

The Theoretical Research of Wake Bubble Light Scattering Characteristics

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Abstract

The movement of ships in water and the cavitation of propeller would bring air into water and form bubble curtain of a lot of bubbles. The sporting bubbles form the wake. The optical characteristics of wake bubbles are analyzed in this paper, especially the light scattering characteristics of bubbles. First of all, based on the study of the bubble distribution characteristic, Mie scattering theory is used to the light scattering of single bubbles. The bubbles of different size's scattering intensity in different angle is studied. On the basis of scattering model of single bubble, the nature of bubble curtains scattering is further discussed. Combined with the image, the theoretical value and experimental value is analyzed and discussed. Establishing Monte Carlo model, light scattering of bubble group is carried on the theory study. Through the simulation calculation, it indicates that it is feasible to use Monte Carlo method to study the light scattering of bubble group, which provides a more practical calculation method for the light detection of wake.

Key words

wake; bubble curtains; Mie scattering; Light scattering characteristic; Monte Carlo simulation

1. Introduction

The optical characteristics of ship wake is primarily a process of light transmission, scattering, refraction, absorption on bubble curtains. After laser incidence on bubble curtains, transmission direction could change and energy will be constantly absorbed and declined. Geometrical optics theory could not fully explain the characteristics of bubble light scattering theory, and the classical Fraunhofer diffraction theory in wave optics is obviously limited, while

the Mie scattering theory which is according to the electromagnetic field theory to calculate could explain the distribution of light energy quite good.

Ship wake bubble density is obviously different from seawater background, and the difference is very big. To analyze using the parameters of optical characteristics in wake field, using the sensitivity of light wave to tiny bubbles in the wake could greatly improve the detection and tracking capability, and provide an important means for the tracking, recognition and target location of underwater vehicles. The bubble distribution and movement is quite complicated in the near field of wake. Owing to turbulence effect, it is easy to cause bubble collision, crushing, aggregation and dissolution. After the turbulence attenuation, bubbles enter far-field region. In this region, bubble distribution is becoming stable. It could be approximately thought that the bubbles of far-field wake zone are not related and the scattering light can be recognized as irrelevant scattering. When light waves spread in medium, the molecules and atoms of medium would do forced vibration under certain light wave frequency, forming vibrating dipoles to launch electromagnetic wave in all directions. In vacuo, these secondary waves are coherent waves which bring about the interference effect, so there would not appear scattering phenomenon. But when the medium is inhomogeneous, there will be no coherent waves, scattering light would appear in all directions and produce the scattering effect. In reality, there are many active particles in seawater, which could bring about light scattering phenomenon.

2. Mie scattering theory of single bubbles

According to the research above we could get that there are a large number of bubbles in water, the radius of bigger bubbles could largen with its speed in the rising process, they would burst after rising to the water surface. The radius of small bubbles became smaller as time goes until they finally disappear or exist in the water not floating to the surface of wake. There exists a critical radius of about $44.2\mu\text{m}$ between big bubbles and small bubbles, the bubbles near the critical radius have the longest survival time. The experiment obtains that the bubbles with the radius of $40\text{-}50\mu\text{m}$ in ship wake field have the longest survival time, and their number density is the largest.

In the light scattering process of single bubble, the classical Mie scattering theory could be adopted. Mie scattering theory is using the Lorenz electromagnetic field theory into studying electromagnetic field's absorption and scattering of particles with small radius. It is the exact solutions of scattering the homogeneous spherical particle got under plane monochromatic light irradiation which Maxwell Electromagnetic Wave Equations are carried on the strict

mathematical derivation under certain boundary conditions. It was put forward by MieG in 1907. The details are shown in figure 1.

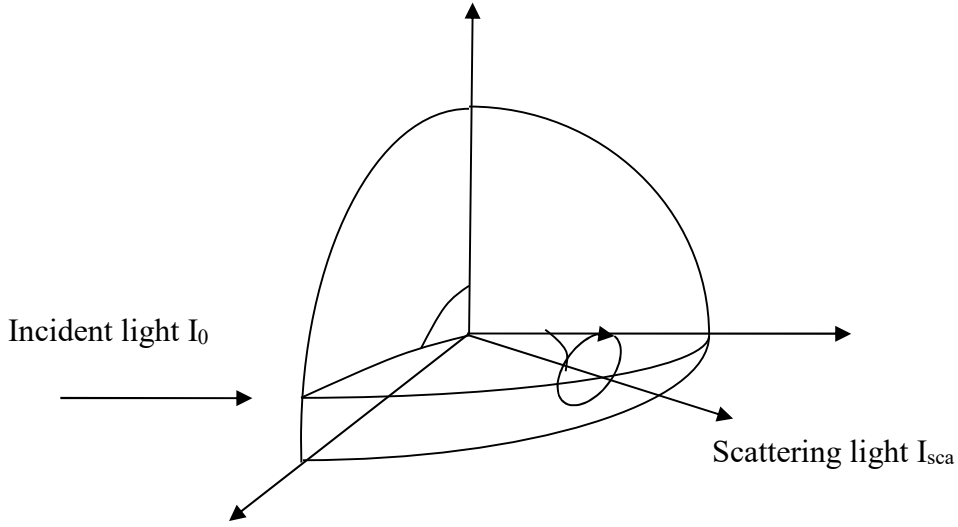


Figure 1: light scattering figure of single bubbles

When the unpolarized monochromatic light irradiation of the wavelength λ among medium, at the observation point with the distance of r and of scattering angle θ , the scattering intensity is:

$$I_{sca} = I_0 \frac{\lambda^2}{8\pi^2 r^2} \left[|s_1(\theta)|^2 \sin^2 \varphi + |s_2(\theta)|^2 \cos^2 \varphi \right] \quad (1)$$

s_1, s_2 in the formula are amplitude value function, which could be expressed as follows:

$$s_1(\theta) = \sum_{n=1}^{\infty} \frac{2n+1}{n(n+1)} [a_n \pi_n(\cos \theta) + b_n \tau_n(\cos \theta)]$$

$$s_2(\theta) = \sum_{n=1}^{\infty} \frac{2n+1}{n(n+1)} [a_n \tau_n(\cos \theta) + b_n \pi_n(\cos \theta)] \quad (2)$$

a_n, b_n in the formula are Mie scattering coefficient, the calculation formula is:

$$a_n = \frac{m \varphi_n(m\alpha) \varphi'_n(\alpha) - \varphi_n(\alpha) \varphi'_n(m\alpha)}{m \varphi_n(m\alpha) \xi'_n(\alpha) - \xi_n(\alpha) \varphi'_n(m\alpha)}$$

$$b_n = \frac{\varphi_n(m\alpha) \varphi'_n(\alpha) - m \varphi_n(\alpha) \varphi'_n(m\alpha)}{\varphi_n(m\alpha) \xi'_n(\alpha) - m \xi_n(\alpha) \varphi'_n(m\alpha)}$$

(3)

φ_n, ξ_n are Bessel function and Hankel function, φ'_n, ξ'_n are the derivative of its independent variable.

$$\pi_n(\cos\theta) = \frac{p_n^{(1)}(\cos\theta)}{\sin\theta} = \frac{dp_n(\cos\theta)}{d(\cos\theta)} \qquad \tau_n(\cos\theta) = \frac{dp_n^{(1)}(\cos\theta)}{d\theta}$$

(4)

p_n in the formula is Legendre function, $p_n^{(1)}$ is the associated Legendre function of $m=1$.

3. Light scattering characteristics of wake cross section

Based on Mie scattering theory, it is assumed that bubbles are in the irrelevant circumstances, here we study scattering intensity of the bubbles with different sizes at each scattering angle, and light scattering characteristics of bubble curtain at wake cross section, analyze and discuss its scattering results.

3.1 Single bubble scattering

When the incident light wavelength is $0.6328\mu\text{m}$, bubble radius $a=15.1\mu\text{m}$, seawater refractive index $m=1.442$, the scattering of single bubble is as follows:

X-coordinate is scattering angle (θ), ordinate is the logarithm of a certain angle scattering intensity and incident intensity ratio (10 is the bottom), $j=1$ the corresponding dashed line is the vertical situation of incident light and the scattering plane, $j=2$ the corresponding actual line is the parallel situation of incident light and the scattering plane.

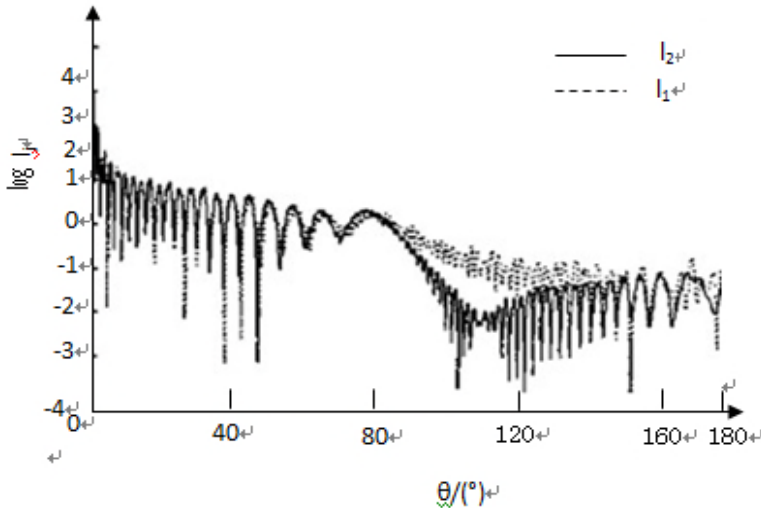


Figure 2: Mie scattering of small bubbles

The shape and trend of Figure 2 is similar to the obtained figure from which Stround and Marston(1994) used the incident light of the same wavelength to calculate the scattering of

bubbles with the same radius, and the calculation results also clearly show that the backward scattering is enhancing, which is consistent with the actual result.

3.2 Bubble curtain scattering of wake cross section

The 6th Bureau of United States Defense Commission carried on a calculation on the basis of the wake measured data of 15 sailing destroyers using sonar of 1946. Figure 3 is the calculation results when wake age is $t=3\text{min}$.

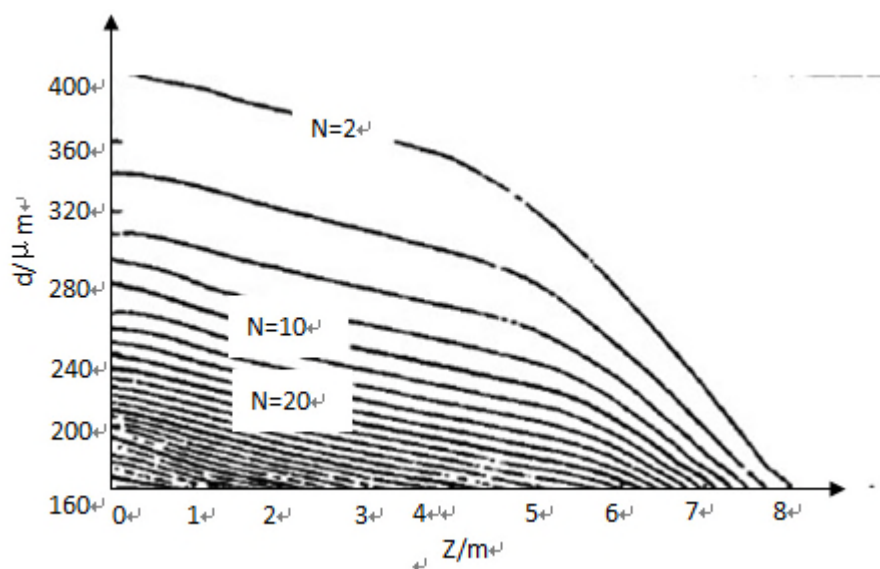


Figure 3: The relationship between wake bubble density and depth (Z) when $t=3\text{ min}$ (bubble diameter d)

According to the given data of ship wake above to calculate the scattering characteristics of wake bubble curtains, we could get the scattering situation the bubble group gives to underwater laser at the wake depth of $z=1, 3, 5$ and 6.5 m .

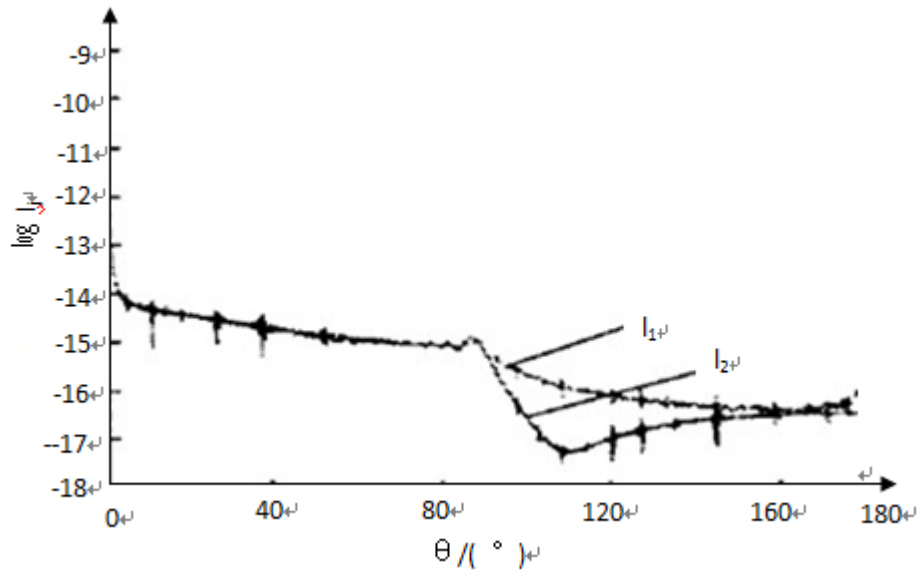


Figure 4: The wake bubble scattering when $t = 3$ min, $z = 1$ m

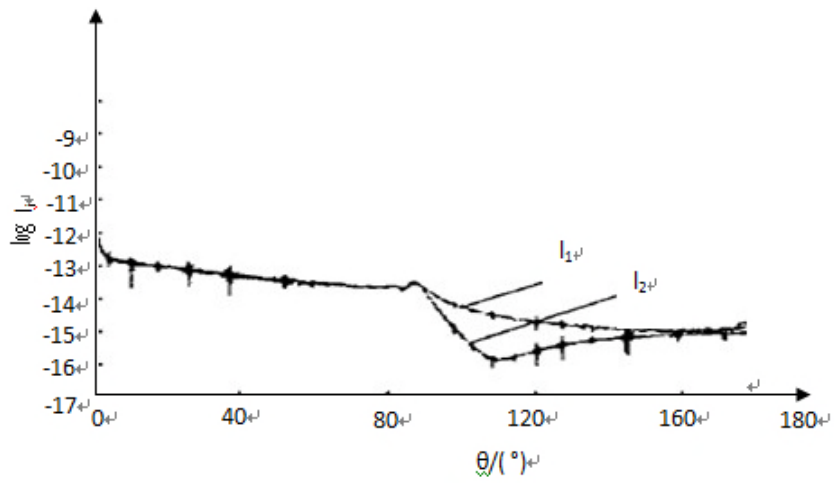


Figure 5: The wake bubble scattering when $t = 3$ min, $z = 3$ m

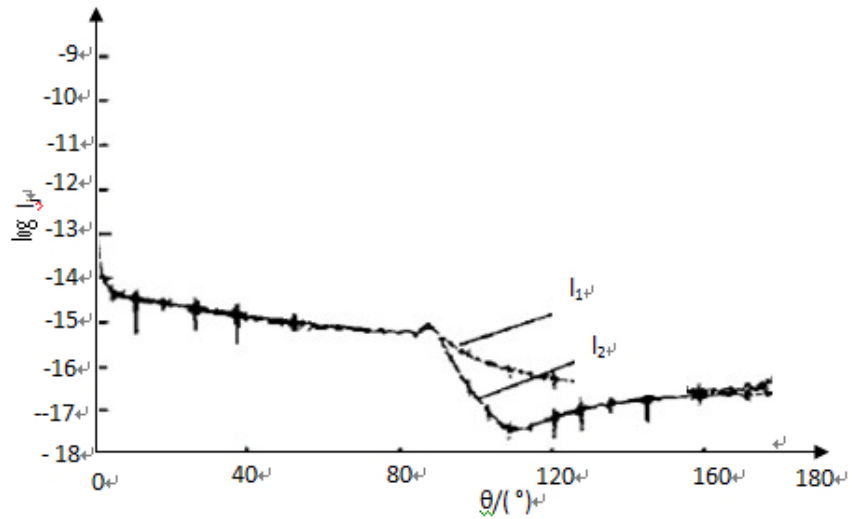


Figure 6: The wake bubble scattering when $t = 3$ min, $z = 5$ m

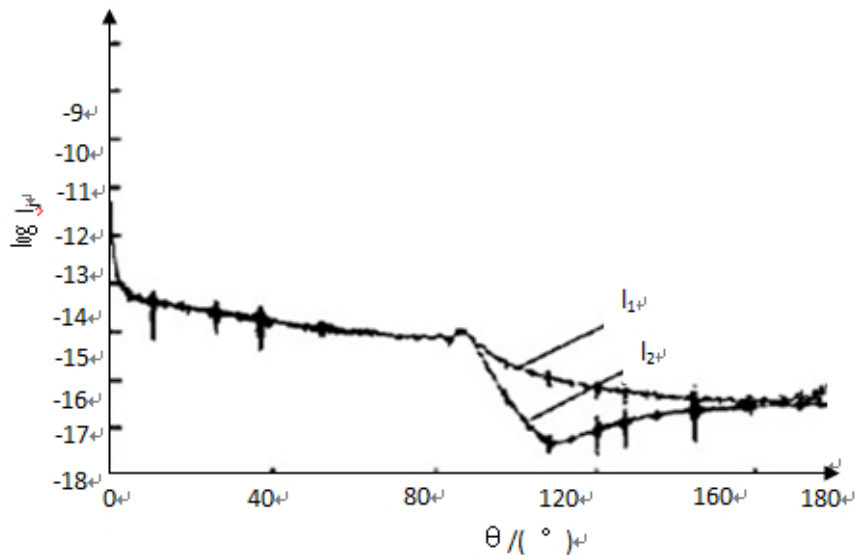


Figure 7: The wake bubble scattering when $t = 3$ min, $z = 6.5$ m

From the scattering calculation of single bubble, the backward scattering of bubbles in water is obvious enhancing at about 180° , the backward scattering enhancement of bubble curtains is more obvious, the closer to the center of wake, the stronger the ship wake bubbles to light scattering intensity, which means it is feasible to use light scattering characteristics to detect and identify the wake.

The scattering characteristics of wake bubbles on light is not only studying single bubbles, what is more, to master the light scattering characteristics of bubble curtains. To study the light scattering characteristics of bubble curtains, the first problem that needs solving is bubble size, classification, distribution law of number density, bubble rising law and the bubble attenuation law. Ocean has complex and changeful environment, in addition to a large number of bubbles in wake, there will be all kinds of bubbles in the surrounding background, such as plankton, dust,

and the bubbles produced by all kinds of perturbation (including atmospheric flow, all kinds of vibration, acoustic, turbulence, etc.). The important condition of studying light scattering is to carry on the classification and identification of different bubbles, analyze bubble size, the law of movement and number density. In addition, the bubble deformation, division, combination and attenuation of bubbles should be fully considered. Therefore, establishing a reasonable mathematical model, on the basis of the Mie scattering theory to study and analyze, the accurate light scattering characteristics of wake bubbles could be obtained, and a more perfect theoretical system could be formed.

4. The establishment of bubble curtain Monte Carlo model

On real environment, there is not only a bubble in water, but also a large number of bubbles constructing bubble curtains. Currently, there is no complete and precise theoretical system to use into the research of bubble curtains, most laboratories and researchers used single scattering approximation method based on Mie scattering theory of single bubble. Although this method is simple, the scattered light influence of twice or more than twice is not taken into consideration, single bubble independent scattering is regarded as the premise, and the particle albedo is limited by the optical thickness. Monte Carlo simulation method adopts probability statistics principle and random numbers, considering the multiple scattering situation, which is closer to the actual situation. Therefore, the results could reflect better the real characteristics of light scattering.

4.1 The description of Monte Carlo method

The process of Monte Carlo simulating photon movement is the transmission process that bubble curtains infinitely narrow photon beam after the incidence into water, tracking each photon's track in the water, establishing a random output value when encountering a bubble, and the output value of this time is regarded as the input of next encounter, in turn, until the final output state.

The calculation process is as follows

The first step: Establish the coordinate, according to the incident conditions to determine the initial track position.

The second step: Determine the photons travel direction and the position of next collision at random.

The third step: Analyze the absorption and scattering distribution of photons at the determined location

The fourth step: Back to the second step, carry on the loop calculation until the photon survival probability decreases to a certain value or disappears leaving its tracking range, the details are shown as follows:

Return to the first step to record another photon, and track all the photons that are set

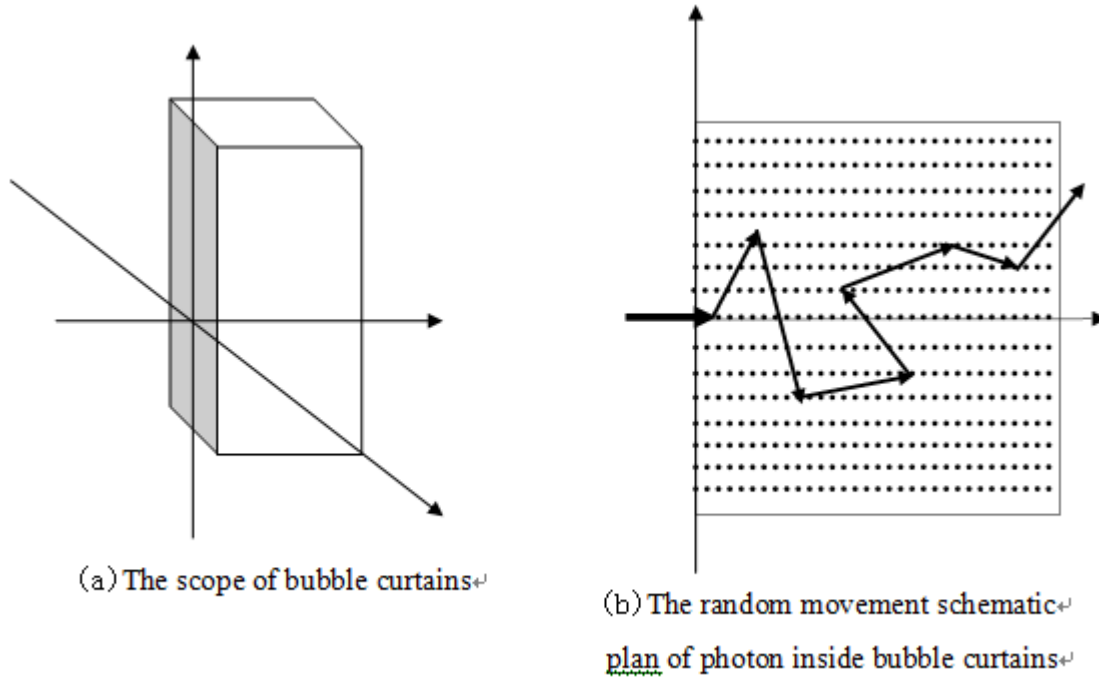


Figure 8: The sporting schematic plan of bubble curtains and photons at bubble curtains

4.2 Determine the step size of photon advance.

The survival probability of a photon passing the scattering at location L is:

$$W = \exp(-\rho\sigma_s L) \tag{5}$$

σ_s is the scattering cross section of single bubble, thus we could get, the sampling function of photons between two bubbles whose collision distance is L is:

$$L = -\ln \eta / (\rho\sigma_s) \tag{6}$$

η is the random number between (0,1), considering the collision bubble's absorption to light, calculate the step, we could get:

$$L = -\ln \eta / (\rho\sigma_t) \tag{7}$$

And $\sigma_t = \sigma_s + \sigma_a$ is the total scattering cross section of bubbles, σ_a is the absorption cross section of bubbles. Therefore, the location of m+1th collision point could be expressed as follows:

$$\begin{aligned}x_{m+1} &= x_m + U_{m+1}L \\y_{m+1} &= y_m + V_{m+1}L \\z_{m+1} &= z_m + W_{m+1}L\end{aligned}\tag{8}$$

In the formula, $(U_{m+1}, V_{m+1}, W_{m+1})$ is the direction cosine of photonic movement.

4.3 Determine the scattering direction of photon.

Photon scattering direction would change due to the collision. If the scattering angle is θ_m after the photon happens collision m times, azimuth angle is φ_m , through coordinate transformation, the new direction cosine after photon collision could be obtained as follows:

$$\begin{aligned}U_{m+1} &= \frac{\sin \theta_m}{\sqrt{1-W_m^2}}(U_m W_m \cos \varphi_m - V_m \sin \varphi_m) + U_m \cos \theta_m \\V_{m+1} &= \frac{\sin \theta_m}{\sqrt{1-W_m^2}}(V_m W_m \cos \varphi_m - U_m \sin \varphi_m) + V_m \cos \theta_m \\W_{m+1} &= -\sin \theta_m \cos \theta_m \sqrt{1-W_m^2} + W_m \cos \theta_m\end{aligned}\tag{9}$$

If the movement direction of the photon is close to the axis z, the new direction cosine is:

$$\begin{aligned}U_{m+1} &= \sin \theta_m \cos \varphi_m \\V_{m+1} &= \sin \theta_m \sin \varphi_m \\W_{m+1} &= W_m \cos \theta_m\end{aligned}\tag{10}$$

It is generally recognized that corner φ_m is evenly distributed among $(0, 2\pi)$.

After collision, the direction distribution of space scattering angle satisfies Phase function approximate formula of Henye-Greestein.

$$B(\mu) = (\sigma_s / \sigma_t)(1 - \mu^2)(1 + \mu^2 - 2g\mu)^{\frac{3}{2}}\tag{11}$$

And $\mu = \cos \theta$, $g = \bar{\mu}$ is the average value of μ , becomes the unsymmetrical factor of light scattering, when $0 \leq g \leq 1$, $g=0$, it is indicated the scattering the medium to photons is isotropic, $g=1$ indicates forward scattering, g could be calculated by the following formula:

$$g = \langle \cos \theta \rangle = \frac{4\pi R^2}{\alpha^2 \sigma_s^2} \left[\sum_n \frac{n(n+2)}{n+1} \operatorname{Re}\{a_n a_{n+1}^* + b_n b_{n+1}^*\} + \sum_n \frac{2n+1}{n(n+1)} \operatorname{Re}\{a_n b_n^*\} \right]$$

(12)

The scattering cross section of the bubble is:

$$\sigma_s = \frac{2\pi}{k^2} \sum_{n=1}^{\infty} (2n+1)(|a_n|^2 + |b_n|^2), \text{ and } k = \frac{2\pi}{\lambda} \text{ is the wave number.}$$

(13)

From formula (11), the sampling function of μ could be obtained as follows:

$$\mu = \begin{cases} \left[(1+g^2) - (1-g^2)(1-g-2g\xi)^{-2} \right] / 2g & g \neq 0 \\ \mu = 2\xi - 1 & g = 0 \end{cases}$$

(14)

In the formula, ξ is the random number between (0, 1), from $\mu = \cos \theta$ we could get scattering angle θ .

If the times $m+1$ of a photon collision is out of bubble groups, these bubbles are out of the statistical scope, and $(U_{m+1}, V_{m+1}, W_{m+1})$ is the emergent direction of photons.

The process above is simulating angular distribution of bubble scattering intensity. The scattering angle of 0~180° is divided into 181 statistical intervals, count the photons falling into each interval, and then the corresponding estimated value could be obtained. All the values divided the total incident photon number, the relative intensity distribution could be obtained as follows:

$$P = \frac{I_i}{I_0} = \frac{N_i}{\sum_{i=0}^{180} N_i}$$

(15)

N_i indicates the photon number falling into the i th interval. Figure 9 is the normalized intensity distribution of bubbles with different radius. From figure 9 we could get the bubbles with the radius of several decade microns have obvious backscatter. While the bubble size being reduced to several microns, the backscatter changes weak and disappears gradually. When the refractive index of medium is $n(=1.33, 1.4, 1.6)$, the normalized intensity distribution of backscatter is shown in figure 10. It is analyzed that the bigger the refractive index is, the stronger the backscatter is. Actually, the refractive index of fresh water is much smaller than the refractive index of sea water. Therefore, it is easier to use light to detect wake bubbles in seawater.

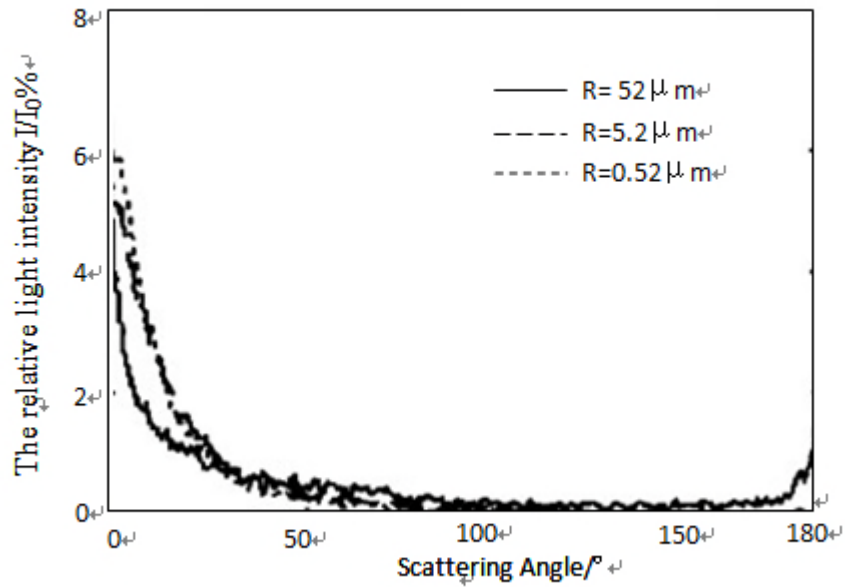


Figure 9: The normalization light intensity distribution while the semidiameter of the bubble is different

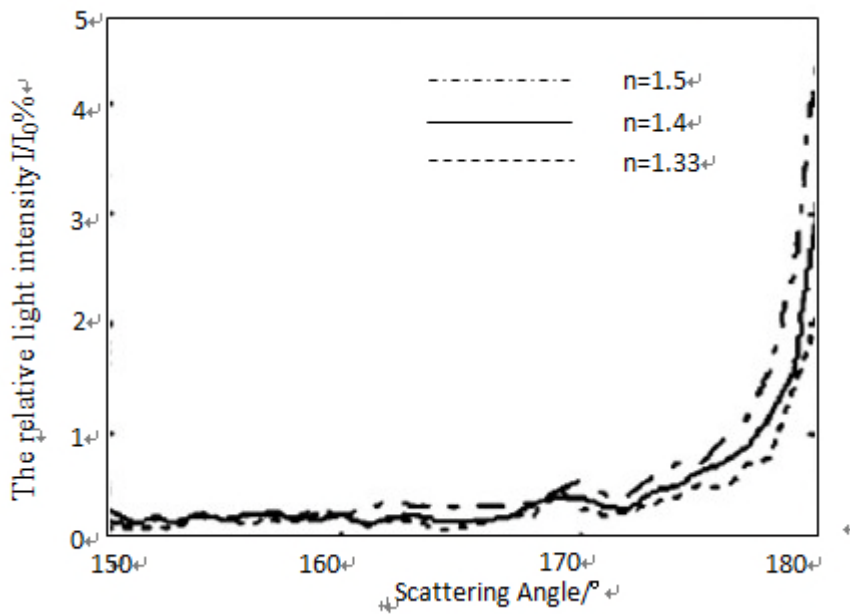


Figure 10: The normalization light intensity distribution while water refractive index is different

5. Results and discussion

(1) First of all, Mie scattering theory of single bubbles is introduced. Based on Mie scattering theory, the bubbles of different size's scattering intensity in different angle is studied.

The light scattering characteristics of bubble curtain on wake cross section are analyzed. It is indicated that the backscattering of single bubble becomes obvious reinforcing at about 180° , the closer to the center of wake, the stronger the ship wake bubbles to light scattering intensity. Combined with the image to analyze and discuss, the experimental value is in good accordance with the theoretical value, which means it is feasible to use light scattering characteristics to detect and identify the wake.

(2) Through establishing the Monte Carlo model to carry on the light scattering simulation calculation of bubble curtains, it proved it is a more practical calculating method to use the Monte Carlo model to study the bubble curtain light scattering. Scattering intensity mainly concentrated on the front part. The research shows that the increase of water refractive index would increase light backscatter ratio to some extent.

6. Conclusion

In this paper, the movement of ships in water and the cavitation of propeller would bring air into water and form bubble curtain of a lot of bubbles. The sporting bubbles form the wake. The optical characteristics of wake bubbles are analyzed in this paper, especially the light scattering characteristics of bubbles, which provides a more practical calculation method for the light detection of wake.

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