

## **Research on Cooperation Innovation among Enterprises in the Strategic Emerging Industrial Cluster**

\*Li Xing

\*Department of management science, Management science and engineering, Guizhou University of finance and economics, China, Huaxi university town, Huaxi District, Guiyang City(netlixing2008@126.com)

### **Abstract**

Based on the game theory and complex science method, we analyzed the cooperation innovation among enterprises in the strategic emerging industry cluster. Firstly, we analyzed the decision-making process of cooperative innovation among enterprises. Secondly, we analyzed the promoting mechanism of cooperative innovation among enterprises from the aspect of overall rationality and individual rationality. And we concluded that In a dynamic cooperation alliance in the industrial cluster, when each cluster enterprise participating in cooperation take the optimal strategy according to the current time and corresponding conditions, the alliance's overall profit value with time is changing constantly, and the change degree of profit in each instant depends on two aspects: one is the discounted value of instant pay of the cluster alliance at each time.

**Keywords:** Game theory, strategic emerging industrial cluster, cooperation innovation, overall rationality, individual rationality

### **1. Introduction**

In the post-crisis era, with the rise of a new round of revolution of science and technology in the world, it has become an important issue of common concern from all walks of life how to play the "engine" effect and "leverage" effect of the strategic emerging industries, and cultivating and developing its unique competitive advantage. Fostering the strategic emerging industry cluster is an important way to promote the spanning development of regional economic and transformation. And the formation of collaborative innovation network has great significance for

acquiring the innovation resources, and sharing the achievements, and improving the efficiency of innovation, and it is also one of the important organizational forms to promote the continuous development of the strategic emerging industries. Therefore, it has important research significance to advance the continuous accumulation and dynamic evolution of the synergy innovation network capability of the strategic emerging industries.

Research on the strategic emerging industries, the scholars in our country mainly make a research from the aspects of development situation and countermeasure, the selection and evaluation of the industry, the measurement of innovation efficiency, and collaborative innovation and so on. Recent representative scholars and their views are as follows: Xing-zhi Xiao (2010) analyzed the influence factors and the development way of strategic emerging industries, as well as the growth of the strategic emerging industry and low carbon industry based on the realization form and development model of the of industrial upgrading. Zhi-yang Liu (2010) pointed out the factors affecting the leading design of the emerging industry based on the design principles of strategic emerging industries, which are the governance capability of alliance, financial capital, the base of initial user, the superiority of system technology, scalability and the function of the government. Xing-zhi Xiao, Xie Li (2011) used SFA model to estimate the innovation efficiency of China's strategic emerging industries, and then analyzed the influence path of the enterprise scale and the different ways on the industrial innovation efficiency through the model of Tobit, and the influence degree of different property right structure on the innovation efficiency. Guo Juan (2011) discussed the endogenous mechanism the growth of strategic emerging industries from the Angle of independent innovation. Xiaonan Qiao, Hong-sheng Li(2011) made an in-depth analysis of the growth mechanism and the inherent law of China's strategic emerging industries. Suqin Wu (2012) analyzed the influence of energy consumption on the strategic emerging industries in Hunan province, the influence of different energy consumption influence on strategic emerging industries in Hunan province, and the relationship between the strategic emerging industries and industrial waste emissions in Hunan province. Deng-ke Yu, Guoping Tu and Chen Hua (2012) put forward the single-core, multi-core and star pattern of coordinated development of industry cluster, and the strategy of promoting the coordinated development of strategic emerging industry cluster. Yiwen Yang (2012) analyzed the factors affecting the development of strategic emerging industry including the market power of the distribution innovative of product, operational efficiency, and the preferences degree of user or consumer for innovative products. Guiquan Zhang (2012) measured the competitiveness of the strategic emerging industry of China's three major economic circles since the new century by

using the method of principal of component analysis. Xiuting Wang (2012) established the collaborative innovation network model of the strategic emerging industry based on the perspective of system science, and analyzed the four abilities of the synergy innovation network of strategic emerging industries and its dynamic evolution direction and strategy based on life cycle phase. Li Yuan (2013) analyzed market mechanism and double driving mechanism of the government of the growth of strategic emerging industries in China, and measured the innovation efficiency of China's strategic emerging industries. Ruijuan Wang (2013) analyzed the development orientation of the strategic emerging industry, and put forward the suggestion of realize the coordinated development mechanism of the strategic emerging industry.

To sum up, the current domestic academic research related to strategic emerging industry is still at the stage of exploration and demonstration stage, and the research literature about the development problems of the strategic emerging industry from the perspective of cluster is very few. In addition, although some scholars analyzed the collaborative innovation of strategic emerging industry, there is no in-depth analysis about the conditions of ensuring the success of collaborative innovation, as well as the benefit allocation problem of among the synergy innovation subjects. Based on this, we use the game theory and complex science method to analyze the cooperation innovation mechanism and benefit distribution mechanism among enterprises in the strategic emerging industry cluster in this paper, and the research achievements will have a certain theoretical guidance and reference to the development of strategic emerging industry in China.

## **2. Analysis of the decision-making process of cooperative innovation**

### **2.1 Motivation analysis of collaborative innovation**

Motivation of cooperative innovation among enterprises in the strategic emerging industry cluster of mainly lies in the following three aspects:

(1) The imperfection of the cluster prompts the collaborative innovation among enterprises in the cluster, and then promoting the sustainable development of the cluster. With the strengthening of global competition, it not only requires the frequent interaction among enterprises in the cluster to achieve resource sharing and mutual learning, but also needs to the multi-dimensional and multi-level links constantly with the enterprises and institutions outside of the cluster, so that they can open up new markets, and expand the space of cluster innovation in order to obtain the complementary resource. Therefore, it is only through interaction and cooperation among the cluster enterprises to promote the sustainable development of industrial cluster more effectively.

(2) It can create resources within a cluster, and own the rights of using the resources within the cluster to enhance its own competitive advantage through the cooperative innovation among enterprises in the cluster. We know that the innovation of emerging enterprises needs a variety of resources, and purchasing these resources needs to pay a larger cost, but we can get the resources through collaborative innovation. Cluster has the structural integration ability of resources, and this ability is the main source of the cluster innovation as well as gaining competitive advantage. The cluster innovation and competitive advantage is derived from resources endowment and the resource integration ability of cluster. The enterprise must rely on the resources to engage in the cluster innovation activities and acquire the innovation achievement. Specialization by cluster can make up for the lack of its own resources, and can play their strengths, as well as focusing on their core expertise, and eventually it can enhance the technology innovation ability of enterprises in the cluster.

(3) A cluster is a kind of club product, cooperation among the enterprises in the cluster can create a new club. From a certain perspective, the cluster is equivalent to a set of organization or club to converge the resource, cluster enterprises gathered for some purpose. Cluster can provide enterprises with a variety of products and services such as resource not only conform to the principle of benefit, and the efficiency of using the resource is higher, and it is hard to get the resources outside of the cluster, or to get with a relatively high cost, so from this perspective, the cluster improved the welfare of the enterprise. Cooperation innovation among enterprises in the cluster can create a new club.

## **2.2 Choice of the innovation ways of cluster enterprise**

In the any stage of the development of cluster enterprise, no matter the independent innovation or the imitation innovation for the cluster enterprise, it all needs the corresponding resources of the human, material and financial to support, and the capital investment depends on the above stage of cluster enterprise income and its own capital. Here, we assume that the income gained by the cluster enterprise  $i$  is  $\pi_i(t-1)$  at time  $t-1$ , and he owns the capital (human, technology, etc.) is  $G_i$ , and the cluster enterprise  $i$  needs to purchase a capital  $C_i(t-1)$  in order to the innovation behavior at the stage of  $t$  at the same time. If  $\pi_i(t-1) > 0$ , the cluster enterprise  $i$  in phase will input  $a_i\pi_i(t-1)$  as a fund  $H_i$  of research and development at the stage of  $t$ , and the fund of research and development of cluster enterprise can be used in both fund of fundamental innovation  $H_i^1$  and the fund of imitation innovation  $H_i^2$  of the enterprise  $i$ , and the R&D strategy

of cluster enterprise  $i$  at the stage of  $t$  depends on the partition coefficient  $\lambda_i$  of the fundamental innovation fund and imitative innovation fund, therefore,

$$H_i^1(t) = \lambda_i H_i(t) = \lambda_i a_i \pi_i(t-1) \quad H_i^2(t) = (1 - \lambda_i) a_i \pi_i(t-1) \quad (1)$$

Obviously, the R & D strategy is also changing constantly at the different stages of development of cluster enterprise. If we assume that the global market share of the cluster enterprises' products is increasing through radical innovation, then, so more R&D funds will be allocated to the fundamental innovation of the cluster enterprise, otherwise, so more R&D funds will be allocated to the imitation innovation of the cluster enterprise. Therefore, the change of the partition coefficient depends on the product's market share, namely

$$\lambda_i(t+1) = \frac{\eta_i(t) - \eta_i(t-1)}{\eta_i(t-1)} \lambda_i(t) \quad (2)$$

At the same time, we assume that  $IN_i(t)$  and  $IM_i(t)$  represent the expenses for radical innovation and imitation innovation of cluster enterprise  $i$  respectively at the stage of  $t$ , therefore, if  $\pi_i(t-1) > 0$ , then:

$$\begin{aligned} IN_i(t) &= IN_i(t-1) + \lambda_i a_i \pi_i(t-1) \\ IM_i(t) &= IM_i(t-1) + (1 - \lambda_i) a_i \pi_i(t-1) \\ C_i(t) &= C_i(t-1)(1 - \varphi) + (1 - a_i) \pi_i(t-1) \end{aligned} \quad (3)$$

Among them,  $\varphi$  represent the depreciation rate of capital.

Clearly, both fundamental innovation and imitation innovation all need cost, and the larger the company size is, the greater the cost is for innovation. For the cluster enterprise  $i$ , if the cost  $IN_i(t)$  used to fundamental innovation exceeds  $R(C_i(t))$ , that is  $IN_i(t) \geq R(C_i(t))$ , then he will have a chance to get a new capital  $G_i'$ , and the cost  $IM_i(t)$  used to imitation innovation exceeds  $L(C_i(t))$ , that is  $IM_i(t) \geq L(C_i(t))$ , then he will have the opportunity to copy the optimal technology its adjacency enterprise. Therefore, the decision-making mechanism of cluster enterprise  $i$  selecting the radical innovation or imitation innovation is as follows:

$$\begin{aligned} &\text{If } IN_i(t) \geq R(C_i(t)), \text{ then} \\ IN_i(t+1) &= IN_i(t) - R_i(C(t)) \\ G_i(t+1) &= \max \{G_i(t), G_i'(t)\} \\ &\text{If } IM_i(t) \geq L(C_i(t)), \text{ then} \end{aligned} \quad (4)$$

$$\begin{aligned}
G_i(t+1) &= \max\{G_i(t), \max G'_i(t)\} \\
IM_i(t+1) &= IM_i(t) - L_i(C(t))
\end{aligned} \tag{5}$$

### 3. Analysis of promote mechanism of cooperative innovation

According to Professor Yang Rongji and Petrossian's research, a dynamic cooperation innovation model which is consist of enterprises whose amount is  $n$  in the industrial clusters was built in this paper. Supposed that the start time and the end time of the game is  $t_0$  and  $T$ , and the initial state of the game is  $x_0$ , the state space of the game is  $X \in R^m$ ,  $x(t)$  represents the state variable, which allows the state of the track is  $\{x(t), t_0 \leq t \leq T\}$  and  $x_i(t) \in X_i \in R^m$  represents the state variable of company  $i$  at the point time of  $t$ , and this state variable is changing with time.  $s_i \in S$  represents the control of company  $i$ , because it represents a strategic approach to progress along with time, while  $s_i(t)$  indicates the control of enterprise at the time of  $t$ .

In addition, according to the assumption 2, given a discount rate  $r(t)$ , for  $t \in [t_0, T]$ , the reward of every enterprise participating in the cooperation innovation at the time  $t$  after the time  $t_0$  will need to be discounted according to the discount factor  $\exp[-\int_{t_0}^t r(\theta)d\theta]$ . At the time of  $t$ , assuming that the enterprise  $i$  will receive instant reward  $f^i[t, x_i(t), s_i(t)]$  from the investment profit, profit and tax, and at the end time of  $T$ , assumed that company  $i$  will get the final reward  $p^i(x_i(T))$  which represents the present value of the future potential net income calculated according to the various aspects of the situation and the economic potential of the company at the end time of  $T$ . Finally, we use  $x_N(t) = [x_1(t), x_2(t), \dots, x_n(t)]$  indicates the state variables of all enterprises at the time of  $t$  and use  $x_N^0 = [x_1^0, x_2^0, \dots, x_n^0]$  indicates the initial state of all enterprises.

Based on the above analysis, we can conclude that every enterprise's reward includes two aspects: one is instant reward obtained by each at the time of  $t$ ; another is the end reward received at the end of innovation, that is:

$$\begin{aligned}
R_i[x_i(t), s_i(t)] &= \int_{t_0}^T f^i[t, x_i(t), s_i(t)] \exp[-\int_{t_0}^t r(\theta)d\theta] dt + p^i(x_i(T)) \exp[-\int_{t_0}^T r(\theta)d\theta] \\
f^i(\bullet) &\geq 0, p^i(\bullet) \geq 0 \quad i \in [1, 2, \dots, n]
\end{aligned} \tag{6}$$

And the change of the dynamic game status depends on:

$$\frac{dx_i}{dt} = g^i[t, x_i(t), s_i(t)], x_i(t_0) = x_i^0, i \in [1, 2, \dots, n] \quad (7)$$

In such a dynamic cooperative innovation game, we should not only guarantee the collective rationality of the cooperation alliance, but also should meet the individual rationality of every enterprise in the cooperation alliance if we want to make the cooperation to be successful. Collective rationality required that cooperation strategies formulated by participants can lead to the state of Pareto optimal, and individual rationality required that the individual rational of participants must be maintained along the optimal trajectory of the game all the time.

Here, we make a simple definition on overall rationality and individual rationality as follows:

**Definition 1:** in a dynamic alliance game with payment can be transferred in the cluster, if and only if the sum total of the reward of the cluster enterprise of each enterprise engaging in cooperation is equal to the value of the total cooperation alliance, we can call it overall rationality or overall optimum.

**Definition 2:** in a dynamic alliance game with payment can be transferred in the cluster, if and only if the sum total of the reward of the cluster enterprise of each enterprise engaging in cooperation is higher than alone innovation gains, we call it individual rationality, because the compensation of the rational enterprise engaging in cooperation in a cooperation scheme must be not less than their own ensured profits.

### 3.1 Meeting conditions of overall rationality

According to the definition of the overall rationality, we assumed that there were  $K_2 \in N$  cluster enterprises to form a coalition. Therefore, these enterprises can produce synergistic effect in such aspects as capital, technology, resources and talents to. Below, we analyze the payment of the cluster enterprise alliance, and in order to meet the requirements of the overall rationality, we must realize the maximization of the overall yield of (6), which satisfies the following conditions:

$$\begin{aligned} \max \sum_{i \in k_2} R_i[x_i(t), s_i(t)] &= \max \left\{ \int_{t_0}^T \sum_{i=1}^{K_2} f^i(t, x_i(t), s_i(t)) \exp\left[-\int_{t_0}^t r(\theta) d\theta\right] dt \right. \\ &+ \left. \sum_{i=1}^{K_2} p^i(x_i(T)) \exp\left[-\int_{t_0}^T r(\theta) d\theta\right] \right\} \end{aligned} \quad (8)$$

And it subject to the following dynamic systems:

$$\frac{dx_{K_2}(t)}{dt} = g^{K_2}[t, x_{K_2}(t), s_{K_2}(t)] \quad (9)$$

For solving the above optimization model, we mainly refer to the bellman dynamic programming, and then we can get:

**Theorem1.** if the optimal control set  $\{s_{K_2}^*(t, x_{K_2}(t)) = \phi_{K_2}^{(t_0)*}(t, x_{K_2}(t))\}$  and  $\phi_{K_2}^{(t_0)*}(t, x_{K_2}(t)) = [\phi_1^{(t_0)*}(t, x_1(t)), \phi_2^{(t_0)*}(t, x_2(t)), \dots, \phi_{k_2}^{(t_0)*}(t, x_{k_2}(t))]$  is the optimal solution of model (8), if and only if there exists continuous differentiable function  $V^{(t_0)}(t, x(t))$  to meet the following bellman equation in the  $[t_0, T]$ :

$$-V_t^{(t_0)}(t, x(t)) = \max \left\{ \sum_{i=1}^{K_2} f^i[t, x_i(t), s_i(t)] \exp\left[-\int_{t_0}^t r(\theta) d\theta\right] + V_x^{(t_0)} \sum_{i=1}^{K_2} g^i[t, x_i(t), s_i(t)] \right\} \quad (10)$$

Satisfying the boundary conditions:

$$V^{(t_0)}(T, x(T)) = \sum_{i=1}^{K_2} p^i(x_i(T)) \exp\left[-\int_0^T r(\theta) d\theta\right] \quad (11)$$

Among them,  $V^{(t_0)}(t, x(t))$  presents the value function of the cooperation as a whole at time  $t$  and the status is  $x(t)$ .

Proof. At time  $t$  and the status is  $x(t)$ , the optimal value function of overall cooperation can be expressed as:

$$\begin{aligned} V_t^{(t_0)}(t, x(t)) &= \max \left\{ \int_t^T \sum_{i=1}^{K_2} f^i[t, x_i(t), s_i(t)] dt \exp\left[-\int_{t_0}^t r(\theta) d\theta\right] + \sum_{i=1}^{K_2} p^i(x_i(T)) \exp\left[-\int_{t_0}^T r(\theta) d\theta\right] \right. \\ &= \int_t^T \sum_{i=1}^{K_2} f^i[t, x_i^*(t), \phi_i^*(t, x_i^*(t))] dt \exp\left[-\int_{t_0}^t r(\theta) d\theta\right] + \sum_{i=1}^{K_2} p^i(x_i^*(T)) \exp\left[-\int_{t_0}^T r(\theta) d\theta\right] \end{aligned} \quad (12)$$

Satisfying the boundary conditions:

$$V^{(t_0)}(T, x^*(T)) = \sum_{i=1}^{K_2} p^i(x_i(T)) \exp\left[-\int_{t_0}^T r(\theta) d\theta\right]$$

$$\text{And } \frac{dx_i}{dt} = g^i[t, x_i^*(t), \phi_i^*(t, x_i^*(t))]$$

We assume that there exist  $s(t) \in S$  in addition to the optimal control set  $\{s_{K_2}^*(t, x_{K_2}(t)) = \phi_{K_2}^{(t_0)*}(t, x_{K_2}(t))\}$ , and its corresponding state trajectory is  $x(t)$ , so the type (10) implies that the following mathematical relationship is true:

$$\sum_{i=1}^{K_2} f^i[t, x_i(t), s_i(t)] \exp\left[-\int_{t_0}^t r(\theta) d\theta\right] + V_x^{(t_0)}(t, x(t)) \sum_{i=1}^{K_2} g^i[t, x_i(t), s_i(t)] + V_x^{(t_0)}(t, x(t)) \leq 0 \quad (13)$$

$$\sum_{i=1}^{K_2} f^i[t, x_i^*(t), \phi_i^*(t, x_i^*(t))] \exp[-\int_{t_0}^t r(\theta) d\theta] + V_{x^*}^{(t_0)}(t, x_i^*(t)) \sum_{i=1}^{K_2} g^i[t, x_i^*(t), \phi_i^*(t, x_i^*(t))] \quad (14)$$

$$+ V_t^{(t_0)}(t, x^*(t)) = 0$$

Extending the type (13) and type (14) to the range  $[t_0, T]$ , and we can get:

$$\int_{t_0}^T \sum_{i=1}^{K_2} f^i(t, x_i(t), s_i(t)) dt \exp[-\int_{t_0}^t r(\theta) d\theta] + V(T, x(T)) - V(t_0, x(t_0))$$

$$= \int_{t_0}^T \sum_{i=1}^{K_2} f^i(t, x_i(t), s_i(t)) dt \exp[-\int_{t_0}^t r(\theta) d\theta] + \sum_{i=1}^{K_2} p^i(x_i(T)) \exp[-\int_{t_0}^T r(\theta) d\theta] - V(t_0, x(t_0)) \leq 0$$

$$\int_{t_0}^T \sum_{i=1}^{K_2} f^i(t, x_i^*(t), \phi_i^*(t, x_i^*(t))) dt \exp[-\int_{t_0}^t r(\theta) d\theta] + V(T, x^*(T)) - V(t_0, x^*(t_0))$$

$$= \int_{t_0}^T \sum_{i=1}^{K_2} f^i(t, x_i^*(t), \phi_i^*(t, x_i^*(t))) dt \exp[-\int_{t_0}^t r(\theta) d\theta] + \sum_{i=1}^{K_2} p^i(x_i^*(T)) \exp[-\int_{t_0}^T r(\theta) d\theta] - V(t_0, x(t_0)) = 0$$

According to the above type, we can get the following mathematical relationship:

$$\int_{t_0}^T \sum_{i=1}^{K_2} f^i(t, x_i(t), s_i(t)) dt \exp[-\int_{t_0}^t r(\theta) d\theta] + \sum_{i=1}^{K_2} p^i(x_i(T)) \exp[-\int_{t_0}^T r(\theta) d\theta]$$

$$\leq \int_{t_0}^T \sum_{i=1}^{K_2} f^i(t, x_i^*(t), \phi_i^*(t, x_i^*(t))) dt \exp[-\int_{t_0}^t r(\theta) d\theta] + \sum_{i=1}^{K_2} p^i(x_i^*(T)) \exp[-\int_{t_0}^T r(\theta) d\theta]$$

Therefore, the optimal control collection  $\{s_{K_2}^*(t, x_{K_2}(t)) = \phi_{K_2}^{(t_0)*}(t, x_{K_2}(t))\}$  and  $\phi_{K_2}^{(t_0)*}(t, x_{K_2}(t)) = [\phi_1^{(t_0)*}(t, x_1(t)), \phi_2^{(t_0)*}(t, x_2(t)), \dots, \phi_{k_2}^{(t_0)*}(t, x_{k_2}(t))]$  is an optimal solution of the cooperative game.

We can see from the theorem 1:

In a dynamic cooperation alliance in the industrial cluster, when each cluster enterprise participating in cooperation take the optimal strategy according to the current time and corresponding conditions, the alliance's overall profit value with time is changing constantly, and the change degree of profit in each instant depends on two aspects: one is the discounted value of instant pay of the cluster alliance at each time; Secondly, the optimal change of all enterprises' state in the cluster alliance brought the changes of profit value of the whole cluster.

**Theorem2.** For any  $\xi$ , satisfied  $t_0 \leq \xi \leq t \leq T$ , and the following mathematical relationship is established:

$$V^{t_0}(t, x^*(t)) = V^\xi(t, x^*(t)) \exp[-\int_\xi^t r(\theta) d\theta]$$

Proof. According to the proof process of theorem 1, for the optimal control problem of cooperation at time  $t \in [\xi, T]$ , we can get the following bellman equation:

$$-V_t^{(\xi)}(x, x(t)) = \max \left\{ \sum_{i=1}^{K_2} f^i [t, x_i(t), s_i(t)] \exp \left[ -\int_{\xi}^t r(\theta) d\theta \right] + V_x^{(\xi)} \sum_{i=1}^{K_2} g^i [t, x_i(t), s_i(t)] \right\} \quad (15)$$

Satisfying the boundary conditions:

$$V^{(\xi)}(T, x(T)) = \sum_{i=1}^{K_2} p^i(x_i(T)) \exp \left[ -\int_{\xi}^T r(\theta) d\theta \right] \quad (16)$$

Comparing the type (10) and type (15), it is easy to find that the following mathematical relationship is established:

$$\text{For } t_0 \leq \xi \leq t \leq T, \phi_i^{(\tau)*}(t, x_i^*(t)) = \phi_i^{(t_0)*}(t, x_i^*(t))$$

Therefore, according to the above result:

$$\begin{aligned} V_t^{(t_0)}(t, x^*(t)) &= \left\{ \int_t^T \sum_{i=1}^{K_2} f^i [t, x_i^*(t), \phi_i^*(t, x_i^*(t))] dt \exp \left[ -\int_{t_0}^t r(\theta) d\theta \right] \right. \\ &\quad \left. + \sum_{i=1}^{K_2} p^i(x_i^*(T)) \exp \left[ -\int_{t_0}^T r(\theta) d\theta \right] \right\} \\ &= \left\{ \int_t^T \sum_{i=1}^{K_2} f^i [t, x_i^*(t), \phi_i^*(t, x_i^*(t))] dt \exp \left[ -\int_{\xi}^t r(\theta) d\theta \right] + \sum_{i=1}^{K_2} p^i(x_i^*(T)) \exp \left[ -\int_{\xi}^T r(\theta) d\theta \right] \right\} \exp \left[ -\int_{t_0}^{\xi} r(\theta) d\theta \right] \\ &= V^{(\xi)}(t, x^*(t)) \exp \left[ -\int_{t_0}^{\xi} r(\theta) d\theta \right] \end{aligned}$$

We can see from the theorem2:

In a dynamic cooperation alliance in the industrial cluster, under the cases of different initial time, the cluster enterprise participating in the cooperation take the same cooperation strategy at the same time and condition and the discount value of alliance overall profit is equal.

### 3.2 Meeting conditions of individual rationality

Here, we assume that each enterprise in the cluster cooperative innovation at initial time agrees a particular cooperation principle. Therefore, we assume that the pay distribution vector of each cooperative enterprise under the optimal consensus principle is:

$$\eta_N(x_N^0, T - t_0) = [\eta^1(x_N^0, T - t_0), \eta^2(x_N^0, T - t_0), \dots, \eta^n(x_N^0, T - t_0)]$$

That is to say, every enterprise in the cluster cooperation innovation at time  $[t_0, T]$  agreed to receive the rewards  $\eta^i(t_0, T - t_0)$ .

According to the definition of individual rationality, the compensation of rational participants of cluster cooperative innovation in a collaborative scheme must not lower than the income of

their own innovation. Therefore, at time  $t_0$ , and the initial state is  $x_N^0$ , the following conditions must be meet:

$$\eta^i(x_N^0, T-t_0) \geq \delta^i(x_N^0, T-t_0), i \in \{1, 2, \dots, n\}$$

Among them,  $\eta^i(x_N^0, T-t_0)$  represents the compensation of cluster enterprise  $i$  participating in cooperation innovation, and  $\delta^i(x_N^0, T-t_0)$  represents the income of innovation alone of cluster enterprise  $i$ .

About solving  $\eta^i(x_N^0, T-t_0)$ , we can calculate the payment of cluster enterprise  $i$  at time  $t_0$  and the initial state is  $x_N^0$  according to the shapely theorem:

$$\eta^{(t_0)i}(x_N^0, T-t_0) = \sum_{K_2 \in N} \frac{(k_2-1)!(n-k_2)!}{n!} [V^{(t_0)K_2}(x_{K_2}^0, T-t_0) - V^{(t_0)K_2 \setminus i}(x_{K_2 \setminus i}^0, T-t_0)].$$

Among them,  $K_2 \setminus i$  represents enterprise  $i$  does not belong to the enterprise alliance  $K_2$ ,  $V^{(t_0)K_2}(x_{K_2}^0, T-t_0)$  represents the profit of cooperation alliance,  $[V^{(t_0)K_2}(x_{K_2}^0, T-t_0) - V^{(t_0)K_2 \setminus i}(x_{K_2 \setminus i}^0, T-t_0)]$  represents the marginal contribution of enterprise  $i$  on the alliance  $K_2$  after participating the alliance,  $\frac{(k_2-1)!(n-k_2)!}{n!}$  represents the weighted factor of the union.

According to the content mentioned above, the value function of cluster enterprise engaging in innovation alone is the model (6) and (7), and for the solution of model (6) and (7), we refer to the feedback Nash equilibrium solution and the related theorem of scholars Professor Yang Rongji and Petrossian, then we can get:

**Theorem3.** if the optimal control set  $s_{K_1}^{(t_0)*}(t, x_i(t)) = \delta_{K_1}^{(t_0)*}(t, x_i(t))$  and  $\delta_{K_1}^{(t_0)*}(t, x_{K_1}(t)) = [\delta_1^{(t_0)*}(t, x_1(t)), \delta_2^{(t_0)*}(t, x_2(t)), \dots, \delta_{k_i}^{(t_0)*}(t, x_i(t))]$  is feedback Nash equilibrium solution of a non-cooperative game, If and only if there exists continuous differential function satisfying the following partial differential equation at time  $[t_0, T]$ :

$$\begin{aligned} -U_i^{(t_0)i}(t, x_i(t)) &= \max \{f^i[t, x_i(t), s_i(t), \delta_j^{(t_0)*}(t, x_j(t))] \exp[-\int_{t_0}^t r(\theta) d\theta] \\ &+ U_x^{(t_0)i} g^i[t, x_i(t), s_i(t), \delta_j^{(t_0)*}(t, x_j(t))] \} \end{aligned} \quad (17)$$

Satisfying the boundary conditions:

$$U^{(t_0)i}(T, x(T)) = p^i(x_i(T)) \exp[-\int_{t_0}^T r(\theta) d\theta] \quad (18)$$

Among them,  $i \in K_1$ ,  $U^{(t_0)^i}(t, x_i(t))$  represents the value function of enterprise  $i$ .

Proof. The value function of enterprise  $i$  at time  $t$  and the status is  $x(t)$  can be expressed as:

$$\begin{aligned} U_i^{(t_0)}(t, x(t)) &= \int_t^T f^i[t, x_i(t), s_i(t)] dt \exp[-\int_{t_0}^t r(\theta) d\theta] + p^i(x_i(T)) \exp[-\int_{t_0}^T r(\theta) d\theta] \\ &= \int_t^T f^i[t, x_i^*(t), \delta_i^*(t, x_i^*(t))] dt \exp[-\int_{t_0}^t r(\theta) d\theta] + p^i(x_i^*(T)) \exp[-\int_{t_0}^T r(\theta) d\theta] \end{aligned} \quad (19)$$

Satisfying the boundary conditions:

$$U_i^{(t_0)}(T, x^*(T)) = p^i(x_i(T)) \exp[-\int_{t_0}^T r(\theta) d\theta]$$

$$\text{And } \frac{dx_i}{dt} = g^i[t, x_i^*(t), \delta_i^*(t, x_i^*(t))]$$

We assume that in addition to the optimal control set  $\{s_i^*(t, x_i(t)) = \delta_i^{(t_0)*}(t, x_i(t))\}$ , there exists  $s(t) \in S$ , and its corresponding state trajectory is  $x(t)$ , so the type (14) implies that the following mathematical relationship is true:

$$\begin{aligned} f^i[t, x_i(t), s_i(t)] \exp[-\int_{t_0}^t r(\theta) d\theta] + U_{x_i}^{(t_0)}(t, x(t)) g^i[t, x_i(t), s_i(t)] + U_t^{(t_0)}(t, x(t)) &\leq 0 \\ f^i[t, x_i^*(t), \delta_i^*(t, x_i^*(t))] \exp[-\int_{t_0}^t r(\theta) d\theta] + U_{x_i^*}^{(t_0)}(t, x^*(t)) g^i[t, x_i^*(t), \delta_i^*(t, x_i^*(t))] & \\ + U_t^{(t_0)}(t, x^*(t)) &= 0 \end{aligned} \quad (20)$$

Extending the type [20] to the range  $[t_0, T]$ , and we can get:

$$\begin{aligned} &\int_{t_0}^T f^i(t, x_i(t), s_i(t)) dt \exp[-\int_{t_0}^t r(\theta) d\theta] + U(T, x(T)) - U(t_0, x(t_0)) \\ &= \int_{t_0}^T f^i(t, x_i(t), s_i(t)) dt \exp[-\int_{t_0}^t r(\theta) d\theta] + p^i(x_i(T)) \exp[-\int_{t_0}^T r(\theta) d\theta] - U(t_0, x(t_0)) \leq 0 \\ &\int_{t_0}^T f^i(t, x_i^*(t), \delta_i^*(t, x_i^*(t))) dt \exp[-\int_{t_0}^t r(\theta) d\theta] + U(T, x^*(T)) - U(t_0, x^*(t_0)) \\ &= \int_{t_0}^T f^i(t, x_i^*(t), \delta_i^*(t, x_i^*(t))) dt \exp[-\int_{t_0}^t r(\theta) d\theta] + p^i(x_i^*(T)) \exp[-\int_{t_0}^T r(\theta) d\theta] - U(t_0, x(t_0)) = 0 \end{aligned}$$

So we can get the following mathematical relationship:

$$\begin{aligned} &\int_{t_0}^T f^i(t, x_i(t), s_i(t)) dt \exp[-\int_{t_0}^t r(\theta) d\theta] + p^i(x_i(T)) \exp[-\int_{t_0}^T r(\theta) d\theta] \\ &\leq \int_{t_0}^T f^i(t, x_i^*(t), \delta_i^*(t, x_i^*(t))) dt \exp[-\int_{t_0}^t r(\theta) d\theta] + p^i(x_i^*(T)) \exp[-\int_{t_0}^T r(\theta) d\theta] \end{aligned}$$

As can be seen from the theorem 3:

When the cluster enterprise carries on the innovation alone, and other companies are using the optimal strategy according to the current time and corresponding conditions, the value function of enterprise  $i \in K_1$  is changing constantly with time.

## Discussion

From the analysis process above, we analyzed the decision-making process of cooperative innovation among enterprises. Existing researches show that the main advantage of industrial cluster lies in its innovation effect, it can improve the continuous innovation ability of enterprises in the cluster. In fact, the nature of innovation process is a learning process, and the cluster can urge enterprise innovation. Within the industry cluster, the proximity of geographical position provides a convenient for the formal and informal communication among enterprises, so it can accelerate the exchange of information and the transfer speed of knowledge among businesses, and it can prompt companies to obtain new knowledge and knowledge spillover effect. At the same time, the constant interaction among cluster enterprises makes the enterprise in the local cluster understand and learn some good innovative new ideas earlier to other enterprises, and the concept of innovation in the industrial cluster can quickly spread and diffusion. Therefore, the emergence of cluster innovation behavior is the result of cluster interactive learning in the internal and external network. In any development stage of the cluster enterprise, it both to carry on the independent innovation and imitation innovation, all need corresponding to the human, material and financial resources to support, and the capital investment depends on the stage of cluster enterprise income and its own capital.

At the same time, we analyzed the promoting mechanism of cooperative innovation among enterprises from the aspect of overall rationality and individual rationality. And we concluded that in a dynamic cooperation alliance in the industrial cluster, when each cluster enterprise participating in cooperation take the optimal strategy according to the current time and corresponding conditions, the alliance's overall profit value with time is changing constantly, and the change degree of profit in each instant depends on two aspects: one is the discounted value of instant pay of the cluster alliance at each time. In cluster cooperation innovation, since each enterprise participating the cooperation innovation is rational, he often does not come to join in the alliance if the paid form the cooperation innovation cannot meet the individual rationality, and the cooperation will not occur. Therefore, only in the initial stages of cooperation, each enterprise all agreed to the pay distribution adopted by the cooperation league, they will participate in cooperation. In addition, they will tend to loyal cooperation alliance as far as possible in the process of cooperation when allocating the payment according to the principle of optimal consensus, because it is related to their interests. And the principle of optimal consensus specifies how to cooperate among enterprises and how to distribute the income among enterprise.

## Conclusions

We use the game theory and complex science method to analyze the cooperation innovation mechanism and benefit distribution mechanism among enterprises in the strategic emerging industry cluster in this paper. And we received some important conclusions in this paper:

(1) In a dynamic cooperation alliance in the industrial cluster, when each cluster enterprise participating in cooperation take the optimal strategy according to the current time and corresponding conditions, the alliance's overall profit value with time is changing constantly, and the change degree of profit in each instant depends on two aspects: one is the discounted value of instant pay of the cluster alliance at each time; Secondly, the optimal change of all enterprises' state in the cluster alliance brought the changes of profit value of the whole cluster.

(2) In a dynamic cooperation alliance in the industrial cluster, under the cases of different initial time, the cluster enterprise participating in the cooperation take the same cooperation strategy at the same time and condition and the discount value of alliance overall profit is equal.

(3) When the cluster enterprise carries on the innovation alone, and other companies are using the optimal strategy according to the current time and corresponding conditions, the value function of enterprise is changing constantly with time.

## Acknowledgements:

This article is funded by the National Social Science Fund Project(14CJY002), Project of national natural science foundation of china(7127159) ,Science and Technology fund of Guizhou province(qiankeheJ[2013]2089), and Education Department of Natural Science Fund Project of Guizhou province(qianjiaohe KY(2015)378).

## References

1. Calvin Taylor, Beyond Advocacy: Developing an Evidence Base for Regional Creative Industry Strategies, 2006, Cultural Trends, vol. 1, pp 3-18.
2. Cohen.WM, Nelson. R, Walsh J, Links and impacts: The influence of public research on industrial R&D, 2002, Management Science, vol. 1, pp94-96.
3. Deng-ke Yu, Guo Ping Tu, Chen Hua, Research on ways and patterns of the coordinated development of strategic emerging industry cluster, 2012, Science and management of science and technology, vol. 4, pp114-117.

4. F. Malerba, Innovation and the evolution of industries, 2006, *Journal of Evolutionary Economics*, Vol. 16(06), pp3-23.
5. Frantzen D, The causality between R&D and productivity in manufacturing: an international disaggregate panel data study, 2003, *International Review of Applied Economics*, Vol. 17(02), pp31-35.
6. Gassmann. Opening up the innovation process: towards an agenda, 2006, *R&D Management*, vol. 361, pp8-27.
7. Gereffi Q, Humphrey J, Sturgeon T, The governance of global value chains, 2005, *Review of International Political Economy*, vol. 12, pp78-104.
8. H. N. Caroline Lanciano-Morandat, Eric Verdier, Higher Education Systems and Industrial Innovation, 2006, *European journal of social science research*, vol. 19, pp33-41.
9. Jefferson, G. Z. Hu, X. Yu, X. Guan, Ownership, Performance, and Innovation in China's Large-and Medium-size Industrial Enterprise Sector, 2003, *Journal Of Comparative Economics*, vol. 14,( 1), pp89-113.
10. Li Yuan, Growth mechanism of China's strategic emerging industries and empirical research, 2013, Nankai University, pp40-43.
11. M. Dodgson, Asia's national innovation systems: Institutional adaptability and rigidity in the face of global innovation challenges, 2009, *Asia Pac J Magazine*, vol. 26, pp107-111.
12. Rui-juan Wang, Research on coordinated development of emerging strategic industry in Yangtze river delta, 2013, East China normal university, pp7-10.
13. West,J.& Gallagher,S, Challenges of open innovation: the paradox of firm investment in open-source software, 2006, *R&D Management*, vol. 03, pp319-331.
14. Xiu-ting Wang, Research on synergy innovation network model of the strategic emerging industry and the ability of dynamic evolution, 2012, *Forum on Science and Technology in China*, vol.11, pp51-53.
15. Su-qin Wu. Research on the development of strategic emerging industry cluster in Hunan province from the perspective of low-carbon, 2012, Xiangtan university, pp4-7.
16. Xing-zhi Xiao, Xie Li. The empirical analysis of innovation efficiency of China's strategic emerging industry, 2011, *Economic Management*, vol. 11, pp26-29.
17. Yang Rongji, Petrosian. Dynamic cooperation and cutting-edge game theory, 2007, Beijing: China market press, pp61-62, 130-131, 169-170.