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# Finite-time Control of a Class of Uncertain Networked Systems with State Delay and Communication Delay

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## Abstract

In this paper we consider the finite-time control problem for a class of networked systems with state delay and communication delay. The main result provided is a sufficient condition for the design of a state feedback controller which makes the closed loop systems finite-time stable. The sufficient condition is then reduced to a feasibility problem by involving linear matrix inequalities which is dependent on the size of the time delay and can be solved by LMI toolbox in MATLAB. When the LMI is feasible, the explicit expression of the desired finite-time control is also given. A numerical example is presented to illustrate the effectiveness of the proposed method.

## Key words

Networked control systems(NCSs), state delay, communication delay, finite-time control, linear matrix inequalities (LMIs)

## 1. Introduction

Feedback control systems wherein the control loops are closed through a real-time network are called networked control systems (NCSs). The main feature of NCSs is that the components (sensors, controller and actuators) of the systems are connected by a network. Compared with traditional point-ti-point control systems, the NCSs have many advantages such as low cost, reduced wiring, simple installation and maintenance, and high reliability, etc [1-3]. For these reasons, NCSs have been widely applied to many complicated control systems, such as aviation and aerospace fields, and airplane manufacture [4-5].

However, the insertion of the communication network in feedback control loop makes the analysis and design of NCSs complicated. The change of communication architecture induces different forms of time delays between sensors, actuators and controllers. These time delays come from the time sharing of the communication medium as well as the computation time required for physical signal coding and communication processing [6-8]. It is well known that time delays can degrade systems' performance and even cause systems instability. Therefore, the issues of stability analysis and designing controllers for NCSs have received much consideration for decades [1-12].

Much work has been done on the robust control of NCSs over the past ten years. Most of the results in this field relate to stability and performance criteria defined over an infinite time interval. However, the main concern in many practical applications is the behavior of the dynamical systems over a fixed finite time interval [13], for example, large values of the state are not acceptable in the presence of saturations. Therefore, we need to check the unacceptable values that the system state does not exceed a certain threshold during a fixed finite-time interval by giving some initial conditions. The concept of finite-time stability referring to these transient performances of control dynamics dates back to the Sixties, when it was introduced in the control literature [14]. Then, some attempts on finite-time stability can be found by using Lyapunov functional approach [15]. Recently, with the aid of linear matrix inequalities (LMIs) approach, more concepts of finite-time stability have been proposed for linear continuous-time or discrete-time control systems in the literatures [16–20].

But the above papers consider the Lyapunov stability for NCSs, a few results on finite-time stability for NCSs has been reported. Based on this fact, some new methods and approaches should be developed for design controllers of NCSs, which motivates this paper. Inspired by the above literature, in this paper, the attention is focused on the finite-time control of a class of NCSs with state delay and communication delay. The design is divided in two steps: the synthesis condition of the state feedback controller, supposing that the state variables are available, and then the sufficient condition is given in terms of LMIs. Conversely, the approach proposed in this paper leads to LMIs formulation, which gives the opportunity of fitting the finite-time control problem in the general framework of the LMIs approach to the multi-objective synthesis.

Notations: Throughout the paper, $R^n$ denotes the $n$ dimensional Eucliden space.

## 2. Preliminaries

In this paper we consider the following typical NCSs shown in Fig. 1.

$$\dot{x}(t)=(A+\Delta A(t))x(t)+(A_h+\Delta A_h(t))x(t-h)+(B+\Delta B(t))u(t)+(B_1+\Delta B_1(t))\omega(t) \tag{1}$$

where $A, A_h \in R^{n\times n}, B\in R^{n\times m}, B_1 \in R^{n\times l}$ are constant matrices respectively. $x(t)\in R^n$ is the state vector, $u(t)\in R^m$ is the control input, and $h$ denotes systems state delay. $\omega(t)\in R^l$ is the exogenous disturbance satisfying

$$\omega^T(t)\omega(t)\le d,\ \ d\ge 0 \tag{2}$$

$\Delta A(t), \Delta A_h(t)\in R^{n\times n}, \Delta B(t)\in R^{n\times m}, \Delta B_1(t)\in R^{n\times l}$ are unknown matrices representing time-varying parameter uncertainties satisfying

$$\Delta A(t)=D_1F(t)E_1 \qquad \Delta A_h(t)=D_2F(t)E_2$$
$$\Delta B(t)=D_3F(t)E_3 \qquad \Delta B_1(t)=D_4F(t)E_4$$

Where $D_1, D_2, D_3, D_4, E_1, E_2, E_3, E_4$ are known constant matrices with appropriate dimensions, $F(t)$ is unknown matrix with appropriate dimensions satisfying $F^T(t)F(t)\le \boldsymbol{I}$

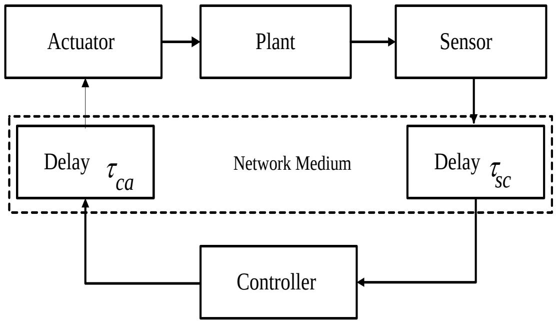


Fig. 1 A typical networked control systems

To simplify the analysis, based on actual engineering background, a full characterization of the NCSs is given by the following assumption.

**Assumption1**. The sensor is time driven; the controller and actuator are event driven. We use $\tau_{sc}$ and $\tau_{ca}$ to represent the sensor-controller and controller-actuator delay, respectively, then the communication delay is given by $\tau=\tau_{sc}+\tau_{ca}$.

Considering the effect of communication delay $\tau$, the above plant model is transformed into an NCSs model

$$\dot{x}(t)=(A+\Delta A(t))x(t)+(A_h+\Delta A_h(t))x(t-h)+(B+\Delta B(t))u(t-\tau)+(B_1+\Delta B_1(t))\omega(t) \tag{3}$$

Concerning NCSs (3), we design a state feedback controller

$$u(t)=Kx(t) \tag{4}$$

Where $K$ is a state feedback gain matrix to be determined later. Then, the resulting closed-loop NCSs follows that

$$\dot{x}(t) = \bar{A}x(t) + \bar{A}_h x(t-h) + \bar{B}Kx(t-\tau) + \bar{B}_1\omega(t) \tag{5}$$

where

$$\bar{A} = A + \Delta A(t) \qquad \bar{A}_h = A_h + \Delta A_h(t)$$
$$\bar{B} = B + \Delta B(t) \qquad \bar{B}_1 = B_1 + \Delta B_1(t)$$

The aim of this paper is to find a sufficient condition which guarantees the existence of a state feedback controller which stabilizes systems (1) over the finite interval $[0,T]$. By selecting the appropriate Lyapunov–Krasovskii function, the main results will be given in the form of LMIs. The general idea of finite-time control can be formalized through the following definitions over a finite-time interval for some given initial conditions [17−20].

**Definition1**. Given three positive scalars $c_1, c_2, T$, with $c_1 < c_2$ and a positive matrix $R$, the time delay NCSs (3) (setting $\omega(t) \equiv 0$) is said to be finite-time stable with respect to $(c_1, c_2, T, R)$, if

$$x^T(0)Rx(0) \le c_1 \Rightarrow x^T(t)Rx(t) < c_2 \quad \forall t \in [0,T] \tag{6}$$

**Discussion1.** Different with the concept of Lyapunov asymptotic stability, finite-time stable is a practical concept used to study the behavior of a system within a finite interval. A system is said to be finite-time stable if, once we fix a finite-time interval, its state remains within prescribed bounds during this time interval. Obviously, a system which is finite-time stable may be not Lyapunov asymptotically stable; conversely a Lyapunov asymptotically stable system could be not finite-time stable if its state exceeds the prescribed bounds.

**Definition2.** (Finite-time control via state feedback). Given three positive scalars $c_1, c_2, T$, with $c_1 < c_2$, a positive definite matrix $R$, the time delay NCSs(3) is finite-time boundedness with respect to $(c_1, c_2, T, R, d)$ if there exists a state feedback controller in the form (4) and the following condition holds

$$x^T(0)Rx(0) \le c_1 \Rightarrow x^T(t)Rx(t) < c_2 \quad \forall t \in [0,T]$$

**Lemma1** [3] For known constant $\varepsilon > 0$ and matrices $D, E, F$ which satisfying $F^T F \le I$, then the following matrix inequality is hold

$$DEF + E^T F^T D^T \le \varepsilon DD^T + \varepsilon^{-1} E^T E$$

**Lemma2** [4] The LMI

$$\begin{bmatrix} Y(x) & W(x) \\ W^T(x) & R(x) \end{bmatrix} > 0$$

is equivalent to

$$R(x) > 0, Y(x) - W(x)R^{-1}(x)W^T(x) > 0$$

Where $Y(x) = Y^T(x), R(x) = R^T(x)$ and $W(x)$ depend on $x$.

The following lemma states a sufficient condition for the finite-time boundedness of a certain linear time-invariant systems in the form

$$\dot{x}(t) = Ax(t) + G\omega(t) \tag{7}$$

which is fundamental to prove the main results in what follows.

**Lemma3**[16] The networked control Systems (7) is finite-time boundedness with respect to $(c_1, c_2, T, R, d)$ if, letting $\tilde{Q}_1 = R^{-(1/2)}Q_1R^{-(1/2)}$, there exist a positive scalar $\alpha$ and two symmetric positive definite matrices $Q_1 \in R^{n\times n}$ and $Q_2 \in R^{l\times l}$ such that

$$\begin{bmatrix} A\tilde{Q}_1 + \tilde{Q}_1A^T - \alpha\tilde{Q}_1 & GQ_2 \\ * & -\alpha Q_2 \end{bmatrix} < 0 \tag{8a}$$

$$\frac{c_1}{\lambda_{\min}(Q_1)} + \frac{d}{\lambda_{\min}(Q_2)} < \frac{c_2 e^{-\alpha T}}{\lambda_{\max}(Q_1)} \tag{8b}$$

Where $\lambda_{\min}(\cdot)$ and $\lambda_{\max}(\cdot)$ indicate the maximum and minimum eigenvalue of the argument, respectively.

## 3. Main Results

In this section, we consider the finite-time control synthesis for NCSs with state delay and communication delay, in terms of LMIs, we obtain the sufficient condition for the finite-time control via state feedback.

**Theorem1.** Given three positive scalars $c_1, c_2, T$, with $c_1 < c_2$, a positive definite matrix $R$, the time delay NCSs (3) is finite-time stabilizable via state feedback with respect to $(c_1, c_2, T, R, d)$, if there exist a scalar $\alpha \geq 0$, positive definite matrices $P \in R^{n\times n}$, $Q \in R^{n\times n}$, $M \in R^{n\times n}$, $S \in R^{l\times l}$, and matrix $K \in R^{m\times n}$ such that the following matrix inequalities hold

$$\begin{bmatrix} \Xi & P\bar{A}_h & P\bar{B}K & P\bar{B}_1 \\ * & -Q & 0 & 0 \\ * & * & -M & 0 \\ * & * & * & -\alpha S \end{bmatrix} < 0 \tag{9a}$$

and

$$\frac{c_1(\lambda_{\max}(\tilde{P}) + h\lambda_{\max}(\tilde{Q}) + \tau\lambda_{\max}(\tilde{M})) + d\lambda_{\max}(S)(1-e^{-\alpha T})}{\lambda_{\min}(\tilde{P})} < c_2 e^{-\alpha T} \qquad (9b)$$

where

$$\Xi = P\bar{A} + \bar{A}^T P + Q + M - \alpha P,\ \tilde{P} = R^{-1/2}PR^{-1/2},\ \tilde{Q} = R^{-1/2}QR^{-1/2},\ \tilde{T} = R^{-1/2}MR^{-1/2}$$

and $\lambda_{\max}(\cdot)$ and $\lambda_{\min}(\cdot)$ indicate the maximum and minimum eigenvalue of the augment, respectively.

The proof is given in Appendix 1.

**Theorem2.** Given three positive scalars $c_1, c_2, T$, with $c_1 < c_2$, a positive definite matrix $R$, the time delay NCSs(3) is finite-time stabilizable via state feedback $u(t) = \bar{K}X^{-1}x(t)$ with respect to $(c_1, c_2, T, R, d)$ if, there exist scalars $\alpha \ge 0,\ \lambda_i > 0,\ i = 1,2,3,4.$, positive definite matrices $X \in R^{n\times n}$, $\bar{Q} \in R^{n\times n}$, $\bar{M} \in R^{n\times n}$, $S \in R^{l\times l}$, and matrix $\bar{K} \in R^{m\times n}$ such that the following matrix inequalities hold:

$$\begin{bmatrix} \Theta & A_h X & B\bar{K} & B_1 & XE_1^T & 0 & 0 & 0 \\ * & -\bar{Q} & 0 & 0 & 0 & XE_2^T & 0 & 0 \\ * & * & -\bar{M} & 0 & 0 & 0 & \bar{K}^T E_3^T & 0 \\ * & * & * & -\alpha S & 0 & 0 & 0 & E_4^T \\ * & * & * & * & -\varepsilon_1 I & 0 & 0 & 0 \\ * & * & * & * & * & -\varepsilon_2 I & 0 & 0 \\ * & * & * & * & * & * & -\varepsilon_3 I & 0 \\ * & * & * & * & * & * & * & -\varepsilon_4 I \end{bmatrix} < 0 \qquad (17a)$$

$$\lambda_1 R^{-1} < X < R^{-1} \qquad (17b)$$

$$\lambda_2 \bar{Q} < \lambda_1 X \qquad (17c)$$

$$\lambda_3 \bar{M} < \lambda_1 X \qquad (17d)$$

$$0 < S < \lambda_4 I \qquad (17e)$$

$$\begin{bmatrix} d\lambda_4(1-e^{-\alpha T}) - c_2 e^{-\alpha T} & \sqrt{c_1} & \sqrt{h} & \sqrt{\tau} \\ * & -\lambda_1 & 0 & 0 \\ * & * & -\lambda_2 & 0 \\ * & * & * & -\lambda_3 \end{bmatrix} < 0 \qquad (17f)$$

Where

$$\Theta = AX + XA^T + \bar{Q} + \bar{M} - \alpha X + \varepsilon_1 D_1 D_1^T + \varepsilon_2 D_2 D_2^T + \varepsilon_3 D_3 D_3^T + \varepsilon_4 D_4 D_4^T$$

The proof is given in Appendix 2.

**Discussion2.** If condition (10a) in Theorem 2 is satisfied with $\alpha = 0$, then systems(3) is also asymptotically stable in the sense of Lyapunov. Moreover in this case the finite-time properties are guaranteed for all $T > 0$.

## 4. Numerical Example

With the introduction of network, the control signal exchanged through network in NCSs. The temperature control system for polymerization reactor is a inertia link with time delay. The state space model of polymerization reactor is usually written as

$$\begin{aligned} &\dot{x}_1(t) = x_2(t) \\ &\dot{x}_2(t) = -a_1 x_1(t) - a_2 x_2(t) + bu(t) \\ &y(t) = x_1(t) \end{aligned}$$

It is impossible to avoid the uncertainty and time delay. We consider the uncertain networked control systems as following

$$\dot{x}(t) = (A + \Delta A(t))x(t) + (A_h + \Delta A_h(t))x(t-h) + (B + \Delta B(t))u(t-\tau) + (B_1 + \Delta B_1(t))\omega(t)$$

where

$$A = \begin{bmatrix} 0 & 0.5 \\ 0 & 0.1 \end{bmatrix},\ A_h = \begin{bmatrix} 0 & 0.1 \\ 0.1 & 0 \end{bmatrix},\ B = \begin{bmatrix} 0 \\ 1 \end{bmatrix},\ B_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix},\ D_1 = \begin{bmatrix} 0.1 & 0.2 \\ -0.2 & 0 \end{bmatrix},$$

$$D_2 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix},\ D_3 = \begin{bmatrix} 0.05 & 0 \\ 0 & 0.05 \end{bmatrix},\ D_4 = \begin{bmatrix} -0.5 & 0 \\ 0 & 0.3 \end{bmatrix},\ F(t) = \sin t,$$

$$E_1 = \begin{bmatrix} 0.01 & -0.04 \\ 0 & 0.3 \end{bmatrix},\ E_2 = \begin{bmatrix} 0 & 0.1 \\ 0.02 & 0.1 \end{bmatrix},\ E_3 = \begin{bmatrix} 0 & 0 \\ 0.1 & 0.1 \end{bmatrix}, E_4 = \begin{bmatrix} 0 & 0.09 \\ 0.09 & 0 \end{bmatrix},$$

$$T = 5,\ h = 0.2,\ \tau = 0.1,\ \omega(t) = 0.5\cos t,\ x(t) = \begin{bmatrix} x_1^T(t) & x_2^T(t) \end{bmatrix}^T$$

In this case, we choose $c_1 = 0.25,\ c_2 = 0.5,\ T = 8,\ R = I_2$. By Solving the LMIs (10), we can obtain

$$X = \begin{bmatrix} 0.8323 & 0.4744 \\ 0.764 & 0.8926 \end{bmatrix}, \bar{Q} = \begin{bmatrix} 2.5921 & -0.8455 \\ -0.8995 & 0.7221 \end{bmatrix}, \bar{M} = \begin{bmatrix} 1.3342 & -0.2329 \\ -0.2679 & 1.0443 \end{bmatrix}$$

$$S = \begin{bmatrix} 2.64238 & -0.9654 \\ -0.9983 & 1.0557 \end{bmatrix}, \bar{K} = [-6.5321\ \ 4.9827], \alpha = 0.2167, \lambda_1 = 0.5484,$$

$$\lambda_2 = 1.5374, \lambda_3 = 0.2466, \lambda_4 = 0.8934$$

Then the state feedback controller is

$$u(t) = \bar{K}X^{-1}x(t) = [-6.5321\ \ 4.9827]x(t)$$

We select the initial condition

$$x(0) = \begin{bmatrix} 0.5 \\ -0.5 \end{bmatrix}$$

With the state feedback finite-time controller (4) in Theorem 2, the simulation results are shown in figs. 2.

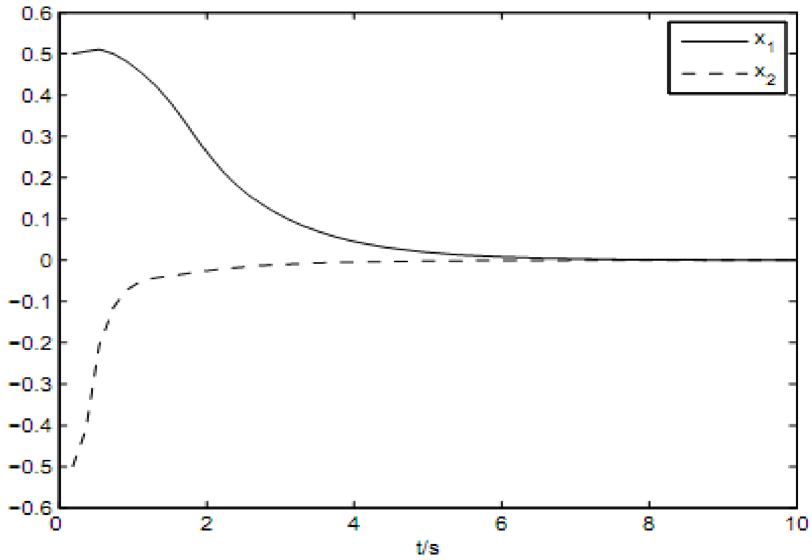


Fig.2. State responses of systems (3)

From the Fig.2, it is clear to see that the design method of state feedback finite-time control for NCSs (3) is effective.

## 5. Conclusion

A general theoretical result involving the Lyapunov functional gives a general sufficient condition for the finite-time stability of NCSs. Nevertheless, this result is not practical and cannot be used to stabilize in finite time a large class of linear systems. This is the reason why we consider the finite-time state feedback control problem has been investigated for a class of NCSs with state delay and communication delay in this paper. First of all, we have given the definition of finite-time stability of NCSs. Then, combined with LMIs techniques, we have provided a sufficient condition guaranteeing finite-time stability via state feedback. This condition has been turned into an optimization problem involving LMIs. A numerical example is provided to demonstrate the effectiveness of the proposed approach.

**6. Acknowledgments**

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## Appendix 1 Proof of Theorem 1

**Proof.** For given symmetric positive definite matrix $P, Q, M$, we construct the following Lyapunov–Krasovskii function:

$$V(x(t)) = x^T(t)Px(t) + \int_{t-h}^{t} x^T(\theta)Qx(\theta)d\theta + \int_{t-\tau}^{t} x^T(\theta)Mx(\theta)d\theta \tag{11}$$

The time derivative of $V(x(t))$ along the trajectories of systems (5) is given by

$$\begin{aligned}\dot{V}(x(t)) =& x^T(t)(P\bar{A} + \bar{A}^T P + Q + M)x(t) + 2x^T(t)P\bar{A}_h x(t-h) + 2x^T(t)P\bar{B}Kx(t-\tau) \\ &+ 2x^T(t)P\bar{B}_1\omega(t) + x^T(t-h)Qx(t-h) + x^T(t-\tau)Mx(t-\tau) \\ =& \begin{bmatrix} x(t) \\ x(t-h) \\ x(t-\tau) \\ \omega(t) \end{bmatrix}^T \Pi \begin{bmatrix} x(t) \\ x(t-h) \\ x(t-\tau) \\ \omega(t) \end{bmatrix}\end{aligned}$$

where

$$\Pi = \begin{bmatrix} P\bar{A} + \bar{A}^T P + Q + M & P\bar{A}_h & P\bar{B}K & P\bar{B}_1 \\ * & -Q & 0 & 0 \\ * & * & -M & 0 \\ * & * & * & 0 \end{bmatrix}$$

From condition (9a), we have

$$\dot{V}(x(t)) < \alpha x^T(t)Px(t) + \alpha\omega^T(t)S\omega(t) < \alpha V(x(t)) + \alpha\omega^T(t)S\omega(t) \tag{12}$$

Multiplying (12) by $e^{-\alpha t}$, we can obtain

$$e^{-\alpha t}\dot{V}(x(t)) - e^{-\alpha t}\alpha V(x(t)) < \alpha e^{-\alpha t}\omega^T(t)S\omega(t)$$

Furthermore

$$\frac{d}{dt}(e^{-\alpha t}V(x(t))) < \alpha e^{-\alpha t}\omega^T(t)S\omega(t)$$

By integrating the above inequality from $0$ to $t$, with $t \in [0, T]$, it follows that

$$e^{-\alpha t}V(x(t)) - V(x(0)) < \int_0^t \alpha e^{-\alpha\theta}\omega^T(\theta)S\omega(\theta)d\theta \tag{13}$$

Noting that $\alpha \geq 0$, $\tilde{P} = R^{-1/2} P R^{-1/2}$, $\tilde{Q} = R^{-1/2} Q R^{-1/2}$, and $\tilde{M} = R^{-1/2} M R^{-1/2}$, we can obtain the following relation:

$$
\begin{aligned}
x^T(t)Px(t) &\leq V(x(t)) \\
&< e^{\alpha t} V(x(0)) + \alpha d \lambda_{\max}(S) e^{\alpha t} \int_0^t e^{-\alpha\theta} d\theta \\
&< e^{\alpha t}[x^T(0)Px(0) + \int_{-h}^0 x^T(\theta)Qx(\theta)d\theta + \int_{-\tau}^0 x^T(\theta)Mx(\theta)d\theta + d\lambda_{\max}(S)(1-e^{-\alpha t})] \\
&< e^{\alpha t}[x^T(0)R^{1/2}\tilde{P}R^{1/2}x(0) + \int_{-h}^0 x^T(\theta)R^{1/2}\tilde{Q}R^{1/2}x(\theta)d\theta \\
&\quad + \int_{-\tau}^0 x^T(\theta)R^{1/2}\tilde{M}R^{1/2}x(\theta)d\theta + d\lambda_{\max}(S)(1-e^{-\alpha t})]
\end{aligned} \tag{14}
$$

$$
\begin{aligned}
&< e^{\alpha t}[\lambda_{\max}(\tilde{P})x^T(0)Rx(0) + \lambda_{\max}(\tilde{Q})\int_{-h}^0 x^T(\theta)Rx(\theta)d\theta \\
&\quad + \lambda_{\max}(\tilde{M})\int_{-\tau}^0 x^T(\theta)Rx(\theta)d\theta + d\lambda_{\max}(S)(1-e^{-\alpha t})] \\
&< e^{\alpha T}[c_1(\lambda_{\max}(\tilde{P}) + h\lambda_{\max}(\tilde{Q}) + \tau\lambda_{\max}(\tilde{M})) + d\lambda_{\max}(S)(1-e^{-\alpha t})]
\end{aligned}
$$

On the other hand, it yields

$$x^T(t)Px(t) = x^T(t)R^{1/2}\tilde{P}R^{1/2}x(t) \geq \lambda_{\min}(\tilde{P})x^T(t)Rx(t) \tag{15}$$

Putting together (14) and (15) we have

$$x^T(t)Rx(t) < \frac{e^{\alpha T}[c_1(\lambda_{\max}(\tilde{P}) + h\lambda_{\max}(\tilde{Q}) + \tau\lambda_{\max}(\tilde{M})) + d\lambda_{\max}(S)(1-e^{-\alpha T})]}{\lambda_{\min}(\tilde{P})} \tag{16}$$

The condition (9b) and inequality (16) imply,

$$x^T(t)Rx(t) \leq c_2, \ \forall\, t \in [0, T].$$

This completes the proof. Therefore, the proof follows.

## Appendix 2 Proof of Theorem 2

**Proof.** Now we prove that the inequality (9a) is equivalent to the inequality (10a). Inserting (5) into (9a), we have

$$
\begin{bmatrix}
\Xi & PA_h + P\Delta A_h(t) & PBK + P\Delta B(t)K & PB_1 + P\Delta B_1(t) \\
* & -Q & 0 & 0 \\
* & * & -M & 0 \\
* & * & * & -\alpha S
\end{bmatrix} < 0
$$

where

$$\Xi = PA + A^T P + Q + M - \alpha P + P\Delta A(t) + \Delta A^T(t)P$$

The above inequality is equivalent to

$$\begin{bmatrix} PA+A^TP+Q+M-\alpha P & PA_h & PBK & PB_1 \\ * & -Q & 0 & 0 \\ * & * & -M & 0 \\ * & * & * & -\alpha S \end{bmatrix}$$

$$+\begin{bmatrix} PD_1F(t)E_1+E_1^TF^T(t)D_1^TP & PD_2F(t)E_2 & PD_3F(t)E_3K & PD_4F(t)E_4 \\ * & 0 & 0 & 0 \\ * & * & 0 & 0 \\ * & * & * & 0 \end{bmatrix}<0$$

With Lemma1, we obtain that the above inequality is equivalent to

$$\begin{bmatrix} \Delta & PA_h & PBK & PB_1 \\ * & -Q+\frac{1}{\varepsilon_2}E_2^TE_2 & 0 & 0 \\ * & * & -M+\frac{1}{\varepsilon_3}K^TE_3^TE_3K & 0 \\ * & * & * & -\alpha S+\frac{1}{\varepsilon_4}E_4^TE_4 \end{bmatrix}<0$$

where

$$\Delta = PA+A^TP+Q+M-\alpha P+\varepsilon_1PD_1D_1^TP+\varepsilon_2PD_2D_2^TP+\varepsilon_3PD_3D_3^TP+\varepsilon_4PD_4D_4^TP+\frac{1}{\varepsilon_1}E_1^TE_1$$

With lemma2, we know that the above inequality is equivalent to

$$\begin{bmatrix} \Lambda & PA_h & PBK & PB_1 & E_1^T & 0 & 0 & 0 \\ * & -Q & 0 & 0 & 0 & E_2^T & 0 & 0 \\ * & * & -M & 0 & 0 & 0 & K^TE_3^T & 0 \\ * & * & * & -\alpha S & 0 & 0 & 0 & E_4^T \\ * & * & * & * & -\varepsilon_1I & 0 & 0 & 0 \\ * & * & * & * & * & -\varepsilon_2I & 0 & 0 \\ * & * & * & * & * & * & -\varepsilon_3I & 0 \\ * & * & * & * & * & * & * & -\varepsilon_4I \end{bmatrix}<0$$

where

$$\Lambda = PA+A^TP+Q+M-\alpha P+\varepsilon_1PD_1D_1^TP+\varepsilon_2PD_2D_2^TP+\varepsilon_3PD_3D_3^TP+\varepsilon_4PD_4D_4^TP$$

Pre-and post-multiplying the above inequality by block-diagonal matrix diag$\{P^{-1}, P^{-1}, P^{-1}, I, I, I, I, I\}$, we know the inequality (9a) is equivalent to

$$\begin{bmatrix} \Sigma & A_h P^{-1} & BKP^{-1} & B_1 & P^{-1}E_1^T & 0 & 0 & 0 \\ * & -P^{-1}QP^{-1} & 0 & 0 & 0 & P^{-1}E_2^T & 0 & 0 \\ * & * & -P^{-1}MP^{-1} & 0 & 0 & 0 & P^{-1}K^T E_3^T & 0 \\ * & * & * & -\alpha S & 0 & 0 & 0 & E_4^T \\ * & * & * & * & -\varepsilon_1 I & 0 & 0 & 0 \\ * & * & * & * & * & -\varepsilon_2 I & 0 & 0 \\ * & * & * & * & * & * & -\varepsilon_3 I & 0 \\ * & * & * & * & * & * & * & -\varepsilon_4 I \end{bmatrix} < 0 \quad (17)$$

where

$$\Sigma = AP^{-1} + P^{-1}A^T + P^{-1}QP^{-1} + P^{-1}MP^{-1} - \alpha P^{-1} + \varepsilon_1 D_1 D_1^T + \varepsilon_2 D_2 D_2^T + \varepsilon_3 D_3 D_3^T + \varepsilon_4 D_4 D_4^T$$

By letting $X = P^{-1}, \bar{K} = KP^{-1}, \bar{Q} = P^{-1}QP^{-1}, \bar{M} = P^{-1}MP^{-1}$, the inequality (17) is equivalent to inequality (10a).

On the other hand, we denote

$$\tilde{X} = R^{-1/2}XR^{-1/2}, \tilde{Q} = R^{-1/2}QR^{-1/2}, \tilde{M} = R^{-1/2}MR^{-1/2}$$

Consider that $R$ is the positive-definite matrix and

$$\lambda_{\max}(X) = \frac{1}{\lambda_{\min}(P)} \quad , \ \lambda_{\min}(X) = \frac{1}{\lambda_{\max}(P)}$$

Now inequalities (10b-10e) imply that

$$1 < \lambda_{\min}(\tilde{P}), \lambda_{\max}(\tilde{P}) < \frac{1}{\lambda_1}, \lambda_{\max}(\tilde{Q}) < \frac{\lambda_1}{\lambda_2}\lambda_{\max}(\tilde{P}), \lambda_{\max}(\tilde{M}) < \frac{\lambda_1}{\lambda_3}\lambda_{\max}(\tilde{P}), \lambda_{\max}(S) < \lambda_4 \quad (18)$$

With the Schur Lemma2, we know the inequality (10f) is equivalent to

$$d\lambda_4(1 - e^{-\alpha T}) - c_2 e^{-\alpha T} + \frac{c_1}{\lambda_1} + \frac{h}{\lambda_2} + \frac{\tau}{\lambda_3} < 0 \quad (19)$$

With (18), the condition (9b) follows that

$$\frac{c_1(\lambda_{\max}(\tilde{P}) + h\lambda_{\max}(\tilde{Q}) + \tau\lambda_{\max}(\tilde{M})) + d\lambda_{\max}(S)(1 - e^{-\alpha T})}{\lambda_{\min}(\tilde{P})} < d\lambda_4(1 - e^{-\alpha T}) + \frac{c_1}{\lambda_1} + \frac{h}{\lambda_2} + \frac{\tau}{\lambda_3} \quad (20)$$

Inserting the inequality (19) into (20), the inequality (9b) is satisfied. This completes the proof.

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