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Two Warehouse Inventory Model for Imperfect Quality Items with Allowable Proportionate Discount

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Abstract

In this paper, a two-warehouse inventory model having a percentage defective items is formulated and solved by introducing proportionate discount for imperfect quality items, where the warehouses possessa fixed capacity in contrast with the traditional inventory model which is assumed to have unlimited capacity. For the present model it is assumed that the inventory costs in rented warehouse(RW) is higher than the own warehouse(OW). The aim of this paper is to find the optimal lot size to maximize the total profit for the developed model by introducing the proportionate rate of discount for items with imperfect quality.Numerical example is provided to illustrate the results of the proposed model.The concavity of the profit function is shown graphically. Sensitivity analysis is conducted toshow the applicability of the proposed model by changing the values of different parameters presentin the derived mathematical expressions for both the actual lot size and the profit.

Key words

EOQ, imperfect quality, two warehouse, proportionate discount, inventory.

1. Introduction

Economic order quantity (EOQ) models containing a random fraction of imperfect quality items with a known probability distribution and all unit quantity discounts for defective items, have attracted the attention of researchers. Due to the presence of defective items, the supplier finds less profit than the actual one and simultaneously their unintentional supply to customers may charge the suppliers to lose goodwill. In practice, for the defective items the supplier usually offers defective quantity discounts to encourage the customers. Therefore, before the defective items goes to the market, it is very important for the decision makers to have a thorough screening of the whole lot and then to introduce a fixed / proportionate discount for the identified defective items present in each lot. In reality, such situations indicate to order more items than the actual demand. But the limited available storage space for the inventory creates a crucial situation. Therefore, due to the limited storage space (Own warehouse (OW)), an additional storage (rented warehouse (RW)) is assumed to be available with abundant space to hold the larger stock.

A good number of researchers have started focusing on two warehouse inventory problems. Hartely[1]was the first researcher who proposed the two warehouse inventory model where the items in RW are first transferred to OW to meet the demand until the stock level present in RW drops to zero for which the items of OW are released. Sarma [2] developed the Hartely [1] model to cover the fixed transportation cost, independent of the quantity transferred from the RW to the OW. Goswami and Chaudhuri [3] established an inventory model with linear time dependent demand and where the transportation cost depends on the quantity to be transported. Bhunia and Maiti [4] established a two ware-house deterministic inventory model by considering the level of inventory dependent on the rate of consumption. Later, Bhunia and Maiti [6] developed their model Bhunia and Maiti [4] by taking the time dependent linear demand with shortages and constant deterioration rate. Kar etal., [9] discussed two separate storage inventory model with a fixed and finite time horizon and the demand is considering linearly time dependent. Zhou and Yang [11] developed a two ware-house deterministic inventory model where the items with stock-level dependent demand. Lee [12] formulated first-in-first-out (FIFO) dispatching two warehouse inventory model with deteriorating items. Yang [13] extended two-warehouse inventory model for deteriorating items with shortages are allowed under the partial backlogging and inflationary effect. Chung and Huang [14] developed an inventory model by considering two ware-houses for deteriorating items under permissible delay in payments with limited storage capacity. Das et al. [15] discussed a two ware-house inventory model with supply chain under credibility measure. Dyeetal., [16] provided a two ware- house deterministic inventory model where the rate of deterioration of the items are different in rented ware house and own warehouse. The other extensions of the article related to this research area are given by Benkherouf [5], Zhou [7], Yang [10], Niu and Xie [17], Rong, Mahapatra, and Maiti [18] etc.

Chung et al., [19] developed a two warehouse inventory model with imperfect quality items by emphasising on the limited available storage space as well as a fixed discount taken for the defective units present in each lot of the inventory. Lin and Chin [20] developed a model by considering two types of errors(type I and type II)with imperfect quality and penalty costs under screening errors. In their model they investigated the screening errors and estimate the optimal solution with the effects of percentage of defective items. A two warehouse inventory model is developed by Jaggi et al. [21] with definite percentage of defective items and are of deteriorating in nature. Saxena etal. [22] presented a two warehouse production inventory model for deteriorating items with inflation under reliability consideration. The main objective of this paper is to optimize the total related cost for reliable production process. Sheikh and Patel [26] developed a two warehouse inventory model with different deterioration rates where the demand is taken to be a linear function of time, holding cost is also time dependent and shortages are backlogged completely. Rajendra kumar etal., [25] established a deterioration inventory model for two warehouses with shortages in owned warehouse only and the excess demand is partially backlogged. First the demand is fulfilled from the inventory in rented warehouse and after that the inventory in owned warehouse has been used. Kaliraman etal.[24] presented a two warehouse inventory model for deteriorating items with constant deterioration rate. In the present work exponential demand rate is considered with permissible delay in payment. The rented warehouse is provided with better facility for the stock than the owned warehouse. The objective of this model is to find the best replenishment policies for minimizing the total appropriate inventory cost. Jaggi, etal.[23] studied a two warehouse inventory model that jointly considers the imperfect quality items, deterioration, and one level of trade credit. Mandal and Giri [27] proposed a two warehouse integrated vendor-buyer model where the vendor extends a quantity discount offer to the buyer to clear the old stocks and to earn modest profit from large sales volume. Patro et al. [28] developed a single ware house inventory model in which the total profit is maximized using proportionate discount under learning effect for the defective items.

So far the researchers used proportionate discount for percentage of defectives, after doing a 100% screening, present in each lot of the single warehouse EOQ model to find the total profit as well as the optimal order lot size. But, the above mentioned papers did not include the proportionate discount for percentage of defectives in two warehouse inventory models.

Therefore, in this paper we develop a two warehouse inventory model with each lot having the percentage of defectives (known by a 100% screening) and finally reduce it to the classical EOQ model and then estimate the cycle wise total profit and the lot size by introducing the proportionate discount for the defective items present in each lot of the inventory. Finally, a numerical example is provided to illustrate the results of the proposed model and the sensitivity analysis is conducted to show the applicability of the model by changing the values of different parameters present in the derived mathematical expressions for the actual lot size and the profit.

2. Notations and Assumptions

The following fundamental assumptions and notations are being considered from K.J.Chunget al. [19], for developing the mathematical model in this paper:

Notations:

z	size of the order quantity
m	own warehouse storage capacity
D	demand rate
C_p	unit cost of variable
C_k	ordering cost
C_h	holding cost
p	defective items of percentage in z
$f(p)$	probability density function(p.d.f) of p
pz	is the amount of defectives with drawn from inventory
S_g	selling price of unit wise good quality item
w	screening rate
C_s	screening cost of one unit item
C_{h_R},	holding costs for items in the rented warehouse
C_{h_w}	holding costs for items in the own warehouse, respectively, $C_{h_R} \geq C_{h_w}$
t_{rw}	$= \frac{z-m}{w}$, screening time required to screen the inventory in rented warehouse
t_{ow}	$= \frac{m}{w}$, screening time required to screen the inventory in own warehouse
t_0	time to use up of the rented warehouse
T	cycle length

Assumptions

1. The lot size z delivered instantaneously.
2. By the whole of z units, m units are kept in OW and $z-m$ units in RW.

3. If $z \leq m$, z units are only kept in OW. Simultaneously, the items are screened with unit screening cost C_s in OW and RW.
4. Known probability density function for percentage of defectives present in a lot.
5. Fixed selling price of good quality items.
6. Single batch selling of defective items at a proportionate discounted price.
7. Shipment wise 100% inspection of items.
8. Shortages are not allowed.
9. Lead time is zero.

3. The Mathematical Model

Case 1: When $z \geq m$.

Then the items of good quality with respect to rented warehouse is

$$(z-m)-(z-m)p=(1-p)(z-m) \tag{1}$$

And the items of good quality items with respect to own warehouse is

$$m-pm=(1-p)m \tag{2}$$

to avoid shortages the number of good items, with respect to rented warehouse and own warehouse are at least equal to the demands during times t_{rw} and t_{ow}, that is

$$(1-p)(z-m) \geq Dt_{rw} \tag{3}$$

$$(1-p)m \geq Dt_{ow} \tag{4}$$

Substituting $t_{rw} = \dfrac{z-m}{w}$ and $t_{ow} = \dfrac{m}{w}$, in Eq. (3) and Eq.(4) we have,

$$p \leq 1-\frac{D}{w} \tag{5}$$

Case 2: When $z < m$.

Then $z - zp = \left(1 - p\right) z$ (6)

According to K. J. Chung et al. [19], we have

$$p \le 1 - \frac{D}{w} \tag{7}$$

From Eq. (5) and (7), we conclude that

$$p \le 1 - \frac{D}{w}$$

All products screen in OW and RW simultaneously at the same time.

$TC\left(z\right)$=fixed ordering cost + unit cost of variable + screening cost with regard to cycle wise lot size + holding cost

$$TC\left(z\right) = C_k + C_p z + C_s\, z + C_h \times \left(\frac{z(1-p)T}{2} + \frac{p\, z^2}{w} \right) \tag{8}$$

$TR\left(z\right) =$ total sales with regard to good quality items $+$ total sales with regard to imperfect type of items.

$$TR(z) = S_g\, z(1-p) + \sum_{i=1}^{z\,p} (S_g - (1 - \frac{z\,p - i}{z\,p})(\frac{TR(z) - TC(z)}{z}))$$

$$TR\left(z\right) = \frac{2S_g\, z^2 + \left\{ C_k + C_p z + C_s\, z + C_h \times \left(\frac{z(1-p)T}{2} + \frac{p\, z^2}{w} \right) \right\} (zp+1)}{2z + (z\,p + 1)} \tag{9}$$

For computation of $TC\left(z\right)$, there are arises two cases.

Case 1:

When $z \le m$. We have

$$ETPU_1(z)=\left\lfloor \frac{2DS_g\,z-2DC_k-2DC_p\,z-2DC_s\,z}{2z+zE[p]+1}E\left(\frac{1}{1-p}\right)-\frac{C_{h_w}z^2\left(1+E[p]\right)}{2z+zE[p]+1}\right\rfloor \tag{10}$$

Case 2:

When $z \geq m$. We have the following two sub-cases:

Case A: when $t_0 \geq t_{ow}$. Then

$TC\left(\bar{y}\right)=$ purchasing cost +screening cost + carrying cost+holding cost

$$=C_k+C_pz+C_s\,z+C_{h_R}\left\lfloor \frac{t_0\left(z-m\right)\left(1-p\right)}{2}+p\left(z-m\right)t_{rw}\right\rfloor$$

$$+C_{h_w}\left\lfloor mt_{ow}+m\left(t_0-t_{ow}\right)\left(1-p\right)+\frac{m\left(T-t_0\right)\left(1-p\right)}{2}\right\rfloor \tag{11}$$

Case B: when $t_0 < t_{ow}$. Then

$$TC(z)=C_k+C_pz+C_s\,z+C_{h_R}\left\lfloor \frac{t_0\left(z-m\right)\left(1-p\right)}{2}+p\left(z-m\right)t_{rw}\right\rfloor$$

$$+C_{h_w}\left\lfloor mt_0+\left(t_{ow}-t_0\right)pm+\frac{\left(T-t_0\right)\left(m-pm\right)}{2}\right\rfloor \tag{12}$$

where $t_0=\dfrac{\left(z-m\right)\left(1-p\right)}{D}$, $t_{rw}=\dfrac{z-m}{w}$, $t_{ow}=\dfrac{m}{w}$ and $T=\left(\dfrac{\left(1-p\right)z}{D}\right)$

From Eq. (11) and Eq. (12) in both the cases after substituting the value of t_0,t_{rw},t_{ow} and T we get as follows:

$$TC\left(z\right)=C_k+C_pz+C_s\,z+C_{h_R}\left\lfloor \frac{\left(z-m\right)^2\left(1-p\right)^2}{2D}+\frac{p\left(z-m\right)^2}{w}\right\rfloor \tag{13}$$

$$+C_{h_w}\left\lfloor \frac{m\,z(1-p)^2}{D}-\frac{m^2(1-p)^2}{D}+\frac{m^2p}{w}\right\rfloor$$

$$TC(z) = C_k + C_p z + C_s z + C_{h_R}\left[\frac{(z-m)^2(1-p)^2}{2D} + \frac{p(z-m)^2}{w}\right]$$

$$+C_{h_w}\left[\frac{mz(1-p)^2}{D} - \frac{m^2(1-p)^2}{D} + \frac{m^2 p}{w}\right] \tag{14}$$

Eq. (13) and (14) are same each other. So, we conclude the total cost per unit time is given as follows:

$$TC(z) = C_k + C_p z + C_s z + C_{h_R}\left[\frac{(z-m)^2(1-\bar{p})^2}{2D} + \frac{p(z-m)^2}{w}\right]$$

$$+C_{h_w}\left[\frac{m z(1-p)^2}{D} - \frac{m^2(1-p)^2}{D} + \frac{m^2 p}{w}\right] \tag{15}$$

Consequently, the cycle wise total profit $TP(z)$ is the difference between the cycle wise total revenue $TR(z)$ and the cycle wise total cost $TC(z)$.

$$TP(z) = TR(z) - TC(z)$$

$$TP(z) = \frac{2S_g z^2 - 2z\left\{\begin{array}{l} C_k + C_p z + C_s z + C_{h_R}\left[\frac{(z-m)^2(1-p)^2}{2D} + \frac{p(z-m)^2}{w}\right] \\ +C_{h_w}\left[\frac{m z(1-p)^2}{D} - \frac{m^2(1-p)^2}{\bar{D}} + \frac{m^2 p}{w}\right]\end{array}\right\}}{2z + z p + 1} \tag{16}$$

The unit time wise total profit is obtained by dividing the cycle wise total profit $TP(z)$ by the cycle length T and the unit time wise total profit $TPU(z)$ is given as follows.

$$TPU(z) = TP(z)/T$$

$$TPU(z) = \frac{2DS_g z - 2DC_k - 2DC_p z - 2DC_s z}{(2z + z p + 1)(1-p)} - \frac{C_{h_R}(z-m)^2(1+p)}{(2z + zp + 1)}$$

$$+\frac{C_{h_w}(1-3p)m^2}{(2z + z p + 1)} - \frac{2zmC_{h_w}(1-p)}{(2z + z p + 1)} \tag{17}$$

Since p is the percentage of defective items, which is a random variable with a known probability density function $f(p)$, the expected value of Eq. (17), $ETPU_2(z)$, is given as

$$ETPU_2(z)=\frac{2DS_g\,z-2DC_k-2DC_p\,z-2DC_s\,z}{(2z+z\,E[p]+1)}E\left(\frac{1}{1-p}\right)-\frac{C_{h_R}(z-m)^2(1+E[p])}{(2z+zE[p]+1)}$$

$$+\frac{C_{h_w}m^2(1-3E[p])}{(2z+zE[p]+1)}-\frac{2zmC_{h_w}(1-E[p])}{(2z+zE[p]+1)} \qquad (18)$$

Combining the Eqs. (10) and (18), we have

$$ETPU(z)=\begin{cases} ETPU_1(z) & if \quad z\le m \qquad (19a)\\ ETPU_2(z) & if \quad z\ge m \qquad (19b)\end{cases} \qquad (19)$$

And $ETPU_1(m)=ETPU_2(m)$ (20)

Therefore, $ETPU(z)$ is continuous on $z>0$. Eqs. (19a) and (19b) yield the first and second derivatives of $ETPU_1(z)$ and $ETPU_2(z)$, respectively.

$$ETPU_1'(z)$$

$$\left(\frac{1}{(2z+zE[p]+1)^2}\right)\left[\begin{array}{l}\left(2DS_g-2DC_p-2DC_s+4DC_k+2DC_kE[p]\right)E\left(\frac{1}{1-p}\right)-2C_{h_w}z^2-C_{h_w}z^2\left(E[p]\right)^2\\ -3C_{h_w}z^2E[p]-2C_{h_w}z-2C_{h_w}zE[p]\end{array}\right]$$

(21)

$$ETPU_1''(z)=-\left(\frac{2}{(2z+zE[p]+1)}\right)$$

$$\left[\left(2+E[p]\right)\left(2DS_g-2DC_p-2DC_s+4DC_k+2DC_kE[p]\right)E\left(\frac{1}{1-p}\right)+C_{h_w}+C_{h_w}E[p]\right]<0$$

(22)

$$ETPU_2'(z)=\left(\frac{1}{(2z+zE[p]+1)^2}\right)$$

$$\left[\begin{array}{l}\left(2DS_g-2DC_p-2DC_s+4DC_k+2DC_kE[p]\right)E\left(\frac{1}{1-p}\right)\\ -2C_{h_R}z^2-3C_{h_R}z^2E[p]-2C_{h_R}z+2C_{h_R}m+2C_{h_R}m^2+3C_{h_R}m^2E[p]\\ -C_{h_R}z^2(E[p])^2-2C_{h_R}zE[p]+2C_{h_R}mE[p]+C_{h_R}m^2(E[p])^2-2C_{h_w}m^2\\ +5C_{h_w}m^2E[p]+3C_{h_w}m^2(E[p])^2-2C_{h_w}m+2C_{h_w}mE[p]\end{array}\right] \quad (23)$$

$$ETPU_2''(z)=-\left(\frac{2}{(2z+zE[p]+1)^3}\right)$$

$$\left[\begin{array}{l}\left(2+E[p]\right)\left(2DS_g-2DC_p-2DC_s+4DC_k+2DC_kE[p]\right)E\left(\frac{1}{1-p}\right)\\ +4C_{h_R}m+4C_{h_R}m^2+8C_{h_R}m^2E[p]+6C_{h_R}mE[p]+7C_{h_R}m^2(E[p])^2\\ -4C_{h_w}m^2+8C_{h_w}m^2E[p]+11C_{h_w}m^2(E[p])^2-4C_{h_w}m\\ +2C_{h_w}mE[p]+2C_{h_R}m(E[p])^2+C_{h_R}m^2(E[p])^3+3C_{h_w}m^2(E[p])^3\\ +2C_{h_w}m(E[p])^2+C_{h_R}+C_{h_R}(E[p])\end{array}\right] \quad (24)$$

From Eq. (24) we find that $ETPU_2''(z)<0$ i.e., $ETPU_2(z)$ is concave in z .Again, because $E\left(\frac{1}{1-p}\right)$ is greater than or equal to one and C_{h_R} is more than C_{h_w} ,

Eq. (22) and (24) imply that $ETPU_1(z)$ and $ETPU_2(z)$ are concave on $z>0$. Eq.(19a) and (19b) imply that $ETPU(z)$ is piecewise concave on $z>0$.

We obtain the following values of z_1^* and z_2^* by solving the equations $ETPU_1'(z_1^*)=0$ and $ETPU_1'(z_2^*)=0$

$$z_1^*=\sqrt{\frac{\left(2DS_g-2DC_p-2DC_s+4DC_k+2DC_kE[p]\right)E\left(\frac{1}{1-p}\right)}{2C_{h_w}+3C_{h_w}E[p]+C_{h_w}\left(E[p]\right)^2}} \quad (25)$$

$$z_2^*=\sqrt{\frac{\begin{array}{l}\left(2DS_g-2DC_p-2DC_s+4DC_k+2DC_kE[p]\right)E\left(\frac{1}{1-p}\right)+2C_{h_R}m-2C_{h_R}m^2\\ -3C_{h_R}m^2E[p]+2C_{h_R}m(E[p])+C_{h_R}m^2(E[p])^2-2C_{h_w}m^2+5C_{h_w}m^2E[p]\\ +3C_{h_w}m^2(E[p])^2-2C_{h_w}m+2C_{h_w}mE[p]\end{array}}{2C_{h_R}+3C_{h_R}E[p]+\frac{2C_{h_R}}{z}+C_{h_R}\left(E[p]\right)^2+\frac{2C_{h_R}(E[p])}{z}}}$$

For large value of z, $\frac{1}{z} \to 0$

$$z_2^* = \sqrt{\frac{\begin{array}{l}\left(2DS_g - 2DC_p - 2DC_s + 4DC_k + 2DC_k E[p]\right) E\left(\frac{1}{1-p}\right) + 2C_{h_R} m - 2C_{h_R} m^2 \\ -3C_{h_R} m^2 E[p] + 2C_{h_R} m(E[p]) + C_{h_R} m^2 (E[p])^2 - 2C_{h_w} m^2 + 5C_{h_w} m^2 E[p] \\ +3C_{h_w} m^2 (E[p])^2 - 2C_{h_w} m + 2C_{h_w} m E[p]\end{array}}{2C_{h_R} + 3C_{h_R} E[p] + C_{h_R} \left(E[p]\right)^2}} \tag{26}$$

When $m = 0$ and $C_{h_R} = C_{h_w}$, the above equation reduces to Eq.(25).

$$z_2^* = \sqrt{\frac{\left(2DS_g - 2DC_p - 2DC_s + 4DC_k + 2DC_k E[p]\right) E\left(\frac{1}{1-p}\right)}{2C_{h_w} + 3C_{h_w} E[p] + C_{h_w} \left(E[p]\right)^2}} \tag{27}$$

If imperfect quality is not allowed, then $p = 0$ and $C_s + C_p = S_g$ then Eq. (27) reduces to the traditional EOQ formula which is given as follows:

$$z_2^* = \sqrt{\frac{2DC_k}{C_h}} \tag{28}$$

5. Numerical Results

For an inventory system, we adopted the values of the parameters of K. J. Chung et al.[19]for solving the equations of the model:

The values in the numerical example are:

D =50 000 units/year, C_k =\$100/cycle, C_{h_R} =\$7/unit/year, C_{h_w} =\$5/unit/year, C_s =\$0.5/unit, C_p = \$25/unit, S_g = \$50/unit, and w =1 unit/min= 175200 unit/year,Storage capacity of OW, m =800 units/cycle

As the percentage of defective items p is a random variable and it is distributed uniformly with a known probability density function is given as follows:

$$f(p) = \begin{cases} 25, & 0 \le p \le 0.05 \\ 0 & otherwise \end{cases}$$

By considering the above conditionwe get the expected values of p are given as follows: $E[p] = 0.02$, and $E\left[1/(1-p)\right] = 1.02055$

Eq. (25) and(26) yield $z_1^* = 1514.36955$ and $z_2^* = 1349.69$

And $ETPU(z_2^*)$=1230030

6. Sensitivity Analysis for the Model

Sensitivity analysis is performed by giving a percentage change to the valuesof each of the following parameters by 25%, 15%, -15% and -25%, where change in % of one parameter is considered keeping the remaining parameters unchanged.

Parameters	**Percentage change**	z_2^*	$ETPU(z_2^*)$	**%change in** $ETPU(z_2^*)$
D	+25	1490.75	1538480	25.08
	+15	1435.99	1415090	15.05
	-15	1257.48	1045010	-15.04
	-25	1192.05	921698	-25.07
S_g	+25	1382.07	1861130	51.32
	+15	1369.21	1608810	30.79
	-15	1329.88	851255	-30.79
	-25	1316.52	598738	-51.32
C_k	+25	1476.14	1229140	-0.07
	+15	1426.91	1229490	-0.04
	-15	1267.78	1230610	0.05
	-25	1210.09	1231020	0.08
C_p	+25	1333.21	914384	-25.66
	+15	1339.82	1040640	-15.4
	-15	1359.48	1419420	15.4
	-25	1365.97	1545680	25.66

C_s	+25	1349.36	1223720	-0.51
	+15	1349.49	1226240	-0.31
	-15	1349.89	1233820	0.31
	-25	1350.02	1236350	0.51
C_{h_R}	+25	1259.16	1229860	-0.01
	+15	1291.37	1229920	-0.01
	-15	1424.8	1230160	0.01
	-25	1488.38	1230270	0.02
C_{h_w}	+25	1310.04	1229340	-0.06
	+15	1326.04	1229620	-0.03
	-15	1372.93	1230450	0.03
	-25	1388.21	1230730	0.06
m	+25	1394.54	1230150	0.01
	+15	1375.59	1230120	0.01
	-15	1327	1229930	-0.01
	-25	1313.74	1229840	-0.02

7. Observation

From the above table we observed the following:

i. When the parameters D, S_g and m are increased by 25% and 15%, both z_2^* and $ETPU(z_2^*)$ also increase. Similarly, when D, S_g and m are decreased by 15% and 25%, both z_2^* and $ETPU(z_2^*)$ decrease.

ii. When the parameter C_k is increased by 25% and 15%, z_2^* increases and $ETPU(z_2^*)$ decreases. Similarly, when C_k is decreased by 15% and 25%, z_2^* decrease and $ETPU(z_2^*)$ increases.

iii. When the parameters C_p, C_s, C_{h_R} and C_{h_w} are increased by 25% and 15%, both z_2^* and $ETPU(z_2^*)$ decrease. Similarly, when C_p, C_s, C_{h_R} and C_{h_w} are decreased by 15% and 25%, both z_2^* and $ETPU(z_2^*)$ increase.

So, the decision maker, after analyzing the above results, can plan for the optimal value for total profit, and for other related parameters.

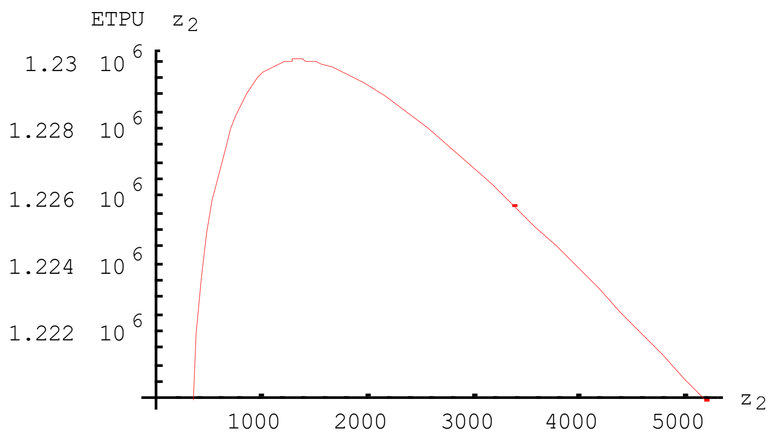

Fig.1. $ETPU(z_2^*)$ vs z_2^* Graphical Analysis for the Expected Total Profit of the Model

The 2D graph (Fig.1) shows the concavity of the expected total profit function $ETPU(z_2^*)$ (obtained with respect to the expected total profit verses the lot size ranging from 1100 to 1570).

Conclusion

In the proposed model,we have discussed a two warehouse inventory model for items with imperfect quality by introducing allowable proportionate discount where each lot contains a percentage of defective items (known by a 100% screening process).The inventory model developed in the present paper is found to be more realistic than the modelby K. J. Chung et al. [19], where there were constant discount for all the imperfect qualityitems and rented warehouse for the storage of the additional units ordered.The expected total profit per unit time in the developed model is more than the K. J. Chung etal.[19] model. Numerical example was considered to illustrate the model.

It is hoped that this paper can provide insights to develop fuzzy imperfect type EOQ/EPQ models with different rate of discounts for two warehouse problems.

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