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MHD Viscoelastic Fluid Flow through a Vertical Flat Plate with Soret and Dufour Effect

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Abstract

MHD viscoelastic fluid flow through a vertical flat plate in the presence of Soret and Dufour effect has been studied. The usual transformations have been applied to obtain the non-dimensional, partial coupled non-linear momentum, temperature and concentration equations. Explicit finite difference technique has been applied to solve the problem numerically. To obtain the restriction of parameters, stability and convergence criteria have been analyzed. The numerical computations have been carried out for fluid velocity, temperature concentration, local shear stress, Nusselt number, Sherwood number, average shear stress, average Nusselt number and average Sherwood number. The obtained results have been illustrated graphically. Finally a qualitative comparison has been shown in tabular form.

Key words

MHD, Viscoelastic fluid, Soret number, Dufour number, Explicit finite difference, Stability analysis.

1. Introduction

Newton's law of fluid dynamics has divided the fluid into two groups. One type is Newtonian fluid which has some scope to testify a fluid to be a Newtonian fluid or not. So many fluids in industrial process and in nature show unexpected and interesting flow patterns which outside the scope the Newtonian fluid dynamics. This type of fluids is known as non-Newtonian fluid.

The most common non-Newtonian fluid is viscoelastic fluid. This research paper is concerned with viscoelastic fluids. It has two properties, one is viscous property and another one is elastic property and so it is named as viscoelastic fluid. Its importance is increasing day by day due to its many engineering, biological, industrial and chemical aspects. Some common viscoelastic fluids are engine oils, paints, honey, shampoo, ointments, gels, molten plastics, blood and so on. These appear in many industrial process, chemical reaction and pharmaceutical industries.

Some of the researchers have studied about its nature, some studied its structure or deformation, some studied its flow patterns (i.e., velocity, temperature, density, pressure and etc.), and some investigated the effects of various flow parameters on this fluid. The analysis of viscoelastic fluids was started about 1950. Oldroyd [1], Beard and Walters [2] and Rajagopal et al. [3] are considered the Pioneer of viscoelastic fluids (second grade fluids), who have developed the boundary layer theory for the second grade fluids. This boundary layer theory for the second grade fluids has motivated many researchers to really explore this kind of fluids with various conditions. Rajagopal et al. [3] studied boundary layer flow of a viscoelastic fluid over a stretching sheet. Dandapat [4] investigated flow and heat transfer in a viscoelastic fluid over a stretching sheet. Kumar et al. [5] studied flow and heat transfer of a viscoelastic fluid over a flat plate with a magnetic field and a pressure gradient.

Dash et al. [6] investigated finite difference analysis of hydromagnetic flow and heat transfer of an elastico-viscous fluid between two horizontal parallel porous plates. Islam and Alam [7] have investigated Dufour and Soret Effects on unsteady MHD Free Convection and mass transfer fluid flow through a porous medium in a rotating system. Khan et al. [8] studied convective heat transfer in the flow of visco-elastic fluid in a porous medium past a stretching sheet. Attia and Ewis [9] studied unsteady MHD Couette flow with heat transfer of a viscoelastic fluid under exponential decaying pressure gradient. Recently Gbadeyan et al. [10] examined heat and mass transfer for Soret and Dufour's effect on mixed convection boundary layer flow over a stretching vertical surface in a porous medium filled with a viscoelastic fluid in the presence of magnetic field. Hossain and Alam [11] solved the Gbadeyan et al. [10]'s model by implicit finite difference method for unsteady case.

Our objective is to extend the work of Hossain and Alam [11] for unsteady two dimensional case and to solve the problem numerically by explicit finite difference method. Also, this work has been done for Soret and Dufour effects in the presence of magnetic field on two dimensional unsteady flow of an incompressible viscoelastic fluid through a vertical flat plate.

2. Mathematical Formulation

The fluid is assumed to be incompressible, unsteady and laminar flow of a viscoelastic fluid moving through a vertical flat plate in the presence of magnetic field. The fluid is also experienced of Soret and Dufour effects. The positive x coordinate is measured along the plate in the direction of fluid motion and the positive y coordinate is measured normal to the plate. A uniform magnetic field $\mathbf{B}$ is imposed to the plate $(y = 0)$ to be acting along the $y-$axis which is assumed to be electrically non-conducting. Assumed that $\mathbf{B} = (B_x, B_y, B_z) = (0, B_0, 0)$ is the magnetic field vector and $B_x B_y, B_z$ be the components of the magnetic field in the x, y, z direction respectively. Also it is assumed that the uniform velocity of the fluid is U_0 with the wall temperature T_w and wall concentration C_w along the plate. T_∞ and C_∞ represent the temperature and concentration outside the boundary layer. The physical model of this problem is shown in Fig. 1.

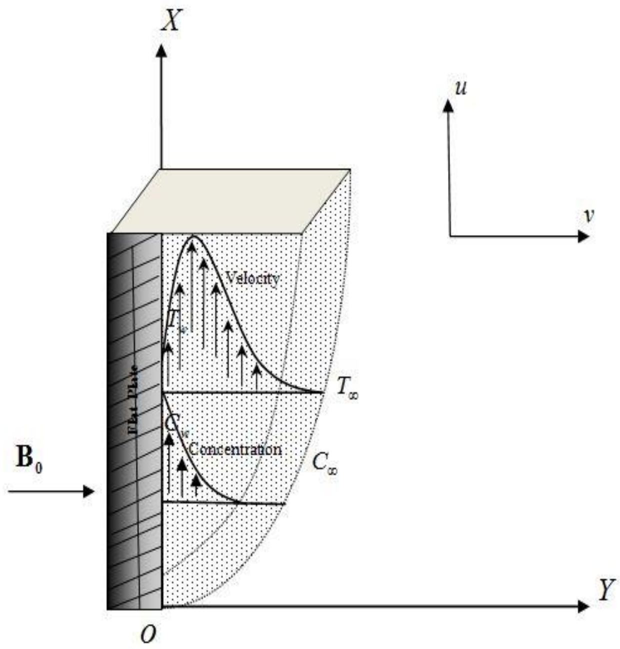


Fig. 1: Physical model with coordinate system

On the basis of the above state assumptions and the physical model of the fluid motion is governed by the given equations;

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = g\beta(T - T_\infty) + g\beta^*(C - C_\infty) - \frac{1}{\rho}\sigma B_0{}^2 u + \upsilon\frac{\partial^2 u}{\partial y^2} + \kappa_0\left[\frac{\partial^3 u}{\partial t\partial y^2} + u\frac{\partial^3 u}{\partial x\partial y^2} + v\frac{\partial^3 u}{\partial y^3} + \frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2} + \frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y\partial x}\right] \tag{2}$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{\kappa}{\rho c_p}\frac{\partial^2 T}{\partial y^2}+\frac{\sigma {B_0}^2}{\rho c_p}u^2+D_m\frac{\partial^2 C}{\partial y^2} \tag{3}$$

$$\frac{\partial C}{\partial t}+u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_m\frac{\partial^2 C}{\partial y^2}+D_T\frac{\partial^2 T}{\partial y^2} \tag{4}$$

The boundary conditions are;

$$u=U_0, v=0, T=T_w, C=C_w \text{ at } y=0$$
$$u\to 0,\ \frac{\partial u}{\partial y}\to 0,\ T\to T_\infty,\ C\to C_\infty \text{ as } y\to\infty \tag{5}$$

The following dimensionless variables that are used to obtained dimensionless governing equations (1)- (4) as;

$$X=\frac{xU_0}{\upsilon}, Y=\frac{yU_0}{\upsilon}, U=\frac{u}{U_0}, V=\frac{v}{U_0}, \tau=\frac{tU_0^2}{\upsilon}, \theta=\frac{T-T_\infty}{T_w-T_\infty}, \phi=\frac{C-C_\infty}{C_w-C_\infty}$$

The following dimensionless equations have been obtained by using the above non-dimensional quantities;

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \tag{6}$$

$$\frac{\partial U}{\partial \tau}+U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=G_r\theta+G_m\phi-MU+\frac{\partial^2 U}{\partial Y^2}+K\left[\frac{\partial^3 U}{\partial \tau\partial Y^2}+U\frac{\partial^3 U}{\partial X\partial Y^2}\right.$$
$$\left.+\frac{\partial U}{\partial X}\frac{\partial^2 U}{\partial Y^2}+\frac{\partial U}{\partial Y}\frac{\partial^2 U}{\partial Y\partial X}+V\frac{\partial^3 U}{\partial Y^3}\right] \tag{7}$$

$$\frac{\partial \theta}{\partial \tau}+U\frac{\partial \theta}{\partial X}+V\frac{\partial \theta}{\partial Y}=\frac{1}{P_r}\frac{\partial^2 \theta}{\partial Y^2}+ME_cU^2+D_u\frac{\partial^2 \phi}{\partial Y^2} \tag{8}$$

$$\frac{\partial \phi}{\partial \tau}+U\frac{\partial \phi}{\partial X}+V\frac{\partial \phi}{\partial Y}=\frac{1}{S_c}\frac{\partial^2 \phi}{\partial Y^2}+S_r\frac{\partial^2 \theta}{\partial Y^2} \tag{9}$$

where, G_r is the Grashof number, G_m is the modified Grashof number, M is the Magnetic parameter, K is the dimensionless viscoelastic parameter, P_r is the Prandtl number, E_c is the Eckert number, D_u is the Dufour number, S_r is the Soret number and S_c denotes the Schmidt number.

The corresponding non dimensional boundary conditions are;

$$U=1, V=0, \theta=1, \phi=1 \text{ at } Y=0 \tag{10}$$

$U \to 0$, $V \to 0$, $\theta \to 0$, $\phi \to 0$ as $Y \to \infty$

3. Shear Stress, Nusselt Number and Sherwood Number

From the velocity, the effects of various parameters on the local and average shear stress have been calculated. Local shear stress $\tau_L = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0}$ and average shear stress $\tau_A = \mu\int\left(\frac{\partial u}{\partial y}\right)_{y=0} dx$ which are proportional to $\left(\frac{\partial U}{\partial Y}\right)_{Y=0}$ and $\int_0^{100}\left(\frac{\partial U}{\partial Y}\right)_{Y=0} dX$ respectively. From the temperature field, the effects of various parameters on the local and average heat transfer coefficients have been investigated. Local Nusselt number, $N_{uL} = \mu\left(-\frac{\partial T}{\partial y}\right)_{y=0}$ and average Nusselt number, $N_{uA} = \mu\int\left(-\frac{\partial T}{\partial y}\right)_{y=0} dx$ which are proportional to $\left(-\frac{\partial \theta}{\partial Y}\right)_{Y=0}$ and $\int_0^{100}\left(-\frac{\partial \theta}{\partial Y}\right)_{Y=0} dX$ respectively. From the concentration field, the effects of various parameters on the local and average mass transfer or Sherwood number coefficients have been analyzed. Local Sherwood number, $S_{hL} = \mu\left(-\frac{\partial C}{\partial y}\right)_{y=0}$ and average Sherwood number, $S_{hA} = \mu\int\left(-\frac{\partial C}{\partial y}\right)_{y=0} dx$ which are proportional to $\left(-\frac{\partial \phi}{\partial Y}\right)_{Y=0}$ and $\int_0^{100}\left(-\frac{\partial \phi}{\partial Y}\right)_{Y=0} dX$ respectively.

4. Numerical Analysis

The governing non-dimensional continuity, momentum, energy and concentration equations (6)-(9) respectively with associated boundary conditions have been solved numerically by explicit finite difference technique. The rectangular region within the boundary layer is divided by some mash of lines parallel to X and Y axes where $X-$axis is taken along the plate and $Y-$axis is normal to the plate as shown in Fig. 2. Here, the plate of

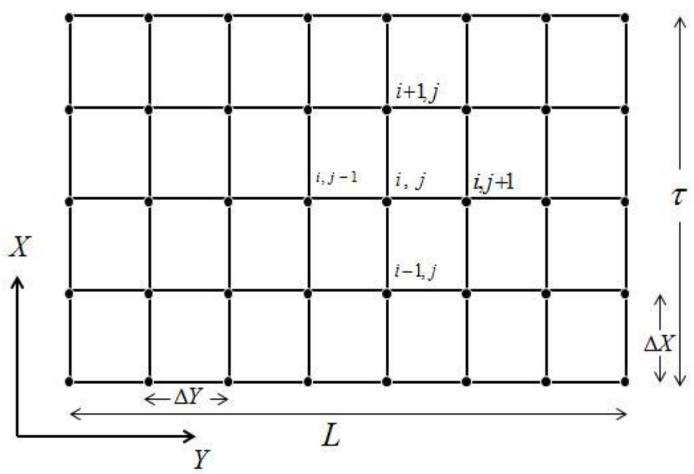


Fig. 2: Finite difference grid system

height $X_{\max} = (100)$ i.e. X varies from 0 to 100 and it is assumed that the maximum length of boundary layer is $Y_{\max} = (35)$ as corresponding to $Y \to \infty$ i.e. Y varies from 0 to 35 have been considered. Consider $m = 150$ and $n = 150$ in X and Y directions. It is assumed that ΔX, ΔY are constant mesh sizes along X and Y directions respectively and taken as follows,

$\Delta X = 0.67(0 \le x \le 100); \Delta Y = .23(0 \le y \le 35)$ with the smaller time-step, $\Delta\tau = 0.005$.

Let U', V', θ' and ϕ' denote the values of U, V, θ and ϕ at the end of a time-step respectively. An appropriate set of finite difference equations have been obtained as;

$$\frac{U'_{i,j} - U'_{i-1,j}}{\Delta X} + \frac{V'_{i,j} - V'_{i,j-1}}{\Delta Y} = 0 \tag{11}$$

$$\begin{aligned}
\frac{U'_{i,j} - U_{i,j}}{\Delta\tau} + U_{i,j}\frac{U_{i,j} - U_{i-1,j}}{\Delta X} + V_{i,j}\frac{U_{i,j+1} - U_{i,j}}{\Delta Y} &= G_r\theta'_{i,j} + G_m\phi'_{i,j} - MU_{i,j} + \frac{U_{i,j+1} - 2U_{i,j} + U_{i,j-1}}{(\Delta Y)^2} \\
&+ K\left[\frac{U'_{i,j+1} - 2U'_{i,j} + U'_{i,j-1} - U_{i,j+1} + 2U_{i,j} - U_{i,j-1}}{\Delta\tau(\Delta Y)^2}\right. \\
&+ U_{i,j}\frac{U_{i,j+1} - 2U_{i,j} + U_{i,j-1} - U_{i-1,j+1} + 2U_{i-1,j} - U_{i-1,j-1}}{\Delta X(\Delta Y)^2} \\
&+ V_{i,j}\frac{U_{i,j+2} - 3U_{i,j+1} + 3U_{i,j} - U_{i,j-1}}{(\Delta Y)^3} + \frac{U_{i,j} - U_{i-1,j}}{\Delta X}\frac{U_{i,j+1} - 2U_{i,j} + U_{i,j-1}}{(\Delta Y)^2} \\
&\left.+ \frac{U_{i,j+1} - U_{i,j}}{\Delta Y}\frac{U_{i,j+1} - U_{i,j} - U_{i-1,j+1} + U_{i-1,j}}{\Delta Y\Delta X}\right]
\end{aligned} \tag{12}$$

$$\begin{aligned}
\frac{\theta'_{i,j} - \theta_{i,j}}{\Delta\tau} + U_{i,j}\frac{\theta_{i,j} - \theta_{i-1,j}}{\Delta X} + V_{i,j}\frac{\theta_{i,j+1} - \theta_{i,j}}{\Delta Y} &= \frac{1}{P_r}\frac{\theta_{i,j+1} - 2\theta_{i,j} + \theta_{i,j-1}}{(\Delta Y)^2} + ME_cU_{i,j}{}^2 \\
&+ D_u\frac{\phi_{i,j+1} - 2\phi_{i,j} + \phi_{i,j-1}}{(\Delta Y)^2}
\end{aligned} \tag{13}$$

$$\begin{aligned}
\frac{\phi'_{i,j} - \phi_{i,j}}{\Delta\tau} + U_{i,j}\frac{\phi_{i,j} - \phi_{i-1,j}}{\Delta X} + V_{i,j}\frac{\phi_{i,j+1} - \phi_{i,j}}{\Delta Y} &= \frac{1}{S_c}\frac{\phi_{i,j+1} - 2\phi_{i,j} + \phi_{i,j-1}}{(\Delta Y)^2} \\
&+ S_r\frac{\theta_{i,j+1} - 2\theta_{i,j} + \theta_{i,j-1}}{(\Delta Y)^2}
\end{aligned} \tag{14}$$

The initial and boundary conditions with the finite difference scheme are;

$$U^n_{i,0} = 1,\ V^n_{i,0} = 0,\ \theta^n_{i,0} = 1,\ \phi^n_{i,0} = 1 \tag{15}$$

$U^n_{i,L} = 0$, $V^n_{i,L} = 0$, $\theta^n_{i,L} = 0$, $\phi^n_{i,L} = 0$ where $L \to \infty$.

The numerical values of the local shear stress, Nusselt number and Sherwood number are evaluated by five-point approximate formula for the derivatives and then the average shear stress, Nusselt number and Sherwood number are calculated by the use of the Simpson's $\frac{1}{3}$ integration formula.

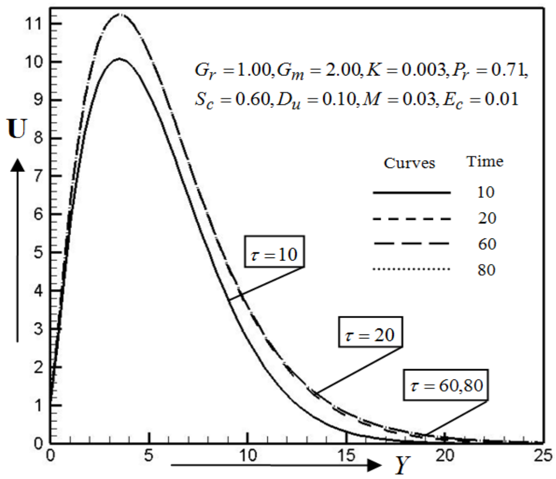
The stability conditions of the problem are as furnished below as;

$$U\frac{\Delta\tau}{\Delta X}+|V|\frac{\Delta\tau}{\Delta Y}+\frac{2}{P_r}\frac{\Delta\tau}{(\Delta Y)^2}\le 1 \text{ and } U\frac{\Delta\tau}{\Delta X}+|V|\frac{\Delta\tau}{\Delta Y}+\frac{2}{S_c}\frac{\Delta\tau}{(\Delta Y)^2}\le 1$$

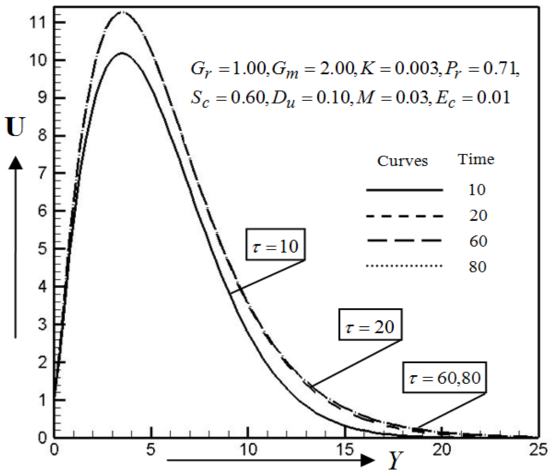
When $\Delta\tau$ and ΔY approach to zero then the problem will be converged. The convergence criteria of the problem are $P_r \ge 0.19$ and $S_c \ge 0.19$.

5. Results and Discussion

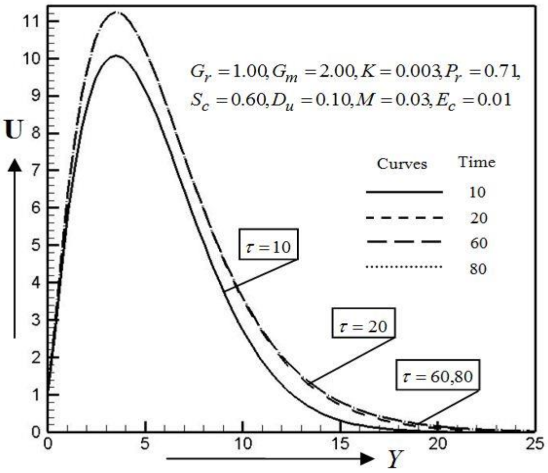
Analytically it can not be solved the obtained non-linear, coupled partial differential equations. For this reason explicit finite difference technique has been applied to solve those equations numerically. To obtain the steady-state solutions of the computations, the calculation has been carried out up to dimensionless time, $\tau = 120$. The velocity, temperature and concentration profiles do not show any change after dimensionless time, $\tau \ge 60$. Therefore the dimensionless time, $\tau \ge 60$ is the steady state solution for this problem. Also for more accurate results (numerical and graphical) different mesh size has been considered and shown graphically in Fig. 3(a-c). It has been seen that the graphical representation of various dimensionless parameters have been reached its highest convergence when mesh size is taken as $m = 150$ and $n = 150$. Therefore various flow parameters have been illustrated graphically in Figs. (4-7) with dimensionless time $\tau = 60$ and mesh size $m = 150$ and $n = 150$.



3(a)



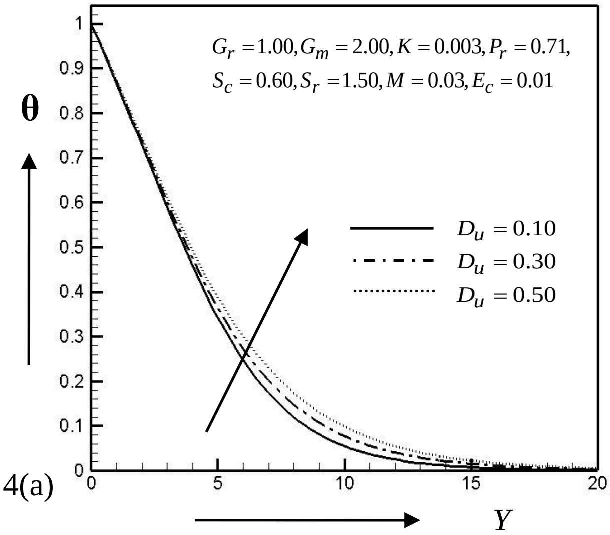
(b)



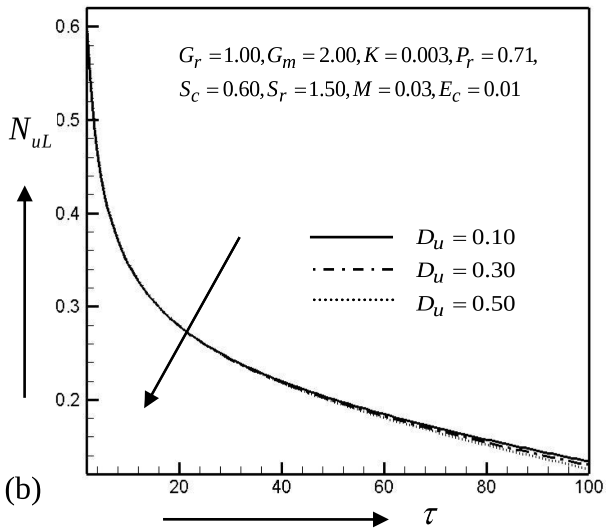
(c)

Fig.3. Illustration of steady state solution for $S_r = 2.00$ at different time intervals with mesh size (a) $m = 100$ and $n = 100$ (b) $m = 130$ and $n = 130$ (c) $m = 150$ and $n = 150$.

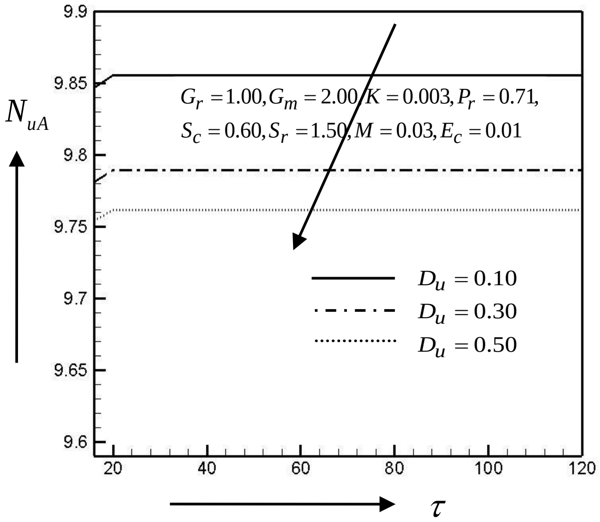
In Fig. 4(a-c) it has been shown the effect of Dufour number D_u on fluid temperature and Nusselt number and Sherwood number. Thermal boundary layer is found to increase in Fig. 4 (a) with the increase of Dufour number D_u.



4(a)



(b)



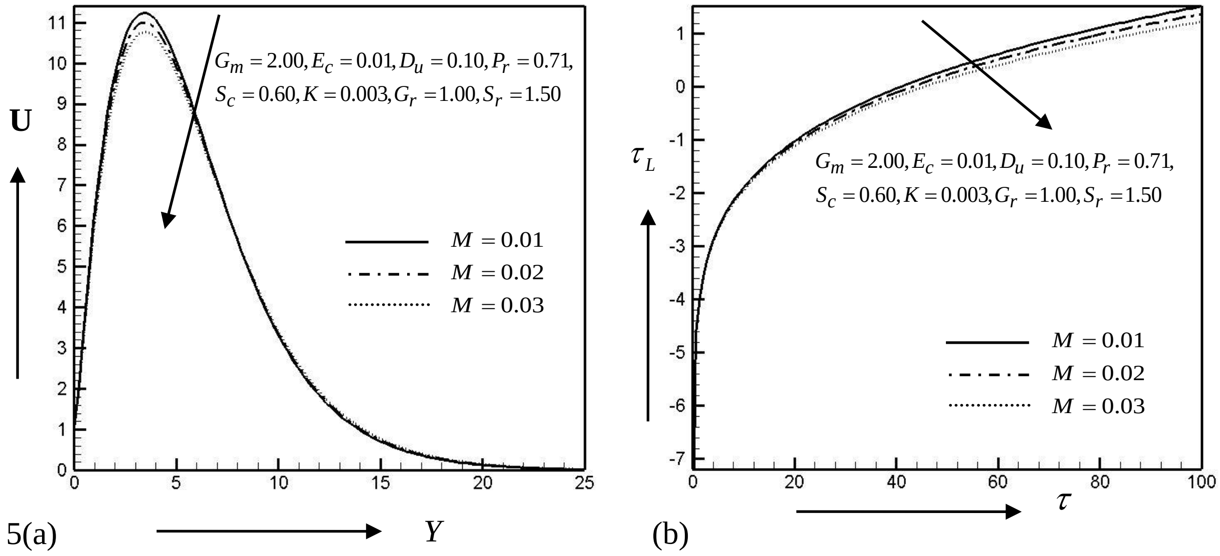
(c)

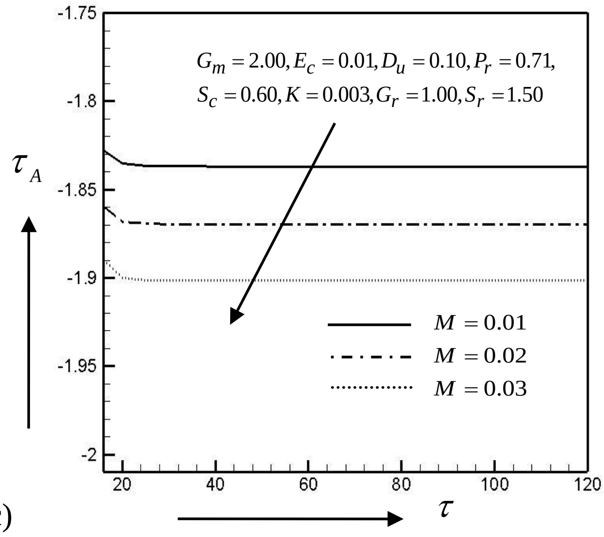
Fig.4 Illustration of (a) Velocity profiles (b) Local shear stress (c) Average shear stress for different values of Magnetic parameter M .

But the Nusselt number and average Nusselt number decrease with the increase of Dufour number D_u those have been shown in Fig. 4(b) and 4(c) respectively.

The velocity profiles, local shear stress and average shear stress for various values of Magnetic parameter M have been illustrated in Fig. 5(a-c). In those figures, fluid velocity, local shear stress and average shear stress decrease with the increase values of Magnetic parameter M .

The effect of Soret number S_r on fluid concentration, Sherwood number and average Sherwood number has been shown in Fig. 5 (a-c). It is observed that fluid concentration increases as Soret number S_r increase. On the other hand, the Sherwood number and average Sherwood number have been decreased with the increase of Soret number S_r .

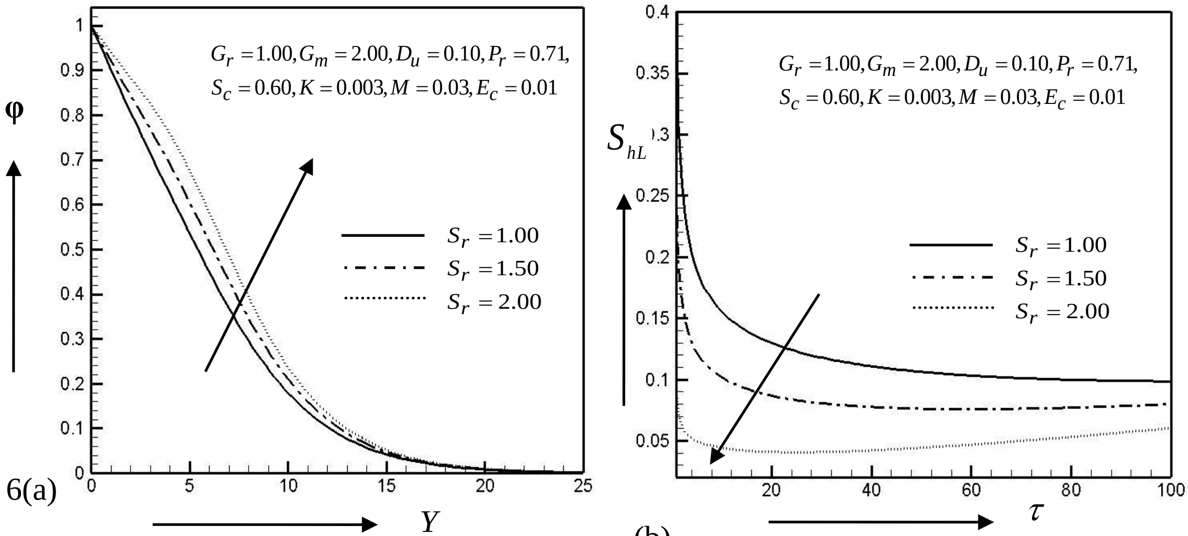




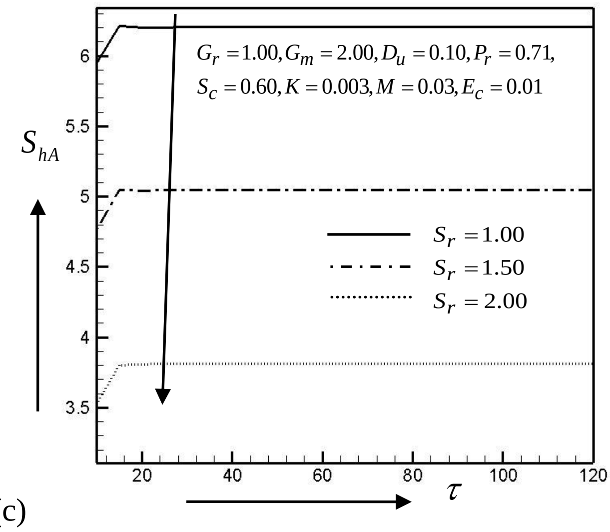
(c)

Fig.5. Illustration of (a) Velocity profiles (b) Local shear stress (c) Average shear stress for different values of Magnetic parameter M .

Viscoelastic parameter K is the key parameter in this problem and the effect of this parameter has been shown in Fig. 7(a-c). In Fig. 7(a), with the increase of Viscoelastic parameter K the fluid velocity has been decreased. Similar effect has been shown in Fig. 7 (b) and Fig. 7(c) that the local shear stress and average shear stress decrease with the increase of Viscoelastic parameter K .



6(a) (b)



(c)

Fig.6 Illustration of (a) Concentration profiles (b) Sherwood number (c) Average Sherwood number for different values of Soret number S_r.

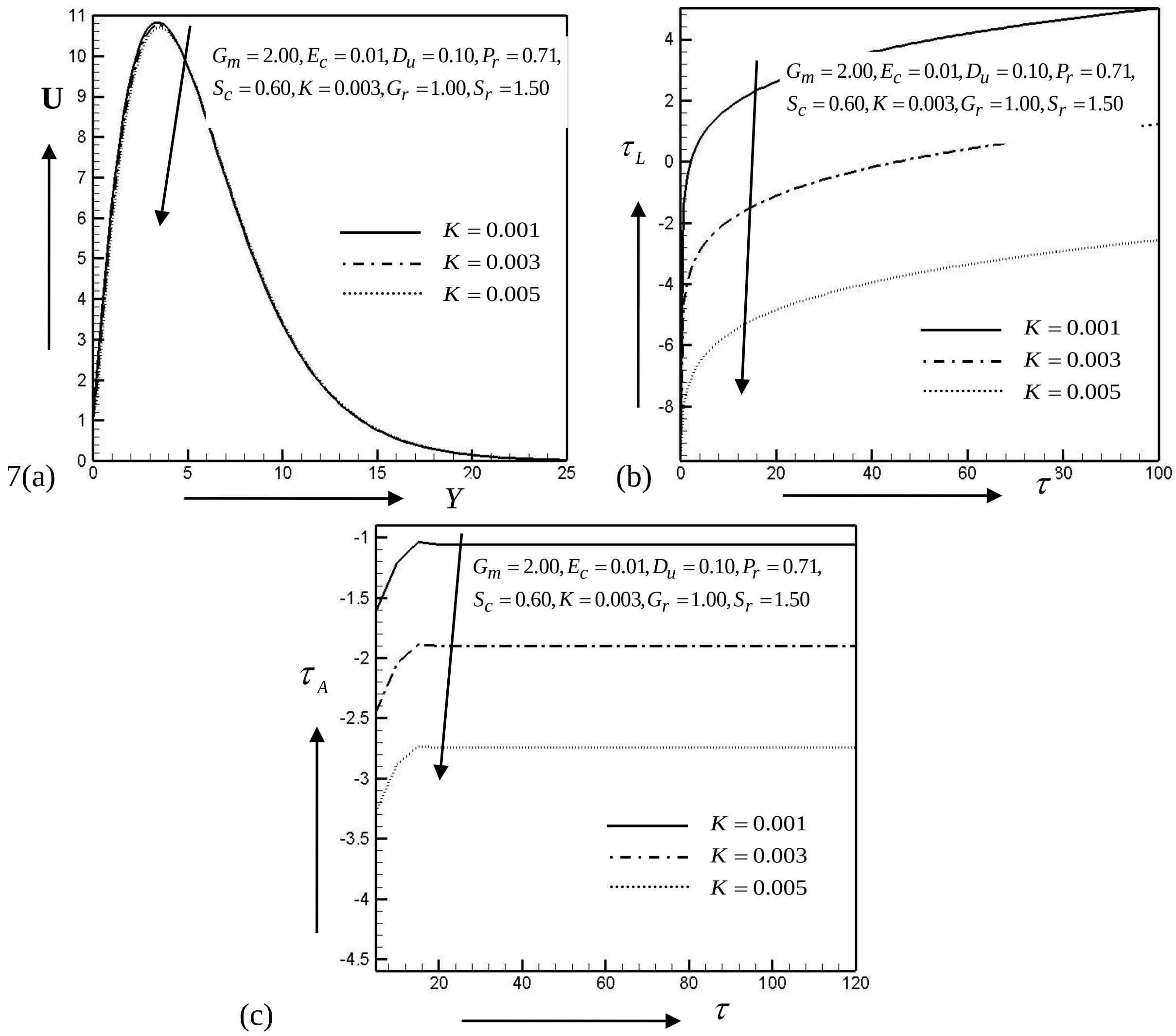


Fig.7. Illustration of (a) Velocity profiles (b) Local shear stress (c) Average shear stress for different values of Magnetic parameter K .

6. Comparison

Qualitative comparison of the present results with previous results (Gbadeyan et al. [10]) are presented in tabular form. The accuracy of the present results is qualitatively good in case of all the respective flow parameters. Other results are not shown for brevity.

Table 1. Qualitative comparison of the present results with the previous results

Increased Parameter	Pervious results given by Gbadeyan et al. [10]			Present results		
	$F'(\eta)$	$\theta(\eta)$	$\phi(\eta)$	U	θ	ϕ
D_u	Inc.	Inc.	Dec.	Inc.	Inc.	Dec.
M	Dec.	Inc.		Dec.	Inc.	
S_r		Dec.	Inc.		Dec.	Inc.
K		Dec.				Dec.

7. Conclusions

Viscoelastic fluid flow with Soret and Dufour effects through a vertical flat in the presence of magnetic field has been taken into account. The resulting governing systems of equations are solved by explicit finite difference method. The results are discussed for different values of important parameters Dufour number D_u, Magnetic parameter M, Soret number S_r and Viscoelastic parameter K.

The important findings of this investigation from the above mentioned graphical representation are listed below;

1. Fluid velocity, local shear stress, average shear stress have been decreased with the increase of Viscoelastic parameter K and Magnetic parameter M.
2. Fluid Temperature and concentration have been increased with the increase of Dufour number D_u and Soret number S_r respectively.
3. Nusselt number and average Nusselt number of the fluid have been decreased with the increase of Dufour number D_u.
4. Sherwood number and average Sherwood number of the fluid have been decreased with the increase of Soret number S_r.

Acknowledgement

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