Chemical Reaction Effect of MHD Micropolar Fluid Flow between two Parallel Plates in the Presence of Heat Source/Sink

*A.K. Dash, **S.R. Mishra, ***B.P. Acharya

*Department of Mathematics, Ravenshaw University, College square, Cuttack, Odisha, India. (ashisdash8247@gmail.com)

**Department of Mathematics, Siksha ‘O’ Anusandhan Deemed to be University, Khandagiri, Bhubaneswar-751030, Odisha, India. (satyaranjan_mshr@yahoo.co.in)

***Department of Mathematics, Utkal University, Bhubaneswar, Odisha, India

Abstract

This investigation intended to study the electrically conducting micropolar fluid on steady incompressible flow between two parallel plates along with heat generation i.e. source/absorption i.e. sink and first order chemical reaction. The non-dimensional form of the nonlinear coupled ordinary differential equations is obtained by the help of similarity technique and then Runge–Kutta method along with shooting technique is used to solve these transformed governing equations. It is necessary to discuss in detail about the distributions of the parameters that govern the flow phenomena graphically and further the numerical computation for rate of shear stress, rate of heat and mass transfer are presented using tables. It is remarked that Dufour and heat source favor in enhancing the fluid temperature.

Key words

Micropolar fluid, MHD, Heat source, Chemical reaction, Numerical solution.

1. Introduction

In recent years the study of dynamics of micropolar fluid has been the subject of interest for many scientists for its increasing applications in processing industries. In 1966, Eringen
introduced the theory of micropolar fluids to describe the motion of fluids with microstructure taking into consideration the local motion of the particles inside the volume element of the fluid. In his model of micropolar fluids, he developed the equations of motion together with appropriate constitutive equations. It is observed that most of the fluids used in chemical and allied processing applications are non-Newtonian. Application of non-Newtonian fluid is immense in various fields like biology, physiology, technology and industry. So a large studies has been taken place to analyze and understand such fluids.

Eringen [1] studied the effect of inertial spin, local rotary inertia and the couple stresses suitably provides a model for the non-Newtonian behavior observed in the suspended fluids and blood, polymers, paints, lubricants etc. Simple problems on the flow of such fluids were studied by a number of researchers and a review of such work was given by Ariman et al. [2]. Deka et al. [3] investigated the effect of first order homogeneous chemical reaction on the process of an unsteady flow past an infinite vertical plate with a constant heat and mass transfer. Ahraf et al. [4] numerically investigated the flow of micropolar fluids between a porous and non-porous disk. Further, they have studied the flow of micropolar fluids in a channel with porous wall [5].

Recently, Pal and Biswas [6] studied effect of heat sink on magneto-thermal radiative convective oscillatory flow of micropolar fluid. Mishra et al. [7] investigated flow of heat and mass transfer MHD free convection in a micropolar fluid with heat source. Srinivas Raju et al. [8] examined heat absorption effects both analytically (Laplace transform method) and numerically (Finite Element Method) on unsteady MHD free convection flow over exponentially moving vertical plate. Other recent studies focus on micropolar flow with heat source/sink effects with different geometries are Alam et al. [9] (stretching/shrinking wedge), Mishra et al. [10], Tripathy et al. [11]. Recently, it is also important to study radiation absorption effect in these flows, since Dombrovsky and Sazhin [12] have suggested that, radiation absorption is applicable in vaporization process of n-decane, combustion process in diesel engines and planetary atmosphere [13].

In recent years, natural convection flows with radiative, chemically reactive heat transfer have mobilized some interest, along with viscous dissipation and ohmic (Joule) heating has also prominent these days, in applications of materials fabrication operations, which interested us to invoke joule dissipation term which is conventionally neglected on the premise that under normal conditions the Eckert number is small based on an order of magnitude analysis. Gebhart [14] presented one of the earliest and most definitive studies of viscous dissipation in natural convection. Rahman [15] studied the effects of viscous dissipation and Joule heating in convective flows of micropolar fluid. Haque et al. [16] examined the micropolar fluid behaviours
on steady MHD free convection flow with Joule heating and viscous dissipation. Dash and Acharya [24] have presented a model for entropy generation analysis of MHD micropolar fluid flow. Recently, Sheri and Shamshuddin [17] have addressed the problem of coupled heat and mass transfer in magnetohydrodynamic micropolar flow with both viscous dissipation and chemical reaction effects. Rout et al. [18] employed Runge-Kutta fourth order with shooting technique to investigate chemical reaction effect on MHD free convection flow in a micropolar fluid.

In the above investigations, the micropolar transport in oscillatory inclined porous plate have been excluded. The boundary layer flows adjacent to inclined plates (or indeed other geometries) have however received less attention. Rahman et al. [19] analyzed heat transfer in micropolar fluid along an inclined permeable plate with variable fluid properties. Recently, Ajaz and Elangovan [20] investigated the action of alternating electric field with the effect of inclined magnetic field on the oscillatory flow of micropolar fluid. Other recent studies focused on micropolar fluid dynamics along inclined surface include the works by Aurangzaib et al. [21] and Srinivasacharya and Himabindu [22].

In view of above investigations, we have extended the work of Srinivasacharya and Mekonnen [23] by introducing the magnetic field effect on the flow of micropolar fluid through two parallel plates in the presence of heat source/sink and first order chemical reaction in the flow heat and mass transfer equations respectively which was not discussed earlier. The transformed nonlinear differential equations are solved by employing numerical method like Runge-Kutta forth order method associated with shooting technique. The present result is compared with the result of [23] in a particular case.

2. Mathematical Formulation and Analysis

Mixed convection flow of a steady, incompressible micropolar fluid between two parallel plates distance \( h \) apart has been considered in this paper. The flow direction is along \( x - \) axis and the \( y - \) axis is perpendicular to the plates. The temperature of the fluid at the lower plate \( y = 0 \) is \( T_1 \) and concentration \( C_1 \), whereas the temperature and concentration at the upper plate \( y = h \) are \( T_2 \) and \( C_2 \) respectively. Transverse magnetic field is applied with field strength \( B_0 \). In addition, the heat source/sink and chemical reaction are also taken into account in energy and mass transfer equations respectively. The transpiration cross-flow velocity \( v_0 \) remains constant, where \( v_0 < 0 \) is the velocity of suction and \( v_0 > 0 \) is the velocity of injection.
With the above assumptions and Boussinesq approximations with energy and concentration, the equations governing the flow of an incompressible micropolar fluid are [23]:

\[ v = v_0 = \text{constant} \] (1)

\[
\rho v_0 \frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + (\mu + k) \frac{\partial^2 u}{\partial y^2} + k \frac{\partial \Gamma}{\partial y} - \sigma B_0 u + \rho g \beta_T (T - T_1) + \rho g \beta_c (C - C_1)
\] (2)

\[
\rho j v_0 \frac{\partial \Gamma}{\partial y} = \gamma \left( \frac{\partial^2 \Gamma}{\partial y^2} - 2k \rho g \Gamma - k \frac{\partial u}{\partial y} \right)
\] (3)

\[
c_p \rho v_0 \frac{\partial T}{\partial y} = k_f \frac{\partial^2 T}{\partial y^2} + (\mu + k) \left( \frac{\partial u}{\partial y} \right)^2 + \gamma \left( \frac{\partial \Gamma}{\partial y} \right)^2 + 2k \left( \Gamma^2 + \frac{\partial u}{\partial y} \right) + \frac{DK_T}{C_s} \frac{\partial^2 C}{\partial y^2} - q (T - T_x)
\] (4)

\[
v_0 \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} + \frac{DK_T}{T_m} \frac{\partial^2 C}{\partial y^2} - Kc^* (C - C_m)
\] (5)

where \( u \) is velocity components in the \( x \)- directions, \( \Gamma \) is microrotation, \( \rho \) and \( j \) are the fluid density and gyration parameter, \( \mu, \kappa \) and \( \gamma \) are the material constants (viscosity coefficients), \( g \) is the acceleration due to gravity, \( p \) is pressure, \( \beta_T \) is the coefficient of thermal expansion, \( \beta_c \) is the coefficient of solutal expansion, \( k_f \) the coefficient of thermal conductivity, \( D \) is the mass diffusivity, \( c_p \) is the specific heat of fluid, \( C_s \) is the concentration susceptibility, \( T_m \) is the mean fluid temperature, and \( K_T \) is the thermal diffusion ratio.

The boundary conditions are:

\[
u = 0, \quad v = v_0, \quad \Gamma = 0, \quad T = T_1, \quad C = C_1, \quad \text{at} \ y = 0
\]

\[
u = 0, \quad v = v_0, \quad \Gamma = 0, \quad T = T_2, \quad C = C_2, \quad \text{at} \ y = h
\] (6)

Introducing the following non-dimensional variables:

\[
\eta = \frac{y}{h}, \quad u = U_0 f(\eta), \quad \Gamma = \frac{U_0}{h} \omega(\eta), \quad \theta(\eta) = \frac{T - T_1}{T_2 - T_1}, \quad \phi(\eta) = \frac{C - C_1}{C_2 - C_1}
\] (7)

in Eqs. (1)–(5), we get the following non-linear system of ordinary differential equations
\[
\frac{1}{N-1} f'' - R f' + \frac{N}{1-N} w' + \frac{Gr}{Re} \theta + \frac{Gc}{Re} \phi - Mf' - A = 0
\] (8)

\[
\frac{2-N}{m_p^2} w'' - a_j \frac{1}{N} R w' - (2w + f') = 0
\] (9)

\[
\theta'' - R Pr \theta' + \frac{Br}{1-N} \left[ f''^2 + \frac{N(2-N)}{m_p^2} w'^2 + 2N(w^2 - wf') \right] + Du Pr \phi'' + Pr \beta \theta = 0
\] (10)

\[
\frac{1}{Sc} \phi'' - R \phi' + Sr \theta'' - \gamma \phi = 0
\] (11)

where primes denote differentiation with respect to \( \eta \), \( Sc = \frac{V}{D} \) is the Schmidt number, \( Pr = \frac{\mu c_p}{k_f} \)

is the Prandtl number, \( Re = \frac{\rho U_0 h}{\mu} \) is the Reynolds number, \( Sr = \frac{DK_r(T - T_i)}{v T_0 (C_2 - C_1)} \) is the Soret number, \( Du = \frac{DK_r(C_2 - C_1)}{v C_s C_r(T_2 - T_1)} \) is the Dufour number, \( R = \frac{\rho v_0 h}{\mu} \) is the suction/injection parameter, \( N = \frac{k}{\mu + k} \) is coupling number, \( Gr = \frac{g \rho^2 \beta_c (T_2 - T_1) h^3}{\mu^2} \) is temperature Grashof number, \( Gc = \frac{g \rho^2 \beta_c (C_2 - C_1) h^3}{\mu^2} \) is the mass Grashof number, \( A = \frac{h^2}{\mu U_0} \frac{dP}{dx} \) is the constant pressure gradient, \( m_p^2 = \frac{h^2 k(2 \mu + k)}{\gamma (\mu + k)} \) is the micropolar parameter, \( a_j = \frac{j}{h^2} \) is the micro-inertial density parameter, \( Br = \frac{\mu U_0^2}{k_f (T_2 - T_1)} \) is the Brinkman number.

Boundary conditions (6) in terms of \( f, \omega, \theta \) and \( \phi \) become

\[
\begin{align*}
    f &= 0, \omega = 0, \theta = 0, \phi = 0 \quad \text{at} \quad \eta = 0 \\
    f &= 0, \omega = 0, \theta = 1, \phi = 1, \quad \text{at} \quad \eta = 1
\end{align*}
\] (12)

3. Physical Quantities of Interest

The major physical quantities of interest are the skin friction coefficient \( C_f \), Nusselt number \( Nu \), and Sherwood number \( Sh \), which are defined as

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\[ C_j = \frac{\tau_w}{p u^2_w/2}, \quad Nu = \frac{xq_w}{k(T_w - T_{\infty})}, \quad Sh = \frac{xq_m}{D(C_u - C_{\infty})} \] (14)

where surface shear stress, surface heat and mass flux are defined as

\[ \tau_w = \left[ (\mu + k) \frac{\partial u}{\partial y} + k \omega \right] _{y=0}, \quad q_w = -k \left( \frac{\partial T}{\partial y} \right) _{y=0}, \quad q_m = -D \left( \frac{\partial C}{\partial y} \right) _{y=0} \] (15)

Using the non-dimensional variables (8), we get from Eqs. (14) and (15) as

\[ \frac{1}{2} C_j \text{Re}_y^2 = \left( 1 + \frac{K}{2} \right) f'(0), \quad \frac{Nu}{\text{Re}_y^2} = -\theta'(0), \quad \frac{Sh}{\text{Re}_y^2} = -\phi'(0) \] (16)

4. Results and Discussion

In order to insight the physical significance of the pertinent physical parameters characterizes the flow phenomena of an electrically conducting micropolar fluid between two parallel plates are discussed. The numerical solutions are obtained to exhibit the effects of emerging physical parameters on the momentum, angular momentum, temperature and concentration distributions through graphs. Also, the numerical computation of the shear stress, for both the momentum profiles, rate of heat and mass transfer are shown in Table-1.

In the present study, flow of an electrically conducting micropolar fluid between two parallel plates subject to transverse magnetic field in the presence of uniform heat source/sink and first order chemical reaction has been investigated. The aim of the investigation is to bring out the effects of additional parameters introduced such as magnetic parameter \( M \), heat generation / absorption \( \beta \) parameter and the destructive / generative chemical reaction \( \gamma \) besides the other parameters characterizes the flow phenomena. For the verification the present result is compared with the result of Srinivasacharya and Mekonnen [23] in a particular case.

Figs.2 and 3 illustrate the effect of magnetic parameter on the flow phenomena in the absence of heat source and chemical reaction and the presence of other physical parameters and \( N > 1 \). In the absence of \( M \) (\( M=0 \)) present result compared with the result of Srinivasacharya and Mekonnen [23] and found that the present result is in good agreement. It is interesting to note that an increase in magnetic parameter enhances the velocity profile at all points in the velocity boundary layer. Also, near the first plate (\( \eta < 0.2 \)), microrotation profile increases as magnetic

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parameter increases whereas reverse effect is encountered after that \((\eta > 0.2)\) i.e. an increase in magnetic parameter micorotatin profile decelerates.

![Fig.2. Effect of \(M\) on velocity profiles for \(N = 2.5\) and \(R = 2\), \(Pr = 0.71\), \(Gr = 0.2\) , \(Gc = 2\), \(Sc = 2\), \(\gamma = 0\), \(Re = 1\), \(a = 1\), \(aj = 0.001\), \(mp = 1\), \(Br = 1\), \(Du = 1\), \(Sr = 0.2\), \(\beta = 0\)
](image)

Figs. 4 and 5 exhibit the effect of magnetic parameter on velocity and microrotation profiles respectively for \(N < 1\). It is observed that in both the profile the effect is reverse as that of Figs.2 and 3. The velocity profile retards as magnetic parameter increases. It is due to the Lorentz force which is a resistive force generates due to the interaction of conducting fluid and magnetic field, retards the velocity profile. It is also remarked that the microrotation profile has opposite characteristics about the middle layer of the channel i.e. \((\eta = 0.5)\). The profile decelerates before the region \(\eta < 0.5\) and then enhances after that region to meet the inadequate boundary conditions.
Fig. 4. Effect of $M$ on velocity profiles for $N = 0.5$ and $R = 2, Pr = 0.71, Gr = 0.2, \text{ and } Gc = 2,$

$Sc = 2, \gamma = 0, Re = 1, a = 1, aj = 0.001, m_p = 1, Br = 1, Di = 1, Sr = 0.2, \beta = 0.$

Fig. 5 Effect of $M$ on microrotation profiles for $N = 0.5$ and $R = 2, Pr = 0.71, Gr = 0.2, \text{ and } Gc = 2,$

$Sc = 2, \gamma = 0, Re = 1, a = 1, aj = 0.001, m_p = 1, Br = 1, Di = 1, Sr = 0.2, \beta = 0.$
Fig.6. Effect of $Gr$ and $Gc$ on velocity profile for $n=0.5$ and $R=2, Pr=0.71, Sc=2, M=0.5$,
$\gamma = 0, Re = 1, a = 1, aj = 0.001, mp = 1, Br = 1, Du = 1; Sr = 0.2; \beta = 0$

Fig.7. Effect of $Gr$ and $Gc$ on microrotation profile for $N=0.5$ and $R=2, Pr=0.71, Sc=2, M=0.5$,
$\gamma = 0, Re = 1, a = 1, aj = 0.001, mp = 1, Br = 1, Du = 1; Sr = 0.2; \beta = 0$

The effects of thermal and mass buoyancy parameters on both the velocity and microrotation profiles are exhibited in Figs.6 and 7 in the presence of magnetic and other parameters shown in the caption for $N=0.5$. It is seen that an increase in thermal buoyancy and mass buoyancy parameter velocity profile increases significantly. For the higher value of thermal buoyancy parameter i.e. $Gr=2.0$, the velocity becomes maximum (Fig.6). From Fig.7 it is clear that the effect of these buoyant forces behaves adversely as that of velocity in Fig.6 within the region $\eta = 0.65$ and then trend becomes reversed. Hence, it is concluded that higher values of $Gr$ and $Gc$ favours in an increase in the velocity profile.

Fig.8 exhibits the effects of Prandtl number and heat generation (source)/ absorption (sink) parameter on the temperature profile. In case of water ($Pr=0.71$), increase in source enhances the
fluid temperature whereas sink opposes it. That is, sink has reverse effect on the temperature profile. Further, it is interesting to note that increase in Prandtl number increases the fluid temperature. In a particular case in the absence of heat source ($\beta = 0$) the present result is in good agreement with that of Srinivasacharya and Mekonnen [23].

Figs. 9 illustrates the effect of Dufour and Soret number on the temperature profile in the presence of magnetic, heat source and other fixed values of the pertinent parameters for $N = 0.5$. It is noteworthy that the increase in Dufour number the fluid temperature increases. The presence of source plays an important role to enhance the temperature of the fluid at all points in the thermal boundary layer.

![Graph](image-url)

**Fig.8.** Effect of $Pr$ and $\beta$ on temperature profile for $N = 0.5$ and

$$R = 2, Sc = 2, Gr = 0.2, Gc = 2, M = 0.5, \gamma = 0, Re = 1, a = 1, aj = 0.001, mp = 1, Br = 1, Du = 1; Sr = 0.2$$
Fig. 9 Effect of $Du$ on temperature profile for $N = 0.5$ and

$$R = 2, Sc = 2, Gr = 0.2, Pr = 0.71, Gc = 2, M = 0.5, \gamma = 0, Re = 1, a = 1, aj = 0.001, mp = 1, Br = 1, Sr = 0.2; \beta = 1$$

Fig. 10 Effect of $Sr$ on Concentration Profile for $N = 0.5$ and

$$R = 2, Sc = 2, Gr = 0.2, Pr = 0.71, Gc = 2, M = 0.5, \gamma = 0, Re = 1, a = 1, aj = 0.001, mp = 1, Br = 1, Du = 1; \beta = 1$$

Fig.10 presents the effect of Soret on the concentration profile in the absence of chemical reaction parameter. It is observed that the fluid concentration increases as an increase in Soret
number. Presence of heavier species i.e. high value of Schmidt number inclusion with Soret number favours to increase the fluid concentration in its concentration boundary layer.

Fig.11 exhibits the effects of Schmidt number and Chemical reaction on the concentration profile in the presence of magnetic and other fixed values of physical parameters for \( N = 0.5 \). In the present case we discuss the effect of destructive chemical reaction (\( \gamma > 0 \)), no chemical reaction (\( \gamma = 0 \)) and constructive chemical reaction (\( \gamma < 0 \)). It is clear to note that higher value of chemical reaction i.e. in case of destructive chemical reaction the concentration of the fluid decelerates where as in case of constructive the adverse effect is observed. Also in the absence of chemical reaction the fluid concentration becomes linear. Further the heavier species also decelerates the fluid concentration at all points in the concentration boundary layer.

![](image)

Fig.10. Effect of \( Sc \) and \( \gamma \) on Concentration Profile for \( N = 0.5 \) and 

\[ R = 2, Sc = 2, Pr = 0.2, Gr = 0.71, Gc = 2, M = 0.5, Re = 1, a = 1, aj = 0.001, mp = 1, Br = 1; Du = 1; \beta = 1 \]

Tab 1. Skin Friction Coefficient, Nusselt Number and Sherwood Number for 

\[ R = 2, Sc = 2, Pr = 0.71, Re = 1, a = 1, aj = 0.001, mp = 1, Br = 1, Du = 1 \]

<table>
<thead>
<tr>
<th>( M )</th>
<th>( N )</th>
<th>( Gr )</th>
<th>( Gc )</th>
<th>( \beta )</th>
<th>( \gamma )</th>
<th>( f'(0) )</th>
<th>( -\phi'(0) )</th>
<th>( -\phi'(0) )</th>
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</thead>
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<td>0</td>
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<td>0.2</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>-0.157892</td>
<td>0.8208708</td>
<td>0.1562794</td>
</tr>
<tr>
<td>0.5</td>
<td>0.5</td>
<td>0.2</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>-0.155528</td>
<td>0.82056</td>
<td>0.1563524</td>
</tr>
<tr>
<td>0.5</td>
<td>2.5</td>
<td>0.2</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0.6699169</td>
<td>0.7885034</td>
<td>0.1667105</td>
</tr>
</tbody>
</table>
The numerical computations of rate of shear stress i.e. skin friction coefficient, rate of heat and mass transfer are obtained and presented in Table-1 for different values of the pertinent parameters and fixed values of other parameters characterizes in the flow phenomena. It is observed that the increasing value of magnetic parameter, thermal and solutal buoyancy decreases the skin friction in magnitude whereas an increase in material parameter, $N$ increases the skin friction coefficient. Also all the parameters except heat source and destructive chemical reaction decreases the rate of heat transfer whereas reverse effect is remarked for rate of mass transfer. Further, sink reduces the rate of heat transfer and constructive chemical reaction is desirable to increase the rate of mass transfer.

5. Conclusion

From the above discussion the following conclusions are made:

- An increase in magnetic parameter velocity profile decreases whereas the microrotation profile enhances for $N>1$ but reverse effect is observed for $N<1$.
- Velocity becomes maximum for higher values of buoyant forces.
- Both Dufour and Soret enhance the thermal and concentration boundary layer respectively.
- Both chemical reaction and heavier species retards the concentration profile.
- An increase in $N$ decreases the skin friction coefficient and rate of heat transfer whereas the rate of mass transfer increases.

References


**Nomenclature**

\[ C \quad \text{Fluid concentration} \]

\[ D \quad \text{Coefficient of the mass diffusivity} \]
Pr Prandtl number
\( g \) Acceleration due to gravity
\( M \) Magnetic parameter
\( k \) Material parameter
\( C_p \) Specific molecular diffusivity

**Greek symbols**

\( Sc \) Schmidt number
\( \theta(\eta) \) Dimensionless temperature
\( T \) Fluid temperature
\( \mu \) Dynamic viscosity
\( T_\infty \) Fluid temperature at infinity
\( \delta \) Solutal buoyancy parameter
\( B_0 \) Magnetic flux density
\( \alpha \) Thermal diffusivity
\( Sh \) Sherwood number
\( \sigma \) Electrical conductivity
\( k_i \) Thermal conductivity
\( \lambda \) Thermal buoyancy or mixed convection parameter
\( Nu \) Nusselt number
\( \rho \) Density of the fluid
\( u, v \) Velocity components along x- and y-direction
\( \kappa \) Vortex viscosity or microrotation viscosity
\( j \) Micro-inertia density
\( C_w \) Stretching sheet concentration
\( Ec \) Eckert number
\( T_w \) Stretching sheet temperature
\( x, y \) Coordinates
\( \gamma \) Chemical reaction parameter
\( \nu \) Kinematic viscosity
\( c_i \) Species concentration at upper plate
\( \beta \) heat source/sink parameter
\( \omega \) Angular Velocity or microrotation vector
\( \phi(\eta) \) Non-dimensional concentration parameter