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Optimal Model of Toll Public Road with Management Cost

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Abstract

Operation/management cost (OMC) in toll public road refers to the cost incurred after the build-operate-transfer (BOT) concession period. In the previous research, the operation or management cost is neglected or assumed to be fixed or given during the concession period. The impact of the OMC is investigated from three aspects: length of concession period, toll charge and road capacity. The results show that existed BOT optimal model of private firm is suitable to government management based on OMC; that new private firm optimal model is rebuilt with profit and OMC constraint conditions. Comparing with traditional studies the toll charge formulas of governemnt and private firm have one more term related with management. Study can provide new ideas for charging the road in OMC concession period.

Keywords

Toll public road; Operation/management cost; social welfare; private profit

1 Introduction

More and more franchising road, built through built-operate-transfer (BOT), will be due in the future. De Palma et al. assumed that road is will be exacerbated in concession period [1]. Some researching results pointed out that concession period of BOT road equals to road life [2, 3]. These results mean that BOT road cannot be used when it transferred to government. In fact, government will repair it after BOT concession period if it cannot service for the users. If BOT road is transferred to government, it will become public road. Generally speaking, public road is free to the road users, but this kind of public road is usually interstate road which is different from city road. If keeping high service level, government has to spend much money on it. So the best method is to charge it to compensate the operation/management cost (OMC).

In order to make further analysis, we assume that government determines to charge the toll public road. Based on the above description, we name it as toll public road. The concession period of toll public road is named as OMC concession period. In 2015, ministry of transport of the people's republic of China promulgated the documents of revised version of 'Management Regulations of Toll Road', which pointed that the highway will be tolled continuously for compensating the operation/management cost (OMC) of highway. Toll public road is different from the traditional toll road for congestion and loan.

If charging it continuously, government has two choices, one is that government can manage and charge the road by itself, the other is that private firms still manage it. Contrasting to the previous research, the paper contains three primary factors: concession period, toll charge and road capacity. And optimal model will be built with OMC constraint between the public and private firm. Our model is armed at studying the OMC which can distinguish the working efficiency and managing level between government and private firm. In our model, with adding a constraint of OMC, we not only can distinguish toll public road manager, but also determine the toll level and the length of concession period through solving the maximizing social welfare model and maximizing private profit model. We further find some new propositions. Under the assumption that the private firm and the government both have perfect information on the capitalized cost and future travel demand, we will investigate the properties of the OMC and who will recharge the road.

The paper is structured as follows: Section 2 gives literature review. Section 3 introduce some notations and assumptions. Based on considering management cost, Section 4 presents the optimal model of government and private firm, then introduces first-best and second-best contracts via the constrained private firm profit problem. Section 5 concludes the paper with some remarks.

2 Literature review

Most of the existing literature on BOT roads focuses on the toll charge, capacity choice, social welfare and private profit without considering the management cost; the concession period is generally assumed to be given or fixed and the capacity cost per unit period is assumed to be constant [4-6]. In study of private firm managing the BOT road, Verhoef and Rouwendal addressed some important issues of second-best congestion pricing [7]. Tsai and Chu analyzed how a BOT contract may impact the travel demand, toll charge, road capacity and social welfare through considering varied BOT contracting situations [8]. In order to guarantee the roads users benefits, Verhoef investigated how to design a set of regulations to guarantee the social welfare through auctioning the concession period of a road from the angle of government [9]. A road usually plays a role in the network, Xiao and Yang studied toll charge and road capacity among the operation period of a private firm's congestion roads in a network [10]. Yang and Meng examined social welfare gain and profitability of a single new BOT road with elastic demand in a general network using numerical experiments [11]. Yang and Meng further showed that the self-financing result can also apply to a general network under the same assumption in Mohring and Harwitz's model for a single road [12, 13]. Considering the road capacity also as a variable, Guo and Yang first analyzed the BOT contract from three aspects, i.e. concession period, road capacity and toll charge with deterministic demand and homogeneous users [2]. Tan and Yang investigated the properties of Pareto-efficient contracts for road franchising by means of bi-criteria optimization under perfect information [3]. Vincent investigated private supply of two congestible infrastructures under four market structures including a monopoly and three duopolies that differed in how private firm interact [14]. In perspective of guarantee, Feng et. al. built several optimal model to analyze the government guarantee for the private firm [15]. From a social welfare perspective, Rouhani made a broad overview on road pricing [16].

3 Notation and assumptions

Based on the assumption that government determine to charge the road continuously. We assume that a toll public road between two cities is to be maintained and the technical characteristics of it are exogenous. Let $v \geq 0$ be the potential travel demand of the road, c_1 is the road capacity when the road is transferred from private firm to government, $c \geq c_1$ be the road capacity which would not be changed during the toll public road life, and $t(v,c)$ be the link travel time function of travel demand v and road capacity c, and $B(v)$ be the inverse demand function (or the marginal benefit function). Note that v and c are measured in the number of vehicles per unit period. The following condition which satisfies demand-supply equilibrium always holds:

$$B(v) = p + \beta t(v,c)\,, \tag{1}$$

where p is the toll charged to each road user and β is the value of time which is a factor that can convert time into an equivalent monetary cost (only considering the homogenous users).

Condition (1) simply means that the new road travel demand v is determined by the full price for a trip.

From equilibrium condition (1), the toll charge p can be viewed as a function of the road capacity v and travel demand v, mathematically:

$$p(v,c) = B(v) - \beta t(v,c) . \tag{2}$$

The toll p is determined by the demand v uniquely for a given road capacity c. Therefore, it is essentially equivalent to selecting v and c when the toll p and capacity c are decision variables. Hereafter, for the convenient exposition, the demand v is substituted by the toll p. The following basic assumptions are made about $B(v)$ and $t(v,c)$ throughout the paper.

Assumption 1.

a) For any $v \geq 0$, $B(v)$ is a strictly continuously decreasing and differentiable function of v and the function $vB(v)$ is strictly concave in v.

b) $t(v,c)$ is a continuously differentiable function of both v and c, for $v \geq 0$ and $c \geq c_1$; it is a convex and increasing function in v for $c \geq c_1$ and a decreasing function in c for any $v \geq 0$.

Let $\hat{T}$ $(\hat{T} > 0)$ be the road life under consideration of toll public road, namely, the optimal life a road lasts in terms of a reasonable level of service. Let T $(0 < T < \hat{T})$ be the OMC concession period if the private firm gets the road franchising and $\hat{T} - T$ be the post-OMC concession period when the private firm returns the road franchising to the government. Based on above content, we introduce two kinds of unit social surplus $S(v,c)$, $\tilde{S}(c)$ and the unit-time revenue $R(v,c)$. Then the unit-time social surplus $S(v,c)$ during the OMC concession period is determined as:

$$S(v,c) = \int_0^v B(w)dw - \beta vt(v,c) . \tag{3}$$

Under Assumption 1, we can see that $S(v,c)$ is strictly concave in v.

The unit-time social surplus $\tilde{S}(c)$ during the post-OMC concession period is determined as:

$$\tilde{S}(c) = \max_{v_1 \geq 0} S(v_1,c) = \max_{v_1 \geq 0} \int_0^{v_1} B(w)dw - \beta v_1 t(v_1,c) . \tag{4}$$

Eq. (4) implies that the government can choose the optimal traffic volume to maximize the unit social welfare with the fixed or given road capacity during the post-OMC concession period of toll public road.

Correspondingly, the unit-time revenue $R(v,c)$ during the OMC concession period is calculated as:

$$R(v,c) = vp(v,c) = vB(v) - \beta vt(v,c) , \tag{5}$$

where the travel demand v is determined by Eq. (1).

In line with Shi et al. [17], we assume the OMC is composed of two parts: demand-related OMC and capacity-related OMC. Demand-related OMC delegates the operating/managing cost of traffic demand; the capacity-related OMC is related to the maintenance cost of road. Then Let $M_{s,\hat{T}}(v)$ and $M_{g,\hat{T}}(v)$ be demand-related OMC of private firm and the governmental for the whole toll public road life $\hat{T}$. Let $M_{s,T}(v)$ be the private firm's OMC during the OMC concession period of toll public road and $M_{g,\hat{T}-T}(v)$ be the government's OMC during the post-OMC concession period of toll public road. $I(c)$ is the sum of capacity-related OMC and construction cost of the road; then $I(c)$ is a function of the capacity c. For convenience of exposition, we substitute $M_s(v)$ and $M_g(v)$ for $M_{s,\hat{T}}(v)$ and $M_{g,\hat{T}}(v)$ throughout the paper. Then we have the following assumption.

Assumption 2.

a) The government's demand-related OMC function $M_g(v)$ and $M_{g,T}(v)$ are continuously increasing and differentiable with respect to v for $v>0$; the private firm demand-related OMC function $M_s(v)$ and $M_{s,\hat{T}-T}(v)$ are continuously increasing and differentiable with respect to v for $v>0$; the sum of capacity-related OMC and road construction cost function $I(c)$ is continuously increasing and differentiable with respect to c for $c>c_1$;

b) For any given $c>0$, $R(v,c)=vp(v,c)$ is a strictly concave function of v for $v\geq 0$, i.e., $\partial R^2/\partial v^2<0$.

First, we consider the private firm's problem when the government let the private firm manage the road in the OMC concession period. The private firm would choose a combination of the concession period *T*, the road capacity c and the travel demand v to maximize its profit $P_s(v,c)$ during the concession period:

$$P_s(v,c)=TR(v,c)-M_{s,T}(v)-I(c). \tag{6}$$

The first term of Eq. (6) is the total revenue collected by the private firm from the road users during the OMC concession period; the second and the third terms are the OMC of private firm and the sum of the capacity-related OMC and road construction cost respectively, which are both borne by the private firm. With Eq.(5), the problem (6) can be written as:

$$P_s(v,c)=TvB(c)-\beta Tvt(v,c)-M_{s,T}(v)-I(c). \tag{7}$$

With part (b) of Assumption 1 that $t(v,c)$ is convex function, the term $vt(v,c)$ of Eq. (7) is also convex in v for $c > 0$. Based on the strict concavity of $vB(v)$, the private profit function $P_s(v,c)$ is strictly concave in v for $c > 0$ in the OMC concession period *T*.

Next, this paper considers the governmental problem of toll public road. The government usually wants to choose the best combination of variables (T, v, c) to maximize the social welfare during the life of toll public road $\hat{T}$. When the OMC considered, there are two kinds of problems. One problem is that when the road is managed by the government itself during the road life $\hat{T}$, we calculate it with the OMC of government, $M_g(q)$:

$$W_g(v,c) = \hat{T}S(v,c) - M_g(v) - I(c)\ . \tag{8}$$

The other problem is that the government let the private firm to manage the road during the OMC concession period T and the government manages the road by itself during post-OMC concession period $\hat{T} - T$. We calculate the social welfare:

$$W_s(v,c) = TS(v,c) + (\hat{T} - T)\tilde{S}(v,c) - M_{s,T}(v) - M_{g,\hat{T}-T}(v) - I(c)\ , \tag{9}$$

where $W_g(v,c), W_s(v,c)$ are, respectively, the social welfare of road managed by the government and the private firm.

Assumption 3.

a) $t(v,c)$ is homogeneous of degree zero in the v, and c, i.e., $t(\alpha v, \alpha c) = t(v,c)$ for any $\alpha > 0$.

b) Constant return to scale in the OMC of the government and private firm, namely, $M_g(v) = m_g v$ and $M_s(v) = m_s v$; the construction cost is also constant return to scale, namely, $I(v) = kv$, where m_g, m_s denote the constant cost of per unit demand, *k* denote the constant cost of per unit capacity.

Note that, this paper does not adopt an interest rate to calculate future revenues to equivalent present values of the private firm or the government. In fact, the discounting rate does not alter results because that both the social welfare and the private profit in this study are invariant for the same reason [2, 3].

4 Optimal model of toll public road based on management cost

In this section, we look into the properties of the OMC in the problem under perfect information, namely, both the private firm and the government share the common knowledge: travel demand r, and the OMC including $M_g(v), M_s(v)$, $I(c)$. At first, we have the following proposition.

Proposition 1. Under Assumption 1 and 3, considering the OMC in optimal problem, the OMC concession period of private firm will be

$T = 0$ if $m_g < m_s$.

$T = \hat{T}$ if $m_g \geq m_s$.

According to the traditional wisdom, we can get the Proposition 1 easily. In fact, the private firm usually wants to operate the road as long as possible, since they can get more profit through selecting appropriate price, road capacity and OMC saving. From Proposition 1, we know that government will let the private firm to manage road if $m_g > m_s$ with more social welfare and less OMC.

In China, facing expiration of road franchising, almost all of the private firms want to extend the concession period such as Huayu Company of Shenzhen city which manages the Shui-guan highway of Shenzhen. The governmental highway companies also want to extend the concession period such as Highway Company of Shandong province. Niu and Zhang gives the similar proposition with uncertain travel demand of BOT road, however, they do not analyze the impact of OMC of toll public road [18].

Based on above proposition, the social welfare $W_s(T, v, c)$ (Eq. (9)) and the private profit $P_s(T, v, c)$ (Eq. (7)) can be written as

$$W_s(v,c) = \hat{T}S(v,c) - M_s(v) - I(c)\,, \tag{10}$$

$$P_s(v,c) = \hat{T}vB(v) - \beta\hat{T}vt(v,c) - M_s(v) - I(c)\,. \tag{11}$$

4.1 Optimal model with governmental management

If the unit OMC of the government m_g is smaller than the unit OMC of the private firm m_s, the government will manage the road by itself to maximize social welfare through choosing the optimal price and capacity during the toll public road life $\hat{T}$. And maximizing social welfare is the only goal of government. Then the problem is defined as Eq. (8).

Let $(\hat{T}, \tilde{v}_g, \tilde{c}_g)$ be the social optimal (SO) solution that maximize social welfare $W_g(\hat{T}, v, c)$. Then it meets the following first-order optimal conditions:

$$\frac{\partial W_g}{\partial v} = T\left[B(\tilde{v}_g) - \beta t(\tilde{\rho}_g) - \beta\tilde{\rho}_g t'(\tilde{\rho}_g)\right] - m_g = 0 \tag{12}$$

$$\frac{\partial W_g}{\partial c} = T\beta\tilde{\rho}_g^2 t'(\tilde{\rho}_g) - k = 0\,, \tag{13}$$

where the $\tilde{r}_g$ denote the SO volume/capacity ratio when the government manages the new road itself, namely, $\tilde{r}_g = \tilde{q}_g / \tilde{y}_g$. The $\tilde{r}_g$ can be calculated by condition (13),

$$T\beta \tilde{r}_g^2 t(\tilde{r}_g) = k \,, \tag{14}$$

The toll charge for optimal social welfare can be calculated from (12) and (2).

$$\tilde{p}_g = B(\tilde{q}_g) - \beta t(\tilde{r}_g) = \beta \tilde{r}_g t^{'}(\tilde{r}_g) + m_g / T \,. \tag{15}$$

The government can carry out this road capacity and toll charge to approach optimal social welfare. Then, toll charge equals to the congestion externality exactly.

In reality, the government wants to manage the road with zero profit, then the optimal model can be defined as follow,

$$W_g(v,c) = \hat{T}S(v,c) - M_g(v) - I(c) \tag{16}$$

subject to

$$P_g(v,c) = TR(v,c) - M_g(v) - I(c) \tag{17}$$

Using the Lagrange method, the toll charge of zero profit, then road is managed by government, can be computed.

4.2 Optimal model with private firm management

In the section of notation and assumption, we build the unconstrained profit maximizing problem model and unconstrained social welfare maximizing problem. Here, we build the constrained profit maximizing problem model under the assumption that the unit government m_g is bigger than the unit private firm m_s . When managing the road, the private firm will maximize its profit during the OMC concession period through selecting road capacity c , toll charge $p(c)$. So we build the constrained profit maximizing problem model by the total revenue minus total construction cost and private OMC.

$$\max_{y \geq 0, T \geq 0} W_s(v,c) = \hat{T}S(v,c) - M_s(v) - I(c) \tag{18}$$

subject to

$$\hat{T}vp(v,c) - M_s(v) - I(c) \geq \tilde{P}, \tag{19}$$

$$\hat{T}vp(v,c) - M_s(v) - I(c) \leq M_g(v) - M_s(c) \,, \tag{20}$$

$$m_g \geq m_s, v \geq 0, c \geq 0, T \geq 0 \,, \tag{21}$$

where $\tilde{P} \geq 0$ is the minimum profit that the private firm will find acceptable. So even if the $m_g \geq m_s$ and government want private firm to management road, it also satisfies condition (19), namely, if government want attract private firm to manage road, the profit must be bigger than profit margin $\tilde{p}$. The constrained condition (20) means that private profit will not exceed the difference value between the government OMC and private OMC, namely, if the profit is bigger than the difference value, government will management the road by itself.

The condition (20) can be simplified as

$$\hat{T}vp(v,c) - M_g(v) - I(c) \le 0\,. \tag{22}$$

The Eq. (22) tell us that the government let the private firm manage road when its profit is negative which is identical with the condition $m_g > m_s$. With the condition (22) and management cost, our model is different to the traditional optimal models.

Based on above problem, we introduce the definition of first-best problem and second-best problem.

Definition. (first-best and second-best OMC contract). Let $(\hat{T},v,c)$ be an optimal solution to the Eqs. (18)-(21), $(\hat{T},v,c)$ is to be a first-best OMC contract if (v,c) maximize the Eq. (10), otherwise, $(\hat{T},v,c)$ is to be a second-best OMC contract.

The first-best and the second-best OMC contract are important concepts that classify the optimal problem. In fact, the private firm usually reaches the second-best OMC contract with government if the first-best OMC contract cannot be reached.

Proposition 2. It holds for the second-best solution $(\hat{T},v_s,c_s)$, that $\bar{v} \le v_s \le \tilde{v}$.

Proof. Let $(\hat{T},v_s,c_s)$ be the second-best optimal solution. We first take the partial derivatives of social welfare $W_s(\hat{T},v,c)$ and profit $P_s(\hat{T},v,c)$ in demand v and capacity c ,

$$\frac{\partial W_s(\hat{T},v,c)}{\partial v} = TS'(v) = T\left[B(v) - \beta t(v,c) - \beta v\frac{\partial t(v,c)}{\partial v}\right] - m_s\,, \tag{23}$$

$$\frac{\partial W_s(\hat{T},v,c)}{\partial c} = -\left[T\beta q\frac{\partial t(v,c)}{\partial c} + I'(c)\right], \tag{24}$$

and

$$\frac{\partial P_s(\hat{T},v,c)}{\partial v} = TR'(v) = T\left[B(v) + vB'(v) - \beta t(v,c) - \beta v\frac{\partial t(v,c)}{\partial v}\right] - m_s\,, \tag{25}$$

$$\frac{\partial P_s(\hat{T},v,c)}{\partial c} = -\left[T\beta v\frac{\partial t(v,c)}{\partial c} + I'(c)\right]. \tag{26}$$

Let $\tilde{v}$ maximize the social welfare $W_s(T,v,c)$, the term on the right-hand side of Eq. (23) equals zero; i.e.:

$$B(\tilde{v}) - \beta t(\tilde{v},c_s) - \beta\tilde{v}\frac{\partial t(\tilde{v},c_s)}{\partial v} = \frac{m_s}{T}\,. \tag{27}$$

Let $\bar{v}$ maximize the private firm profit $P_s(T,v,c)$, the term on the right-hand side of Eq. (25) equals zero; i.e.:

$$B(\bar{v})+\bar{v}B'(\bar{v})-\beta t(\bar{v},c_s)-\beta\bar{v}\frac{\partial t(\bar{v},c_s)}{\partial v}=\frac{m_s}{T}. \tag{28}$$

Under the assumption 1, there exist $\tilde{v}_s$ and $\bar{v}$ satisfying Eqs. (23) and (25) respectively and $\bar{v}<\tilde{v}_s$.

Comparing Eqs. (23) and (25), we have $\partial W/\partial v>\partial P/\partial v$ under the condition of $B'(v)<0$ for any feasible solution $(\hat{T},v,c)$. If $(\hat{T},v_s,c_s)$ is a second-best solution, $\partial W(\hat{T},v_s,c_s)/\partial v$ and $\partial P(\hat{T},v_s,c_s)/\partial v$ cannot be positive and negative simultaneously, otherwise, increasing or decreasing demand v will increase or decrease social welfare and profit at the same time, which contradicts that $(\hat{T},v_s,c_s)$ is a second-best solution. Thus we have $\partial W(\hat{T},v_s,c_s)/\partial v\ge 0$ and $\partial P(\hat{T},v_s,c_s)/\partial v\le 0$. This completes the proof.

Proposition 2 point out that under the constraint condition and , the private firm will chase to private profit with travel demand v_s.

With equations and , the private toll charge can be written as follow,

$$\tilde{p}_s=\beta\tilde{v}_s\frac{\partial t(\tilde{v}_s,c_s)}{\partial v}+\frac{m_s}{T}. \tag{29}$$

$$\bar{p}_s=\beta\bar{v}\frac{\partial t(\bar{v},c_s)}{\partial v}+\frac{m_s}{T}-\bar{v}B'(\bar{v}). \tag{30}$$

If private firm charges the road users with equation (29), then government will get its social welfare W_s, then private firm will not get its maximized profit. if the private firm charged the road users with equation (30), the private firm will get the maximizing profit $\bar{P}_s$.

4.3 Discussion

Based on above analysis, we can give the following comparison between government and private firm.

Tab. 1 Comparison between Government and Private Firm

Variables and constraint	Government management based on management cost	Private firm management based on management cost
Constraint condition	No constraint or zero profit constraint condition	Profit constraint condition Management cost constraint
Demand	$\tilde{v}_g$	$v_s(\bar{v}_s\le v_s\le\tilde{v}_s)$
Toll charge	$\tilde{p}_g$	$p_s(\tilde{p}_s<p_s<\bar{p}_s)$

With constraint condition management cost (20), the optimal model in the paper is different to traditional model. In OMC concession period, the government can maximize the social welfare with toll charge $\tilde{p}_g = \beta \tilde{v}_g t^{'}(\tilde{v}_g) + m_g / T$, and the correspond travel demand is $\tilde{v}_g$. Comparing to traditional toll, the second term m_g / T is compensating for the management cost. The equations (29) and (30) also have the term about the private firm management cost. However, under the constraint condition, the private firm must manage road with toll charge $p_s(\tilde{p}_s < p_s < \bar{p}_s)$, and the corresponding travel demand is $v_s(\bar{v}_s \le v_s \le \tilde{v}_s)$. Then government and private firm both get the same objective.

5 Conclusions

When either the government or a private firm manages the toll public road, the OMC may have a significant effect on price, capacity and concession period, yet the previous research typically neglected the OMC. Motivated by this, this paper has examined how the OMC influences on the toll public road. We first investigated the social welfare when the government manages the toll public road by itself under the condition that the government OMC is smaller than the private firm OMC. Contrast to this, the social welfare, private profit have been complicated when the governmental OMC is bigger than the private firm OMC. It was found that the government would tend to let a private firm operate a toll public road for the whole life of road, namely the concession period equal to the road life, when the governmental OMC is bigger than the private firm OMC. However, even in this case, the private firm will also consider the profit of the project. Considering the OMC in toll public road raises a series of issues for further investigation and this piece of work is just a beginning point. The adoption of OMC optimal model shall require a serious reconsideration of management level, governmental regulations and so on in the field of transportation infrastructure planning, construction, operations and management in the developing and developed countries. Study can provide new ideas for charging the road after BOT concession period in order to offer high-quality service.

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References

1. A. De Palma, M. Kilani, R. Lindsey, Maintenance, service quality and congestion pricing with competing roads, 2007, Transportation research part B, vol. 41, no. 5, pp.573-591.
2. X. Guo, H.Yang, Analysis of a build-operate-transfer scheme for road franchising, 2009, International journal of sustainable Transportation, vol.3, pp.312-338.
3. Z.J. Tan, H. Yang, X.L. Guo, Properties of Pareto-efficient contracts and regulations for road franchising, 2010, Transportation research part B, vol.44, no.4, pp.415-433.
4. E. Engel, R.D. Fischer, A. Galetovic, Highway franchising: pitfalls and opportunities, 1997, American economic review, vol. 87, no. 2, pp.68-72.
5. E.M.R.A. Engel, R.D. Fischer, A. Galetovic, Least-present-value-of-revenue auctions and highway franchising, 2001, Journal of political economy, vol. 109, no.5, pp.993-1020.
6. R. Lindsey, E.T. Verhoef, Traffic congestion and congestion pricing, 2001, In: Button K J, Hensher D A, Handbook of transport systems and traffic control elsevier science, Oxford, pp. 77-105.
7. E. T. Verhoef, J. Rouwendal, Pricing, capacity choice, and financing in transportation networks, 2004, Journal of regional science, vol.44, no.3, pp.405-435.
8. J. Tsai, C. Chu, The analysis of regulation on private highway investment under a build-operate-transfer scheme. 2003, Transportation, vol.30, pp.221-243.
9. E. T. Verhoef, Second-best road pricing through highway franchising. 2007, Journal of urban economics, vol.62, no.2, pp.337-361.
10. F. Xiao, H. Yang, D. Han, Competition and efficiency of private toll roads, 2007, Transportation research part B, vol.41, no.3, pp.292-308.
11. H. Yang, Q. Meng, Highway pricing and capacity choice in a road network under a build-operate-transfer scheme. 2000, Transportation research part A, vol.34, no.3, pp.207-222.
12. H. Yang, Q. Meng, A note on"highway pricing and capacity choice in a road network under a build-operate-transfer scheme" 2002, Transportation research part A, vol.36, no.7, pp.659-663.
13. H. Mohring, M. Harwitz, Highway benefits: an analytical framework, 1962, Northwestern university press, Evanston, IL.
14. A.C.V.D. Vincent, Serial private infrastructures. 2013, Transportation research part B, vol.56, pp.186-202.

15. Z. Feng, S. Zhang, Y. Gao, Modeling the impact of government guarantees on toll charge, road quality and capacity for Build-Operate-Transfer (BOT) road projects, 2015, Transportation Research Part A, vol.78, pp. 54-67.
16. O. M. Rouhani, Next Generations of Road Pricing: Social Welfare Enhancing, 2016, Sustainability, no.8, pp. 1-15.
17. S. S. Shi, Y. F. Yin and X. L. Guo, Optimal choice of capacity, toll and government guarantee for build-operate-transfer roads under asymmetric cost information, 2016, Transportation Research Part B, vol. 85, pp. 56-69.
18. B.Z. Niu, J. Zhang, Price, capacity and concession period decisions of Pareto-efficient BOT contracts with demand uncertainty, 2013, Transportation Research Part E, vol. 53, pp. 1-14.