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Reduced Order Modelling for Linear Dynamic Systems

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Abstract

A method based on clustering the poles of high order linear dynamic system is proposed for reduced order modelling, where objective is to reduce the system complexity and get an appropriate reduced order model. The poles of the original system are clustered in the s-plane using new technique “Logarithmic pole clustering”, where it preserve the dominant frequency of linear dynamic system and shift non-dominant frequency towards the selected dominant frequency. Also Factor division algorithm is used to maintain the transient and steady state response of the original system with reduced model. Few examples have been taken from literature and their results are compared with proposed one.

Key words

Logarithmic Pole Clustering, Order Reduction, Factor Division Algorithm, Stability, Transfer Function.

1. Introduction

Mathematical representation of a physical system may be of high order differential equation, where analysis is very difficult. So, to analysis such high order system, one may convert it into reduced order model, where analysis may become very easy to get desired objectives. Therefore model order reduction techniques play an important role to convert high order system into reduced order model in appropriate manner. Many order reduction methods [1-6] have been suggested in the last few decades to reduce the order of single input, single output [SISO] as well as multi input multi output [MIMO] systems in frequency and time domains, where few order reduction methods are efficient to reduce all kind of systems and some methods [7-8] are not so efficient to preserve the stability of the original system in the reduced order models. So, by combining the algorithm of two or more order reduction methods, numerous mixed order reduction methods [9-13] have been developed in the last few decades.

This paper deals with a mixed reduction method by combing the features of the logarithmic pole clustering and factor division algorithm [14]. The Clustering method was originally suggested by Pal [15] and it has been modified by Vishwakarma [16] known as modified pole clustering, which gives more dominant pole cluster center than Pal [15]. In this paper, A seven step algorithm [16], based on inverse distance criterion [IDM] is formulated in a single step to generate more dominant frequency for denominator polynomial of a reduced model, which preserve the important characteristics of the original system. In the proposed algorithm, the system frequencies are kept in a k_{th} group for k_{th} model order reduction, where each group of frequencies are replaced by logarithmic pole cluster centers i.e. all system frequencies of a selected group, shift towards the most dominant selected system frequency of a particular group (pole cluster) and generate a new dominant system frequency, which retain the main characteristics of a stable original high order system. The dominant frequency selection i.e. logarithmic pole clustering based proposed method is computationally simple, efficient, and also capable to retain the stability of the original system. The suggested method has been compared through two performance indices i.e. Integral of Square of Error (ISE) and Integral of the Absolute magnitude of Error (IAE) with existing order reduction techniques in MATLAB environment.

2. Problem Statements

The problem statements for SISO system is given as:

Let the transfer functions of high order SISO system of the order 'n' is represented as-

$$G_n(s) = \frac{N_n(s)}{D_n(s)} = \frac{a_0 + a_1 s + a_2 s^2 + \ldots + a_{n-1} s^{n-1}}{b_0 + b_1 s + b_2 s^2 + \ldots + b_n s^n} \tag{1}$$

Let the transfer function of the reduced order SISO system of the order 'k' is represented as-

$$R_k(s) = \frac{N_k(s)}{D_k(s)} = \frac{c_o + c_1 s + c_2 s^2 + \ldots + c_{k-1} s^{k-1}}{d_o + d_1 s + d_2 s^2 + \ldots + d_k s^k} \tag{2}$$

This paper transforms the original system (1) into reduced order model (2), such that it preserves the significant features of the original systems.

3. Descriptions of the Method

To find the reduced order model from original high order system, the method is described through following sub-sections.

3.1 Denominator polynomial for reduced order model

For the k^{th} – order reduced model, denominator polynomial is determined using proposed logarithmic pole clustering technique.

To make k^{th} – group of poles for k^{th} – model order reduction, few important points are mentioned below.

- Pole cluster should be made separately for real and complex poles.
- Each cluster must contain all the poles of the left half s -plane or right half s –plane only.
- Pole cluster should not be a combination of poles of both left half s -plane and right half s – plane.

Let i^{th} – cluster have $'r'$ real poles i.e. $\sigma_1, \sigma_2, \ldots \sigma_r$,

where, $|\sigma_1| < |\sigma_2| < |\sigma_3| < \ldots < |\sigma_r|$ Then a logarithmic pole cluster center for any selected group of pole, can be calculated using a formula given as

$$\sigma_{Li} = -\left\{ |\sigma_1| + \left[\log_{10} \left\{ 1 + \frac{|\sigma_1| + |\sigma_2| + \ldots |\sigma_r|}{k \times r} \right\} \div (r \times n) \right] \right\} \quad (3)$$

Here $'k'$ is the number of orders for reduced model, $'r'$ is number of poles in a selected group of poles and $'n'$ is the number of orders for original system. Also $'\sigma_1'$ is the most dominant pole of the selected pole group, other non-dominant poles i.e. $\sigma_2, \sigma_3 \cdots$ shift towards the most dominant pole.

Let, j^{th} - cluster has $'\Phi'$ complex conjugates poles. So for complex conjugate poles, same formula (3) is used for real and imaginary poles separately. Hence, j^{th} - logarithmic pole cluster is formulated as

$\Phi_{Lj} = A_{Lj} \pm jB_{Lj}$, where

$$\overset{*}{\Phi}_{Lj} = A_{Lj} + jB_{Lj} \quad \textit{and} \quad \overset{\bullet}{\Phi}_{Lj} = A_{Lj} - jB_{Lj}$$

Few cases may occur while synthesizing k^{th} – order denominator polynomial $D_k(s)$:

Case-1: If all logarithmic pole clusters centers are real, the $D_k(s)$ may be written for k^{th} – order as $D_k(s) = (s - \sigma_{L1})(s - \sigma_{L2})\ldots(s - \sigma_{Lk})$ (4)

Case-2: If all logarithmic pole clusters centers are complex conjugates, $D_k(s)$ is as follows

$$D_k(s) = (s - \overset{*}{\Phi}_{L1})(s - \overset{\bullet}{\Phi}_{L1})\ldots(s - \overset{*}{\Phi}_{Lk/2})(s - \overset{\bullet}{\Phi}_{Lk/2}) \quad (5)$$

Case-3: If $(k-2)$ logarithmic pole clusters centers are real and a pair of logarithmic pole clusters centers is complex conjugate, the denominator polynomial $D_k(s)$ is written as

$$D_k(s) = (s - \sigma_{L1})(s - \sigma_{L2})\ldots(s - \sigma_{L(k-2)})(s - \overset{*}{\Phi}_{L1})(s - \overset{\bullet}{\Phi}_{L1}) \quad (6)$$

3.2 Numerator coefficients for reduced order model

Numerator coefficients are calculated as follows

Let $k^{th} -$ order reduced model is written as

$$R_k(s) = \frac{N_k(s)}{D_k(s)} \quad (7)$$

where $D_k(s)$ is already obtained from Section 3.1.

To calculate coefficients for the numerator polynomial $N_k(s)$, $G_n(s)$ may be taken as.

$$G_n(s) = \frac{N_n(s) \times D_k(s)}{D_n(s) \times D_k(s)} = \frac{N_n(s) \times D_k(s) / D_n(s)}{D_k(s)} \quad (8)$$

Hence, numerator polynomial $N_k(s)$ is determined by the series expansion [14] of

$$\frac{N_n(s) \times D_k(s)}{D_n(s)} = \frac{\sum_{i=0}^{n+k-1} \alpha_i s^i}{\sum_{i=0}^{n} b_i s^i} \text{(about } s = 0 \text{, up to the term of the order } s^{k-1}\text{)} \quad (9)$$

Using the Routh recurrence formula, the numerator coefficients are calculated as follows

$$\left.\begin{aligned}
c_0 &= \frac{\alpha_0}{b_0}\left\langle\begin{matrix}\alpha_0 & \alpha_1 & \dots & \alpha_{k-1}\\ b_0 & b_1 & \dots & b_{k-1}\end{matrix}\right.\\
c_1 &= \frac{\beta_0}{b_0}\left\langle\begin{matrix}\beta_0 & \beta_1 & \dots & \beta_{k-2}\\ b_0 & b_1 & \dots & b_{k-2}\end{matrix}\right.\\
c_2 &= \frac{\gamma_0}{b_0}\left\langle\begin{matrix}\gamma_0 & \gamma_1 & \dots & \gamma_{k-3}\\ b_0 & b_1 & \dots & b_{k-3}\end{matrix}\right.\\
&\vdots \quad \vdots\\
&\vdots \quad \vdots\\
c_{k-2} &= \frac{u_0}{b_0}\left\langle\begin{matrix}u_0 & u_1\\ b_0 & b_1\end{matrix}\right.\\
c_{k-1} &= \frac{v_0}{b_0}\left\langle\begin{matrix}v_0\\ b_0\end{matrix}\right.
\end{aligned}\right\} \tag{10}$$

where

$$\left.\begin{aligned}
\beta_i &= \alpha_{i+1} - c_0 b_{i+1} \qquad i = 0,1,2,\dots k-2\\
\gamma_i &= \beta_{i+1} - c_1 b_{i+1} \qquad i = 0,1,2,\dots k-3\\
&\vdots\\
&\vdots\\
v_0 &= u_1 - c_{k-2} b_1
\end{aligned}\right\} \tag{11}$$

Finally numerator polynomial $N_k(s)$ with coefficients $(c_0, c_1, c_2 \dots c_{k-1})$ is written as

$$N_k(s) = c_o + c_1 s + c_2 s^2 + \dots + c_{k-1} s^{k-1} \tag{12}$$

4. Examples

Three different systems have been considered from the literature and represented in the form of mathematical model to reduce the complexity and dimensions of original system into a proper reduced model. To understand the proposed method, first example is solved in details and remaining examples are described with results and significant features. To explore the goodness of the proposed method, performance indices i.e. an Integral Square Error (ISE) and an Integral of the Absolute magnitude of Error (IAE) are calculated between the natural response of the reduced and original models in MATLAB environment, where lower value of ISE and IAE shows good performance of proposed reduced order models.

$$\text{ISE} = \int_0^\infty \left[y(t) - y_k(t)\right]^2 dt \tag{13}$$

$$\text{IAE} = \int_0^\infty \left| y(t) - y_k(t) \right| dt \tag{14}$$

where $y(t)$ is the unit step response of original system and $y_k(t)$ is the unit step response of reduced system.

Example 1

Consider the 7th order single input single output (SISO) supersonic jet engine inlet investigated by Telescu [17], which is given by

$$G(z) = \frac{2.0434z^6 - 4.9825z^5 + 6.57z^4 - 5.8189z^3 + 3.636z^2 - 1.4105z + 0.2997}{z^7 - 2.46z^6 + 3.433z^5 - 3.333z^4 + 2.546z^3 - 1.584z^2 + 0.7478z - 0.252}$$

For model order reduction as per proposed method, the above discrete transfer function is sample at 0.01, and is normalized in s-domain as follows

$$G(s) = \frac{25s^6 + 421.6s^5 + 10200s^4 + 95820s^3 + 870800s^2 + 3089000s + 10430000}{s^7 + 13.78s^6 + 612.7s^5 + 4730s^4 + 88120s^3 + 328100s^2 + 2894000s + 3020000}$$

The following two pole clusters are possible in this example as:

$[(-1.1512)]$ and $[(-1.0733 \pm j7.0487), (-2.0554 \pm j11.9457), (-3.1857 \pm j18.4813)]$

The logarithmic pole cluster centers are obtained as

Real pole cluster

$$\sigma_{L1} = \sigma_1 = -1.1512$$

Complex pole cluster

$$A_{L1} = -\left\{ \left|-1.0733\right| + \left[\log_{10}\left\{ 1 + \frac{\left|-1.0733\right| + \left|-2.0554\right| + \left|-3.1857\right|}{3 \times 3} \right\} \div (3 \times 7) \right] \right\} = -1.0847$$

$$B_{L1} = -\left\{ \left|-7.0487\right| + \left[\log_{10}\left\{ 1 + \frac{\left|-7.0487\right| + \left|-11.9457\right| + \left|-18.4813\right|}{3 \times 3} \right\} \div (3 \times 7) \right] \right\} = -7.08265$$

As per proposed method in Section 3.1, $\phi_{L1} = (A_{L1} \pm jB_{L1})$

Hence $\phi_{L1} = (-1.0843 \pm j7.08265)$

So denominator polynomial is obtained as

$$D_3(s) = (s + \sigma_{L1})(s + \phi_{L1})$$

$$D_3(s) = s^3 + 3.3198s^2 + 53.836s + 59.1021$$

Now using Section 3.2 of the proposed method, we calculate numerator coefficients as

$$c_0 = 203.8359\left\langle \begin{matrix} 616.4 & 744.1 & 252.4 \\ 3.024 & 2.894 & 0.328 \end{matrix}\right.$$

$$c_1 = 50.99\left\langle \begin{matrix} 154.1986 & 185.54 \\ 3.024 & 2.894 \end{matrix}\right.$$

$$c_2 = 12.556\left\langle \begin{matrix} 37.97 \\ 3.024 \end{matrix}\right.$$

Therefore numerator polynomial is obtained as

$$N_3(s) = 12.569s^2 + 50.9801s + 203.784$$

Using the proposed method, the 3^{rd} – order reduced model is synthesized as

$R_3(s) = \dfrac{12.556s^2 + 50.99s + 203.8359}{s^3 + 3.3198s^2 + 53.836s + 59.1021}$ with the performance error indices ISE=0.2457 and IAE=0.6061.

To analyze the performance of reduced order models, step input response and step impulse response of the proposed model is compared with the original high order system and models obtained by Telesue [17] and Othman [3] in Fig.1a and 1b respectively.

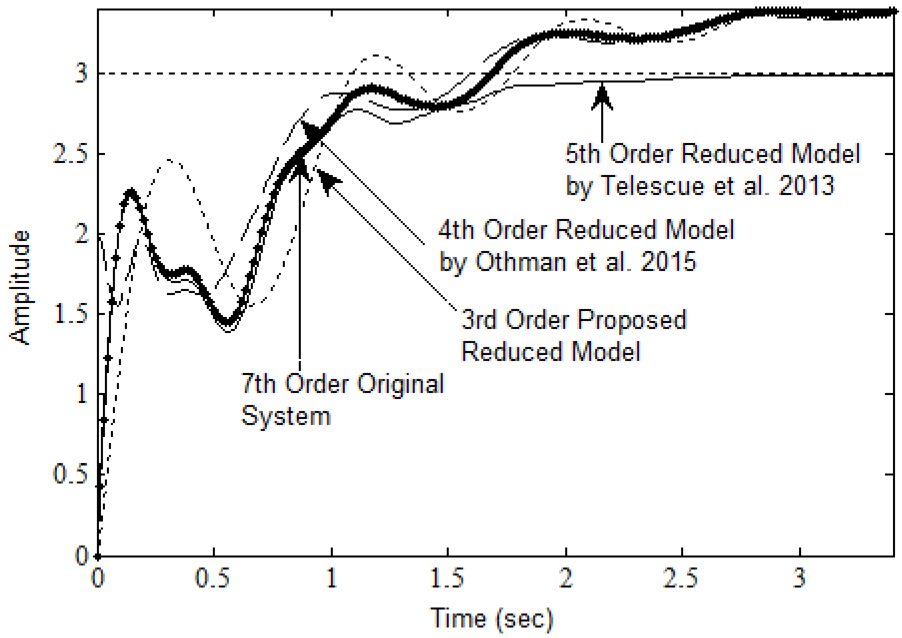

Fig.1a. Step responses of the full and reduced order models for Example 1

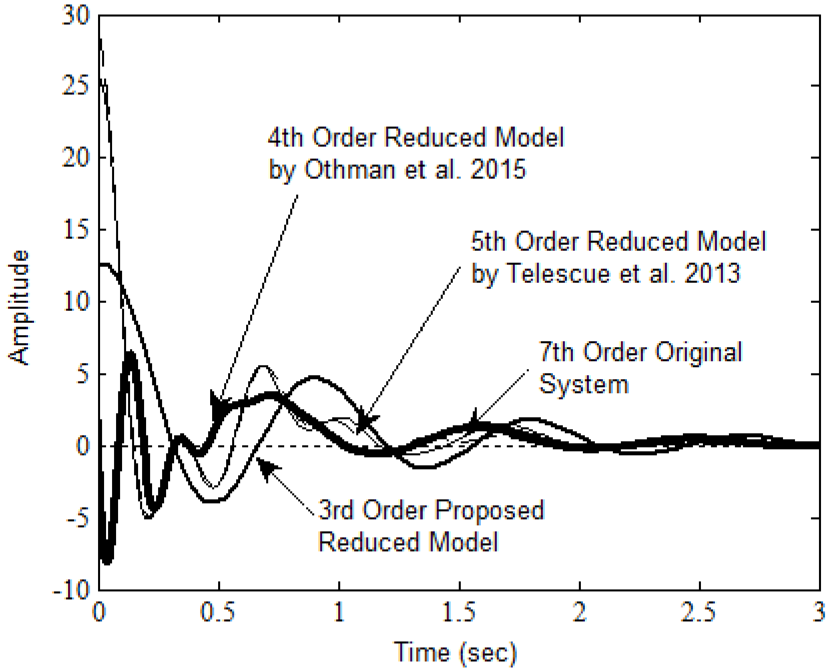

Fig.1b. Impulse responses of the full and reduced order models for Example 1

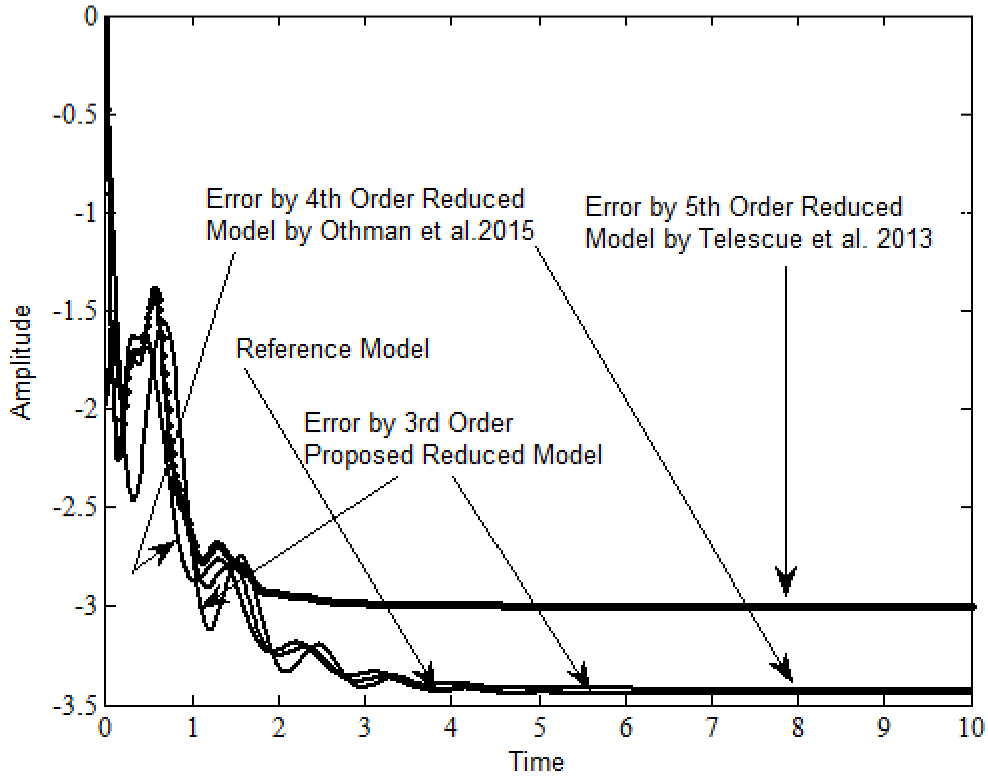

Fig.1c. Error graph for the different reduced order models for Example 1

. From Fig. 1a and 1b, it is clear that response of proposed reduced model follow the response pattern of original high order system even order of reduced model is too small as compared to Telesue [17] and Othman [3]. Also fifth order reduced model obtained by Othman [3] have error index i.e. ISE=0.157 and IAE= 0.5403 and fourth order reduced model obtained by Telescue [17] have ISE=1.539 and IAE=3.643, where third order proposed reduced model have ISE=0.2457 and IAE=0.6061 which is closer to fifth order model of Othman [3]. Error response is shown in Fig. 1c, where it is clear that proposed model has low error value.

Example 2

Consider a system of eight-order from Mukherjee [18].

$$G_8(s) = \frac{18s^7 + 514s^6 + 5982s^5 + 36380s^4 + 122664s^3 + 222088s^2 + 185760s + 40320}{s^8 + 36s^7 + 546s^6 + 4536s^5 + 22449s^4 + 67284s^3 + 118124s^2 + 109584s + 40320}$$

To synthesize second order reduced model, two pole clusters are: $(-1,-2,-3,-4)$ and $(-5,-6,-7,-8)$.

Using Section 3.1 of the proposed method, logarithmic pole cluster centers are determined as

$$\sigma_{L1} = -\left\{|-1| + \left[\log_{10}\left\{1 + \frac{|-1|+|-2|+|-3|+|-4|}{2\times 4}\right\} \div (4\times 8)\right]\right\} = -1.011$$

$$\sigma_{L2} = -\left\{|-5| + \left[\log_{10}\left\{1 + \frac{|-5|+|-6|+|-7|+|-8|}{2\times 4}\right\} \div (4\times 8)\right]\right\} = -5.0196$$, hence $\sigma_{L1} = -1.011$ and $\sigma_{L2} = -5.0196$

Therefore $D_2(s)$ may be written as

$$D_2(s) = (s + \sigma_{L1})(s + \sigma_{L2})$$

Now using Section 3.2 of the proposed method the numerator $N_2(s)$ is obtained as-

$$c_0 = 5.074 \left\langle \begin{matrix} 204600 & 1186000 \\ 40320 & 109584 \end{matrix} \right.$$

$$c_1 = 15.623 \left\langle \begin{matrix} 629926.429 \\ 40320 \end{matrix} \right.$$

Hence, $N_2(s) = c_0 + c_1 s$

Finally second order reduced model is written as

$$R_2(s) = \frac{15.6232s + 5.0744}{s^2 + 6.0306s + 5.0748}$$

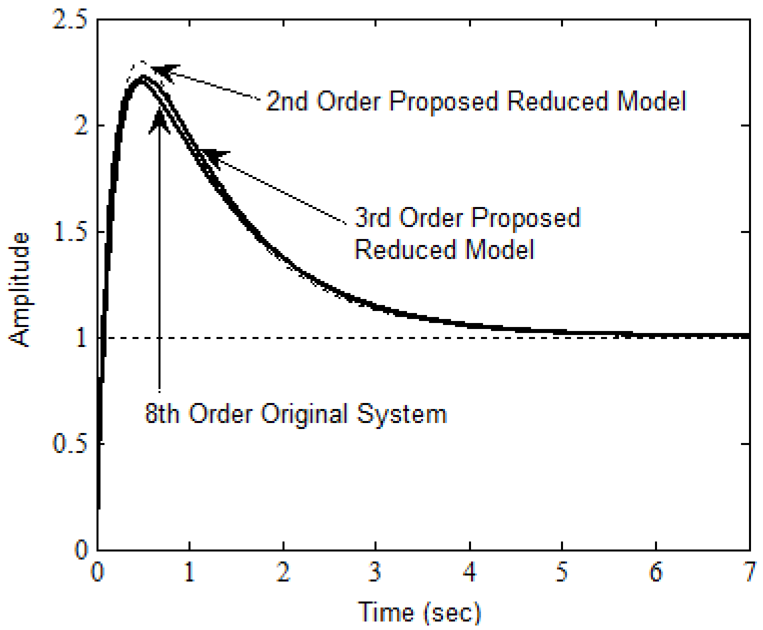

Fig.2. Comparison of step responses for Example 2

TABLE 1: Performance error i.e. Integral Square Error (ISE), Integral of the Absolute magnitude of Error (IAE) Comparison of the proposed method for Example 2

Reduction Methods	**Reduced Model**	**ISE**	**IAE**
Proposed Method	$R_2(s) = \frac{14.82s + 5.074}{s^2 + 6.031s + 5.075}$	0.00832	0.1583
	$R_3(s) = \frac{14.722s^2 + 61.96037s + 18.358}{s^3 + 10.0481s^2 + 27.2941s + 18.358}$	0.00682	0.1310
C.B.Vishwakarma[16]	$R_2(s) = \frac{16.51145s + 5.45971}{s^2 + 6.19642s + 5.45971}$	0.0140	0.1972
G.Parmar *et al.*[18]	$R_2(s) = \frac{24.11429s + 8}{s^2 + 9s + 8}$	0.0481	0.3006
Mukherjee *et al.*[4]	$R_2(s) = \frac{11.3909s + 4.4357}{s^2 + 4.2122s + 4.4347}$	0.05689	0.4572
Mittal *et al.* [1]	$R_2(s) = \frac{7.0908s + 1.9906}{s^2 + 3s + 2}$	0.2689	0.8054
Prasad and Pal [12]	$R_2(s) = \frac{17.98561s + 500}{s^2 + 13.24571s + 500}$	1.4584	1.000

Krishnamurthy [19]	$R_2(s)=\dfrac{155658.6152s+40320}{65520s^2+75600s+40320}$	1.6533	2.4090
Hutton [20]	$R_2(s)=\dfrac{1.98955s+0.43184}{s^2+41.17368s+0.43184}$	1.9171	10.0702

Similarly for third order reduced model, three pole clusters are selected as $(-1,-2),(-3,-4,-5)$ and $(-6,-7,-8)$

Using proposed method, third order reduced model may be synthesize as

$$R_3(s)=\frac{14.722s^2+61.96037s+18.358}{s^3+10.0481s^2+27.2941s+18.358}$$

The step responses of the reduced models $R_2(s)$ and $R_3(s)$ are compared with the original model $G_8(s)$ as shown in the Fig.2. The method is also compared through error index between proposed and existing methods [1, 4, 12, 16, 18, 19 & 20] in the Table 1.

Example 3

Consider an eight order system by J. Pal [15].

$$G_8(s)=\frac{N(s)}{D(s)}$$

Where,

$$N(s)=19.82s^7+429.26156s^6+4843.8098s^5+45575.892s^4+241544.75s^3$$
$$+905812.05s^2+1890443.1s+842597.95$$

$$D(s)=s^8+30.41s^7+358.4295s^6+2913.8638s^5+18110.567s^4$$
$$+67556.983s^3+173383.58s^2+149172.19s+37752.826$$

Using Section 3.1 and 3.2 of the proposed method, 4^{th} –order reduced model is obtained as

$$R_4(s)=\frac{19.8240s^3+18.7966s^2+724.8086s+1170.5}{s^4+9.702s^3+23.51s^2+122s+52.45}$$

TABLE 2: Performance error i.e. Integral Square Error (ISE), Integral of the Absolute magnitude of Error (IAE) Comparison of the proposed method for Example 3

Reduction Methods	Reduced Models	ISE	IAE

Proposed Method	$R_4(s)=\dfrac{19.8240s^3+18.7966s^2+724.8086s+1170.5}{s^4+9.702s^3+23.51s^2+122s+52.45}$	13.29	7.385
J.Pal [1]	$R_4(s)=\dfrac{61.27s^3+242.8s^2+2390s+3153}{s^4+12.78s^3+42.77s^2+268.2s+141.3}$	41.54	12.47

The step response and error index i.e. ISE response of the reduced model $R_4(s)$ is compared with the original system $G_8(s)$ and model obtained by Pal [15] in the Fig.3a and 3b respectively. The performance error indices ISE and IAE are calculated and shown in the Table 2.

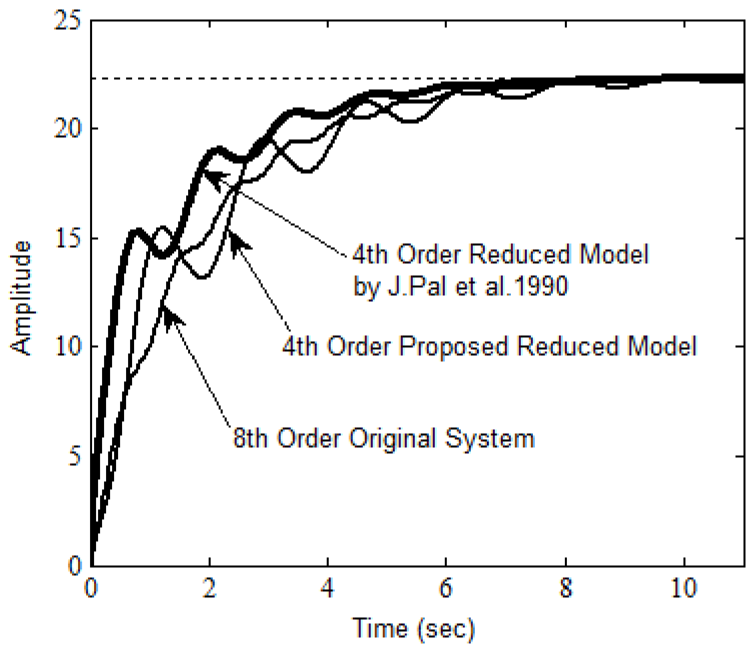

Fig.3a. Step responses of the full and reduced order models for Example 3

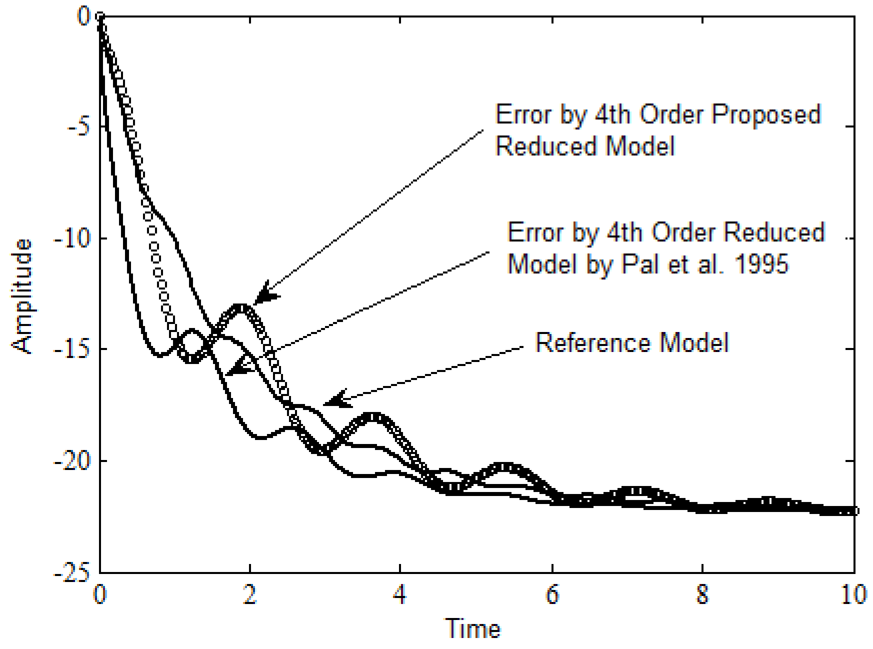

Fig.3b. Error Graph for the different reduced order models for Example 3

Conclusion

A mixed reduction method based on the logarithmic pole clustering technique and factor division algorithm has been proposed in this paper. The reduced order denominator polynomial is obtained by using logarithmic pole clustering technique and the coefficients of numerator are determined by factor division algorithm. The proposed method has been tested on three different systems having real and complex poles. The matching of the step and impulse responses is quite good in Fig 1a and 1b of the Example 1. Also models obtained by proposed method in Example 2 and 3 have better response matching with original system as shown in Fig 2 and 3. Performance of models obtained by proposed method and models obtained by different literatures have been checked via performance error indices which speak the beauty of the method. The suggested mixed method may be used for controller design and designed controllers via reduced order model may apply in full order power system directly.

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