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Modeling of Fluid Flow and Heat Transfer inside a Saturated Porous Conduit at Constant Surface Heat Flux

*Hassan Adeeb AlMasa'deh, **Hamzeh Mustafa Duwairi

*Fire Safety Engineering Department, Prince Al-Hussein Bin Abdullah II Academy of Civil Protection, Amman, Jordan (hassana83@yahoo.com)
**Mechanical Engineering Department, School of Engineering, The University of Jordan Amman, Jordan (Duwairi@ju.edu.jo)

Abstract

The fluid flow and heat transfer inside a saturated porous impermeable conduit at constant surface heat flux was modeled analytically and numerically. The present model was depicted the fluid flow and the temperature distribution through both of the entrance region and the fully developed region. The Nusselt number was found in both regions subjected to constant surface heat flux boundary condition. The present modeling was developed by using; the Continuity, Momentum and Energy equations. The equations were in two dimensions - cylindrical coordinate, both of Darcy model and Forchheimer model were studied, and the Nusselt number was found ,analytically and numerically by using finite deference scheme, according to the conditions of constant surface heat flux, as approximately decaying exponentially along the conduit length until it reaches to the value (8). Furthermore, the temperature profile was depicted.

Key words

Porous Media, Darcy Model, Forchheimer Model, Forced Convection Heat Transfer, Nusselt Number.

1. Introduction

The fluid flow and heat transfer inside a saturated porous conduit had a great interest in past decades in order to develop the heat transfer rate equations, due to their relevance in scientific, technological and industrial applications, such as nuclear reactors cooling, combustion technology, fuel cells, vehicle's radiator, heat exchanger in building, oil and gas flow in reservoirs, etc.

The best method of increase the heat transfer rate with optimum pressure drop is by using the porous material in heat exchanger, either as saturated filled or partially filled in the fluid inside heat exchangers. The using of porous materials enhanced the heat transfer rates, because of; the large surface area they provide for heat transfer. Many researchers have been interested in studying the importance of heat transfer rate, the fluid flow and heat transfer inside a saturated porous conduit were studied theoretically and experimentally.

The pressure loss and forced convective heat transfer in an annulus filled with aluminum foam was studied experimentally at constant heat flux condition, and more significant inertia forces are expected, through high porosity medium by Noh et al. [1]. The effect of a porous medium on forced convection of a reciprocating curved channel; the physical model was taken as two vertical channels and one horizontal, furthermore the porous medium was partially filled on the top surface had studied by Shung Fu et al. [2]. The forced convection gaseous slip flow in circular porous micro-channels; The Darcy-Brinkman-Forchheimer model was used to model the fluid flow inside the porous medium, to give the relation between several parameters and their action on the velocity slip and the temperature gradient at the wall, had done by Haddad et al. [3]. The variable conductivity in forced convection for a tube with porous media: A perturbation solution; Was found the values of Nusselt number are between 4.36 and 8, by Jamal-Abad et al. [4]. The heat transfer from walls to porous medium is more effectively convected to water at lower porosity than at higher porosity, the heat transfer rates are increased by the increasing of the Reynolds number and decreased by the increasing of porosity. Were investigated by Abhilash K V, Kumar [5]. The effect of velocity was more evident if the gradient-porosity was existing in radial direction than it was exist in axial direction, and the heat transfer is more considerably than non porous case, were argued numerically and investigated by Wang et. al. [6].

In this study a new model of fluid flow and heat transfer inside a saturated porous conduit at constant surface heat flux, by using the Darcy and Forchheimer models, is going to be laid, through both of entrance and fully developed regions. So a new theoretical results for the coefficient of heat transfer to the flow inside a saturated porous conduit were found, for the both regions.

2. Mathematical Formulation and Solution

When an external power source as a pump, blower or a fan generate the fluid motion, it is leading to the forced convection heat transfer rate. The velocity of the fluid leads to transport a significant amount of the heat energy which enhances the heat transfer rate, (the higher the velocity the higher the heat transfer rate). The convection heat transfer rate can be expressed by Newton's law of cooling as:

$$q = h(\Delta T) \qquad [W/m^2] \tag{1}$$

However, present study was concentrated on forced convection heat transfer problems inside a circular conduit filled with a porous media, the conduit is completely saturated with porous media, in two dimensional, the fluid is incompressible. The continuity, momentum and energy equations were solved simultaneously for intention to Darcy and Forchheimer's effects on fluid flow and heat transfer inside a saturated porous conduit. A theoretical model was built up by using continuity, momentum and energy equations, and it was solved due to the constant surface heat flux boundary condition, through entrance and fully developed region.

Consider a long saturated porous conduit as was depicted in figure (1). The conduit was subjected to forced convection. The following assumptions were made for building up the mathematical model:

- The flow is steady, laminar, incompressible, forced fluid flow, and two - dimensions in cylindrical coordinate.
- The fluid and solid matrix are everywhere in local thermodynamics equilibrium.
- All the physical properties of the fluid are isotropic and homogeneous.
- The temperature of the fluid is everywhere below the boiling point.

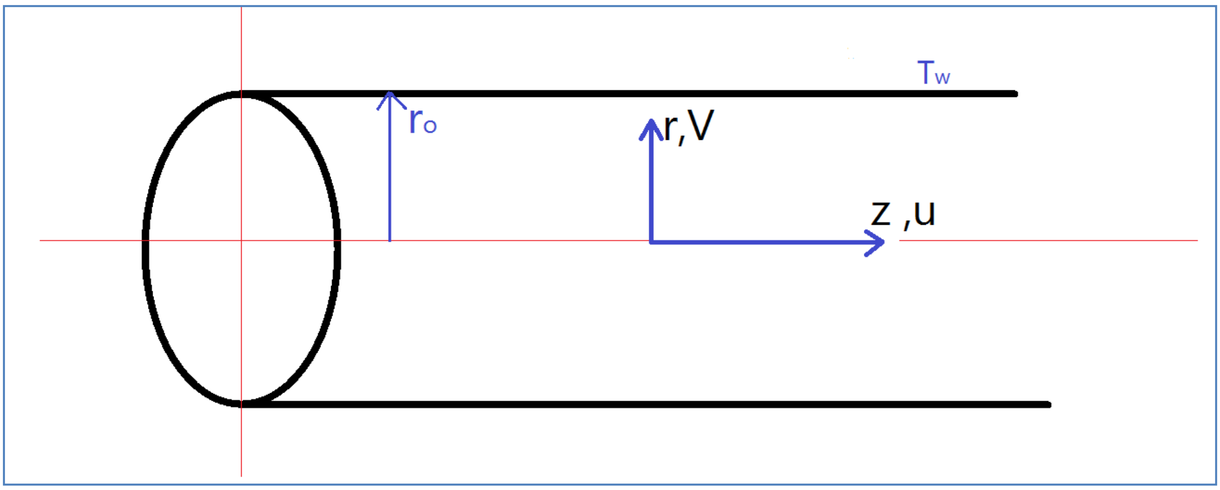

Fig. 1. Schematic diagram represents the conduit coordinate

Where r and z, are the normal and axial coordinate, respectively, and their corresponding velocities are v and u, respectively. The problem is described by the following boundary conditions as: the velocity in $\mathrm{r}-$direction was neglected; i.e. $\mathrm{v}=0$, and there is no heat transfer in r direction at the center; i.e. $\partial \mathrm{T}/\partial \mathrm{r}=0$ at $\mathrm{r}=0$. Under these assumptions, the governing equations can be written as (Nield and Bejan, 2006):

The continuity equation is:

$$\frac{\partial \mathrm{u}}{\partial \mathrm{z}}+\frac{1}{\mathrm{r}}\frac{\partial(\mathrm{vr})}{\partial \mathrm{r}}=0 \qquad (2)$$

Neglects the velocity component in the radial direction; thus $\mathrm{v}=0$, the equation (2) is reduced to:

$$\frac{\partial \mathrm{u}}{\partial \mathrm{z}}=0 \qquad (3)$$

The momentum equation is:

$$\mathrm{u}+\frac{\mathrm{C_F}\sqrt{\mathrm{K}}}{\vartheta}\mathrm{u}^2=-\frac{\mathrm{K}}{\mu}\frac{\mathrm{dp}}{\mathrm{dz}} \qquad (4)$$

If the second term in left hand side of equation (4) is neglected, the Darcy model is obtained. Likewise, when both terms in equation (4) are represented the Forchheimer model is obtained.

The energy equation is:

$$\rho\, \mathrm{c_p}\left(\mathrm{u}\frac{\partial \mathrm{T}}{\partial \mathrm{z}}+\mathrm{v}\frac{\partial \mathrm{T}}{\partial \mathrm{r}}\right)=\mathrm{k}\left(\frac{\partial^2 \mathrm{T}}{\partial \mathrm{z}^2}+\frac{\partial^2 \mathrm{T}}{\partial \mathrm{r}^2}+\frac{1}{\mathrm{r}}\frac{\partial \mathrm{T}}{\partial \mathrm{r}}\right) \qquad (5)$$

Note that, for fully developed region $\frac{\partial \mathrm{u}}{\partial \mathrm{z}}=0$;

To proceed further with this study, the governing equations should be solved, so the wall boundary conditions on temperature should be specified at entrance and fully developed regions. Consideration will be given to the case where the heat flux at wall is constant; i.e. $\mathrm{q_w}=$ constant.

The velocity profile, temperature profile and the Nusselt number are studied at entrance region and fully developed region, with respect to the both of Darcy and Forcheimmer models.

2.1 Through Entrance Region

A long suction of conduit is considered in which there is no heat transfer prior to the conduit section in which heat transfer takes place. In many such cases, the velocity profile is then essentially fully developed before the heat transfer occurs, and it is then only the temperature that is developing, i.e. there is only a "thermal entrance region". This is illustrated in figure (2).

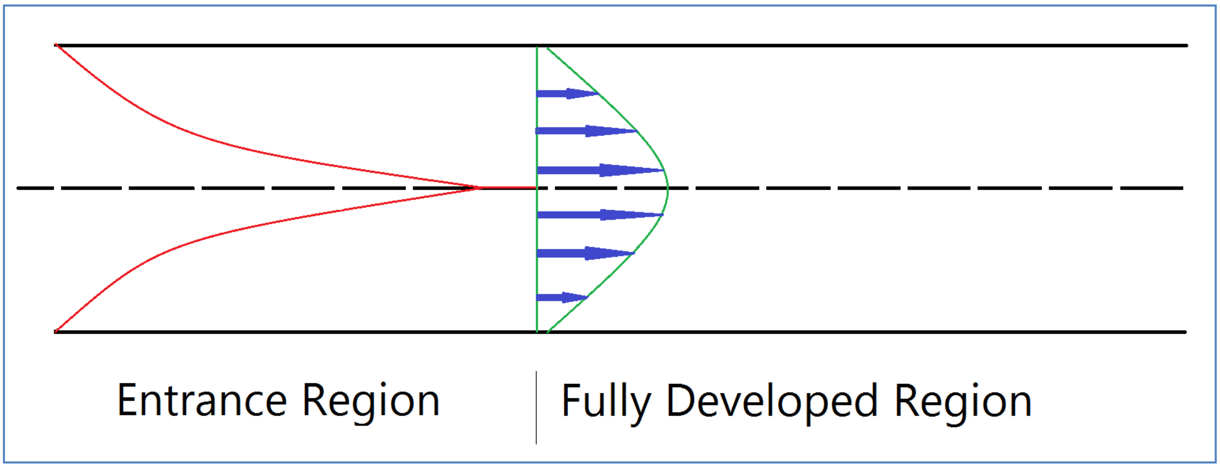

Fig. 2. Schematic diagram represents thermal entrance region inside the conduit

Because of; there is no heat transfer in the initial portion of the conduit flow, the fluid will have a uniform temperature at the point at which heat transfer starts, i.e.; at $z = 0 : \ T = T_e$.

Attention will be given to thermally developing flow in a conduit. In this case, the velocity profile, which is not changing with z, is given. i.e.; $u = $ constant. The temperature profile is changing with distance z along the conduit, the temperature being assumed to be governed from the energy equation (5).

The velocity component in radial direction v, being zero because of; the velocity field is fully developed. The diffusion of heat in axial direction will be neglected compared to that in the radial direction, i.e. $\partial^2 T / \partial z^2 = 0$. So the equation (5) governing the temperature is assumed to have the form:

$$u \frac{\partial T}{\partial z} = \left(\frac{\vartheta}{Pr}\right)\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r}\right) \tag{6}$$

Where Pr is the Prandtl number which equal to (ϑ/α) , ϑ is the kinematic viscosity and α is the thermal diffusivity. The following dimensionless parameters are used:

$$\theta = \frac{(T - T_e)}{q_w D/k}, \qquad U = \frac{u}{u_m}, \qquad R = \frac{r}{D}, \qquad Z = \frac{z}{D} \tag{7}$$

Where θ denotes for the dimensionless temperature, T_e is the temperature at entrance point, U is dimensionless velocity, u_m is the mean velocity in the conduit, $D = 2r_o$ is the diameter of the conduit, Z and R are the dimensionless variables which represent for the length and the radius of the conduit, respectively. $Re = u_m D/\vartheta$ is the Reynolds number based on the mean velocity, $Pr = \vartheta/\alpha$ is the Prandtl number, and $Pe = RePr$ is the Peclet number.

In terms of the dimensionless variables, the energy equation (6) governing the developing temperature field. Substituting the dimensionless terms in equation (7) and $u_m = Re\vartheta/D$ into the equation (6), and because of; the aim is to solve for $\partial\theta/\partial R$, the temperature at the entrance T_e, is out of the conduit line, so it is a constant, then it's derivation equal zero i.e.; $\partial T_e/\partial Z = 0$ and $\partial T_e/\partial R = 0$, and the velocity was eliminated because of; it is constant $u = u_m$, i.e. $U = 1$, then by ridding of the similar terms, the energy equation (6) is reduced to:

$$(RePr)\frac{\partial\theta}{\partial Z} = \frac{\partial^2\theta}{\partial R^2} + \frac{1}{R}\frac{\partial\theta}{\partial R} \tag{8}$$

The following dimensionless conditions, at the beginning of the thermally developing region, can be written as:

The initial condition is:

$$\text{at} \quad Z = 0 \quad : \quad \theta = 0 \tag{9}$$

And the boundary conditions are obtained as:

$$\text{at} \quad R = 0.5 : \quad \frac{\partial\theta}{\partial R} = 1 \tag{10}$$

$$\text{at} \quad R = 0 \quad : \quad \frac{\partial\theta}{\partial R} = 0 \tag{11}$$

In uniform surface heat flux, the dimensionless temperature at wall can be defined as:

$$\theta_w = \frac{T_w - T_e}{q_w D/k} \tag{12}$$

So the local Nusselt number can be found from the equation (12) as:

$$Nu_D = \frac{1}{\theta_w} = \frac{1}{\theta|_{R=0.5}} \tag{13}$$

where is Nu_D is the local Nusselt number, and θ_w is the dimensionless temperature at the wall.

In the case of uniform surface heat flux and from the equation of dimensionless temperature (7), the temperature equation can be rewritten as:

$$T = \theta\left(\frac{q_w D}{k}\right) + T_e \tag{14}$$

Note that, the mean temperature, T_m can be found as:

$$T_m = \frac{\int_0^{r_o} u\, T\, 2\pi r dr}{\int_0^{r_o} u\, 2\pi r dr} \tag{15}$$

By substituting the dimensionless parameters in the equation (7) and the equation (14) into the equation (15), it can be represented as:

$$T_m - T_e = U8\left(\frac{q_w D}{k}\right)\int_0^{0.5} \theta\, RdR \tag{16}$$

Substituting (U = 1) as previously assumption, constant velocity, into the equation (16), so it leads to:

$$\frac{T_m - T_e}{(q_w D/k)} = 8\int_0^{0.5} \theta\, RdR \tag{17}$$

By using the equation (12) with the following mean variables such as: θ_m and T_m instead of θ_w and T_w respectively, the equation (12) can be represented for the mean temperature as dimensionless form at any value of Z, as:

$$\theta_m = \frac{T_m - T_e}{q_w D/k} \tag{18}$$

Substitute the equation (18) into the equation (17) to become as:

$$\theta_m = 8\int_0^{0.5} \theta \, RdR \tag{19}$$

The Nusselt number which based on the difference between the wall and the mean temperatures $(T_w - T_m)$ can be found from the equation (18) as:

$$Nu_m = \frac{1}{(\theta_w - \theta_m)} \tag{20}$$

The value of the Nusselt number was found exponential decay until reach to 8.

2.2 Through Fully Developed Region

Because of; the mass flow rate remains constant along the pipe as dose, by assumption, the velocity gradient at fully developed region was eliminated; i.e. $\partial u/\partial z = 0$; then the continuity equation (2) is reduced to:

$$\frac{\partial(vr)}{\partial r} = 0 \tag{21}$$

So the terms vr is a constant denoted by c, and because of the radial velocity at wall equal zero i.e. $v = 0$, then the constant c leads to zero i.e. $c = 0$.

The momentum equation at $r = r_o$ can be represented as:

$$u + \frac{C_F\sqrt{K}}{\vartheta} u^2 = -\frac{K}{\mu}\frac{\partial p}{\partial z} \qquad z - \text{momentum} \tag{22}$$

$$v + \frac{C_F\sqrt{K}}{\vartheta} v^2 = -\frac{K}{\mu}\frac{\partial p}{\partial r} \qquad r - \text{momentum} \qquad (23)$$

But from the continuity equation it was known that, $v = 0$, so $r - \text{momentum}$ equation (23) is reduced to:

$$\frac{\partial p}{\partial r} = 0 \qquad (24)$$

It leads to; $P = \text{constant}$, and $r - \text{momentum}$ was negligible;

Then it was needed to solve for u from $z - \text{momentum}$ equation (24), if the second term was neglected the Darcy velocity equation can be obtained as:

$$u_D = -\frac{K}{\mu}\frac{\partial p}{\partial z} \qquad (25)$$

If the both terms in the left hand side of the equation (24) were considered, the Forchheimer velocity equation was appeared, and due to find the velocity component in $z - \text{direction}$; u ,the equation (24) can be written in the general form as $ax^2 + bx + c = 0$, so it was solved for x; as $x = \left(-b \pm \sqrt{b^2 - 4ac}\right)/2a$. Hence the Forchheimer velocity can be presented as:

$$u_F = \frac{\vartheta}{C_F}\left[-\frac{1}{2\sqrt{K}} \pm \sqrt{\frac{1}{4K} - \frac{C_F\sqrt{K}}{\rho\vartheta^2}\frac{dp}{dz}}\right] \qquad (26)$$

It was noted that, the positive velocity (u) is constant, because it is independent variable, and it was the area average velocity for both fluid and solid in saturated porous media conduit.

The negative solution of (u) was negligible, because it is meaningless.

For simplicity, the dimensionless term $\left(\left(C_F\sqrt{K}\right)/(\rho\vartheta^2)\right)(dp/dz)$ can be denoted by , as:

$$H = \frac{C_F\sqrt{K}}{\rho\vartheta^2}\frac{dp}{dz} \qquad (27)$$

So, the Forchheimer velocity equation is reduced to:

$$u_F = \frac{\vartheta}{C_F}\left[-\frac{1}{2\sqrt{K}} \pm \sqrt{\frac{1}{4K} - H}\right] \tag{28}$$

The energy equation (5) can be written in the form:

$$u\frac{\partial T}{\partial z} = \alpha\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r}\right) \tag{29}$$

The term $\partial T/\partial z$ can be replaced by another one, which came from a temperature profile such as:

$$\frac{T_w - T}{T_w - T_c} = G\left(\frac{r}{r_o}\right) \tag{30}$$

where G is any function does not depend of the distance along the pipe (z) for fully developed region as depicted in figure (3):

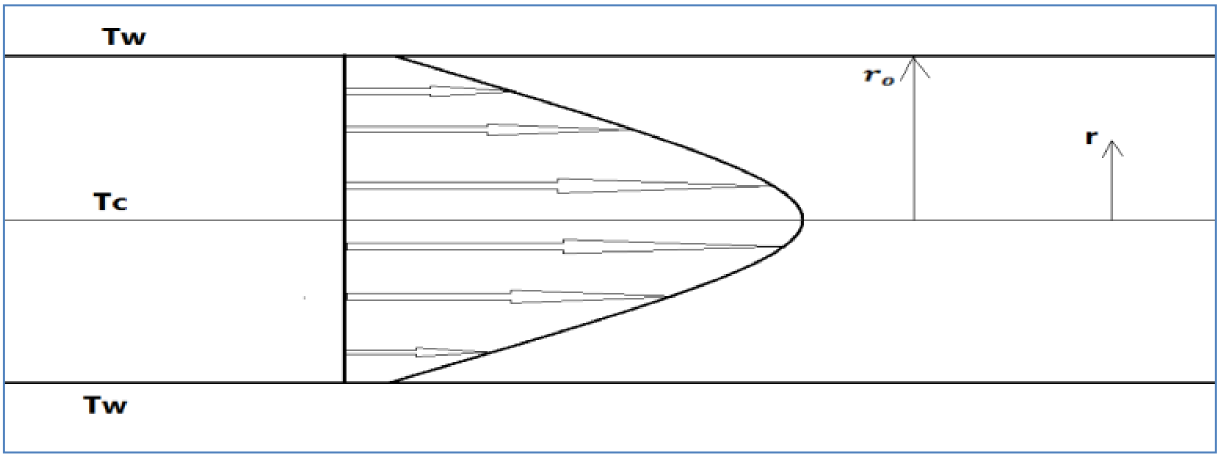

Fig. 3. Schematic diagram represents the temperature profile through the fully developed region inside the conduit

Differentiate equation (30) with respect to z, and the result was gotten as:

$$\frac{\partial T}{\partial z} = \frac{\partial T_w}{\partial z} - \frac{(T_w - T)}{(T_w - T_c)} * \left(\frac{\partial T_w}{\partial z} - \frac{\partial T_c}{\partial z}\right) \tag{31}$$

To proceed further with the solution of the case of constant surface heat flux, and variable wall temperature T_w i.e. $[q_w = \text{constant at } r = r_o]$, so the wall boundary conditions on temperature should be specified as following.

By using Fourier's law, the heat transfer rate at the wall is given as

$$q_w = +k\left.\frac{\partial T}{\partial r}\right|_{r=r_o} \tag{32}$$

The positive sign arises because r is measured from the center toward the wall, whereas the heat flux q_w , is taken as positive in the inward direction, i.e. in the wall to fluid direction.

This equation can be written by using the definition of the function G as on equation (30), and it is differentiated with respect to r at $r = r_o$, to leads to:

$$\left.\frac{\partial T}{\partial r}\right|_{r=r_o} = -(T_w - T_c)\left.\frac{\partial G}{\partial r}\right|_{r=r_o} \tag{33}$$

Substituting the equation (33) into the equation of heat transfer rate (32), which leads to:

$$q_w = -k\,(T_w - T_c)\left.\frac{\partial G}{\partial r}\right|_{r=r_o} \tag{34}$$

By assumption where is the heat transfer rate is uniform i.e. $(T_w - T_c)$ is constant, so it can be shown that:

$$\frac{dT_w}{dz} = \frac{dT_c}{dz} \tag{35}$$

Substituting equation (35) into the equation (31), it leads to:

$$\frac{\partial T}{\partial z} = \frac{dT_w}{dz} \tag{36}$$

Substituting equation (36) into the energy equation (29), it can be reduced to:

$$u\frac{dT_w}{dz} = \alpha\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r}\right) \tag{37}$$

Which has to be solved to give the temperature distribution by using the Forchheimer or the Darcy velocities which were given in the equations (28) and (25), respectively.

A. By Using Forchheimer Model

It can be seen that, the energy equation (29) can be written for Forchheimer flow by substituting the Forchheimer equation (28) into the energy equation (37) to obtain the following:

$$\frac{\vartheta}{C_F}\left[-\frac{1}{2\sqrt{K}} \pm \sqrt{\frac{1}{4K} - H}\right]\left[\frac{dT_w}{dz}\right] = \alpha \left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)\right] \tag{38}$$

The equation (38) can be integrated, subject to the boundary conditions to give the variation of T with r.

The following boundary conditions can be used for the solution:

$$\text{at} \quad r = 0: \qquad \frac{\partial T}{\partial r} = 0 \tag{39}$$

Note that; this boundary condition was becoming from the requirement that, the profile be symmetrical about the center line (at $r = 0$), and:

$$q_w = \text{constant} \tag{40}$$

The equation (38) can be solved analytically, by using the boundary condition in equation (39) to give the variation of T with r.

Equation (38) was integrated with respect to the radial coordinate (r), from (0) to (r) ,by using the boundary condition in the equation (39), so the equation (38) leads to:

$$\frac{\partial T}{\partial r} = \frac{\vartheta}{\alpha\, C_F}\frac{dT_w}{dz}\left[-\frac{1}{2\sqrt{K}} \pm \sqrt{\frac{1}{4K} - H}\right] * \frac{r}{2} \tag{41}$$

Equation (41) was integrated again with respect to the radial coordinate (r), from (r) to (r_o), to become as:

$$T = T_w - \frac{\vartheta}{\alpha\, C_F}\frac{dT_w}{dz}\left[-\frac{1}{2\sqrt{K}} \pm \sqrt{\frac{1}{4K} - H}\right] * \frac{{r_o}^2 - r^2}{4} \tag{42}$$

This is the temperature distribution for fully developed laminar pipe flow when the heat flux at the surface is uniform (constant). It can be written in terms of the specified uniform surface heat flux q_w, by noting that when the equation (42) is used to give the value of $(\partial T/\partial r)|_{r=r_o}$ in equation (32), so differentiate the equation (42) with respect to r and replace r by r_o, to get the following:

$$\left.\frac{\partial T}{\partial r}\right|_{r=r_o} = +\frac{\vartheta}{2\,\alpha C_F}\frac{dT_w}{dz}\left[-\frac{1}{2\sqrt{K}} \pm \sqrt{\frac{1}{4K} - H}\right] * r_o \tag{43}$$

Substituting the equation (43) into the conduction heat transfer equation (32) to leads to:

$$\frac{2q_w}{kr_o} = \frac{\vartheta}{\alpha\, C_F}\left[-\frac{1}{2\sqrt{K}} \pm \sqrt{\frac{1}{4K} - H}\right]\frac{dT_w}{dz} \tag{44}$$

Substituting equation (44) into the equation of the temperature distribution (42) to get the following:

$$T - T_w = +\frac{2q_w}{kr_o} * \frac{r^2 - {r_o}^2}{4} \tag{45}$$

The center line temperature T_c (i.e. at r = 0), can be obtained from the equation (45), by replacing T, r by T_c , zero , respectively, to get the following:

$$T_w - T_c = \frac{q_w r_o}{2k} \tag{46}$$

Therefore, the temperature distribution can be written in the form as for fully developed flow equation (30).

Substituting equations (45-46) into the equation (30), to get the form of the temperature distribution for fully developed flow through a conduit with constant surface heat flux such as:

$$G = \frac{\frac{-2q_w}{kr_o} * \frac{r^2 - r_0{}^2}{4}}{\frac{q_w r_o}{2k}}$$

$$G = \frac{r_0{}^2 - r^2}{r_o{}^2} \quad (47)$$

In fully developed flow, it is usually convenient to utilize the mean fluid temperature, T_m, rather than the center line temperature in defining the Nusselt number. So the mean or bulk temperature is given by:

$$T_m = \frac{1}{\dot{m}} \int_A \rho u \, TdA \quad (48)$$

Where *A* is the cross section area of the conduit; $= \pi r^2$ and $\dot{m}$ is the mass flow rate inside the conduit, so the mass flow rate can be written as:

$$\dot{m} = \int_A \rho u \, dA$$

$$\dot{m} = \rho u \int_0^{r_o} 2\pi r \, dr$$

$$\dot{m} = \rho u \, \pi r_o{}^2 \quad (49)$$

Substitute equation (49) into the equation (48) to get the following:

$$T_m = \frac{2}{r_o{}^2} \int_A Tr \, dr \quad (50)$$

Substitute the equation (42) into the equation (50) to get the following:

$$T_m = \frac{T_w r^2}{r_o{}^2} + \frac{q_w}{kr_o{}^3} \frac{r^4}{4} - \frac{q_w}{kr_o} \frac{r^2}{2} \quad (51)$$

It is noted that, the Nusselt Number is equal:

$$Nu = \frac{hD}{k} \tag{52}$$

Substituting that: $D = 2r_o$ and $h = q_w/(T_{w-}T_m)$ into the equation (52) to become as:

$$Nu = \frac{2r_o q_w}{k(T_{w-}T_m)} \tag{53}$$

Note that; T_m at $r = r_o$ can be obtained by substitute r_o instead of r into the equation (51) to become as:

$$T_m|_{r=r_o} = \left\{T_w - \frac{q_w}{k}\frac{r_o}{4}\right\} \tag{54}$$

By substituting the equation (54) into the equation (53), the Nusselt number can be found as: $Nu = 8$

B. By Using Darcy Model

It also can be seen that, the energy equation (29) can be also written for Darcy flow by substituting the Darcy velocity equation (25), into the energy equation (37) to obtain the following:

$$\left[-\frac{K}{\mu}\frac{\partial p}{\partial z}\right]\left[\frac{dT_w}{dz}\right] = \alpha\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r}\right) \tag{55}$$

The equation (55) can be integrated, subject to the same boundary conditions in the equations (39-40) to give the variation of T with r. So the equation (55) leads to:

$$\frac{\partial T}{\partial r} = \left[-\frac{K}{\alpha\mu}\frac{\partial p}{\partial z}\right]\left[\frac{dT_w}{dz}\right] * \frac{r}{2} \tag{56}$$

The equation (56) can be integrated again to become as:

$$T = T_w + \left[\frac{K}{\alpha\mu}\frac{\partial p}{\partial z}\right]\left[\frac{dT_w}{dz}\right] * \frac{{r_0}^2 - r^2}{4} \tag{57}$$

This is the temperature distribution for fully developed laminar pipe flow when the surface heat flux is uniform (constant). It can be written in terms of the specified uniform surface heat flux q_w, by noting that, the equation (57) can be used to give the value of $(\partial T/\partial r)|_{r=r_o}$ in the equation (32), so differentiate the equation (57) with respect to r , and replacing r by r_o, to get the following:

$$\left.\frac{\partial T}{\partial r}\right|_{r=r_o} = \left[-\frac{K}{2\alpha\mu}\frac{\partial p}{\partial z}\right]\left[\frac{dT_w}{dz}\right] * r_0 \tag{58}$$

Substituting the equation (58) into the equation (32) to leads to:

$$\frac{2q_w}{kr_0} = \left[-\frac{K}{\alpha\mu}\frac{\partial p}{\partial z}\right]\left[\frac{dT_w}{dz}\right] \tag{59}$$

Substituting equation (59) into the equation (57) to get the same form of equation (46).

The same procedure was repeated from equation (49) to (55), and the Nusselt number can be found by substituting the equation (55) into the equation (54), so the result can be evaluated as: $Nu = 8$

3. Numerical Solutions

The equation (8) was solved numerically by using finite difference method according to the conditions in the equations (9-11).

A series of grid lines in the axial and normal directions i.e. in Z and R coordinate, respectively, are built. A uniform steps were used in both directions such that ΔZ and ΔR, respectively.

The 2-D conduit space was divided as shown in figure (4).

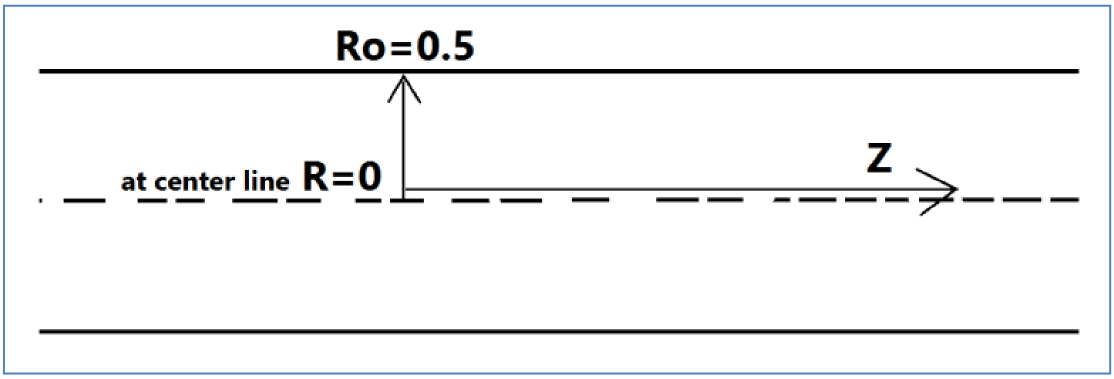

Fig. 4.a

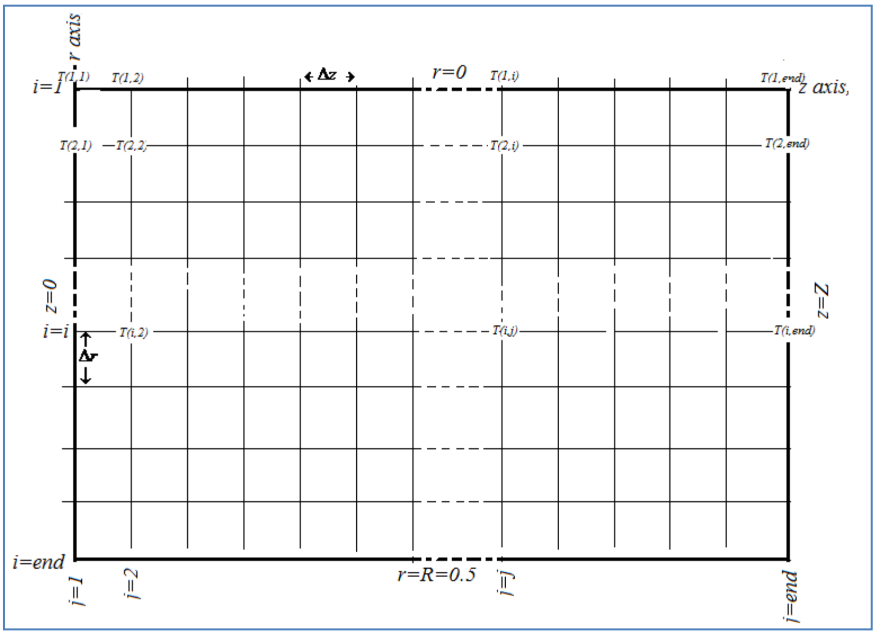

Fig. 4.b

Fig. 4. Schematic diagram represents the space direction and dividing inside the conduit

The equation (8) was written in the discrete form, such as $\frac{\partial\theta}{\partial Z}$ was approximated as:

$$\left.\frac{\partial\theta}{\partial Z}\right|_{(Z,R)} = \frac{\theta(\ \ Z,R)-\theta(Z-\Delta Z,R)}{\Delta Z} \tag{60 }$$

or

$$\left.\frac{\partial\theta}{\partial Z}\right|_{(i,j)} = \frac{\theta(i,j)-\theta(i,j-1)}{\Delta Z} \tag{61}$$

$\frac{\partial\theta}{\partial R}$ was approximated as:

$$\left.\frac{\partial\theta}{\partial R}\right|_{(Z,R)} = \frac{\theta(Z,R)-\theta(Z,R-\Delta R)}{\Delta R} \tag{62 }$$

or

$$\left.\frac{\partial\theta}{\partial R}\right|_{(i,j)} = \frac{\theta(i,j)-\theta(i-1,j)}{\Delta R} \tag{63 }$$

And $\frac{\partial^2\theta}{\partial R^2}$ was approximated as:

$$\left.\frac{\partial^2\theta}{\partial R^2}\right|_{(Z,R)} = \frac{\theta(Z,R+\Delta R)-2\theta(Z,R)+\theta(Z,R-\Delta R)}{(\Delta R)^2} \tag{64 }$$

or

$$\left.\frac{\partial^2\theta}{\partial R^2}\right|_{(i,j)} = \frac{\theta(i+1,j)-2\theta(i,j)+\theta(i-1,j)}{(\Delta R)^2} \qquad (65)$$

Where:

$$R = (i-1).\Delta R \qquad (66)$$

Thus the partial differential equation (8) in discrete form becomes:

$$\frac{\theta(i,j)-\theta(i,j-1)}{\Delta Z} = \frac{\theta(i+1,j)-2\theta(i,j)+\theta(i-1,j)}{(\Delta R)^2} + \frac{\theta(i,j)-\theta(i-1,j)}{(i-1).(\Delta R)^2} \qquad (67)$$

And from equation (67):

$$\theta(i,j)-\theta(i,j-1) = \frac{\Delta Z}{(\Delta R)^2}[\theta(i+1,j)-2\theta(i,j)+\theta(i-1,j)] + \frac{\Delta Z}{(i-1).(\Delta R)^2}[\theta(i.j)-\theta(i-1,j)] \qquad (68)$$

or:

$$\theta(i,j)-\theta(i,j-1) = \frac{\Delta Z}{(\Delta R)^2}\theta(i+1,j) - 2\frac{\Delta Z}{(\Delta R)^2}\theta(i,j) + \frac{\Delta Z}{(\Delta R)^2}\theta(i-1,j) + \frac{\Delta Z}{(i-1).(\Delta R)^2}\theta(i.j) - \frac{\Delta Z}{(i-1).(\Delta R)^2}\theta(i-1,j) \qquad (69)$$

So, it becomes as:

$$\theta(i,j) + 2\frac{\Delta Z}{(\Delta R)^2}\theta(i,j) - \frac{\Delta Z}{(i-1).(\Delta R)^2}\theta(i.j) = \theta(i,j-1) + \frac{\Delta Z}{(\Delta R)^2}\theta(i+1,j) + \frac{\Delta Z}{(\Delta R)^2}\theta(i-1,j) - \frac{\Delta Z}{(i-1).(\Delta R)^2}\theta(i-1,j)$$

$$\left[1+2\frac{\Delta Z}{(\Delta R)^2}-\frac{\Delta Z}{(i-1).(\Delta R)^2}\right]\theta(i,j)=\theta(i,j-1)+\frac{\Delta Z}{(\Delta R)^2}\theta(i+1,j)+\left[\frac{\Delta Z}{(\Delta R)^2}-\frac{\Delta Z}{(i-1).(\Delta R)^2}\right]\theta(i-1,j) \quad (70)$$

Or, in abbreviated form, the iterative equation becomes:

$$\alpha(i)\theta(i,j)=\theta(i,j-1)+\beta.\theta(i+1,j)+\gamma(i)\theta(i-1,j) \quad (71)$$

Where:

$$\alpha(i)=1+\frac{2\Delta Z}{(\Delta R)^2}-\frac{\Delta Z}{(i-1).(\Delta R)^2} \quad (72)$$

$$\beta=\frac{\Delta Z}{(\Delta R)^2} \quad (73)$$

And

$$\gamma(i)=\frac{\Delta Z}{(\Delta R)^2}-\frac{\Delta Z}{(i-1).(\Delta R)^2} \quad (73)$$

So, by using the boundary and initial conditions in equations (9-11), it can be concluded that all the elements of the first column of matrix θ are zeros.

And it was noted that about the second column; Where $\theta(\text{end}+1,2)$, which will appear in the last equation, is assumed equal to $\theta(\text{end},2)$ as practically they are equal as the step in R is very small. So it can be written as Matrix form as:

$$\begin{bmatrix} \alpha(2)-\gamma(2) & -\beta & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ -\gamma(3) & \alpha(3) & -\beta & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & -\gamma(4) & \alpha(4) & -\beta & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & -\gamma(5) & \alpha(5) & -\beta & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & 0 & -\gamma(6) & \alpha(6) & -\beta & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\gamma(\text{end}-2) & \alpha(\text{end}-2) & -\beta & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\gamma(\text{end}-1) & \alpha(\text{end}-1) & -\beta \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\gamma(\text{end}) & \alpha(\text{end}) & -\beta \end{bmatrix} \begin{bmatrix} \theta(2,2) \\ \theta(3,2) \\ \theta(4,2) \\ \theta(5,2) \\ \theta(6,2) \\ \vdots \\ \vdots \\ \vdots \\ \theta(\text{end}-2,2) \\ \theta(\text{end}-1,2) \\ \theta(\text{end},2) \end{bmatrix}$$

$$=\begin{bmatrix} -\gamma(2)\Delta R \\ 0 \\ 0 \\ 0 \\ 0 \\ \vdots \\ \vdots \\ \vdots \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (74)$$

For the third column, it can be written in the matrix form as:

$$\begin{bmatrix} \alpha(2)-\gamma(2) & -\beta & 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 \\ -\gamma(3) & \alpha(3) & -\beta & 0 & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & -\gamma(4) & \alpha(4) & -\beta & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & 0 & -\gamma(5) & \alpha(5) & -\beta & 0 & \dots & 0 & 0 & 0 \\ 0 & 0 & 0 & -\gamma(6) & \alpha(6) & -\beta & \dots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\gamma(end-2) & \alpha(end-2) & -\beta & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\gamma(end-1) & \alpha(end-1) & -\beta \end{bmatrix} \begin{bmatrix} \theta(2,3) \\ \theta(3,3) \\ \theta(4,3) \\ \theta(5,3) \\ \theta(6,3) \\ \vdots \\ \vdots \\ \vdots \\ \theta(end-1,3) \\ \theta(end,3) \end{bmatrix} = \begin{bmatrix} \theta(2,2)-\gamma(2)\Delta R \\ \theta(3,2) \\ \theta(4,2) \\ \theta(5,2) \\ \theta(6,2) \\ \vdots \\ \vdots \\ \vdots \\ \theta(end-2,2) \\ \theta(end-1,2) \end{bmatrix} \quad (75)$$

For the last column, it can be written as the matrix form:

$$\begin{bmatrix} \alpha(2)-\gamma(2) & -\beta & 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 \\ -\gamma(3) & \alpha(3) & -\beta & 0 & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & -\gamma(4) & \alpha(4) & -\beta & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & 0 & -\gamma(5) & \alpha(5) & -\beta & 0 & \dots & 0 & 0 & 0 \\ 0 & 0 & 0 & -\gamma(6) & \alpha(6) & -\beta & \dots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\gamma(end-2) & \alpha(end-2) & -\beta & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\gamma(end-1) & \alpha(end-1) & -\beta \end{bmatrix} \begin{bmatrix} \theta(2,end) \\ \theta(3,end) \\ \theta(4,end) \\ \theta(5,end) \\ \theta(6,end) \\ \vdots \\ \vdots \\ \vdots \\ \theta(end-1,end) \\ \theta(end,end) \end{bmatrix} = \begin{bmatrix} \theta(2,end-1)-\gamma(2)\Delta R \\ \theta(3,end-1) \\ \theta(4,end-1) \\ \theta(5,end-1) \\ \theta(6,end-1) \\ \vdots \\ \vdots \\ \vdots \\ \theta(end-2,end-1) \\ \theta(end-1,end-1) \end{bmatrix} \quad (76)$$

Each of these matrices equations (74-76) were solved to find the unknown elements of the θ matrix. A program was built by using Matlab software, to describe the temperature variation at all points inside the conduit, such as it was illustrated in the matrix (77) below.

$$\begin{bmatrix} \theta(1,1) & \theta(1,2) & \theta(1,3) & \dots & \theta(1,\text{end}-1) & \theta(1,\text{end}) \\ \theta(2,1) & \theta(2,2) & \theta(2,3) & \dots & \theta(2,\text{end}-1) & \theta(2,\text{end}) \\ \theta(3,1) & \theta(3,2) & \theta(3,3) & \dots & \theta(3,\text{end}-1) & \theta(3,\text{end}) \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \theta(\text{end}-1,1) & \theta(\text{end}-1,2) & \theta(\text{end}-1,3) & \dots & \theta(\text{end}-1,\text{end}-1) & \theta(\text{end}-1,\text{end}) \\ \theta(\text{end},1) & \theta(\text{end},2) & \theta(\text{end},3) & \dots & \theta(\text{end},\text{end}-1) & \theta(\text{end},\text{end}) \end{bmatrix} \quad (77)$$

So, the study was concluded by measuring the temperature at each point in the diameter with uniformed steps in R direction along the whole length of the tube. Where the partial differential is parabolic in behavior and is explicit finite difference numerical technique is used.

The thermal circuit of the conduit is best described by conservation principles explained in the governing equations where all mass, momentum and energy must be satisfied at each and every point of the grid.

4. Results and Discussions

It was noted that, the same answer was obtained, because of the difference between both models was appeared in the energy equation and by the definition of the heat transfer rate equation's as it is latent in the equations (45) and (59) for Forchheimer and Darcy models, respectively.

The dimensionless temperature profile is depicted in figure (5) with the dimensionless length of the conduit ($\mathrm{Z-coordinate}$) at selective values of the dimensionless radius of the conduit (R), and figure (6) is present the dimensionless temperature with (R) at selective values of (Z).

It is clear that, from figures (5,6), the temperature increases in the direction of downstream; this is due to continuous heating of the fluid in the entrance region, and it is revealed that; the minimum temperature is found at the center of the conduit.

From figure (5), it also found that, the length of the entrance region in the case of constant surface heat flux is very teeny, and it could not be possible to compute or present it in the figure, so its value is of infinitesimal value.

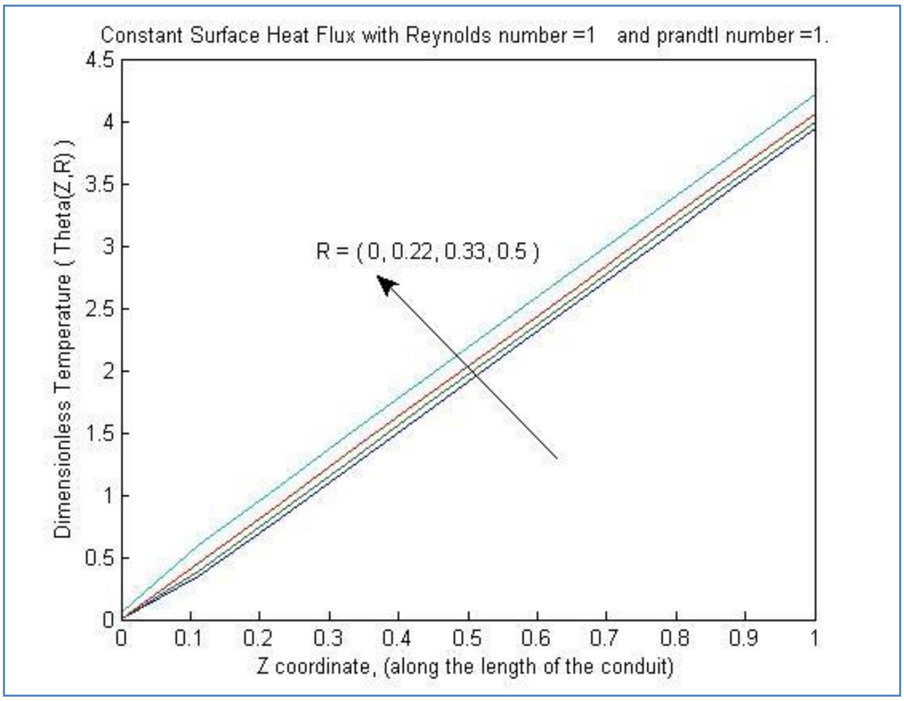

Fig. 5. Dimensionless temperature profile in entrance region for constant surface heat flux along the conduit

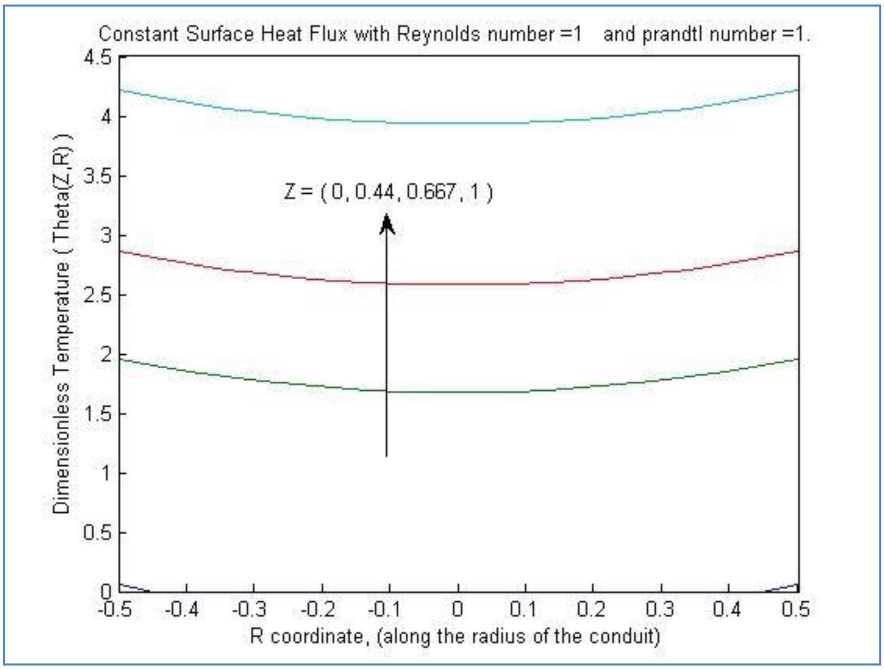

Fig. 6. Dimensionless temperature profile in entrance region for constant surface heat flux with radius of the conduit.

Figure (7.a) depicted the local Nusselt number along the conduit length, for selective values of Reynolds numbers, and figure (7.b) is for selective values of Prandtl numbers. Where the local Nusselt numbers is computed with respect to the difference between the wall and the entrance temperatures.

It is clear that, the local Nusselt number is approximately exponentially decay until it reached zero, this is due to decreasing the difference between the wall temperature and the entrance temperature along the conduit. Also it is clear that, when Reynolds number or Prandtl numbers are increased the local Nusselt number is shifted up.

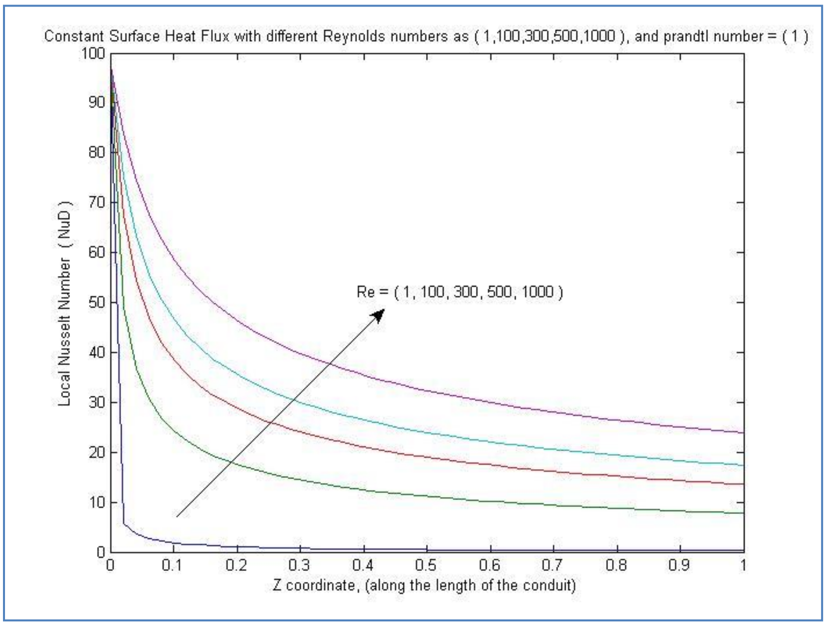

Fig. 7.a

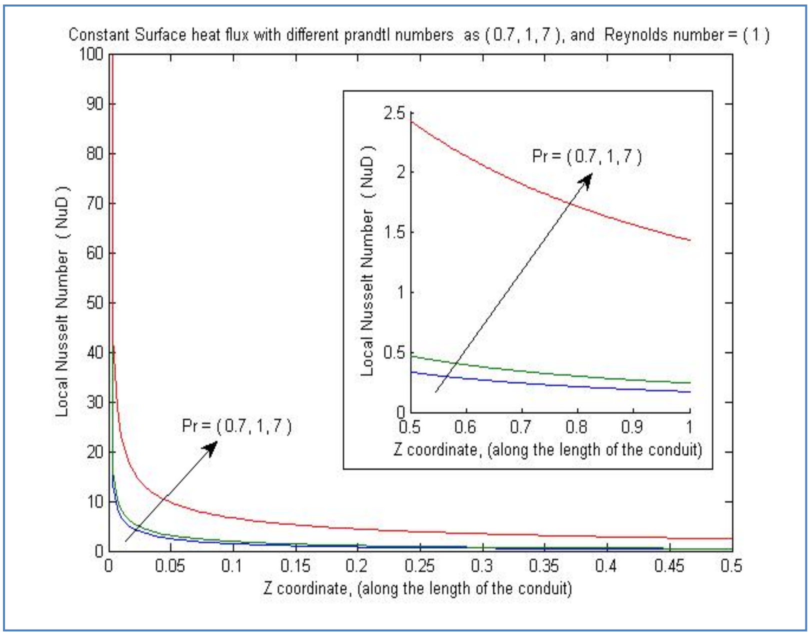

Fig. 7.b

Fig. 7. Local Nusselt number variation in entrance region for constant surface heat flux along the conduit with selective values of:

a) Reynolds numbers b) Prandtl numbers

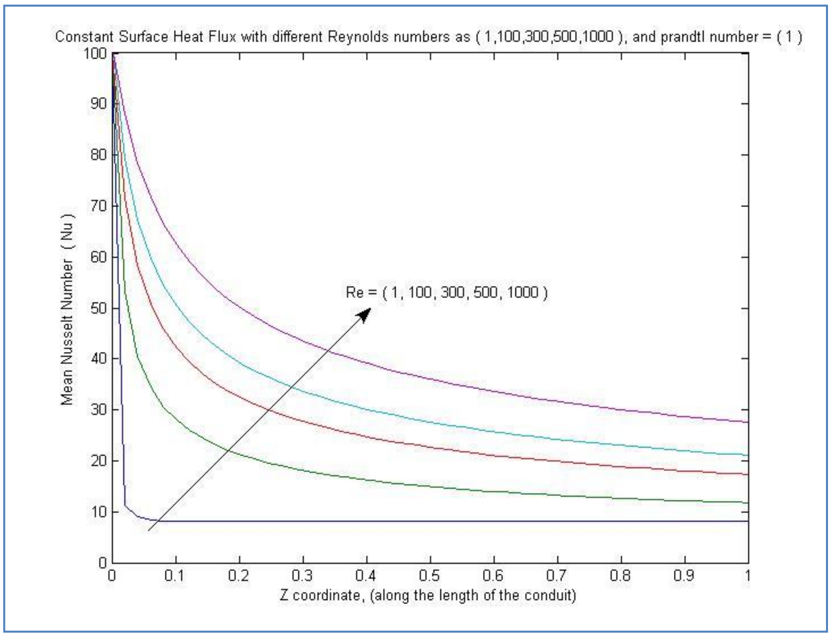

Fig. 8.a

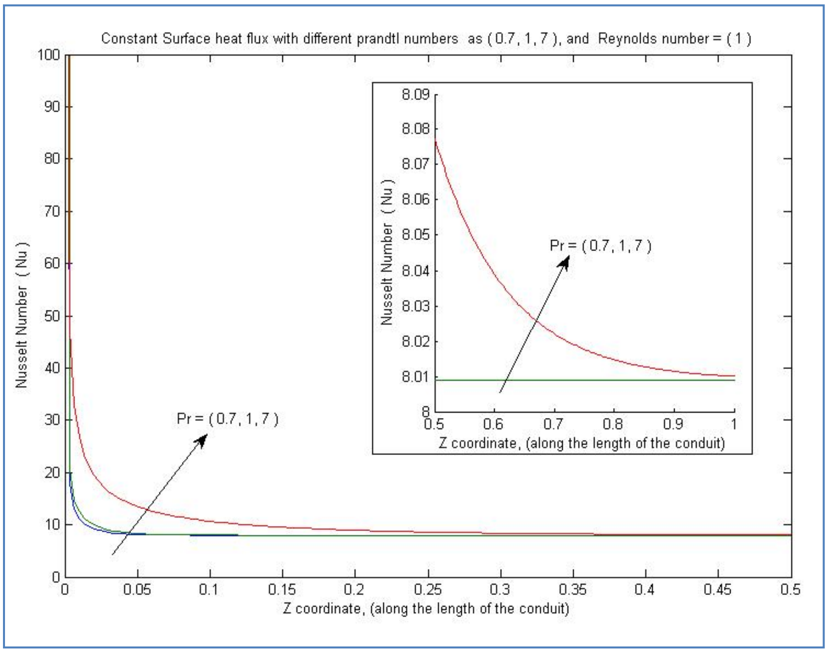

Fig. 8.b

Fig. 8. Nusselt number variation in entrance region for constant surface heat flux along the conduit with selective values of:

a) Reynolds numbers b) Prandtl numbers

Figure (8.a) depicted the Nusselt number along the conduit with unity values of Prandtl numbers, and selective values of Reynolds numbers, while figure (8.b) depicted the Nusselt number along the length of conduit with unity value of Reynolds numbers, and selective values of Prandtl numbers.

The Nusselt numbers is computed with respect to the difference between the mean temperature and the wall temperatures, so figure (8) is presented the Nusselt number as approximately decay exponentially along the conduit length.

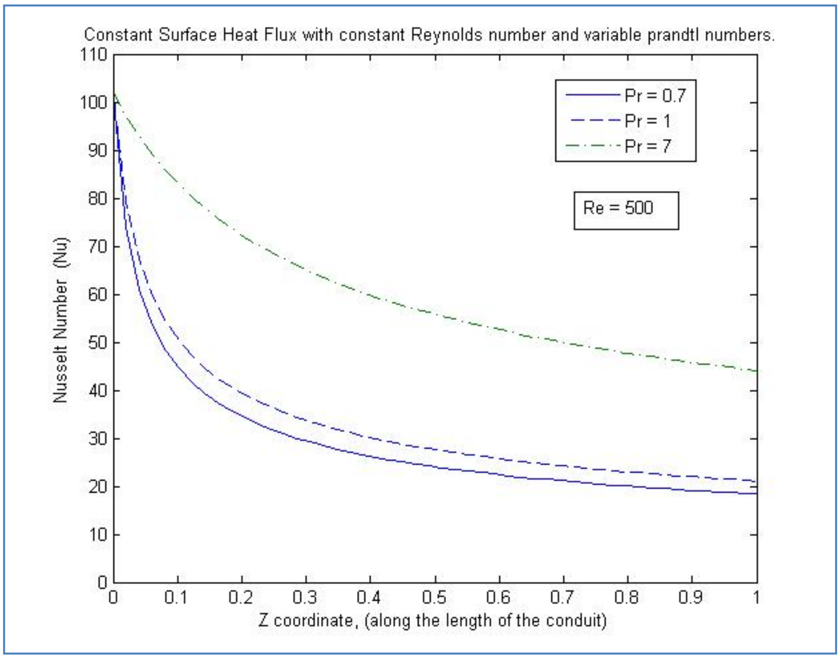

Fig. 9.a

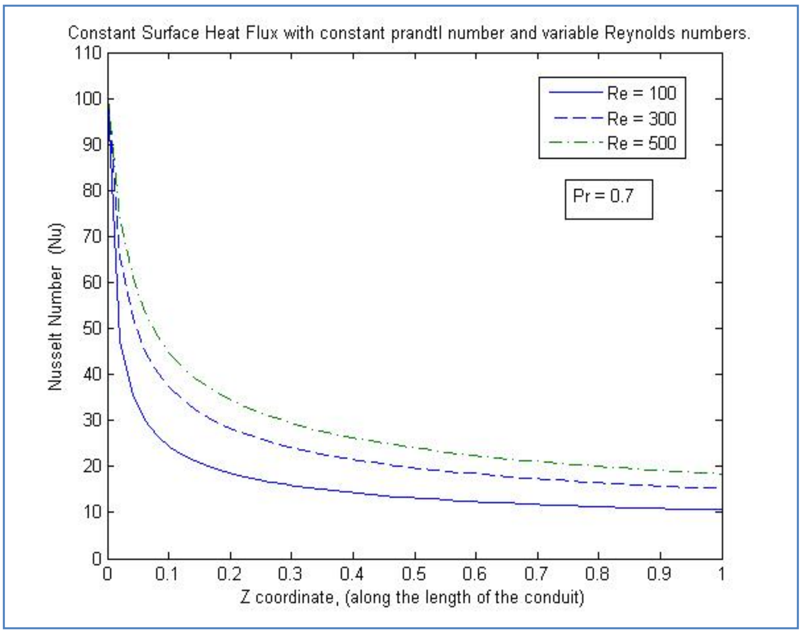

Fig. 9.b

Fig. 9. Nusselt number variation in entrance region for constant surface heat flux along the conduit with :

a) Re=500 and Pr=0.7,1,7

b) Pr=0.7 and Re=100,300,500

Figure (9.a) depicted the Nusselt number along the conduit, with selective values of Prandtl numbers, and constant Reynolds number, while figure (9.b) depicted the Nusselt number along the length of conduit with selective values of Reynolds numbers, and constant value of Prandtl number.

It is clear that from figures (8,9), when the Reynolds number and Prandtl number are increased the Nusselt number is shifted up; this is due to favorable higher inertia forces inside the fluid layers, it is also reveal that, the Nusselt number is increased significantly with higher value of Reynolds number.

The dimensionless temperature at the center of conduit, i.e. at (R = 0) along the conduit is illustrated in figure (10). Figure (10.a) presents the dimensionless temperature at constant value of Reynolds number and selective values of Prandtl numbers, while the figures (10.b) at the constant value of Prandtl number and selective values of the Raynolds numbers.

It is clear that from figure (10); when Reynolds number or Prandtl number are increased the fluid is heated slowly; this is due to high velocity inside boundary layers, and there are not enough time to show the fluid heating effects along the conduit.

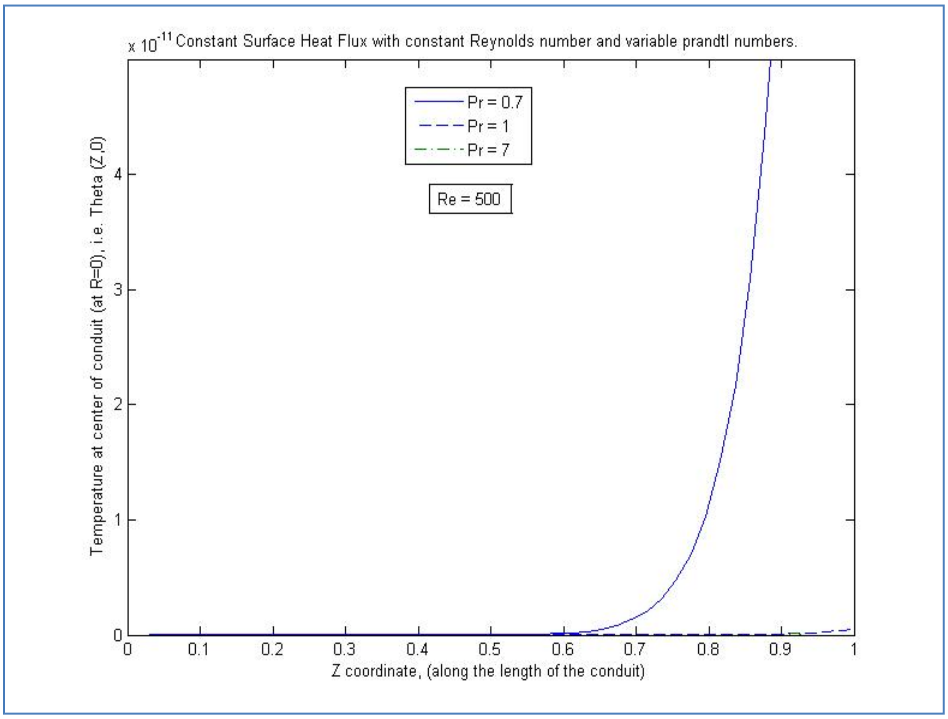

Fig. 10.a

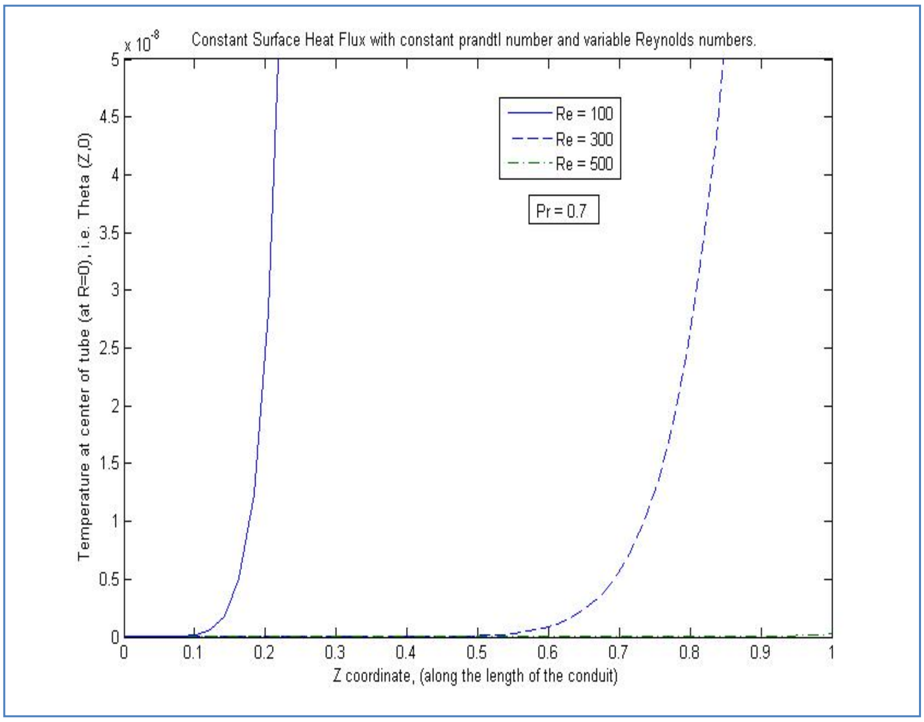

Fig. 10.b

Fig. 10. Dimensionless temperature at center of conduit in entrance region for constant surface heat flux along the conduit with:

a) Re=500 and Pr=0.7,1,7

b) Pr=0.7 and Re=100,300,500

The dimensionless temperature at the end of conduit (i.e. at $Z = 1$) along the radius of the conduit is illustrated in figure (11). Figure (11.a) presents the temperature for constant value of Reynolds number and selective values of Prandtl numbers, while figure (11.b) presents the temperature for constant value of Prandtl number and selective values of Reynolds numbers.

It is reveal that; from figure (11), as the Reynolds number or Prandtl number are increased the temperature near the surface is increased, and the minimum temperature is sticked around the center of the conduit. It is also clear that, when the Reynolds or Prandtl numbers are increased the temperature near the surface is going heated slowly; this is due to increasing the velocity of the fluid inside the conduit.

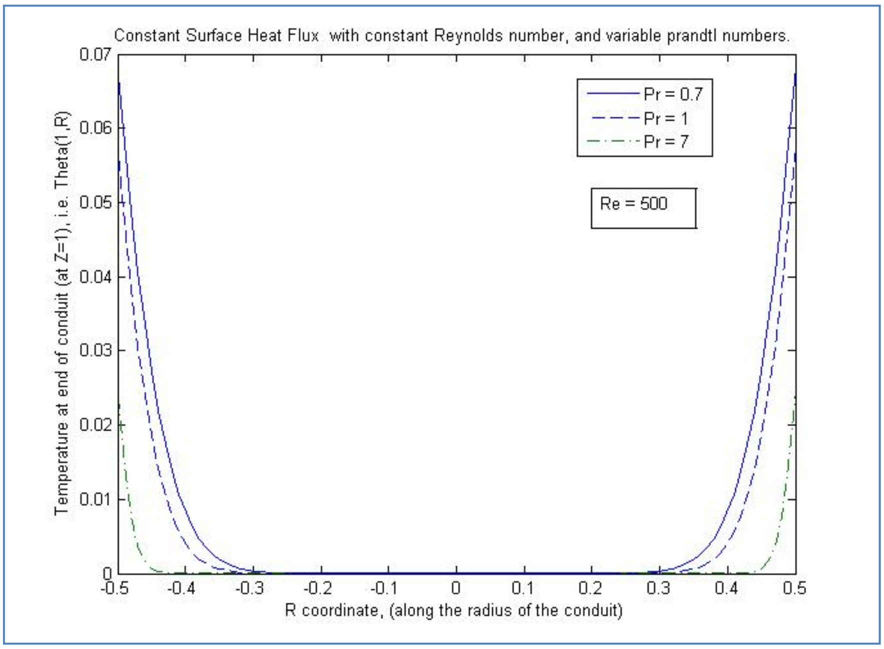

Fig. 11.a

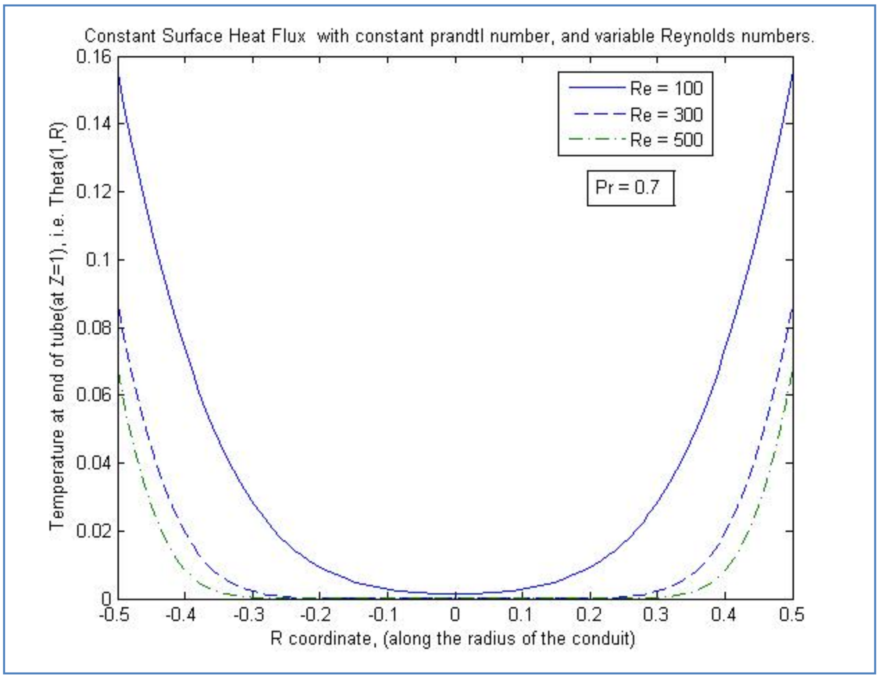

Fig. 11.b

Fig.11. Dimensionless temperature at end of conduit in entrance region for constant surface heat flux along the radius of the conduit with:

a) Re=500 and Pr=0.7,1,7

b) Pr=0.7 and Re=100,300,500

In fully developed region; it is clear that from the equations (45) and (59), the heat flux was defined twice; first equation illustrated the heat flux with respect to the Forchheimer model, and the second one with respect to the Darcy model, respectively. So it is revealed that, from the equation (54) and (55); the velocity is known and implied into the Nusselt number's equation in (54), so the effect of the difference between the both models did not appear in the final results, and the Nusselt number was found analytically for both models, Darcy or Forchheimer identically; as equal to (8). But it is known that; the heat transfer rate, velocity and pressure drop by using Forchheimer model is greater than its in Darcy model.

Conclusion

Through this work, a model of fluid flow and heat transfer inside a saturated porous conduit at constant surface heat flux was investigated. With careful inspection of previous paper, one can conclude that:

1. Using the porous media to increase the rate of heat transfer is very promising.
2. The fluid flow velocity was decreased in the direction of downstream due to using the porous media inside the conduit.
3. The pressure drop was increased due to using the porous media inside the conduit.
4. The Nusselt number was not depend on the flow velocity, if it was constant or in the case of forced flow.
5. The Nusselt number was increased with increasing Reynolds or Prandtl numbers.
6. The heat transfer rate was increased with increasing Reynolds number or the flow velocity.

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NOMENCLATURE

Symbol	Quantity	Unit
A	Cross section area	m^2
C_F	Forchheimer coefficient	–
C_p	Specific heat of the fluid at constant presure	J/(Kg. K)
D	Diameter as $[2r_o]$	m
G	A function does not depend of the distance along the conduit for fully developed region as $[G(r/r_o) = (T_w - T)/(T_w - T_c)]$	–
H	Dimentionless term in Forchheimer model as ; $\left[\frac{c_f\sqrt{K}}{\rho\vartheta^2}\frac{dp}{dz}\right]$	–
h	Heat transfer coefficient	$W/(m^2.K)$
i	Nodes number in axial direction	–
j	Nodes number in normal direction	–
K	Permeability	m^2
k	Thermal conductivity	W/(m. K)
L	Length	m
$\dot{m}$	Mass flow rate	kg/s
Nu	Nusselt number	–
Nu_D	Local Nusselt number	–
p	Pressure	N/m^2
Pe	Peclet number as [RePr]	–
Pr	Prandtl number as $[\vartheta/\alpha]$	–
q	Heat flux per unit area	W/m^2
R	Dimensionless radius of the conduit	–

Re	Reynolds number	$-$
Re_K	Reynolds number based on Permeability as $\left[u_m\sqrt{K}/\vartheta\right]$	$-$
Re_{u_m}	Reynolds number based on mean velocity as $[u_m D/\vartheta]$	$-$
r	Raduis of the conduit	m
r_o	Outer raduis of the conduit	m
T	Temperature	K
T_e	The temperature at the entrance of the conduit	K
T_w	The temperature at the wall of the conduit	K
u	Velocity component in $z-$ direction	m/s
u_m	The mean velocity in the conduit	m/s
v	Velocity component in $r-$ direction	m/s
x	Axial coordinate	$-$
y	Normal coordinate	$-$
Z	The dimension length of the conduit	$-$

Greek Letters

Symbol	Quantity	Unit
α	Thermal diffusivity as $\left[k/\rho C_p\right]$	m^2/s
$\alpha(i)$	A constant in iterative equation number (71)	$-$
$\beta(i)$	A constant in iterative equation number (71)	$-$
$\gamma(i)$	A constant in iterative equation number (71)	$-$
θ	Dimensionless temperature	$-$
μ	Absolute or dynamic viscosity	$kg/(m.s)$
ϑ	Kinematic viscosity as $[\mu/\rho]$	m^2/s
ρ	Fluid density	kg/m^3