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Double Diffusive Free Convection over a Heated Vertical Plate in a Saturated Porous Medium with Soret Effect and Variable Heat Source

M. El haroui, M.Sriti, D.Achemlal

University Sidi Mohamed Ben Abdellah, Polydisciplinary Faculty of Taza,
Engineering Sciences Laboratory, BP.1223, Taza, Morocco
(elharoui_m@yahoo.fr, mmsriti@yahoo.fr, driss_achemlal@yahoo.fr)

Abstract

In the present work, we study numerically the phenomenon of double diffusive free convection induced by a heated vertical plate embedded in a saturated porous medium with lateral mass flux, in the presence of an internal heat source and Soret effect. The system of equations governing the problem is transformed into a dimensionless form by adopting the similarity method. Differential equations thus obtained are solved numerically using a computational code based on the fifth order Runge-Kutta method coupled with the soothing iteration technique. Dimensionless temperature and concentration profiles are presented graphically and discussed in detail for various values of involved parameters. The local Nusselt and Sherwood numbers profiles depending on the suction/injection parameter and Soret number, with and without internal heat source, are also presented and interpreted.

Key words: Double diffusive, free convection, porous medium, Soret effect, fluid suction/injection, heat source.

Nomenclature

x	coordinate along the plate (m)	T	fluid temperature (K)
y	coordinate normal to the plate (m)	C	fluid concentration
(u,v)	velocity components in the x and y directions $(m.s^{-1})$	a	equivalent thermal diffusivity $(m^2.s^{-1})$
g	gravitational acceleration $(m.s^{-2})$	ν	kinematic viscosity $(m^2.s^{-1})$

ρ	fluid density $(Kg.m^{-3})$	Ra_x	local Reynolds number
β_T	thermal expansion coefficient (K^{-1})	Sr	Soret number
β_c	concentration expansion coefficient	Le	Lewis number
K	permeability of the porous medium (m^2)	N	Buoyancy ratio number
C_p	specific heat of fluid $(J.kg^{-1}.K^{-1})$	Nu_x	local Nusselt number.
D_M	coefficient of mass diffusivity $(m^2.s^{-1})$	Sh_x	local Sherwood number
V_w	lateral mass flux	w	wall plate condition
f_w	suction/injection parameter	η	similarity variable
f	dimensionless stream function	∞	infinity plate condition
ψ	stream function	'	derivative with respect to η
θ	dimensionless temperature		
ϕ	dimensionless concentration		
IHS	internal heat source		

1. Introduction

Coupled heat and mass transfer by free convection in a saturated porous medium has attracted considerable attention in the last several decades due to its importance applications in many engineering, geophysical and natural systems of practical interest such thermal energy storage and recoverable systems, geothermal energy utilization, petroleum reservoirs, transport of contaminants in saturated soil, the migration of moisture in fibrous insulation and many others applications. Comprehensive reviews on this topic have been made by many books some of them are Nield and Bejan [1], Ingham and Pop [2] and Vafai [3]. When the heat and mass transfer occurs simultaneously, it has been observed that an energy flux can be produced not only by temperature gradients, but also by concentration gradients. The energy flux caused by a concentration gradients is termed the diffusion-thermo or Dufour effect. On the other hand, mass fluxes can be generated by temperature gradients and this embodies the thermal-diffusion or Soret effect. Hurle and Jackeman [4] argued that for liquid mixtures the Dufour term is indeed very small and thus the Dufour effect may be negligible in comparison to the Soret effect. Murray and Chen [5] are the first who studied experimentally double diffusive free convection in a saturated porous medium, they obtained a good agreement between experiment and theory of linear stability for the critical thermal Rayleigh number R_T . The effect of lateral mass flux (fluid suction/injection) on the free convection boundary layers induced by a heated vertical plate embedded in a saturated porous medium with an exponential decaying heat generation is studied by Postelnicu et al. [6], Ali [7] and Achemlal et al.[8].

The influence of Soret and Dufour effects on the double diffusive free convection over a vertical or horizontal surface embedded in a saturated porous medium and subjected to various heat and concentration situations has the subject of many studies such as: Alam and Rahman [9], Hassan and El-Arbawi [10], Patil et al. [11]. A similarity method have been used by Ferdows et al. [12] for numerical study of Double diffusive free convection along a vertical plate immersed in a saturated porous medium with and without internal heat generation. Postelnicu [13] and Rath et al. [14] have taken into consideration the presence of a magnetic field in the study of double diffusive free convection in a porous medium induced by a vertical surface. The authors found that increasing of the magnetic parameter leads to a widening the thickness of dynamic, thermal and concentration boundary layers. Bennacer et al. [15] have used the Darcy-Brinkman model in their analytical and numerical study of coupled heat and mass transfer in a porous medium with a thermal anisotropy.

In this work, we propose to study, numerically, the double diffusive free convection past an isothermal flat plate, maintained at constant concentration and subjected to a lateral fluid flux (fluid suction/injection), immersed vertically in a homogeneous porous medium saturated with Newtonian and incompressible fluid, taking into account the Soret effect and a variable internal heat generation.

2. Mathematical modeling and similarity analysis

Our study is to investigate the double diffusive free convection induced by a vertical heated flat plate embedded in a saturated porous medium. The plate is maintained at a constant temperature T_w and a constant concentration C_w and it subjected to a lateral fluid flux in the direction normal to the plate and proportional to $x^{-1/2}$. The physical model, the coordinate system and the boundary conditions associated with the problem are shown in Figure 1. The porous medium is considered homogeneous, isotropic and saturated with Newtonian and incompressible fluid, the flow is two dimensional, laminar and stationary. The fluid and medium proprieties, except the fluid density, may be assumed to be constant. The problem is governed by the system of continuity, Darcy, energy and concentration equations based on the Boussinesq approximation:

$$\rho = \rho_\infty \left(1 - \beta_T (T - T_\infty) - \beta_c (C - C_\infty)\right)$$

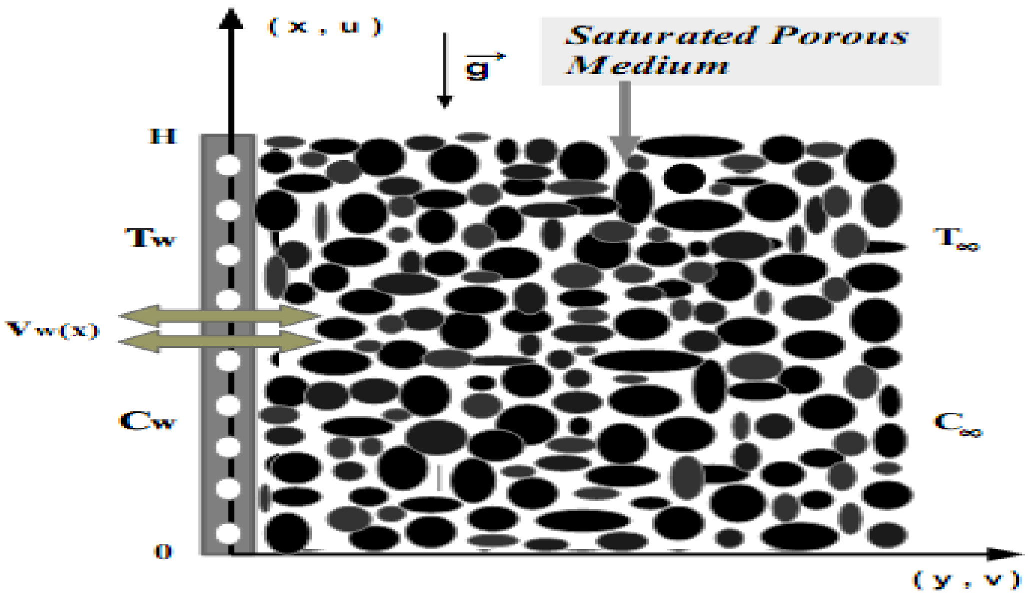

Fig.1. Vertical heated plate in a saturated porous medium.

Taking into account the simplifying assumptions previously mentioned and the Boussinesq approximation, the system of equations describing the problem studied can be written:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u=\frac{Kg}{\nu}\left(\beta_T\left(T-T_\infty\right)+\beta_c\left(C-C_\infty\right)\right) \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=a\frac{\partial^2 T}{\partial y^2}+\frac{\varphi}{\rho C_p} \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_M\frac{\partial^2 C}{\partial y^2}+D_T\frac{\partial^2 T}{\partial y^2} \tag{4}$$

The boundary conditions associated with the problem are:

$$x\geq 0 \quad y=0 \qquad v=v_w(x)\ ,\ T=T_w\ ,\ C=C_w \tag{5}$$

$$x\geq 0 \quad y\to\infty \qquad u=0 \quad ,\quad T=T_\infty\ ,\ C=C_\infty \tag{6}$$

In the above equations, u and v are the Darcian velocity components along the x and y directions, T and C are the fluid temperature and fluid concentration, respectively, φ is an internal heat generation, ρ_∞ is the reference density, ρ , a , g and ρ are, respectively, the kinematic viscosity, thermal diffusivity, gravitational acceleration and fluid density. C_p represents the specific heat at a constant pressure and K is the permeability of the porous medium. D_M is the coefficient of mass diffusion, D_T is a coefficient which quantifies the contribution of the thermal gradient to the mass flux. $V_w(x) = A x^{-1/2}$ is the lateral fluid flux, where A is a positive constant. The continuity of the equation (1) is satisfied by the stream function $\psi(x,y)$ defined by: $u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x}$.

The non-linearity of the model and the complexity of the phenomena encountered (boundary layer, instability, geometry of the porous medium…) make difficult its resolution direct, for this, the similarity method is adopted by posing the following dimensionless variables (8) used by Postelnicu [13]:

$$\begin{aligned} &\eta = \frac{y}{x} Ra_x^{\frac{1}{2}} \ , \ \psi = a\, Ra_x^{\frac{1}{2}} f(\eta) \ , \ \theta = \frac{T - T_\infty}{T_w - T_\infty} \ , \ \phi = \frac{C - C_\infty}{C_w - C_\infty} \\ &Ra_x = \frac{g K \beta_T (T_w - T_\infty)}{\nu a} x \ , \ \varphi = \rho C_p \frac{a (T_w - T_\infty) Ra_x}{x^2} e^{-\eta} \end{aligned} \tag{8}$$

Where η and f are, respectively, the similarity variable and the similarity function. θ and ϕ represents the dimensionless temperature and concentration, Ra_x is the local Rayleigh number and φ is a variable internal heat source.
After development and substitution, we obtain the following dimensionless system of equations (9), (10) and (11) coupled with boundary conditions (12) and (13).

$$f' = \theta + N\phi \tag{9}$$

$$\theta'' = -\left(\frac{1}{2} f\, \theta' + e^{-\eta} \right) \tag{10}$$

$$\phi'' = \frac{Le}{2} (Sr\, (f \theta' + 2 e^{-\eta}) - f \phi') \tag{11}$$

$$\eta = 0 \qquad f = f_w \ , \ \theta = 1 \ , \ \phi = 1 \tag{12}$$

$$\eta \to \infty \quad f' = 0 \ , \ \theta = 0 \ , \ \phi = 0 \tag{13}$$

Where $f_w = -2A \left(\frac{\nu}{a\, g\, \beta_T\ K\, (T_w - T_\infty)} \right)^{1/2}$ is the suction/injection parameter, $Sr = \frac{D_T}{a} \frac{T_w - T_\infty}{C_w - C_\infty}$ is the Soret number, $Le = \frac{a}{D_M}$ is the Lewis number and $N = \frac{\beta_c (C_W - C_\infty)}{\beta_T (T_W - T_\infty)}$ represents the buoyancy ratio number.

The parameters of engineering interest for the problem are the local Nusselt number Nu_x and local Sherwood number Sh_x, which characterize, respectively, the surface heat and mass transfer rate and are defined as:

$$Nu_x = -\frac{x}{(T_w - T_\infty)} \left(\frac{\partial T}{\partial y} \right)_{y=0} \quad = -\theta'(0)\, Ra_x^{1/2} \tag{14}$$

$$Sh_x = -\frac{x}{(C_w - C_\infty)}\left(\frac{\partial C}{\partial y}\right)_{y=0} = -\phi'(0)\,Ra_x^{1/2} \tag{15}$$

3. Numerical results and discussions

The system of equations (9), (10) and (11) coupled with boundary conditions (12) and (13) are solved numerically by fifth order Runge-kutta method coupled with the soothing iteration technique. The calculations were performed for different values of the dimensionless numbers such as: Sr, Le, N, and for suction/injection parameter f_w which correspond to the suction for $f_w < 0$, injection for $f_w > 0$ and impermeable plate when $f_w = 0$, with and without an internal heat source (IHS). To assess the accuracy of our numerical technique, a comparison of the heat and mass transfer at wall for various values of Le and N in the absence of internal heat source and the soret effect, for impermeable plate $(f_w = 0)$ is made with the previously published results of El-Arbawy [10]. The comparison is listed in table 1 and found in excellent agreement.

			$-\theta'(0) = Nu_x\ Ra_x^{-1/2}$		$-\phi'(0) = Sh_x\ Ra_x^{-1/2}$	
Le	N	Sr	El-Arbawy [10]	Present results	El-Arbawy [10]	Present results
1	1	0	0.627556	0.627231	0.627556	0.627231
2	1	0	0.592601	0.592711	0.929544	0.929455
4	1	0	0.558504	0.558532	1.357470	1.357326
6	1	0	0.540770	0.540694	1.684710	1.684625
8	1	0	0.529445	0.529500	1.959950	1.959837
10	1	0	0.521401	0.521333	2.202080	2.202271

Tab.1.Values of $-\theta'(0)$ and $-\phi'(0)$ for various values of Le and N without IHS at $f_w = 0$ and $Sr = 0$

Figures 2 and 3 show the effect of the Soret number on the temperature and concentration profiles in the boundary layer area of an isothermal and impermeable plate for $Le = 1$ and $N = 1$, without IHS. From the figure 2, it is clearly that the presence of the Soret effect has a small influence on the temperature profiles around the plate. From the figure 3, we notice that the increase of the Soret number induces an amplification of the dimensionless concentration profiles in the boundary layer area. On the other hand, we observe that the concentration profiles have a maxima for large values of Sr where the mass transfer is directed from the porous medium to the plate surface, unlike the small values of Sr where the mass flow is transferred in the direct sense (from the plate to the porous medium) .We notice also that the high values of Sr allows to amplifies the concentration boundary layer thickness. The effect of the Soret number on the temperature and concentration profiles is in good concordance with that found by Aouachria et al. [16].

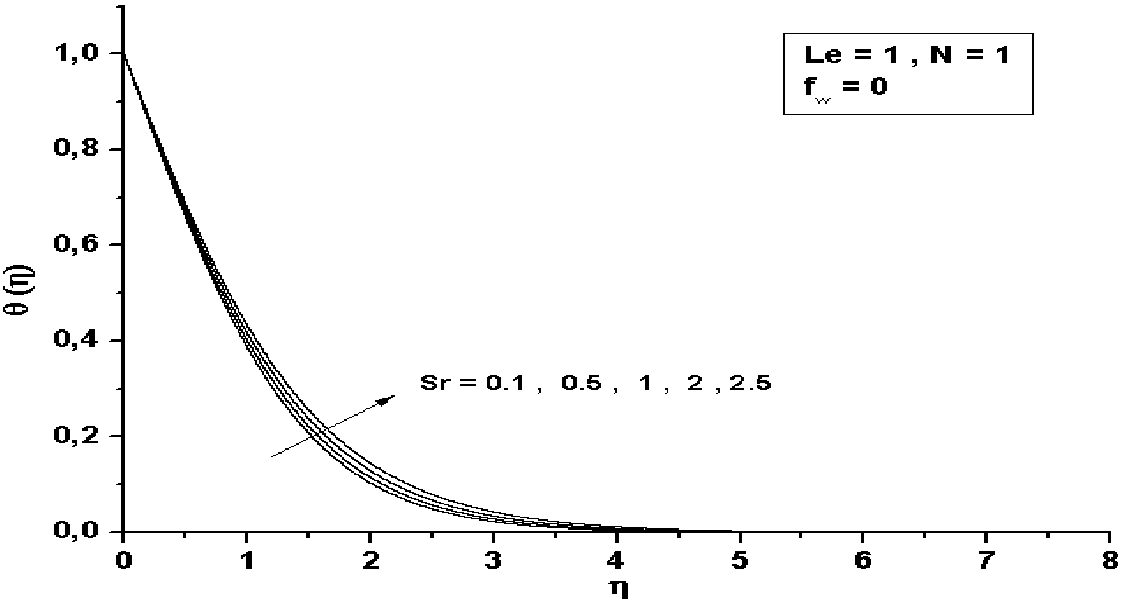

Fig.2. Thermal profiles for various values of Sr at $Le=1$ and $N=1$.

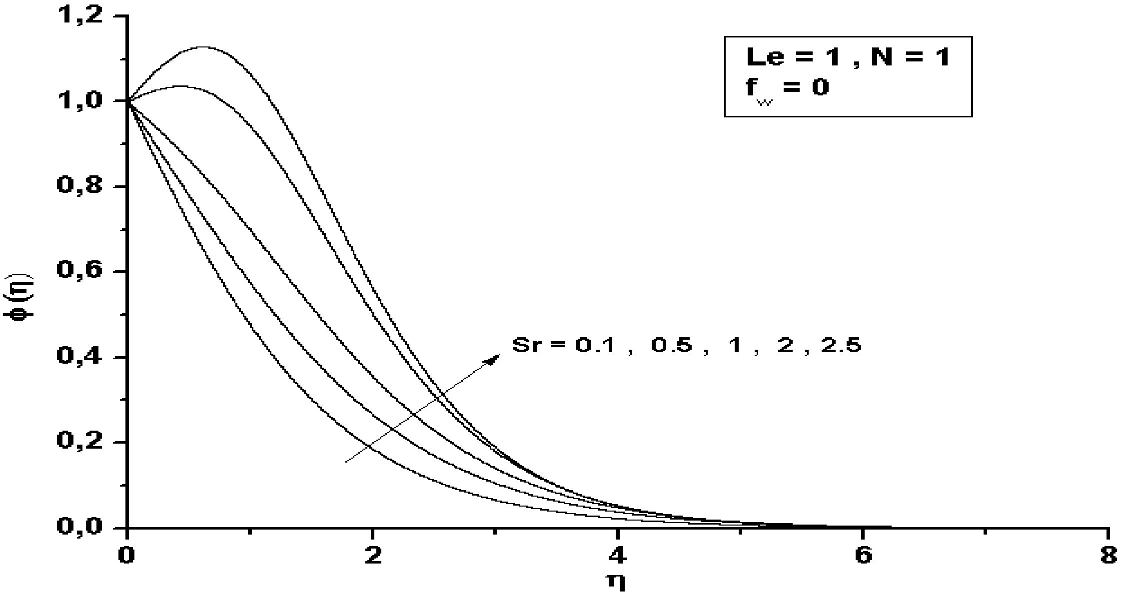

Fig.3. Concentration profiles for various values of Sr at $Le=1$ and $N=1$.

Figure 4 shows the influence of the Lewis number on the temperature profiles in the boundary layer of an isothermal and impermeable plate at $Sr=1$ and $N=1$. From this figure, it is clearly that the variation of the Lewis number does not have a remarkable influence on the profiles. The concentration profiles according to the Lewis number are shown in figure 5 for $N=1$ and $Sr=1$. We observe here that when the Lewis number increases, the profiles are reduced and the high values of Le stabilize the boundary layer concentration. This evolution can be physically justified by the dominance of the thermal diffusion effect compared to the mass diffusion effect for high values of the Lewis number. This is in good agreement with the evolution of the temperature and concentration profiles versus Lewis number found by A. Postelnicu [13].

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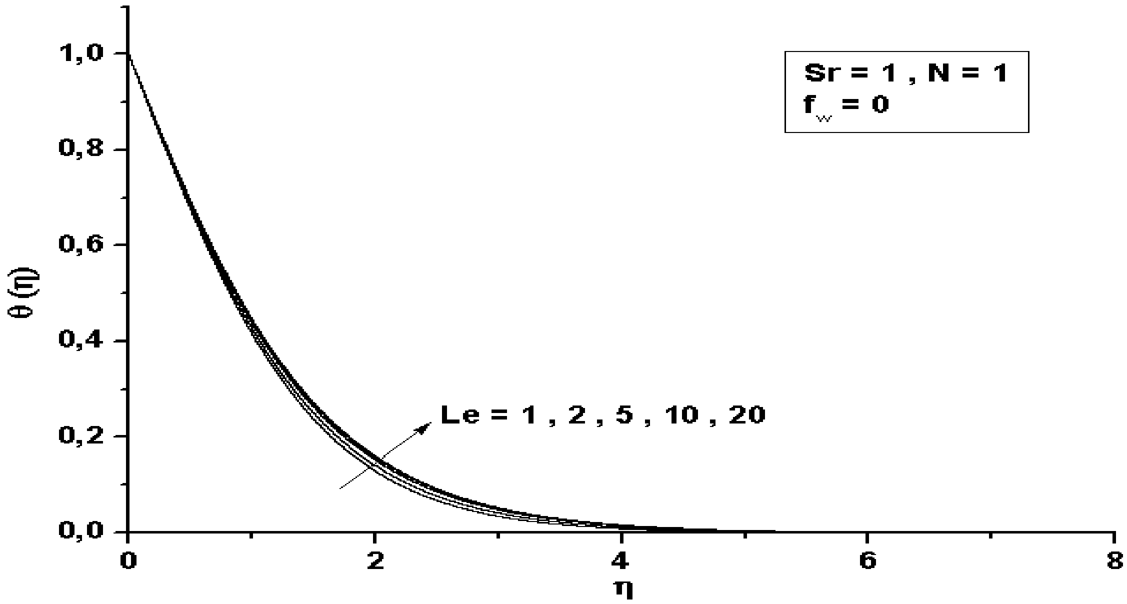

Fig.4. Thermal profiles for various values of Le at $Sr=1$ and $N=1$

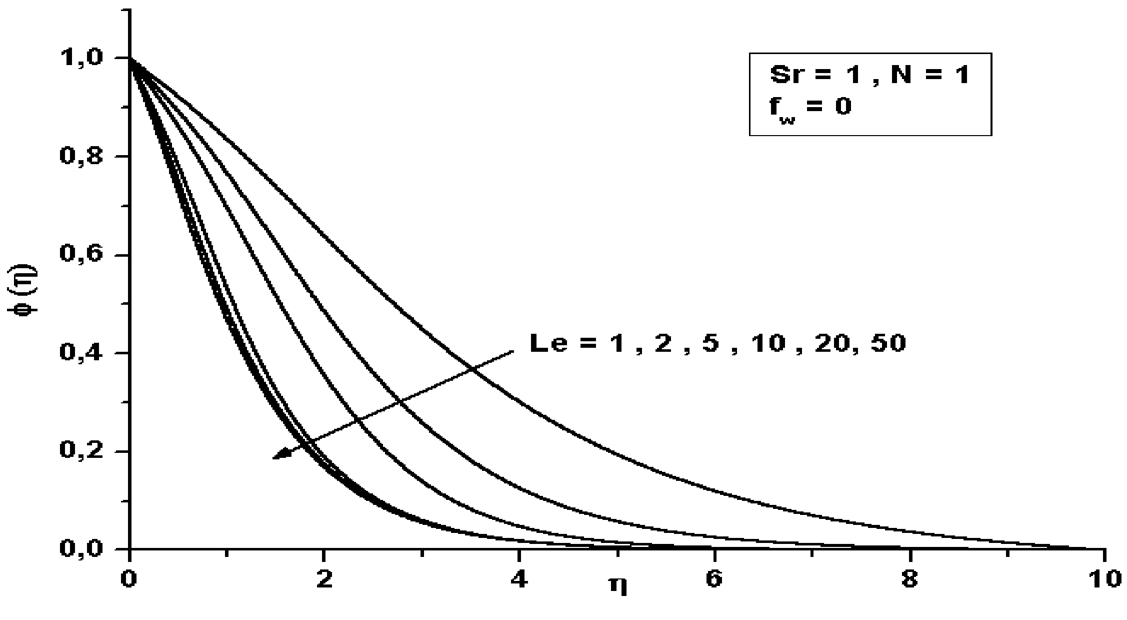

Fig.5. Concentration profiles for various values of Le at $Sr=1$ and $N=1$

Figure 6 presents the dimensionless temperature profiles in the boundary layer area of an impermeable plate for various values of Buoyancy parameter N at $Sr=1$ and $Le=1$. We notice a reduction of the thermal profiles as N increases. This evolution can be explained physically by the dominance of solutal volume forces over the thermal volume forces for the high values of the buoyancy parameter, which leads to a fast cooling in the boundary layer area. Figure 7 projects the dimensionless concentration profiles for various values of the parameter N and for $Sr=1$ and $Le=1$. We observe an almost similar trend of the temperature evolution with an enlargement of the concentration boundary layer thickness compared with the thermal boundary layer thickness. Here, we can say that our result confirms the influence of buoyancy parameter on the profiles in the study made by Ferdows et al. [12].

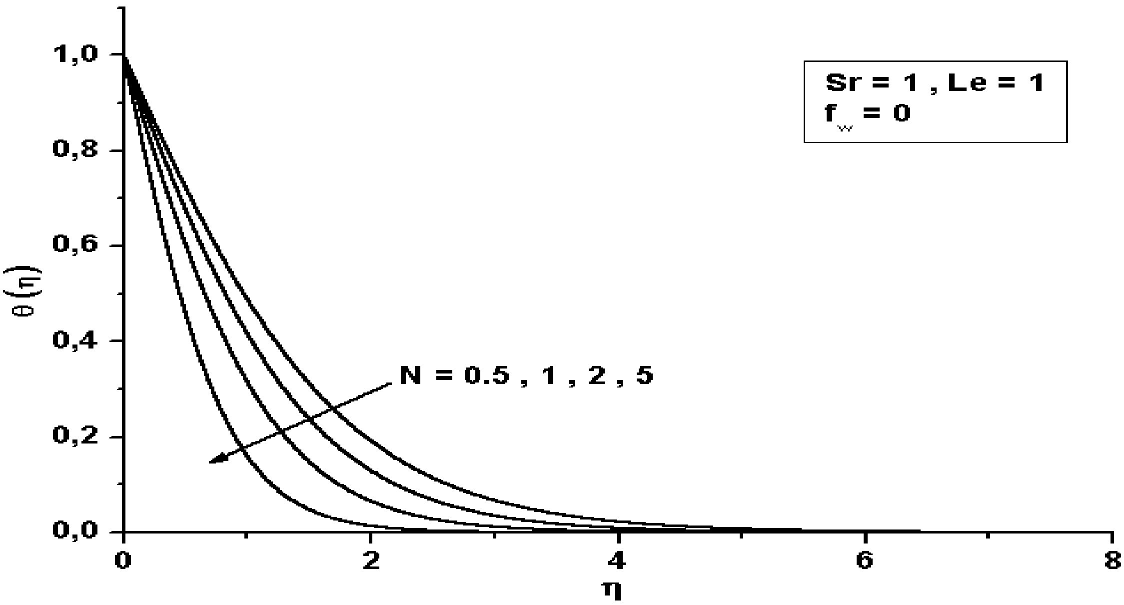

Fig.6. Thermal profiles for various values of N at $Sr=1$ and $Le=1$

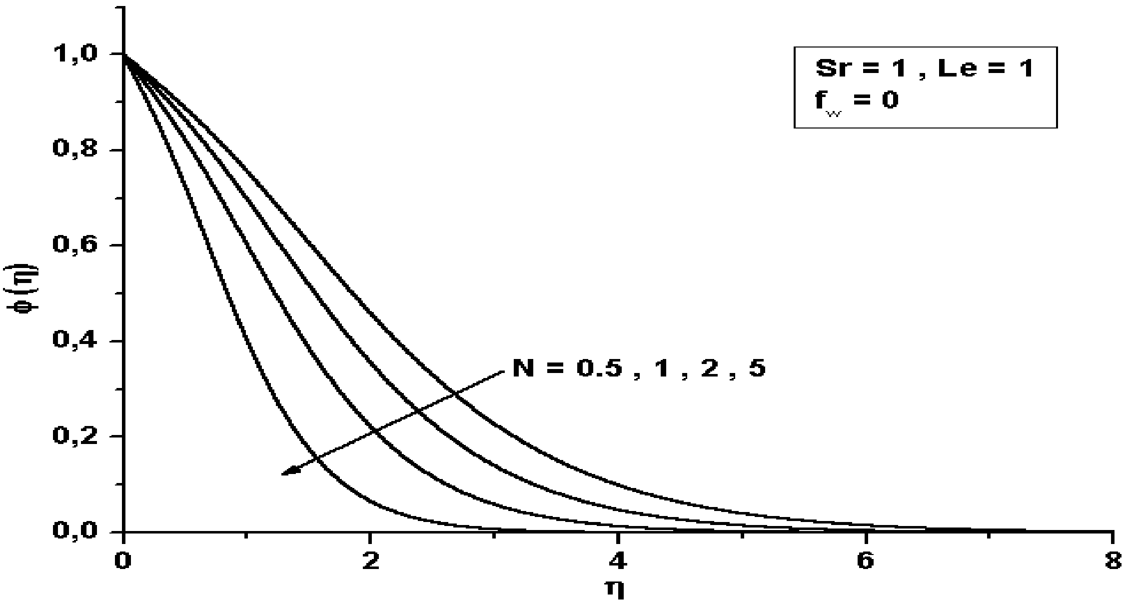

Fig.7. Concentration profiles for various values of N at $Sr=1$ and $Le=1$

The effect of suction/injection parameter f_w on the dimensionless temperature and concentration profiles in the boundary layer area for $Sr=1$, $Le=1$ and $N=1$ is shown in figures 8 and 9, respectively. It is observed that the fluid suction at the plate surface tends to reduce the thickness of the thermal and concentration boundary layer, unlike to the fluid injection effect. This result is in good agreement with those found by Ali et al. [7], Achemlal et al. [8] and Aouachria et al. [16] for an isothermal plate. Moreover, the heat and mass transfers are always directed the plate to the porous medium.

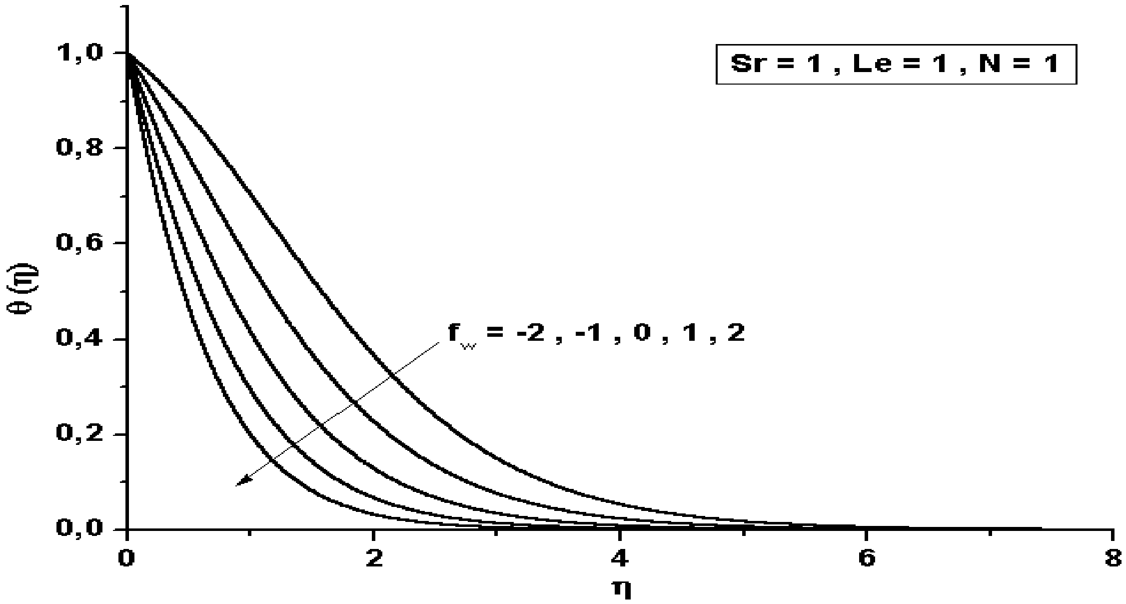

Fig.8. Thermal profiles for various values of f_w at $N=1$, $Sr=1$ and $Le=1$

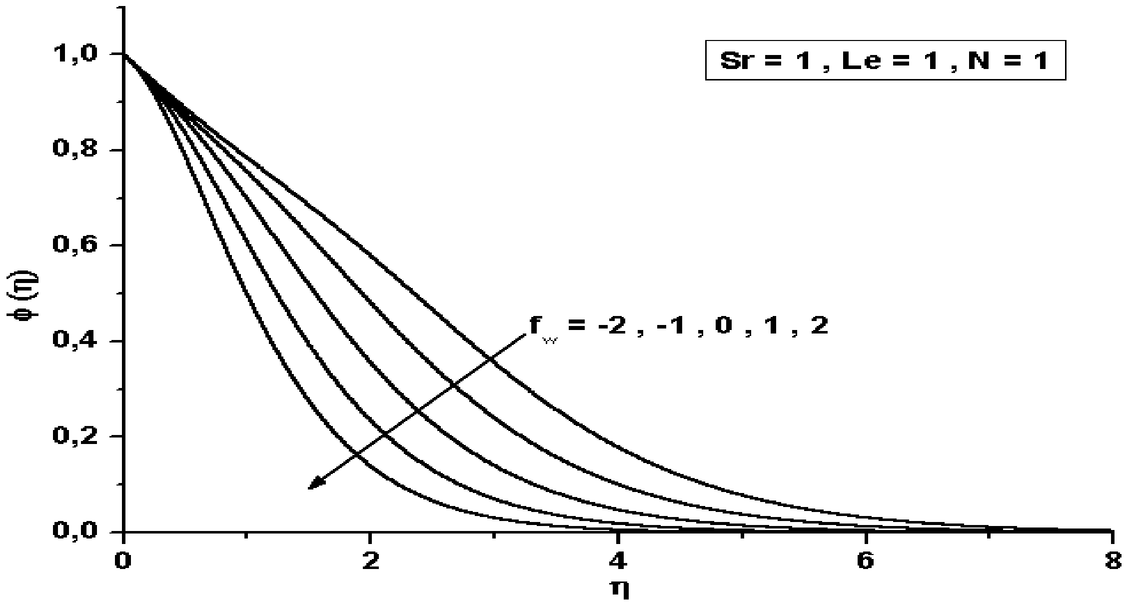

Fig.9. Concentration profiles for various values of f_w at $N = 1$, $Sr = 1$ and $Le = 1$

The evolution of the local Nusselt and Sherwood dimensionless numbers as function of the Soret number with and without internal heat source for an impermeable plate at $Le = 1$ and $N = 1$ is presented, respectively, in figures 10 and 11. From the figure 10, the heat transfer rate at the plate surface increases slightly with increasing Soret number. It also appears that the heat transfer rate is amplified in the presence of an internal heat source (IHS) compared to the case where the IHS is absent. An opposite evolution is observed for the Sherwood number evolution presented in Figure 11 where the mass transfer rate decreases according to the Sr and it is amplified in the presence of IHS in comparison with the case where the IHS is absent.

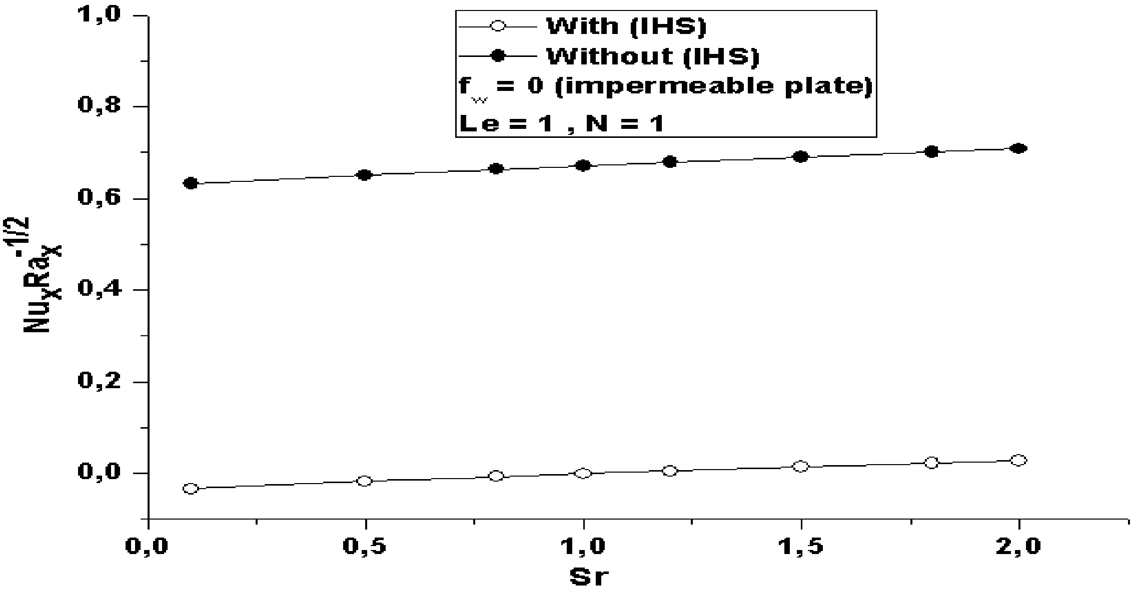

Fig.10. Nu_x versus Sr with and without IHS for $f_w = 0$, $Le = 1$ and $N = 1$

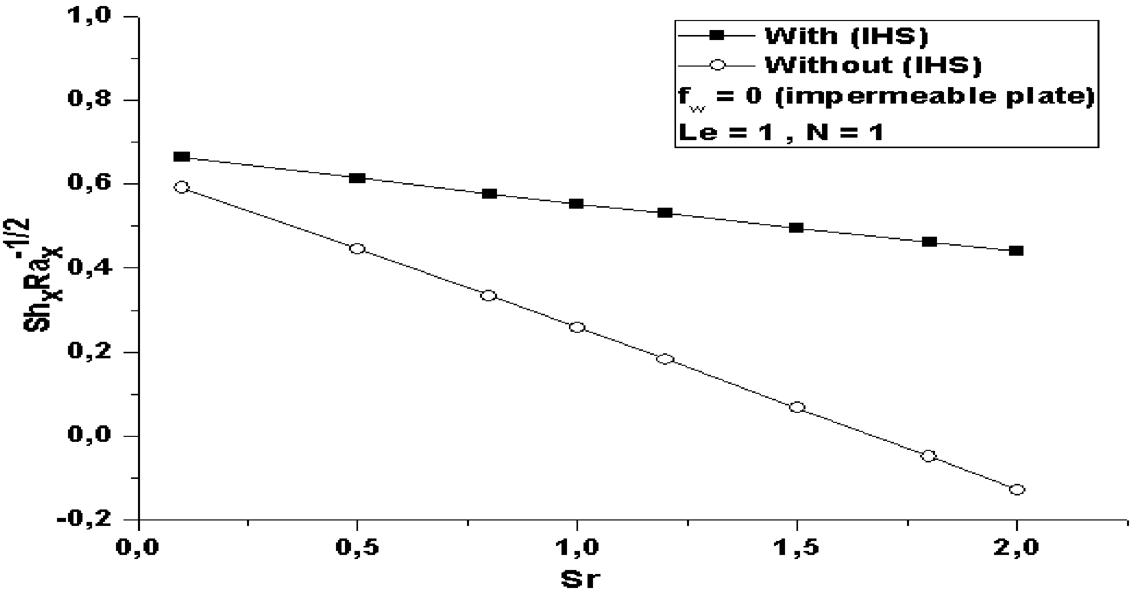

Fig.11. Sh_x versus Sr with and without IHS for $f_w = 0$, $Le=1$ and $N=1$

In figures 12 and 13, we present, respectively, the evolution of the local Nusselt and Sherwood dimensionless numbers depending on the f_w, with and without internal heat source. We observe in figure 12 that the heat transfer rate at the wall increases with the fluid injection and decreases with the fluid suction. On the other hand, the heat transfer rate at the surface appeared important when IHS is absent compared with the case where the IHS is present. Physically, this evolution is normal since the presence of IHS in the medium amplifies the temperature profiles in the boundary layer area, which leads to a decrease of the temperature gradient at the surface. From the figure 13, we see that in the presence of IHS the mass transfer rate to the plate surface increases with fluid injection and decreases with fluid suction, but it is practically constant in the absence of the IHS .

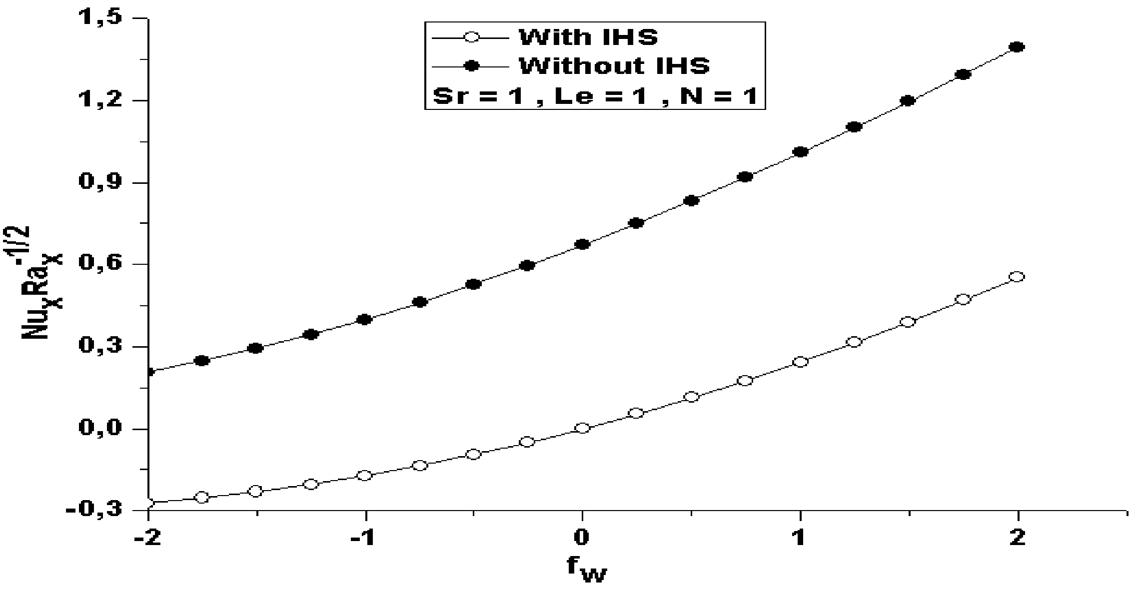

Fig.12. Nu_x versus f_w with and without IHS for $Le=1$ and $N=1$.

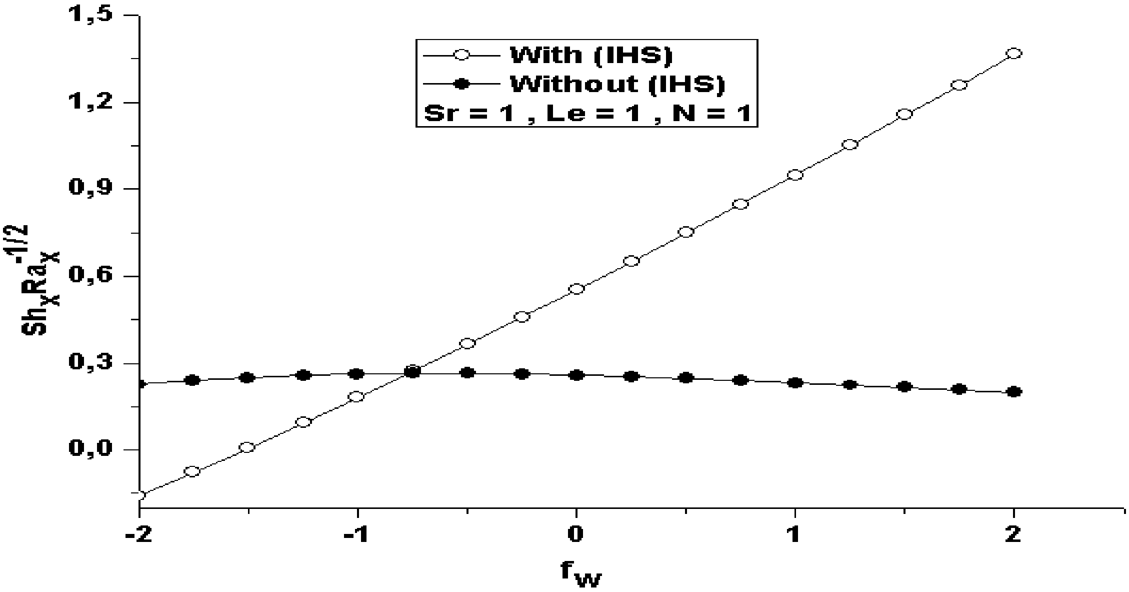

Fig.13. Sh_x versus f_w with and without IHS for $Le=1$ and $N=1$

4. Conclusion

A numerical study has been made of double diffusive free convection near an isothermal vertical plate embedded in a saturated porous medium with lateral mass flux in the presence of Soret effect and an internal heat generation. The effects of some physical parameters on the problem are investigated and presented graphically. The main conclusions of the current analysis are:

- The Soret effect has a slight influence on the temperature profiles, but its influence is more significant on the concentration profiles.
- The high values of the Lewis number stabilizes the concentration boundary layer.
- The dominance of solutal volume forces (N> 1) reduces the thermal and concentration boundary layer area thickness.
- The fluid suction reduces the thermal and concentration profiles in boundary layer area.
- The presence of an internal heat source decreases the heat transfer rate and increases the mass transfer rate at de surface.

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