8	-2.236434699645677	-2.207983015686777	2.8451683958900e-02
0.9	-0.512895321161074	-0.438877301576769	7.4018019584305e-02
1	1.377857673138121	1.520502645502649	1.42644972364528e-01

Table 6. Example 6

x	Exact Value	Approximate Value	Absolute Error
0.1	0.091780451967744	0.091780451970833	3.089001276990189e-12
0.2	0.278654281953035	0.278654287231048	5.278013026632777e-09
0.3	0.474228441488846	0.474228816512148	3.750233019683691e-07
0.4	0.636632986671345	0.636640130869728	7.144198382991185e-06
0.5	0.751102449034015	0.751167469125576	6.502009156095223e-05
0.6	0.817595653557294	0.817957675548001	3.620219907070066e-04
0.7	0.842765777105223	0.844146651241225	1.380874136002e-03
0.8	0.835237962554497	0.838929065323003	3.691102768506e-03
0.9	0.803168295545608	0.809094077290491	5.925781744883e-03
1	0.753238351345508	0.750535232032627	2.703119312881e-03

4. CONCLUSION

The numerical analysis of results between analytical method and DTM shows high degree of accuracy. The method followed is effective and computation is less time consuming. So the prescribed method suggests a better approach for the solution of higher order IVPs.

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