

Chemical Reaction and Heat Source Effects on MHD Flow of Visco-Elastic Fluid past an Exponentially Accelerated Vertical Plate Embedded in a Porous Medium

R. N. Barik^a, G. C. Dash^b, P. K. Rath^c

^aDepartment of Mathematics, Trident Academy of Technology, Infocity, Bhubaneswar-751024, Odisha, India (E-mail: barik.rabinarayan@rediffmail.com)

^bDepartment of Mathematics, S.O.A. University, Bhubaneswar-751030, Odisha, India (E-mail:gcdash45@gmail.com)

^cDepartment of Mathematics, B.R.M. International Institute of Technology, Bhubaneswar-10, Odisha, India (E-mail: pkrath_1967@yahoo.in)

Abstract

An analytical study has been performed to examine the effects of temperature dependent heat source on the unsteady free convective and mass transfer flow of a visco-elastic fluid past an exponentially accelerated infinite vertical plate embedded in a porous medium in the presence of magnetic field and chemical reaction. The novelty of the present study is to analyze the effect of first order chemical reaction on the flow phenomena of a visco-elastic fluid. It is important to note that chemical reaction has decelerating effects on concentration distribution and it is concomitant with increase in S_c i.e for heavier species. The Laplace transform method is used to obtain the expressions for velocity, temperature and concentration. The effects of various parameters occurring into the problem are discussed with the help of graphs.

Key words: MHD, Free convection, Mass diffusion, Visco-elastic fluid, Porous medium, Heat source

1. Introduction

The problem of free-convection and mass transfer flow of an electrically-conducting fluid past an infinite plate under the influence of a magnetic field has attracted interest in view of its application to geophysics, astrophysics, engineering and to the boundary layer control in the field of aerodynamics. The phenomenon of hydromagnetic flow with heat and mass transfer in an electrically conducting fluid past a porous plate embedded in a porous medium has attracted the attention of a good number of investigators because of its varied applications in many engineering problems such as MHD generators, plasma studies, nuclear reactors, oil exploration and geothermal energy extractions.

The study of visco-elastic fluids through porous media has become the basis of many scientific and engineering applications. This type of flow is of great importance in the petroleum engineering concerned with the movement of oil, gas and water through reservoir of oil or gas field to the hydrologist in the study of the migration of underground water and to the chemical engineering for the purification and filtration processes and in the cases like drug permeation through human skin. The principles of this subject are very useful in recovering the water for drinking and irrigation purposes.

The flow of the conducting fluid is effectively changed by the presence of the magnetic field and the magnetic field is also perturbed due to the motion of the conducting fluid. This phenomenon is therefore interlocking in character and the discipline of this branch of science is called magnetohydrodynamic (MHD). Influence of the magnetic field on the non-Newtonian fluid flow has wide applications in chemical engineering, metallurgical engineering, and various industries. Researchers have considerable interest in the study of flow phenomenon between two parallel plates. Because of its occurrence in rheometric experiments to determine the constitutive properties of the fluid, in lubrication engineering, and in transportation and processing encountered in chemical engineering, the flow on non-Newtonian visco-elastic fluid is worthwhile to investigate.

Flow through porous medium is very prevalent in nature and therefore this study has become of the principal interest in many scientific and engineering applications. There is increasing interest in magnetohydrodynamic (MHD) that flows within fluid saturated porous media, because of numerous applications in geophysics and energy related problems, such as thermal insulation of buildings, enhanced recovery of petroleum resources, geophysical flows, packed bed reactors and sensible heat storage beds.

MHD effects on impulsively started vertical infinite plate with variable temperature in the presence of transverse magnetic field were studied by Soundalgekar *et al.* [1]. Jha and Prasad [2] have studied the effects of heat source on MHD free-convection and mass transfer flow through a porous medium. In many industrial applications, the flow past an exponentially accelerated infinite vertical plate plays an important role. This is particularly important in the design of spaceship, solar energy collectors, etc. From this point of view, free convection effects on flow past an exponentially accelerated vertical plate was studied by Singh and Naveen kumar [3]. The skin friction for accelerated vertical plate has been studied analytically by Hossain and Shayo [4]. Jha *et al.* [5] analyzed mass transfer effects on exponentially accelerated infinite vertical plate with constant heat flux and uniform mass diffusion. Muthucumaraswamy *et al.* [6] studied heat transfer effects on flow past an exponentially accelerated vertical plate with variable temperature. Rushi Kumar and

Nagarajan [7] studied the mass transfer effects of MHD free convection flow of incompressible viscous dissipative fluid past an infinite vertical plate. Mass transfer effects on free convection flow of an incompressible viscous dissipative fluid have been studied by Manohar and Nagarajan [8]. Sivaiah *et al.* [9] studied heat and mass transfer effects on MHD free convective flow past a vertical porous plate. Ambethkar [10] studied numerical solutions of heat and mass transfer effects of an unsteady MHD free convective flow past an infinite vertical plate with constant suction and heat source and sink. The Roseland diffusion approximation is used in studying the effect of radiation on free convection from a vertical cylinder embedded in fluid-saturated porous medium by EL-Hakim and Rashad [11].

On the other hand, considerable interest has been developed in the study of the interaction between magnetic fields and the flow of electrically-conducting incompressible visco-elastic fluid due to its wide applications in modern technology. The study of a visco-elastic Pulsatile flow helps to understand the mechanisms of dialysis of blood through an artificial kidney. Singh and Singh [12] studied the MHD flow of an visco-elastic fluid (Walters liquid B') past an infinite horizontal plate for both the classes of impulsively as well as uniformly accelerated motion. Samria *et al.* [13] studied the laminar flow of an electrical-conducting Walters liquid B' , past an infinite non-conducting vertical plate for impulsive as well as uniformly accelerated motion of the plate, in the presence of a transverse magnetic field. Rajesh and Varma [14] studied the effects of thermal radiation on unsteady free convection flow of a visco-elastic fluid over a moving vertical plate with variable temperature in the presence of magnetic field through porous medium. Mohd Kasim *et al.* [15] obtained the free convective boundary layer flow of a visco-elastic fluid in the presence of heat generation. The free convective MHD flow through porous media of a rotating visco-elastic fluid past an infinite vertical porous plate with heat and mass transfer in the presence of chemical reaction have been studied by Rath *et al.* [16]. Sivaraj and Rushi Kumar [17] investigated the unsteady MHD dusty visco-elastic fluid Couette flow in an irregular channel with varying mass diffusion. Recently, the mass and heat transfer effect on MHD flow of a visco-elastic fluid through porous medium with oscillatory suction and heat source have been studied by Mishra *et al.* [18]. Devika *et al.* [19] have studied MHD oscillatory flow of a visco-elastic fluid in a porous channel with chemical reaction.

This paper deals with the analytical study of effects of first order chemical reaction on MHD free convective flow through porous medium of a visco-elastic fluid past an exponentially accelerated infinite vertical plate with variable temperature in presence of heat source. Further, the mass transfer phenomenon, considered in this problem, is associated with chemically reacting

species. The primary objective of the present study is to bring out the effect of chemical reaction in conjunction with other parameters involved in the governing equations characterizing the flow phenomena. Further, it is aimed at discussing the case of absence of chemical reaction, Rajesh [20] as a particular case. The response of chemical reaction in the presence of heat sources is also a matter of interest. Of course, the study is having an inherent limitation not being capable of discussing the case of endothermic reaction

($K_c < 0$).

2. Mathematical formulation of the problem

Let us consider an unsteady free convective heat and mass transfer flow of an electrically conducting incompressible visco-elastic fluid past an infinite vertical plate through porous medium in the presence of heat source and chemical reaction. A magnetic field of uniform strength B_0 is applied transversely to the plate. The induced magnetic field is neglected as the magnetic Reynolds number of the flow is taken to be very small. The flow is assumed to be in x' – direction which is taken along the vertical plate in the upward direction. The y' –axis is taken to be normal to the plate. Initially, the plate and the fluid are at the same temperature T'_∞ with concentration level C'_∞ at all points. When time $t' > 0$, the plate is exponentially accelerated with a velocity $u = u_0 \exp(at')$ in its own plane and the plate temperature is raised linearly with time t' and the level of concentration near the plate is raised to C'_w . The effect of viscous dissipation and ohmic dissipation are assumed to be negligible. The governing equations of flow, heat and mass transfer of visco-elastic fluid (Walters B') with usual Boussinesq's approximation are given by

$$\frac{\partial v'}{\partial y'} = 0 \quad (1)$$

$$\frac{\partial u'}{\partial t'} = \nu \frac{\partial^2 u'}{\partial y'^2} + g\beta(T' - T'_\infty) + g\beta^*(C' - C'_\infty) - \frac{k_0}{\rho} \frac{\partial^3 u'}{\partial y'^2 \partial t} - \frac{\sigma B_0^2 u'}{\rho} - \frac{\nu u'}{K'_p} \quad (2)$$

$$\rho C_p \frac{\partial T'}{\partial t'} = k \frac{\partial^2 T'}{\partial y'^2} + Q'(T'_\infty - T') \quad (3)$$

$$\frac{\partial C'}{\partial t'} = D \frac{\partial^2 C'}{\partial y'^2} - K'_c(C' - C'_\infty) \quad (4)$$

where $\nu, g, \beta, \beta^*, k_0, \rho, \sigma, B_0, K'_p, C_p, Q', D$ and K'_c are kinematic viscosity, acceleration due to gravity, coefficient of thermal expansion, coefficient of mass expansion, dimensional visco-elastic parameter, density of the fluid, electrical conductivity of the fluid, uniform magnetic field,

permeability of the porous medium, specific heat at constant pressure, rate of volumetric heat generation / absorption, molecular diffusivity and the chemical reaction parameter.

The initial and boundary condition of the problem are

$$\left. \begin{aligned} t' \leq 0 : u' = 0, T' = T'_\infty, C' = C'_\infty \text{ for all } y' \\ t' > 0 : u' = u_0 e^{a't'}, T' = T'_\infty + (T'_w - T'_\infty) \frac{u_0^2 t'}{v}, C' = C'_w \text{ at } y' = 0 \\ u' = 0, \quad T' = T'_\infty \quad C' = C'_\infty \text{ as } y' \rightarrow \infty \end{aligned} \right\} \quad (5)$$

On introducing the following non-dimensional quantities

$$\left. \begin{aligned} u = \frac{u'}{u_0}, t = \frac{t' u_0^2}{v}, y = \frac{y' u_0}{v}, T = \frac{T' - T'_\infty}{T'_w - T'_\infty}, G_r = \frac{g \beta v (T'_w - T'_\infty)}{u_0^3} \\ C = \frac{C' - C'_\infty}{C'_w - C'_\infty}, G_e = \frac{g \beta^* v (C'_w - C'_\infty)}{u_0^3}, P_r = \frac{\mu C_p}{k}, S_c = \frac{v}{D} \\ M = \frac{\sigma B_0^2 v}{\rho u_0^2}, S = \frac{Q' v^2}{k u_0^2}, R_c = \frac{k_0 u_0^2}{\rho v^2}, K_p = \frac{u_0^2 K'_p}{v^2}, a = \frac{a' v}{u_0^2}, K_c = \frac{K'_c v^2}{D u_0^2} \end{aligned} \right\} \quad (6)$$

into equations (1) to (4) we get

$$\frac{\partial v}{\partial y} = 0 \Rightarrow v(t) = 0 \text{ (assumption)} \quad (7)$$

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + G_r T + G_e C - R_c \frac{\partial^3 u}{\partial y^2 \partial t} - M u - \frac{u}{K_p} \quad (8)$$

$$P_r \frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial y^2} - S T \quad (9)$$

$$S_c \frac{\partial C}{\partial t} = \frac{\partial^2 C}{\partial y^2} - K_c C \quad (10)$$

Subject to the initial and boundary conditions

$$\left. \begin{aligned} t \leq 0 : u = 0, T = 0, C = 0 \text{ for all } y \\ t > 0 : u = e^{at}, T = t, C = 1 \text{ at } y = 0 \\ u = 0, T \rightarrow 0, C \rightarrow 0, \text{ as } y \rightarrow \infty \end{aligned} \right\} \quad (11)$$

Here $G_r, G_e, P_r, S_c, M, S, R_c, K_p, a$ and K_c are respectively the Grashof number for heat transfer, Grashof number for mass transfer, Prandtl number, Schmidt number, magnetic parameter, heat source parameter, visco-elastic parameter, porosity parameter, accelerating parameter and chemical reaction parameter.

3. Mathematical Solution of the Problem:

In view of inadequacy of the boundary conditions following Beard and Walters [21] on $u(y,t)$ we consider

$$u = u_0 + R_c u_1 + o(R_c^2) \quad (12)$$

Using equation (12) the solutions of the equations (8) - (10) with initial and boundary conditions (11) are obtained by the Laplace Transform Technique and are given by

$$C = \frac{1}{2} \left[\exp(-y\sqrt{K_c}) \operatorname{erfc} \left(\frac{y\sqrt{S_c}}{2\sqrt{t}} - \sqrt{\frac{K_c t}{S_c}} \right) + \exp(y\sqrt{K_c}) \operatorname{erfc} \left(\frac{y\sqrt{S_c}}{2\sqrt{t}} + \sqrt{\frac{K_c t}{S_c}} \right) \right] \quad (13)$$

$$T = \frac{t}{2} \left[\exp(y\sqrt{S}) \operatorname{erfc} \left(\frac{y\sqrt{P_r}}{2\sqrt{t}} + \sqrt{\frac{St}{P_r}} \right) + \exp(-y\sqrt{S}) \operatorname{erfc} \left(\frac{y\sqrt{P_r}}{2\sqrt{t}} - \sqrt{\frac{St}{P_r}} \right) \right] \\ + \frac{yP_r}{4\sqrt{S}} \left[\exp(y\sqrt{S}) \operatorname{erfc} \left(\frac{y\sqrt{P_r}}{2\sqrt{t}} + \sqrt{\frac{St}{P_r}} \right) - \exp(-y\sqrt{S}) \operatorname{erfc} \left(\frac{y\sqrt{P_r}}{2\sqrt{t}} - \sqrt{\frac{St}{P_r}} \right) \right] \quad (14)$$

$$u = A_3 + \frac{cG_r}{d} (A_{14} - T) - \frac{G_r}{2d} (A_2 - A_3 - A_{10} + A_{11}) + \frac{G_c}{2 \left(K_c - M - \frac{1}{K_p} \right)} (A_2 - A_{15} + A_6 - 2c) \\ + \frac{cG_r G_e (S - CP_r)}{d(1 - P_r)} (A_1 - A_9) - \frac{SG_r R_c}{2d(1 - P_r)} (A_2 - A_3) - \frac{R_c G_c (K_c - bS_c)}{\left(K_c - M - \frac{1}{K_p} \right) (1 - S)} (A_4 - A_{12}) \\ - \frac{yR_c}{2} A_5 - \frac{ycR_c G_r}{2d} (A_6 - A_7) + \frac{ybR_c G_c}{2 \left(K_c - M - \frac{1}{K_p} \right)} A_5 + \frac{R_c G_r S}{2d(1 - P_r)} (A_{10} - A_{11}) \quad (15)$$

where

$$\begin{aligned}
A_1 &= \left(\frac{t}{2} + \frac{y}{4\sqrt{M + \frac{1}{K_p} - c}} \right) \exp \left(y \sqrt{M + \frac{1}{K_p} - c} - ct \right) \operatorname{erfc} \left(\frac{y}{2\sqrt{t}} + \sqrt{\left(M + \frac{1}{K_p} - c \right) t} \right) \\
&+ \left(\frac{t}{2} - \frac{y}{4\sqrt{M + \frac{1}{K_p} - c}} \right) \exp \left(-y \sqrt{M + \frac{1}{K_p} - c} - ct \right) \operatorname{erfc} \left(\frac{y}{2\sqrt{t}} - \sqrt{\left(M + \frac{1}{K_p} - c \right) t} \right) \\
A_2 &= \exp \left(-y \sqrt{M + \frac{1}{K_p}} \right) \operatorname{erfc} \left(\frac{y}{2\sqrt{t}} - \sqrt{\left(M + \frac{1}{K_p} \right) t} \right) + \exp \left(y \sqrt{M + \frac{1}{K_p}} \right) \operatorname{erfc} \left(\frac{y}{2\sqrt{t}} + \sqrt{\left(M + \frac{1}{K_p} \right) t} \right) \\
A_3 &= \exp \left(-y \sqrt{M + \frac{1}{K_p} - c} - ct \right) \operatorname{erfc} \left(\frac{y}{2\sqrt{t}} - \sqrt{\left(M + \frac{1}{K_p} - c \right) t} \right) \\
&+ \exp \left(y \sqrt{M + \frac{1}{K_p} - c} - ct \right) \operatorname{erfc} \left(\frac{y}{2\sqrt{t}} + \sqrt{\left(M + \frac{1}{K_p} - c \right) t} \right) \\
A_4 &= \left(\frac{tb+1}{2} - \frac{yb}{4\sqrt{b+M+\frac{1}{K_p}}} \right) \exp \left(bt - y \sqrt{b+M+\frac{1}{K_p}} \right) \operatorname{erfc} \left(\frac{y}{2\sqrt{t}} - \sqrt{\left(b+M+\frac{1}{K_p} \right) t} \right) \\
&+ \left(\frac{tb+1}{2} + \frac{yb}{4\sqrt{b+M+\frac{1}{K_p}}} \right) \exp \left(bt + y \sqrt{b+M+\frac{1}{K_p}} \right) \operatorname{erfc} \left(\frac{y}{2\sqrt{t}} + \sqrt{\left(b+M+\frac{1}{K_p} \right) t} \right) \\
A_5 &= \frac{1}{\sqrt{\pi t}} \left(a + \frac{y^2}{4t^2} - \frac{1}{2t} \right) \exp \left(- \left(M + \frac{1}{K_p} \right) t - \frac{y^2}{4t} \right) \\
&+ \frac{ae^{at} \sqrt{a+M+\frac{1}{K_p}}}{2} \left[\exp \left(-y \sqrt{a+M+\frac{1}{K_p}} \right) \operatorname{erfc} \left(\frac{y}{2\sqrt{t}} - \sqrt{\left(a+M+\frac{1}{K_p} \right) t} \right) \right. \\
&\quad \left. - \exp \left(y \sqrt{a+M+\frac{1}{K_p}} \right) \operatorname{erfc} \left(\frac{y}{2\sqrt{t}} + \sqrt{\left(a+M+\frac{1}{K_p} \right) t} \right) \right]
\end{aligned}$$

$$\begin{aligned}
A_6 &= \frac{\sqrt{M + \frac{1}{K_p}}}{2} \left[\exp \left(-y \sqrt{M + \frac{1}{K_p}} \right) \operatorname{erfc} \left(\frac{y}{2\sqrt{t}} - \sqrt{\left(M + \frac{1}{K_p} \right) t} \right) \right. \\
&\quad \left. - \exp \left(y \sqrt{M + \frac{1}{K_p}} \right) \operatorname{erfc} \left(\frac{y}{2\sqrt{t}} + \sqrt{\left(M + \frac{1}{K_p} \right) t} \right) \right] \\
A_7 &= \frac{\sqrt{M + \frac{1}{K_p} - c}}{2} \left[\exp \left(-y \sqrt{M + \frac{1}{K_p} - c - ct} \right) \operatorname{erfc} \left(\frac{y}{2\sqrt{t}} - \sqrt{\left(M + \frac{1}{K_p} - c \right) t} \right) \right. \\
&\quad \left. - \exp \left(y \sqrt{M + \frac{1}{K_p} - c - ct} \right) \operatorname{erfc} \left(\frac{y}{2\sqrt{t}} + \sqrt{\left(M + \frac{1}{K_p} - c \right) t} \right) \right] \\
A_8 &= \frac{1}{\sqrt{\pi t}} \exp \left(- \left(M + \frac{1}{K_p} \right) t - \frac{y^2}{4t} \right) + \frac{\sqrt{M + \frac{1}{K_p} + b}}{2} \left[\exp \left(bt - y \sqrt{M + \frac{1}{K_p} + b} \right) \operatorname{erfc} \left(\frac{y}{2\sqrt{t}} - \sqrt{\left(M + \frac{1}{K_p} + b \right) t} \right) \right. \\
&\quad \left. - \exp \left(bt + y \sqrt{M + \frac{1}{K_p} + b} \right) \operatorname{erfc} \left(\frac{y}{2\sqrt{t}} + \sqrt{\left(M + \frac{1}{K_p} + b \right) t} \right) \right] \\
A_9 &= \left(\frac{t}{2} + \frac{yP_r}{4\sqrt{S - cP_r}} \right) \exp \left(y\sqrt{S - cP_r} - ct \right) \operatorname{erfc} \left(\frac{y\sqrt{P_r}}{2\sqrt{t}} + \sqrt{\left(\frac{S}{P_r} - c \right) t} \right) \\
&\quad + \left(\frac{t}{2} - \frac{yP_r}{4\sqrt{S - cP_r}} \right) \exp \left(-y\sqrt{S - cP_r} - ct \right) \operatorname{erfc} \left(\frac{y\sqrt{P_r}}{2\sqrt{t}} - \sqrt{\left(\frac{S}{P_r} - c \right) t} \right) \\
A_{10} &= \exp \left(-y\sqrt{S} \right) \operatorname{erfc} \left(\frac{y\sqrt{P_r}}{2\sqrt{t}} - \sqrt{\frac{St}{P_r}} \right) + \exp \left(y\sqrt{S} \right) \operatorname{erfc} \left(\frac{y\sqrt{P_r}}{2\sqrt{t}} + \sqrt{\frac{St}{P_r}} \right) \\
A_{11} &= \exp \left(-y\sqrt{S - cP_r} - ct \right) \operatorname{erfc} \left(\frac{y\sqrt{P_r}}{2\sqrt{t}} - \sqrt{\left(\frac{S}{P_r} - c \right) t} \right) + \exp \left(y\sqrt{S - cP_r} - ct \right) \operatorname{erfc} \left(\frac{y\sqrt{P_r}}{2\sqrt{t}} + \sqrt{\left(\frac{S}{P_r} - c \right) t} \right) \\
A_{12} &= \left(\frac{bt+1}{2} - \frac{ybS_c}{4\sqrt{bS_c - K_c}} \right) \exp \left(bt - y\sqrt{bS_c - K_c} \right) \operatorname{erfc} \left(\frac{y\sqrt{S_c}}{2\sqrt{t}} - \sqrt{\left(b - \frac{K_c}{S_c} \right) t} \right) \\
&\quad + \left(\frac{bt+1}{2} + \frac{ybS_c}{4\sqrt{bS_c - K_c}} \right) \exp \left(bt + y\sqrt{bS_c - K_c} \right) \operatorname{erfc} \left(\frac{y\sqrt{S_c}}{2\sqrt{t}} + \sqrt{\left(b - \frac{K_c}{S_c} \right) t} \right)
\end{aligned}$$

$$\begin{aligned}
A_{13} &= \frac{1}{2} \left[\exp \left(at - y \sqrt{a + M + \frac{1}{K_p}} \right) \operatorname{erfc} \left(\frac{y}{2\sqrt{t}} - \sqrt{a + M + \frac{1}{K_p}} t \right) \right. \\
&\quad \left. + \exp \left(at + y \sqrt{a + M + \frac{1}{K_p}} \right) \operatorname{erfc} \left(\frac{y}{2\sqrt{t}} + \sqrt{a + M + \frac{1}{K_p}} t \right) \right] \\
A_{14} &= \left(\frac{t}{2} + \frac{y}{4\sqrt{M + \frac{1}{K_p}}} \right) \left[\exp \left(y \sqrt{M + \frac{1}{K_p}} \right) \operatorname{erfc} \left(\frac{y}{2\sqrt{t}} + \sqrt{M + \frac{1}{K_p}} t \right) \right] \\
&\quad + \left(\frac{t}{2} - \frac{y}{4\sqrt{M + \frac{1}{K_p}}} \right) \left[\exp \left(-y \sqrt{M + \frac{1}{K_p}} \right) \operatorname{erfc} \left(\frac{y}{2\sqrt{t}} - \sqrt{M + \frac{1}{K_p}} t \right) \right] \\
A_{15} &= \exp \left(bt - y \sqrt{b + M + \frac{1}{K_p}} \right) \operatorname{erfc} \left(\frac{y}{2\sqrt{t}} - \sqrt{b + M + \frac{1}{K_p}} t \right) \\
&\quad + \exp \left(bt + y \sqrt{b + M + \frac{1}{K_p}} \right) \operatorname{erfc} \left(\frac{y}{2\sqrt{t}} + \sqrt{b + M + \frac{1}{K_p}} t \right) \\
A_{16} &= \exp \left(bt - y \sqrt{bS_c - K_c} \right) \operatorname{erfc} \left(\frac{y\sqrt{S_c}}{2\sqrt{t}} - \sqrt{b - \frac{K_c}{S_c}} t \right) \\
&\quad + \exp \left(bt + y \sqrt{bS_c - K_c} \right) \operatorname{erfc} \left(\frac{y\sqrt{S_c}}{2\sqrt{t}} + \sqrt{b - \frac{K_c}{S_c}} t \right) \\
b &= \frac{K_c - M - \frac{1}{K_p}}{S_c - 1}, c = \frac{S - M - \frac{1}{K_p}}{P_r - 1}, d = c \left(S - M - \frac{1}{K_p} \right)
\end{aligned}$$

Sherwood Number (Sh)

The rate of mass transfer (Sh) in non-dimensional form, is obtained from the concentration field as follows:

$$Sh = - \frac{\partial C}{\partial y} \Big|_{y=0} = \sqrt{K_c} \operatorname{erfc} \left(\sqrt{\frac{K_c t}{S_c}} \right) + \sqrt{\frac{S_c}{\pi t}} \exp \left(- \frac{K_c t}{S_c} \right) \quad (16)$$

Nusselt Number (Nu)

The rate of heat transfer (Nu) in the non-dimensional form, is obtained from the temperature field as follows:

$$Nu = -\left. \frac{\partial T}{\partial y} \right]_{y=0} = \frac{2tS + P_r}{2\sqrt{S}} \operatorname{erfc}\left(\sqrt{\frac{St}{P_r}}\right) + \sqrt{\frac{tP_r}{\pi}} \exp\left(-\frac{St}{P_r}\right) \quad (17)$$

Skin Friction (τ)

The Skin friction (τ) in non-dimensional form, is obtained from the velocity field as follows:

$$\begin{aligned} \tau = & -\left[\frac{e^{-\left(M+\frac{1}{K_p}\right)t}}{\sqrt{\pi t}} + \sqrt{M + \frac{1}{K_p}} + ae^{at} \operatorname{erf}\left(\sqrt{\left(M + \frac{1}{K_p} + a\right)t}\right) \right] \\ & + \frac{cG_r}{d} \left[Nu - \sqrt{\frac{t}{\pi}} e^{-\left(M+\frac{1}{K_p}\right)t} + \frac{2\left(M + \frac{1}{K_p}\right) + 1}{2\sqrt{M + \frac{1}{K_p}}} \operatorname{erfc}\left(\sqrt{\left(M + \frac{1}{K_p}\right)t}\right) \right] \\ & + \frac{G_r}{d} \left[\sqrt{M + \frac{1}{K_p}} \operatorname{erf}\left(\sqrt{\left(M + \frac{1}{K_p}\right)t}\right) - \sqrt{M + \frac{1}{K_p}} - ce^{-ct} \operatorname{erf}\left(\sqrt{\left(M + \frac{1}{K_p} - c\right)t}\right) \right] \\ & + \sqrt{S - cP_r} e^{-ct} \operatorname{erf}\left(\sqrt{\frac{S}{P_r} - ct}\right) - \sqrt{S} \operatorname{erf}\left(\sqrt{\frac{St}{P_r}}\right) \\ & + \frac{G_c}{K_c - M - \frac{1}{K_p}} \left[\left(M + \frac{1}{K_p} + b\right) e^{bt} \operatorname{erf}\left(\sqrt{\left(M + \frac{1}{K_p} + b\right)t}\right) - \sqrt{M + \frac{1}{K_p}} \operatorname{erf}\left(\sqrt{\left(M + \frac{1}{K_p}\right)t}\right) \right. \\ & \left. + \sqrt{K_c} \operatorname{erf}\left(\sqrt{\frac{K_c t}{S_c}}\right) - \sqrt{bS_c - K_c} e^{bt} \operatorname{erfc}\left(\sqrt{b - \frac{K_c}{S_c}} t\right) \right] \\ & + \frac{cG_r R_c (S - cP_r)}{d(1 - P_r)} \left[\frac{2c - 2\left(M + \frac{1}{K_p}\right)}{2\sqrt{M + \frac{1}{K_p} - c}} e^{-ct} \operatorname{erf}\left(\sqrt{\left(M + \frac{1}{K_p} - c\right)t}\right) - \sqrt{\frac{t}{\pi}} e^{-\left(M+\frac{1}{K_p}\right)t} \right] \end{aligned}$$

$$\begin{aligned}
& -\frac{2cP_r - 2S - P_r}{2\sqrt{S - cP_r}} e^{-ct} \operatorname{erf}\left(\sqrt{\frac{St}{P_r} - ct}\right) + \sqrt{\frac{tP_r}{\pi}} \exp\left(-\frac{St}{P_r}\right) \\
& -\frac{SG_r R_c}{d(1-P_r)} \left[\sqrt{M + \frac{1}{K_p} - c} e^{-ct} \operatorname{erf}\left(\sqrt{\left(M + \frac{1}{K_p} - c\right)t}\right) - \sqrt{M + \frac{1}{K_p}} \operatorname{erf}\left(\sqrt{\left(M + \frac{1}{K_p}\right)t}\right) \right] \\
& + \frac{R_c G_c (K_c - bS_c)}{\left(K_c - M - \frac{1}{K_p}\right)(1-S_c)} \left[\frac{\left(3b + 2\left(M + \frac{1}{K_p}\right)\right)e^{bt}}{2\sqrt{M + \frac{1}{K_p} + b}} \operatorname{erf}\left(\sqrt{\left(M + \frac{1}{K_p} + b\right)t}\right) + \sqrt{\frac{tb+1}{\pi}} e^{-\left(M + \frac{1}{K_p}\right)t} \right. \\
& \left. - \frac{(3bS_c - 2k_c)e^{bt}}{2\sqrt{bS_c - K_c}} \operatorname{erf}\left(\sqrt{bt - \frac{K_c t}{S_c}}\right) - \left(\sqrt{\frac{tb+1}{\pi}}\right) S_c \exp\left(\frac{K_c t}{S_c}\right) \right] \\
& - \frac{R_c}{2} \left[\left(a - \frac{1}{2t}\right) \frac{e^{-\left(M + \frac{1}{K_p}\right)t}}{\sqrt{\pi t}} + ae^{at} \sqrt{a + M + \frac{1}{K_p}} \operatorname{erf}\left(\sqrt{\left(a + M + \frac{1}{K_p}\right)t}\right) \right] \\
& - \frac{cR_c G_r}{2d} \left[\sqrt{M + \frac{1}{K_p}} \operatorname{erf}\left(\sqrt{\left(M + \frac{1}{K_p}\right)t}\right) - \sqrt{M + \frac{1}{K_p} - c} \operatorname{erf}\left(\sqrt{\left(M + \frac{1}{K_p} - c\right)t}\right) \right] \\
& + \frac{bR_c G_c}{2\left(K_c - M - \frac{1}{K_p}\right)} \left[\frac{e^{-\left(M + \frac{1}{K_p}\right)t}}{\sqrt{\pi t}} + \sqrt{M + \frac{1}{K_p}} e^{bt} \operatorname{erf}\left(\sqrt{\left(M + \frac{1}{K_p} + b\right)t}\right) \right] \\
& + \frac{R_c G_r S}{d(1-P_r)} \left[\sqrt{S - cP_r} e^{-ct} \operatorname{erf}\left(\sqrt{\left(\frac{S}{P_r} - c\right)t}\right) - \sqrt{S} \operatorname{erf}\left(\sqrt{\frac{St}{P_r}}\right) \right]
\end{aligned}$$

4. Results and discussions:

For the purpose of discussion, the effects of permeability of the medium, magnetic parameter, heat source and chemical reaction on free convection and mass transfer flow which arise out of a combined effect of temperature and concentration differences on the flow field are presented. The numerical calculations are carried out and the effects of the parameters are exhibited through the figures 1 to 15.

For the numerical calculations, we have considered $P_r = 0.71$ (air), $P_r = 7.0$ (water); and the values of Schmidt number (S_c) correspond to diffusing chemical species of common interest in air $S_c = 0.60$ (water vapour), $S_c = 0.78$ (NH_3) at approximately $25^\circ C$ and one atmosphere along with some arbitrary values S_c . The values of Grashof number (G_r),

modified Grashof number (G_c) and permeability parameter are taken positive in order to analyse their effects on the flow field. The positive values of G_r correspond to the cooling of the surface and negative values correspond to the heating of the plate. The common feature of the profiles depicted through the figures are of two layer character exhibiting the reverse effects in respect of cooling ($G_r > 0$) and heating ($G_r < 0$) of the surface leading to almost symmetrical profiles about the flow direction. Thus, the effects due to cooling and heating of the surface are opposite to each other for all the parameters. The following discussions are carried out in the presence of exothermic reaction only as $K_c < 0$ is not permissible for real valued flow characteristics.

Velocity distribution

Fig.1 exhibits the effect of chemical reaction on the velocity distribution. The case of absence of chemical reaction ($K_c = 0.0$) Rajesh [17], has been indicated by the curves I, for both heating and cooling of the plate. It is found that the present observation is in good agreement with the earlier in the absence of chemical reaction (K_c). The profiles are symmetrical about the main direction of the flow and asymptotically decrease to ambient state. Further, it is seen that an increase in chemical reaction has a decelerating effect on the velocity field on a cooled plate on the other hand reverse effect is observed in case of heated plate.

From figures 2, 4, 5, 6, 7, 10, and 11, we observe that primary velocity experiences a retarding effect due to increase in time t (fig 2), mass Grashof number G_r (fig 4) and permeability parameter K_p (fig 7). On the other hand, it experiences an accelerating effect for the increasing values of elastic parameter R_c (fig 5), magnetic parameter M (fig 6), Schmidt number S_c (fig 10) and accelerating parameter a (fig 11) in case of cooling of the plate.

Some peculiarities are marked in case of thermal buoyancy parameter (G_r), Prandtl number (P_r) and heat source parameter (S). It is interesting to note that the parameters G_r , P_r , and S have no significant effect on velocity field in the presence of chemical reaction. Thus, it may be concluded that thermal buoyancy force characterized by G_r (fig 3) overrides the chemical reaction parameter ($K_c \neq 0$). Further, the effect of Prandtl number (the ratio of kinematic viscosity and thermal diffusivity) also ignores the presence of chemical reaction in both the cases i.e $P_r > 1$ and $P_r < 1$ (fig 8). The above observation also holds good for heat source parameter (fig 9).

Temperature and concentration distribution

Fig.12 exhibits the temperature distribution in the flow domain. It is interesting to note that for gaseous flow ($P_r = 0.71$), an increase in heat source decreases the temperature at all points but in case of water, effect of heat source is not significant. Higher P_r means momentum diffusivity due to viscosity dominates over the thermal diffusivity of the fluid. Thus, it is concluded that higher momentum diffusivity overrides the effect of heat source so that by increasing strength of heat source no significant difference in temperature distribution is marked.

From fig 13 it is seen that the chemical reaction has a distinct role to play. It is evident from the concentration profiles that presence of chemical reaction has a decelerating effect on concentration distribution and it is concomitant with increase in S_c i.e for heavier species.

Rate of mass transfer (Sh) and heat transfer (Nu)

From the figures 14 and 15 it is clear that the plate experiences a sudden decrease of rate of mass transfer in the beginning of flow, but as the time elapses it decreases asymptotically. Further, it is noticed that an increase in chemical reaction parameter (K_c) and Schmidt number (S_c) leads to increase the rate of mass transfer whereas rate of heat transfer increases with an increase in values of Prandtl number (P_r) and heat source parameter (S). The increase of P_r means slow rate of thermal diffusion. Thus, for the fluid with low diffusivity the rate of heat transfer increases.

Skin friction

The wall shear stress for cooling of the plate and heating of the plate are tabulated in tables 1 and 2 respectively. It is observed that skin friction increases as elastic parameter (R_c) and Schmidt number (S_c) increase, but in all other cases i.e chemical reaction parameter, magnetic parameter, permeability parameter, heat source parameter, Prandtl number, accelerating parameter and time skin friction decreases. Therefore, it may be concluded that the fluids of heavier species, having elastic property, enhance the shearing stress at the plate. It is interesting to note that the skin friction becomes negative for moderately high values of heat source parameter (S) and permeability parameter (K_p).

Table.2 reveals that all the entries of skin friction for heating of the plate are negative except, when $K_p = 1.0$ for which it is positive. This anomalous behaviour due to higher permeability parameter ($K_p = 1.0$) may be attributed to the low resistivity of the porous medium through which the flow takes place. The effects of R_c and S_c remain the same in

case of cooling of the plate also. Only exception is marked in case of chemical reaction which has a reverse effect. As regard to the sign of skin friction, the heat source parameter retains its sign as in case of cooling of the plate. Thus, it is concluded that sign of skin friction remains unaltered due to cooling / heating of the plate in the presence of heat source. One striking feature of skin friction is its response to chemical reaction. It is found that in the presence of destructive reaction ($K_c > 0$) the skin friction experiences a retarding effect due to the cooling of the plate whereas for heating, the reverse effect is observed.

5. Conclusion

- (i) Cooling of the plate and heating of the plate exhibits opposite behaviour and flow reversal occurs in case of heating of the plate.
- (ii) Presence of chemical reaction has a retarding effect on velocity and accelerates the attainment of ambient state.
- (iii) Application of magnetic field to a flow of visco-elastic fluid with higher Schmidt number increases the velocity.
- (iv) Thermal buoyancy, heat source and Prandtl number have no significant contribution to the flow field.
- (v) Fluid with higher conductivity in presence of heat source has a decreasing effect on temperature field.
- (vi) Concentration profile has a similar response as that of temperature to Schmidt number and chemical reaction.
- (vii) Skin friction under the influence of cooling of the plate is reduced due to the flow of reacting species in the presence of magnetic field, porous matrix and heat source.
- (viii) Fluid with low diffusivity gives rise to higher rate of heat transfer.
- (ix) Chemically reacting heavier species accelerate the rate of mass transfer.

Nomenclature

C'_∞ Concentration in the fluid far away from the plate	a Accelerating parameter
C'_w Concentration of the plate	D Chemical Molecular diffusivity
y' Coordinate axis normal to the plate	g Acceleration due to gravity
C Dimensionless concentration	K_p permeability parameter
y Dimensionless coordinate axis normal to the plate	M Magnetic field parameter
u Dimensionless velocity	S Heat Source parameter
B_0 External magnetic field	R_c Visco-elastic parameter
G_c Mass Grashof number	t Dimensionless time
P_r Prandtl number	μ Coefficient of viscosity
S_c Schmidt number	$erfc$ Complementary error function
C' Species concentration in the fluid	ρ Density of the fluid
C_p Specific heat at constant pressure	T Dimensionless temperature
T'_∞ Temperature of the fluid far away from the plate	σ Electric conductivity
T' Temperature of the fluid near the plate	erf Error function
T'_w Temperature of the plate	ν Kinematic viscosity
k Thermal conductivity of the fluid	α Thermal diffusivity
G_r Thermal Grashof number	β^* Volumetric coefficient of expansion with concentration
t' Time	β Volumetric coefficient of thermal expansion
u' Velocity of the fluid in the x' -direction	w Conditions on the wall
u_0 Velocity of the plate	∞ Free stream conditions

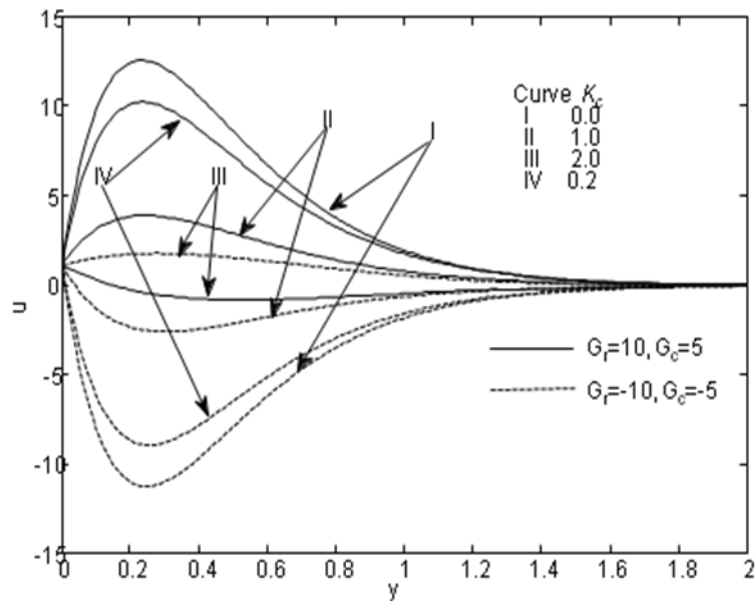


Fig.1 Effect of K_c on velocity profile $t = 0.2, R_c = 0.05, M = 1, K_p = 0.5, P_r = 0.1, S = 4, S_c = 0.78$ and $a = 0.5$

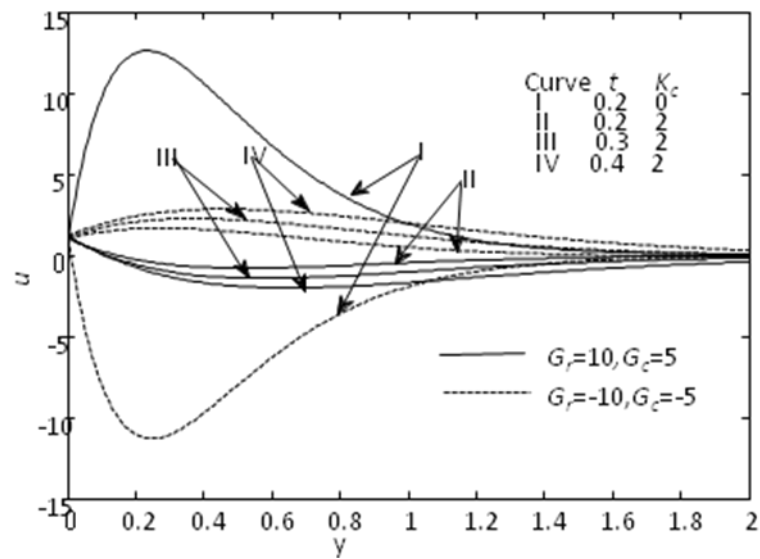


Fig. 2 Effect of t velocity profile for $R_c = 0.05, M = 1, K_p = 0.5, P_r = 0.1, S = 4, S_c = 0.78$ and $a = 0.5$

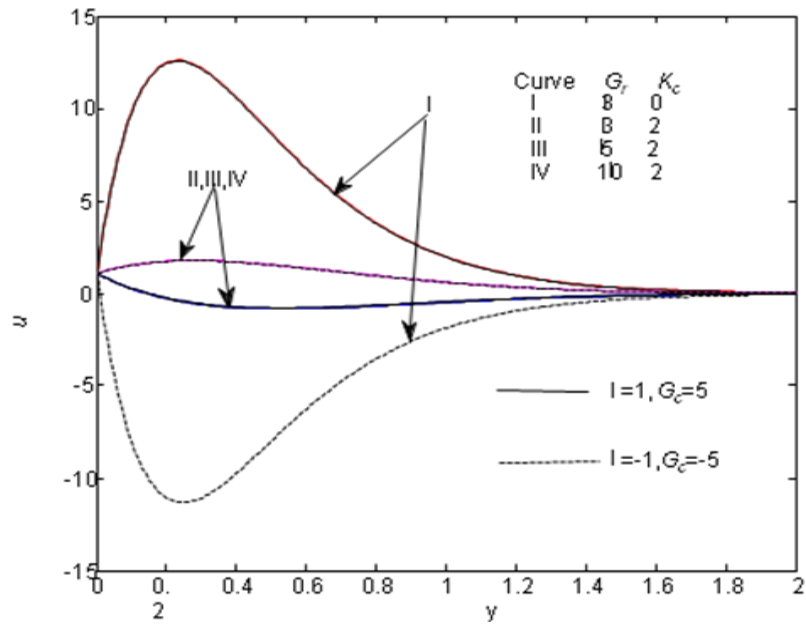


Fig. 3 Effect of G_r on velocity profile for $t = 0.2, R_c = 0.05, M = 1, K_p = 0.5, P_r = 0.1, S = 4, S_c = 0.78$ and $a = 0.5$

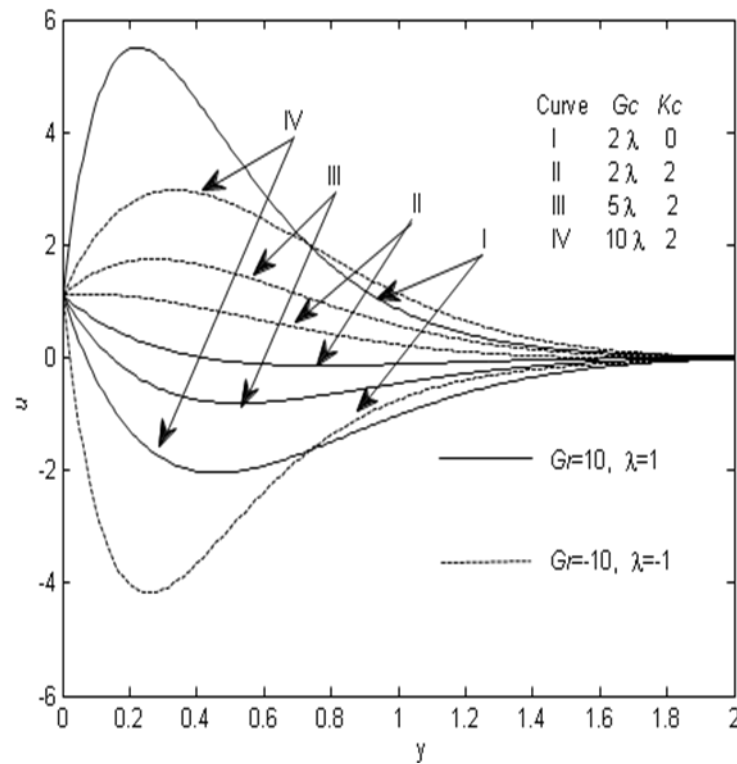


Fig.4 Effect of G_c on velocity profile for $t = 0.2, R_c = 0.05, M = 1, K_p = 0.5, P_r = 0.1, S = 4, S_c = 0.78$ and $a = 0.5$

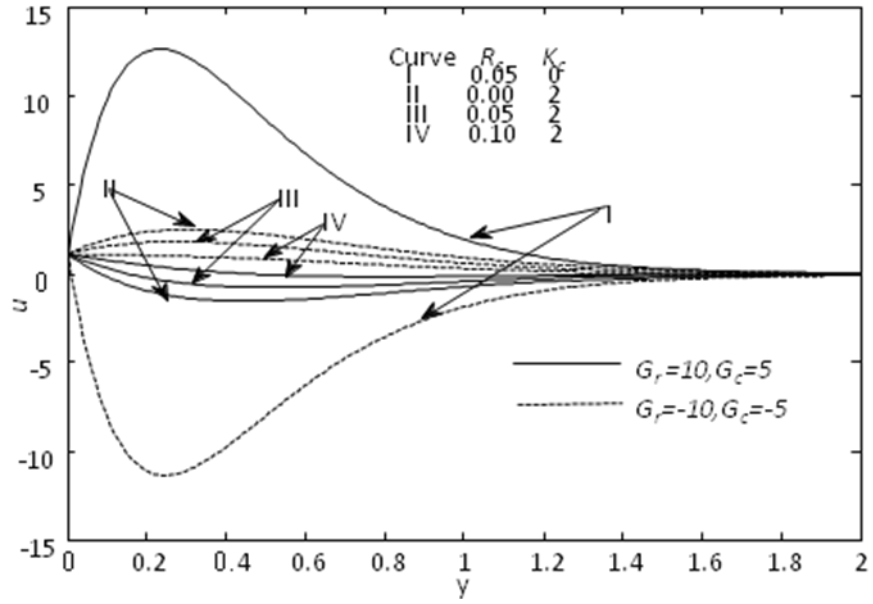


Fig. 5 Effect of R_c on velocity profile for $t = 0.2, M = 1, K_p = 0.5, P_r = 0.1, S = 4, S_c = 0.78$ and $a = 0.5$

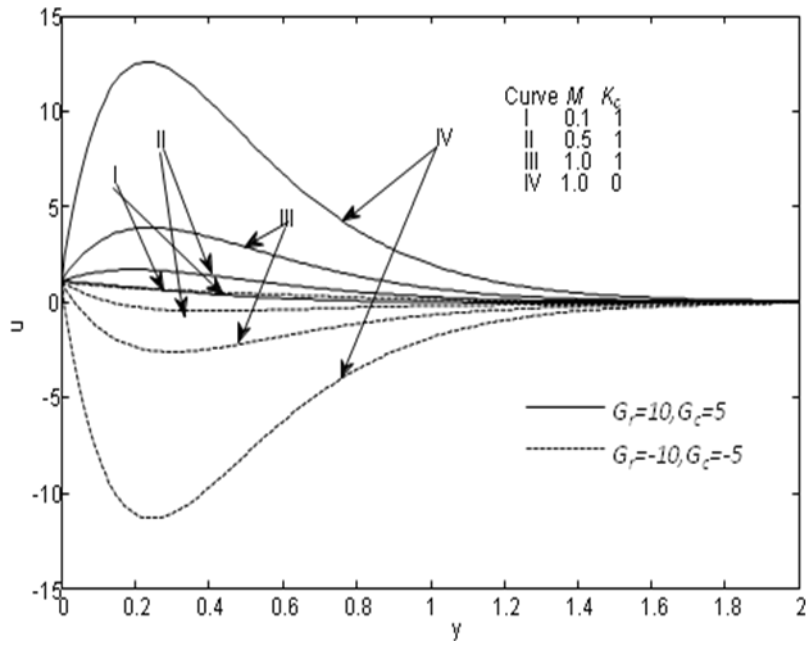


Fig. 6 Effect of M on velocity profile for $t = 0.2, R_c = 0.05, K_p = 0.5, P_r = 0.1, S = 4, S_c = 0.78$ and $a = 0.5$

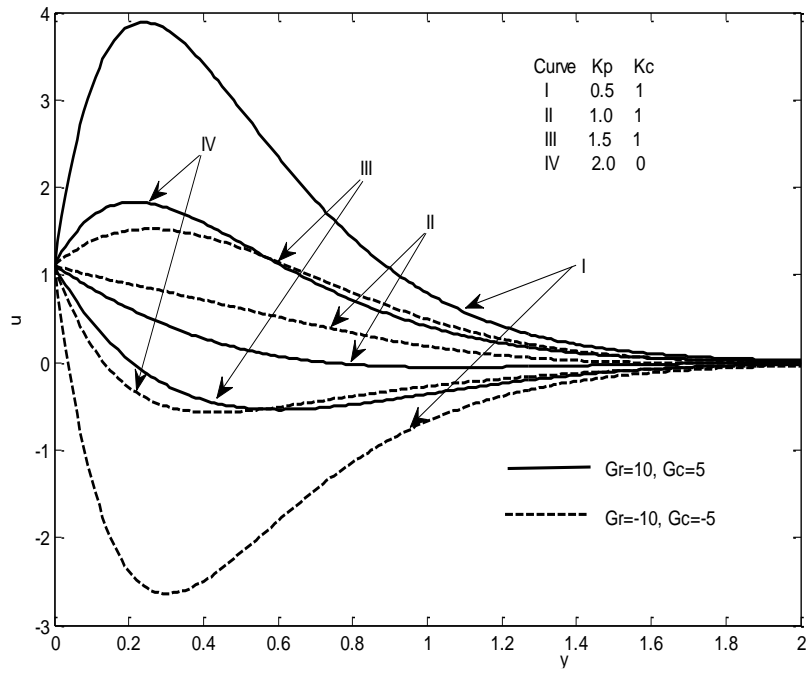


Fig.7 Effect of K_p on velocity profile for $t = 0.2, R_c = 0.05, M = 1, P_r = 0.1, S = 4, S_c = 0.78$ and $a = 0.5$

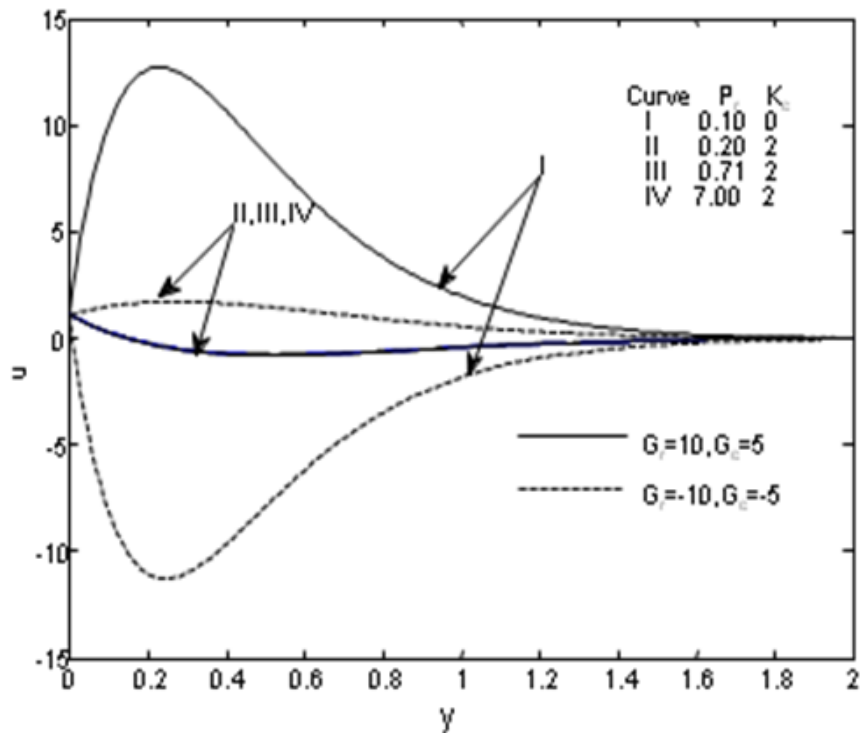


Fig. 8 Effect of P_r on velocity profile for $t = 0.2, R_c = 0.05, M = 1, K_p = 0.5, S = 4, S_c = 0.78$ and $a = 0.5$

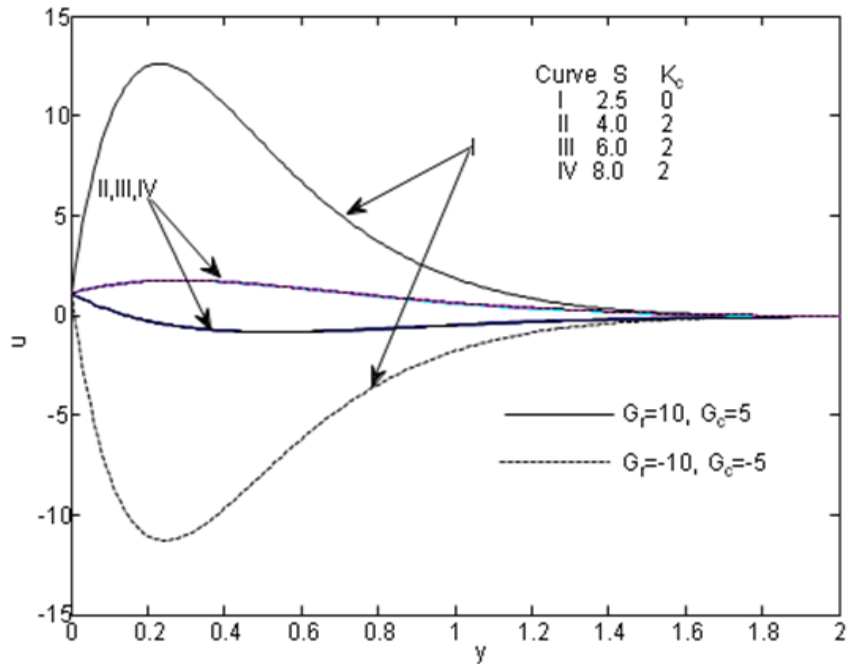


Fig. 9 Effect of S on velocity profile for $t = 0.2, R_c = 0.05, M = 1, K_p = 0.5, P_r = 0.1, S_c = 0.78$ and $a = 0.5$

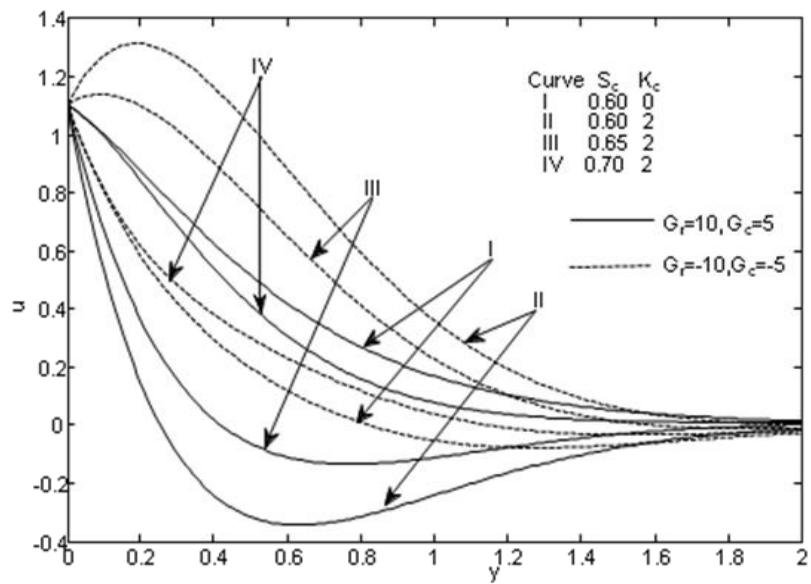


Fig. 10 Effect of S_c on velocity profile for $t = 0.2, R_c = 0.05, M = 1, K_p = 0.5, P_r = 0.1, S = 4$, and $a = 0.5$

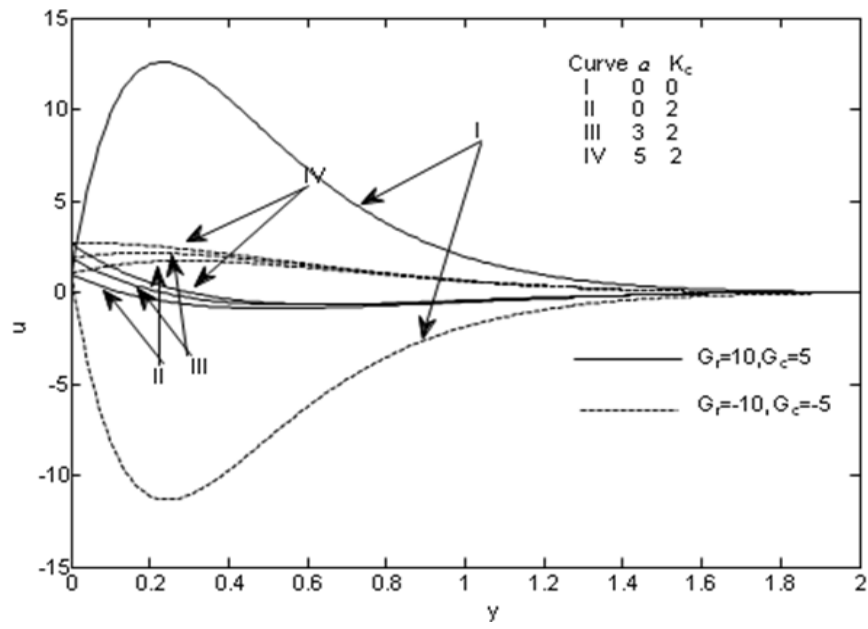


Fig. 11 Effect of a on velocity profile for $t = 0.2, R_c = 0.05, M = 1, K_p = 0.5, P_r = 0.1, S = 4, S_c = 0.78$

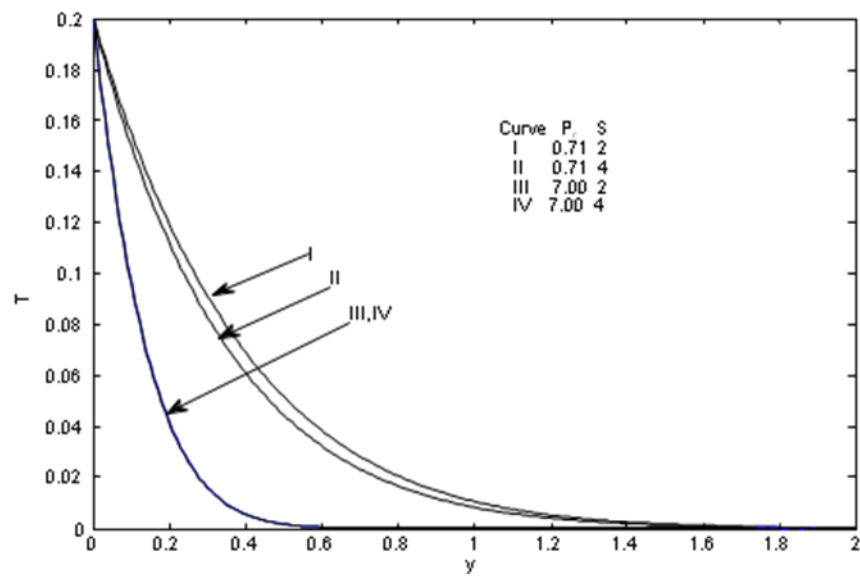


Fig. 12 Temperature profile

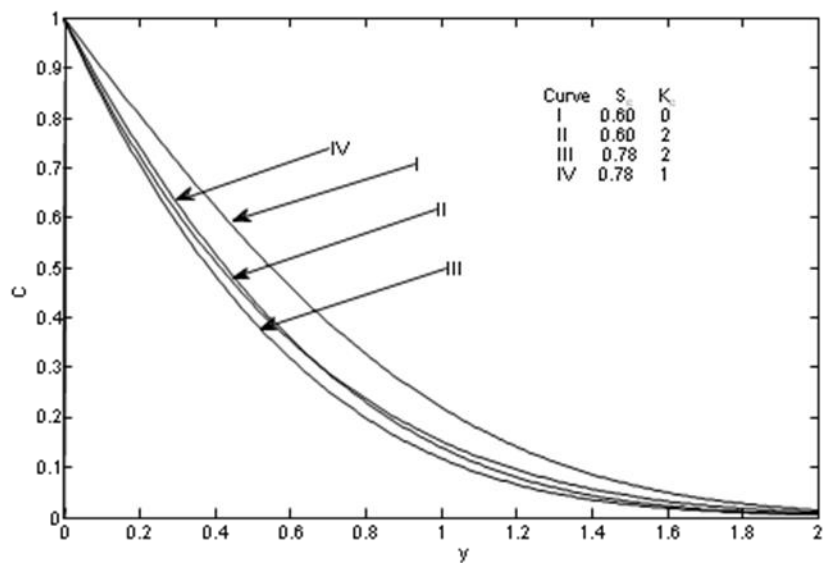


Fig. 13 Concentration profile

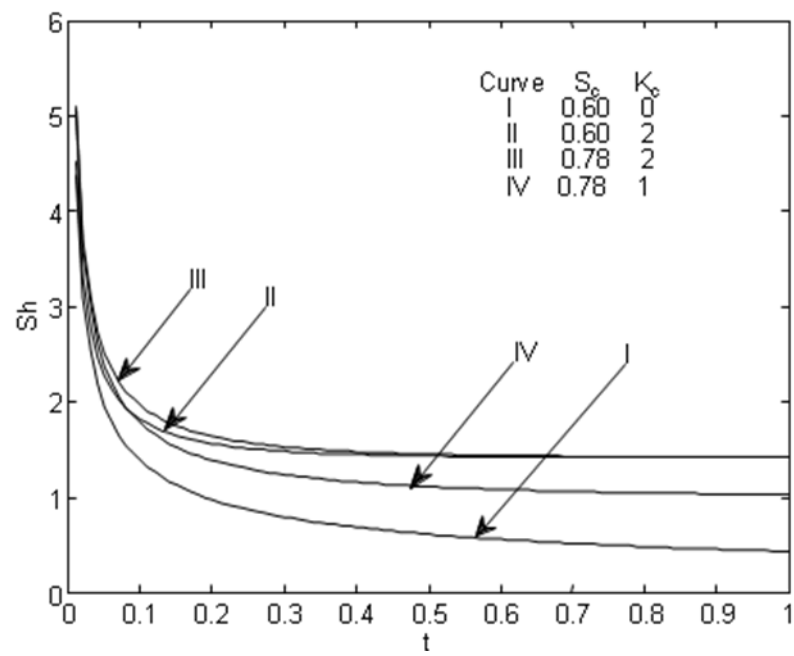


Fig. 14 Rate of mass transfer

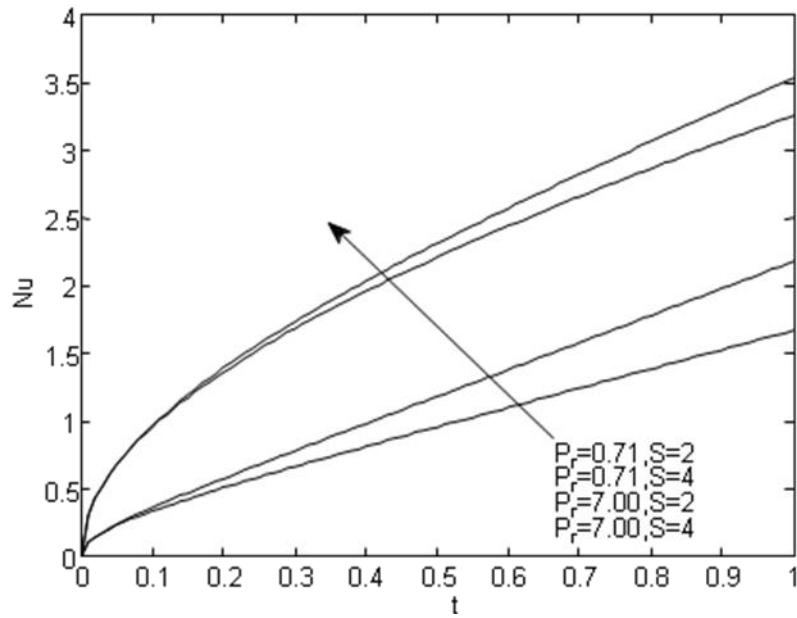


Fig. 15 Rate of heat transfer

Table.1. Skin friction (for cooling of the plate)

Sl.No	R_c	M	K_p	G_r	G_c	P_r	S	K_c	S_c	a	t	τ
I	0.01	0.5	0.5	10	5	0.1	2	0.0	0.6	0.5	0.2	37.17450846
II	0.01	0.5	0.5	10	5	0.1	2	0.1	0.6	0.5	0.2	36.55909659
III	0.05	0.5	0.5	10	5	0.1	2	0.1	0.6	0.5	0.2	37.87697964
IV	0.01	0.2	0.5	10	5	0.1	2	0.1	0.6	0.5	0.2	79.96182642
V	0.01	0.5	1	10	5	0.1	2	0.1	0.6	0.5	0.2	-32.24353127
VI	0.01	0.5	0.5	10	5	0.1	4	0.1	0.6	0.5	0.2	-14.96974079
VII	0.01	0.5	0.5	10	5	0.1	2	2.0	0.6	0.5	0.2	35.2887651
VIII	0.01	0.5	0.5	10	5	0.1	2	0.1	0.6	0.9	0.2	35.57325233
IX	0.01	0.5	0.5	10	5	0.71	2	0.1	0.6	0.5	0.2	9.677038833
X	0.01	0.5	0.5	10	5	0.1	2	0.1	0.78	0.5	0.2	43.80930474
XI	0.01	0.5	0.5	10	5	0.1	2	0.1	0.78	0.5	0.4	69.78006788

Table.2. Skin friction (for heating of the plate)

Sl.No	R_c	M	K_p	G_r	G_c	P_r	S	K_c	S_c	a	t	τ
I	0.01	0.5	0.5	-10	-5	0.1	2	0.0	0.6	0.5	0.2	-12.86534928
II	0.01	0.5	0.5	-10	-5	0.1	2	0.1	0.6	0.5	0.2	-62.5627103
III	0.05	0.5	0.5	-10	-5	0.1	2	0.1	0.6	0.5	0.2	-63.91398731
IV	0.01	0.2	0.5	-10	-5	0.1	2	0.1	0.6	0.5	0.2	-1.07E+02
V	0.01	0.5	1	-10	-5	0.1	2	0.1	0.6	0.5	0.2	3.009234569
VI	0.01	0.5	0.5	-10	-5	0.1	4	0.1	0.6	0.5	0.2	-21.02567422
VII	0.01	0.5	0.5	-10	-5	0.1	2	2.0	0.6	0.5	0.2	-61.2923788
VIII	0.01	0.5	0.5	-10	-5	0.1	2	0.1	0.6	0.9	0.2	-61.00789157
IX	0.01	0.5	0.5	-10	-5	0.71	2	0.1	0.6	0.5	0.2	-86.95270562
X	0.01	0.5	0.5	-10	-5	0.1	2	0.1	0.78	0.5	0.2	-69.81291844
XI	0.01	0.5	0.5	-10	-5	0.1	2	0.1	0.78	0.5	0.4	-1.11E+02

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