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Optimal Pricing and Ordering Policies for Deteriorating Items with Two-Level Trade Credits under Price-Sensitive Trended Demand

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Abstract

In this paper, an inventory model is derived for price-sensitive trended demand. The units in inventory system are subject to deterioration at a constant rate. In this model, the supplier offers a fixed credit period to the retailer and the retailer also offers credit period to the customers which is more practical scenario in the market. Our goal is to maximize the profit of the retailer. The proposed inventory model is validated by a numerical example. Sensitivity analysis is performed to determine critical inventory parameters.

Key words

EOQ, price-sensitive trended demand, two level trade credits, deterioration.

1. Introduction

In the traditional model, the retailer pays for items at the time of delivery. To give credit period is a good strategy to increase profit for the players of the supply chain. Goyal (1985) developed the EOQ model by considering permissible delay in payments. Jamal *et al*. (1997) extended the above model by allowing shortages. Sarker *et al.* (2000) developed supply chain model for deteriorating items incorporating inflation and permissible delay in payment. Chang (2004) analyzed an EOQ model for deteriorating items incorporating inflation and credits-linked to order quantity. Chung and Huang (2003) discussed an EPQ inventory model for optimal cycle time with permissible delay in payment. Teng et al. (2005) developed model to compute optimal pricing and ordering policies when delay in payments is offered.

In previous articles, supplier offers the credit period to the retailer only. But the retailer is not forwarding to customers. Huang (2003) derived an inventory model in which the retailer is getting a credit period M from supplier and passing credit period N to his customers; with N < M. This is known as two-level trade credits financing. Shah et al. (2013 a) analyzed an inventory model with trade credit linked to order quantity for deteriorating items under stock-dependent demand. Shah *et al*. (2013 b) discussed an inventory model with two players in the supply chain for price sensitive trapezoidal demand with net credit scenario for deteriorating inventory.

In this article, we developed inventory system for price-sensitive time dependent demand with two-level trade credits for deteriorating items. Retailer gets credit period M from supplier and passes credit period N to his customers. The aim is to make the total profit maximum of the retailer with respect to pricing and ordering per unit time.

2. Assumptions and Notations

2.1 Notations

$R(s,t)$ Price-sensitive linear demand rate. The functional form is $R(s,t)=a(1+bt)s^{-\eta}$ where $a>0$ is known constant scale demand; $0<b<1$ is the rate at which demand is increasing linearly with time; s is retail price and $\eta>1$ is price mark up

θ Rate of deterioration of units in inventory model; $0<\theta<1$

A Ordering cost/ order

C Purchase cost / unit

s Retail price/unit(decision variable); $s>C$

h Inventory holding cost (without interest charges) /unit/unit time

M Credit period offered by the supplier to the retailer (in years)

N Credit period offered by the retailer to the customer (in years)

I_e Interest earned /\$ per year

I_c Interest charged /\$ for unsold items per year by the supplier; Note: $I_c>I_e$

$I(t)$ Inventory level at time t , $0\le t\le T$

T Cycle time (decision variable)

Q Purchase quantity /cycle (decision variable)

$\pi(s,T)$ Retailer's total profit /unit time

2.2 Assumptions

1. Inventory system deals with single item.
2. The planning horizon is infinite.
3. Inventory system does not allow shortages and lead-time is zero or negligible.
4. The units in inventory are deteriorating at a constant rate 'θ', $0 < \theta < 1$.
5. Retailer gets inflow of revenue in $\left[N, T+N\right]$ by giving credit period N to the customers.
6. When $M \le T+N,$ retailer will pay at M and for the unsold items, the retailer would pay interest during $\left[M, T+N\right]$ at the rate I_c. When $M > T+N,$ the retailer will pay at M and does not pay any interest during the cycle.
7. As cycle begins, the retailer sells items and gets revenue as well as interest till the end of credit period given by supplier. This suggests that for $N \le M,$ the retailer accumulates revenue and earns interest during $\left[N, M\right]$ with rate I_e.

3. Mathematical Model

The inventory level at any instant of time t is governed by the differential equation

$$\frac{dI(t)}{dt} = -R(s,t) - \theta I(t), \ 0 \le t \le T \tag{1}$$

with the boundary conditioned $I(T) = 0$. The solution of differential equation (1) is

$$I(t) = as^{-\eta}\left[\frac{(1+bT)e^{\theta(T-t)} - 1 - bt}{\theta} + \frac{b\left(1-e^{\theta(T-t)}\right)}{\theta^2}\right], \ 0 \le t \le T \tag{2}$$

Using $I(0) = Q$, the retailer's purchase quantity per cycle is

$$Q = I(0) = as^{-\eta}\left[\frac{(1+bT)e^{\theta T} - 1}{\theta} + \frac{b\left(1-e^{\theta T}\right)}{\theta^2}\right] \tag{3}$$

The sales revenue is $SR = s\int_0^T R(s,t)dt = s^{-\eta+1}aT\left[1+\frac{bT}{2}\right]$

The ordering cost is $OC = A$

The purchase cost of Q-units is $PC = CQ$

The holding cost is $HC = h\int_0^T I(t)dt$

For calculating interest charged and earned, we have two cases depending on the duration of M and N.

(A) Suppose $M \geq N$

Case 1 $M \geq T + N$

Since $M \geq T + N$, the retailer has no item left to sell in the inventory system, so the interest charges in the cycle is zero i.e. $IC_1 = 0$. The retailer generates income from the commencement of the cycle and settles the account at time N. So the retailer gets interest at the rate I_e per dollar per year starting from N through M. Therefore, the interest earned per cycle is

$$IE_1 = s\,I_e\left[\int_0^T\int_0^t R(u)\,du\,dt + (M - T - N)\int_0^T R(t)\,dt\right]$$
$$= \frac{a\,s^{-\eta+1} I_e\,T}{6}\left[3(bT+2)(M-N) - T(3+2bT)\right]$$

Case 2 $M \leq T + N$

Here, the retailer does not have adequate fund to settle the account at M as customer will be paying at time $T + N$. So the retailer will have to pay interest charges during $[M, T+N]$ for unsold items at an interest rate I_c per dollar per year. Hence, the interest payable in each cycle is

$$IC_2 = C\,I_c \int_M^{T+N} I(t-N)\,dt = C\,I_c \int_{M-N}^{T} I(t)\,dt$$

Here, the retailer earns interest on the collected revenue during $[N, M]$, which is given by

$$IE_2 = s\,I_e \int_N^M \int_N^{t+N} R(u-N)\,du\,dt = \frac{s^{-\eta+1}\,a\,I_e}{6}\left[T^2(3+bT) + 3b\left(M^2 - N^2\right) + 6(M-N) - 6bMN\right]$$

Hence, the average profit per unit time for retailer is

$$\pi(s,T) = \begin{cases} \pi_1(s,T) = \dfrac{1}{T}\left\{SR - PC - OC - HC - PTI - IC_1 + IE_1\right\}, 0 \leq T \leq M - N & (4) \\ \pi_2(s,T) = \dfrac{1}{T}\left\{SR - PC - OC - HC - PTI - IC_2 + IE_2\right\}, T \geq M - N & (5) \end{cases}$$

(B) Suppose $M \leq N$

Here, the retailer does not get any interest *i.e.* $IE_3 = 0$.

Retailer has to pay interest for all the items. Therefore, the interest charged per cycle is

$$IC_3 = C\,I_c\left[(N-M)Q + \int_N^{T+N} I(t-N)\,dt\right]$$

The average profit per unit time is

$$\pi_3(s,T)=\frac{1}{T}\{SR-PC-OC-HC-PTI-IC_3+IE_3\} \quad (6)$$

The goal is to maximize the average profit per unit time $\pi_i(s,T), i=1,2,3$ with respect to retail price and cycle time. The extremely non-linearity of the objective functions in equations (4 - 6) renders to obtain the closed form solution.

4. Computational Algorithm and Numerical examples

Differentiate $\pi_i(s,T), i=1,2,3$ with respect to s and T and compare with zero and follow the steps given below.

Step 1: Allocate values to all inventory parameters.

Step 2: For $M \geq N$, solve simultaneously $\frac{\partial \pi_1}{\partial s}=0$ and $\frac{\partial \pi_1}{\partial T}=0$ and also $\frac{\partial \pi_2}{\partial s}=0$ and $\frac{\partial \pi_2}{\partial T}=0$.

If $M \geq T+N$, then work out π_1 from equation (4), else π_2 from equation (5). After having (s,T), the retailers purchase quantity can be found using equation (3).

Step 3: For $M < N$, solve $\frac{\partial \pi_3}{\partial s}=0$ and $\frac{\partial \pi_3}{\partial T}=0$ simultaneously and get the average profit per unit time π_3 from equation (6) and purchase quantity using equation (3).

5. Numerical examples

Example 1: Take $A=\$50$ per order, $C=\$6$ per unit, $h=\$2.5$/unit/year, $a=10^4$ units, $b=10\%$, $\eta=1.2$, $I_e=10\%/\$/$ year, $I_c=15\%/\$/$ year, $\theta=0.2, M=0.8$ years and $N=0.3$ years.

Here: π_1 is \$ 4073.49 at $(T=T_1=0.41, s=s_1=\$39.65)$ while π_2 is \$ 4251.27 at $(T=T_2=0.559, s=s_2=\$40.6)$.

So, the retailer's optimum profit is $\pi^*(=\pi_2^*)=\$4251.27$ for purchasing $Q^*=71.46$ units. In figure 1, 3-D plot of the average total profit per unit time demonstrates concavity with respect to retail price and cycle time.

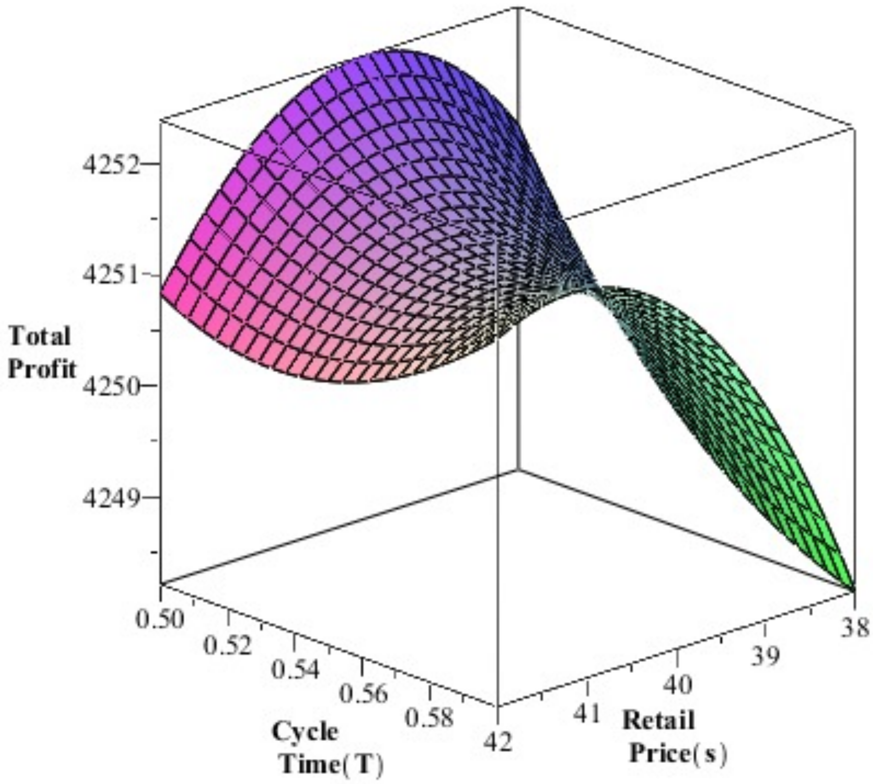

Fig.1 Concavity of total profit for cycle time T and retail price s

Example 2: Take $A = \$50$ per order, $C = \$3$ per unit, $h = \$2.5$/unit/year, $a = 10^4$ units, $b = 10\%$, $\eta = 2.2$, $I_e = 15\% / \$ /$ year, $I_c = 10\% / \$ /$ year, $\theta = 0.2$, $M = 0.7$ year and $N = 0.8$ year. We have $M < N$. Then as step 3 of the above algorithm, we get π_3 is $\$337.79$ at ($T = T_3$=0.46, $s = s_3$=\$7.06).

Now we want to study how variations in inventory parameters influence profit function. For that we carry out sensitivity analysis. We vary the parameters given in example 1 by -40%, -20%, 20% and 40%. The outcome of variations in profit is demonstrated in Figure 2.

Observations

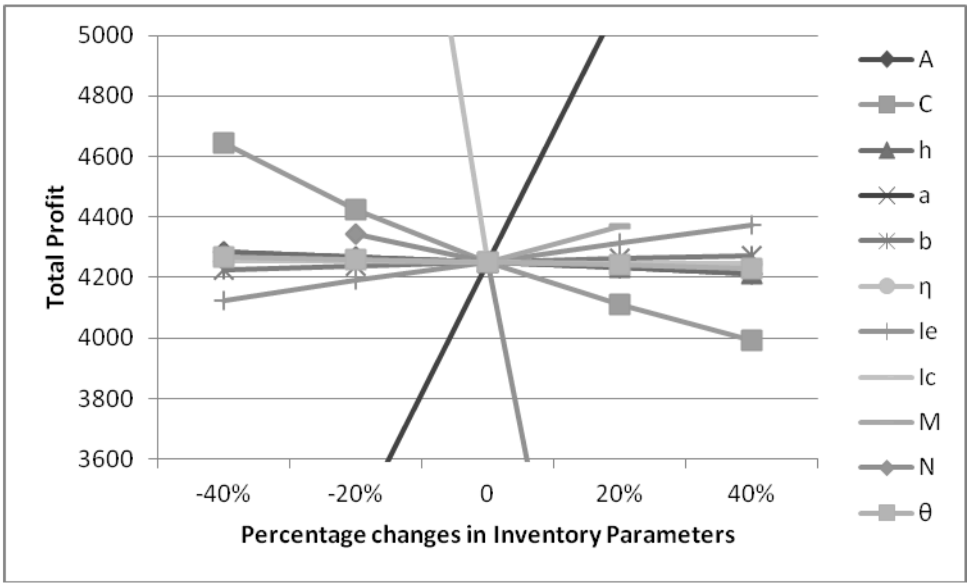

Fig. 2 Sensitivity analysis for total average profit per unit time (π)

From figure 2, it is observed that the average total profit per unit time increases drastically with respect to increase in a, and increases slowly with increase in b, I_e and credit period to the retailer given by the supplier M and decreases highly with increase in A, C, N, η.

This advises that by ordering more and taking advantage of smaller delay period frequently, the retailer can reduce ordering cost.

5. Conclusions

Here, an inventory model is explained which maximizes the total profit of the retailer considering the optimal pricing and ordering strategies. The demand is price-sensitive time dependent increasing. The items in the system have constant rate of deterioration. It is suggested that the retailer should take benefit of delay period by placing smaller orders frequently and some credit should be forwarded to his customers. Due to this retailer can get his account settled earlier and also reduce risk of default from customer.

Further research can be made to study inventory models with stochastic demand, stochastic deterioration rate and different preservation technologies to control deterioration of items & risk analysis.

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