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Thermal Diffusion and Radiation Effects on Three Dimensional MHD Free Convective Flow with Heat and Mass Transfer through a Porous Medium with Periodic Permeability and Chemical Reaction

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Abstract

The effects of thermal diffusion and radiation parameter on three dimensional (3D) MHD free convective flow of a viscous incompressible fluid through a highly porous medium with periodic permeability in the presence of chemical reaction have been studied. Presence of periodic permeability contributed to three-dimensional flow. In view of the periodic permeability, the solutions of the coupled non-linear equations of the main flow are assumed as the superimposition of a perturbed solutions with a small amplitude of variation on the two-dimensional (2D) flow with a small amplitude of variation which is quite justified . The effects of the pertinent parameters are shown with the help of graphs and tables. Interesting and expected results include the significant contribution of buoyancy effect in enhancing the velocity field whereas Lorentz force, endothermic reaction, periodic permeability reduce it. Rate of mass transfer on the surface increases due to Soret number increases but reverse effect is observed in case of radiation parameter and chemical reaction.

Keywords

MHD flow, chemical reaction, thermal diffusion, heat source, radiation

Glossary

B_0 - Uniform magnetic field
C_p - Specific heat at constant pressure
C^* - Concentration
C_w^* - Concentration at plate
C_∞^* - Concentration of fluid far away from the plate
T^* - Temperature
T_w^* - Temperature at plate
T_∞^* - Temperature of fluid far away from the plate
D - Concentration diffusivity
D_1 - Coefficient of thermal diffusivity
G_r - Grashoff number for heat transfer
G_c - Grashoff number for mass transfer
N_u - Nusselt number
P_r - Prandtl number
S_c - Schmidt number
S_0 - Soret number
R_e - Reynolds number
R - Radiation parameter
K_p - Permeability parameter
k - Thermal conductivity
S_h - Sherwood number
L - Wavelength of permeability distribution
U - Free stream velocity
P_∞^* - Pressure in the free stream
v^* - Constant suction velocity
S - Heat source
K_c - Chemical reaction parameter
q_r - Radiative heat flux in the y-direction
g - Acceleration due to gravity
u^*, v^*, and w^* - Components of velocity along x^*, y^* and z^* direction respectively.

Greek Symbols

τ - Skin friction
β - Volumetric coefficient of thermal expansion
β^* - Volumetric coefficient of mass expansion
ρ - Density of the fluid
σ - Electrical conductivity of the fluid
υ - Kinematic viscosity
μ - Viscosity
θ - Dimensionless temperature
ϕ - Dimensionless concentration

1. Introduction

Study of heat and mass transfer play an important role in various industrial applications. More over, in many transport processes heat and mass transfer takes place simultaneously. As a result of which combined buoyancy effects i.e., thermal buoyancy and mass buoyancy affect the flow in the presence of thermal radiation. Hence, radiative heat and mass transfer play an important role in manufacturing industries for the design of fins, steel rolling, nuclear power plants, gas turbines and various propulsion devices for aircraft, missile, satellites, combustion and furnace design, materials processing, energy utilization, remote sensing for astronomy and space exploration, food processing and cryogenic engineering, as well as numerous agricultural, health and military applications. If the temperature of surrounding fluid is rather high, radiation effects play an important role and this situation does exist in space technology. In such cases, one has to take into account the combined effect of thermal radiation and mass diffusion.

The study of MHD flow with mass and heat transfer in the presence of radiation and diffusion has attracted the attention of a large number of scholars due to diverse applications. In astrophysics and geophysics, it is applied to study the stellar and solar structures, radio propagation through the ionosphere, etc. In engineering we find its applications in MHD pumps, MHD bearings etc. The phenomenon of mass transfer is also very common in the theory of stellar structure and observable effects are defectable on the solar surface. In free convection flow the study of effects of magnetic field play a major rule in liquid metals, electrolytes and ionized gases. In power engineering, the thermal physics of hydromagnetic problems with mass transfer have enormous applications. Radiative flows are encountered in many industrial and environmental processes i.e., heating and cooling chambers, fossil fuel combustion energy processes, evaporation from large open water reservoirs, astrophysical flows, solar power technology and space vehicle re-entry. On the other hand, hydromagnetic free convective flows with heat and mass transfer through porous medium have many important applications such as oil, gas production, geothermal energy, cereal grain storage, in chemical engineering for filtration and purification process, in agriculture engineering to study the underground water resource and porous insulation. In view of these applications, the unsteady magneto hydrodynamics incompressible viscous flows past an infinite vertical plate through porous medium have received much attention.

Free convective flow through a porous medium has attracted the attention of several researchers because of its possible application to several geophysical applications. Hossain and Mohammad (1988) studied the effect of Hall current on hydromagnetic free convection flow near an accelerated porous plate. The effects of magnetic field and mass transfer on the free convective flow through a porous medium with constant suction and heat flux has been studied by Acharya *et al*. (2000). Singh *et al*. (2001) have studied free convection in MHD flow of a rotating viscous liquid in porous medium past a vertical porous plate. Rath *et al*. (2001) have studied the flow and heat transfer of an electrically conducting visco-elastic fluid between two horizontal squeezing/stretching plates. Ece (2005) has studied the free convection flow about a cone under mixed thermal boundary conditions and a magnetic field. Dash *et al*. (2009) have studied the unsteady free convective MHD flow through porous media in a rotating system with fluctuating temperature and concentration. As the importance of chemical reaction in the field of MHD came into society than many researchers started working in this field. MHD flow through a porous medium past a stretched vertical permeable surface in the presence of heat source/sink and a chemical reaction has been studied by Dash *et al*. (2008). Free convection heat and mass transfer from a horizontal cylinder of elliptic cross section in micropolar fluids has been studied by Chang (2006). Afify (2004) has studied the effect of a chemical reaction on the free convective flow and mass transfer of a viscous incompressible and electrically conducting fluid over a stretching surface. Kar et al. (2013) have studied the diffusion thermo effect on free convection and mass transfer MHD flow in a vertical channel in the presence of chemical reaction.

Singh *et al.* (1995) have studied the effect of mass transfer on 3D unsteady forced and free convective flow passed an infinite vertical plate with periodic suction. Three dimensional free convective flow and heat transfer through porous medium with periodic permeability has been studied by Singh and Sharma (2002).

Guria *et al*. (2009) have studied the three dimensional free convective flow in a vertical channel with a porous medium. Three dimensional free convective flow with heat and mass transfer through a porous medium with periodic permeability has been studied by Vershney and Singh (2005). The effects of thermal radiation and mass transfer on MHD flow past a vertical oscillating plate with variable temperature and variable mass diffusion have studied by Vijay Kumar and Varma (2011). Kesavaiah *et al*. (2013) have studied the effects of radiation and free convection currents on unsteady couette flow between two vertical parallel plates with constant heat flux and heat source through porous medium. Three dimensional MHD free convective flow with heat and mass transfer through a porous medium with periodic permeability and chemical reaction have studied by Rath *et al*. (2013).

The main objective of the present study is to analyse the effects of thermal diffusion and radiation on MHD free convective flow of viscous incompressible fluid through a highly porous medium with periodic permeability giving rise to a three-dimensional flow in the presence of chemical reaction.

2. Mathematical formulation of the problem

Considering the flow of a viscous fluid through a highly porous medium bounded by an infinite vertical porous plate with thermal diffusion and chemical reaction. The plate lying vertically on the x^*-z^* plane with x^*-axis is taken along the plate in the vertical upward direction (Fig. 1). The y^*-axis is taken normal to the plate and directed into the fluid flowing laminarly with a uniform free stream velocity U. The permeability of the medium may not be uniform. Therefore, it is assumed to be periodic.

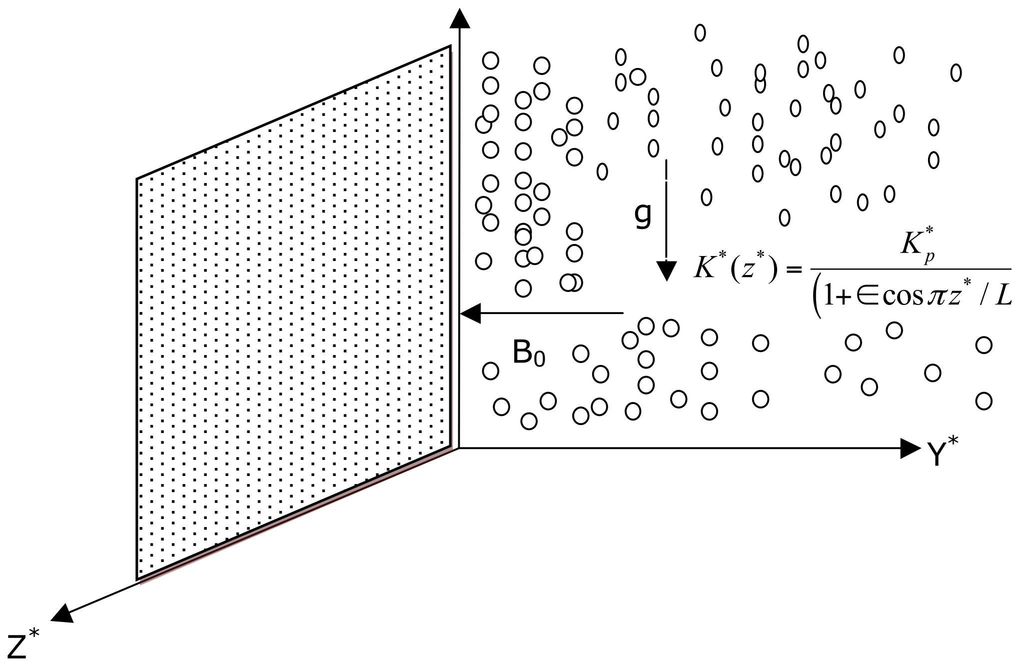


Fig. 1 Schematic diagram of the flow

$$K^*(z^*) = \frac{K_p^*}{\left(1+\in\cos\pi z^*/L\right)} \tag{1}$$

where K_p^* is the mean permeability of the medium. L is the wave length of permeability distribution and $\in(<<1)$ is the amplitude of the permeability variation. The problem becomes three-dimensional due to such a permeability variation. All fluid properties are assumed constant except that the influence of the density variation with temperature and concentration is considered only in the body force term.

Thus, with the above assumptions the velocity components by u^*, v^*, w^* in the directions of x^*, y^*, z^* respectively, the temperature T^* and concentration C^*, are governed by the following equations:

$$\frac{\partial v^*}{\partial y^*} + \frac{\partial w^*}{\partial z^*} = 0, \tag{2}$$

$$v^*\frac{\partial u^*}{\partial y^*} + w^*\frac{\partial u^*}{\partial z^*} = g\beta\left(T^* - T_\infty^*\right) + g\beta^*\left(C^* - C_\infty^*\right) + \upsilon\left(\frac{\partial^2 u^*}{\partial y^{*2}} + \frac{\partial^2 u^*}{\partial z^{*2}}\right)$$

$$-\frac{\upsilon}{K^*}\left(u^* - U\right) - \frac{\sigma B_0^2\left(u^* - U\right)}{\rho} \tag{3}$$

$$v^*\frac{\partial v^*}{\partial y^*} + w^*\frac{\partial v^*}{\partial z^*} = -\frac{1}{\rho}\frac{\partial p^*}{\partial y^*} + \upsilon\left(\frac{\partial^2 v^*}{\partial y^{*2}} + \frac{\partial^2 v^*}{\partial z^{*2}}\right) - \frac{\upsilon v^*}{K^*} \tag{4}$$

$$v^*\frac{\partial w^*}{\partial y^*} + w^*\frac{\partial w^*}{\partial z^*} = -\frac{1}{\rho}\frac{\partial p^*}{\partial z^*} + \upsilon\left(\frac{\partial^2 w^*}{\partial y^{*2}} + \frac{\partial^2 w^*}{\partial z^{*2}}\right) - \frac{\upsilon w^*}{K^*} - \frac{\sigma B_0^2 w^*}{\rho} \tag{5}$$

$$v^*\frac{\partial T^*}{\partial y^*} + w^*\frac{\partial T^*}{\partial z^*} = \frac{k}{\rho C_p}\left(\frac{\partial^2 T^*}{\partial y^{*2}} + \frac{\partial^2 T^*}{\partial z^{*2}}\right) - \frac{1}{\rho C_p}\frac{\partial q_r}{\partial y^*} + S^*(T^* - T_\infty^*), \tag{6}$$

$$v^*\frac{\partial C^*}{\partial y^*} + w^*\frac{\partial C^*}{\partial z^*} = D\left(\frac{\partial^2 C^*}{\partial y^{*2}} + \frac{\partial^2 C^*}{\partial z^{*2}}\right) + D_1\left(\frac{\partial^2 T^*}{\partial y^{*2}}\right) - K_c^*\left(C^* - C_\infty^*\right) \tag{7}$$

where g, $\beta, \beta^*, P^*, K^*, S^*, K_c^*, \rho, \sigma, \upsilon, \mu, k, C_p, D$ and D_1 are acceleration due to gravity, volumetric coefficient of thermal expansion, volumetric coefficient of mass expansion, pressure, permeability parameter, heat source parameter, chemical reaction parameter, density, electric conductivity, kinematic viscosity, viscosity, thermal conductivity, specific heat at constant pressure, concentration diffusity and coefficient of thermal diffusivity respectively.

The corresponding boundary conditions are :

$$y^* = 0;\ u^* = 0,\ v^* = -V,\ w^* = 0,\ T^* = T_w^*,\ C^* = C_w^*$$

$$y^* \to \infty;\ u^* \to U,\ w^* \to 0,\ p^* \to P_\infty^*,\ T^* \to T_\infty^*,\ C^* \to C_\infty^* \tag{8}$$

where T_w^* and C_w^* are the temperature and concentration of the plate, T_∞^* and C_∞^* are the temperature and concentration of the fluid far away from the plate, P_∞^* is a constant pressure in the free stream and V > 0 is a constant and the negative sign indicates that suction is towards the plate.

The local radiant for the case of an optically thin gray gas is expressed by

$$\frac{\partial q_r}{\partial y^*} = -4a^*\sigma\left(T_\infty^{*4} - T^{*4}\right) \tag{9}$$

where σ and a^* are the Stefan-Boltzmann constant and the mean absorption coefficient respectively. Following Vijay Kumar and Varma (2011), we assume that the temperature difference within the flow are sufficiently small and that T^{*4} may be expressed as a linear function of the temperature. This is obtained by expanding T^{*4} in a Taylor series about T_∞^* and neglecting the higher order terms, thus we get

$$T^{*4} \cong 4T_\infty^{*3}T^* - 3T_\infty^{*4} \tag{10}$$

From equation (9) and (10), equation (6) reduces to

$$v^*\frac{\partial T^*}{\partial y^*} + w^*\frac{\partial T^*}{\partial z^*} = \frac{k}{\rho C_p}\left(\frac{\partial^2 T^*}{\partial y^{*2}} + \frac{\partial^2 T^*}{\partial z^{*2}}\right) + \frac{1}{\rho C_p}16a^*\sigma T_\infty^{*3}(T_\infty^* - T^*) + S^*(T^* - T_\infty^*) \tag{11}$$

Introducing the following non-dimensional quantities,

$$y = \frac{y^*}{L}, z = \frac{z^*}{L}, u = \frac{u^*}{U}, v = \frac{v^*}{V}, w = \frac{w^*}{V}$$

$$p = \frac{p^*}{\rho V^2}, \theta = \frac{T^* - T_\infty^*}{T_w^* - T_\infty^*}, \phi = \frac{C^* - C_\infty^*}{C_w^* - C_\infty^*}, S = \frac{S^* L}{V}, K_c = \frac{K_c^* L}{V} \tag{12}$$

Equations (2) to (5), (7) and (11) are reduced to the following forms

$$\frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{13}$$

$$v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}=G_rR_e\theta+G_cR_e\phi+\frac{1}{R_e}\left(\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)$$

$$-\frac{(u-1)(1+\in\cos\pi z)}{R_eK_p}-\frac{M^2(u-1)}{R_e}, \tag{14}$$

$$v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}=-\frac{\partial p}{\partial y}+\frac{1}{R_e}\left(\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right)-\frac{(1+\in\cos\pi z)v}{R_eK_p} \tag{15}$$

$$v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}=-\frac{\partial p}{\partial z}+\frac{1}{R_e}\left(\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right)-\frac{(1+\in\cos\pi z)w}{R_eK_p}-\frac{M^2 w}{R_e} \tag{16}$$

$$v\frac{\partial \theta}{\partial y}+w\frac{\partial \theta}{\partial z}=\frac{1}{R_eP_r}\left(\frac{\partial^2 \theta}{\partial y^2}+\frac{\partial^2 \theta}{\partial z^2}\right)+\left(S-\frac{R}{R_eP_r}\right)\theta \tag{17}$$

$$v\frac{\partial \phi}{\partial y}+w\frac{\partial \phi}{\partial z}=\frac{1}{R_eS_c}\left(\frac{\partial^2 \phi}{\partial y^2}+\frac{\partial^2 \phi}{\partial z^2}\right)+\frac{S_0}{R_e}\frac{\partial^2 \theta}{\partial y^2}-K_c\phi \tag{18}$$

where

$$G_r=\frac{\upsilon g\beta\left(T_w^*-T_\infty^*\right)}{UV^2},\ G_c=\frac{\upsilon g\beta^*\left(C_w^*-C_\infty^*\right)}{UV^2},\ R_e=\frac{VL}{\upsilon},\ P_r=\frac{\mu C_p}{k},$$

$$S_c=\frac{\upsilon}{D},\quad K_p=\frac{K_p^*}{L^2},\quad M=\left(\frac{\sigma}{\mu}\right)^{1/2}B_0L,\quad R=\frac{16a^*L^2\sigma T_\infty^{*3}}{k},\quad S_0=\frac{D_1\left(T_w^*-T_\infty^*\right)}{\upsilon\left(C_w^*-C_\infty^*\right)},$$

represent Grashoff number for heat transfer, Grashoff number for mass transfer, Reynolds number, Prandtl number, Schmidt number, permeability parameter, Hartmann number, Radiation parameter and Soret number respectively.

The corresponding boundary conditions are

$$\left.\begin{aligned}&y=0;\ u=0,\ v=-1,\ w=0,\ \theta=1,\ \phi=1\\&y\to\infty;\ u\to1,\ w\to1,\ p\to p_\infty,\ \theta\to0,\ \phi\to0\end{aligned}\right\} \tag{19}$$

In order to solve we assume the solutions of the following form because the amplitude $\in(<<1)$ of the permeability variation is very small:

$$\begin{aligned}
u(y,z)&=u_0(y)+\in u_1(y,z)+\in^2 u_2(y,z)+\dots\dots\dots\\
v(y,z)&=v_0(y)+\in v_1(y,z)+\in^2 v_2(y,z)+\dots\dots\dots\\
w(y,z)&=w_0(y)+\in w_1(y,z)+\in^2 w_2(y,z)+\dots\dots\dots\\
p(y,z)&=p_0(y)+\in p_1(y,z)+\in^2 p_2(y,z)+\dots\dots\dots\\
\theta(y,z)&=\theta_0(y)+\in\theta_1(y,z)+\in^2 \theta_2(y,z)+\dots\dots\dots\\
\phi(y,z)&=\phi_0(y)+\in\phi_1(y,z)+\in^2 \phi_2(y,z)+\dots\dots\dots
\end{aligned} \tag{20}$$

when $\in = 0$, the problem reduces to two-dimensional free convective flow through a porous medium with constant permeability which is governed by the following equations :

$$\frac{dv_0}{dy} = 0, \tag{21}$$

$$\frac{d^2u_0}{dy^2} - v_0 R_e \frac{du_0}{dy} - \left(M^2 + \frac{1}{K_p}\right)u_0 = -G_r R_e^2 \theta_0 - G_c R_e^2 \phi_0 - \left(M^2 + \frac{1}{K_p}\right), \tag{22}$$

$$\frac{d^2\theta_0}{dy^2} - v_0 R_e P_r \frac{d\theta_0}{dy} + \left(SR_e P_r - R\right)\theta_0 = 0 \tag{23}$$

$$\frac{d^2\phi_0}{dy^2} - v_0 R_e S_c \frac{d\phi_0}{dy} - R_e S_c K_c \phi_0 + S_0 S_c \frac{d^2\theta_0}{dy^2} = 0 \tag{24}$$

The corresponding boundary conditions become

$$y = 0;\ u_0 = 0,\ v_0 = -1,\ \theta_0 = 1,\ \phi_0 = 1,$$

$$y \to \infty;\ u_0 \to 1,\ p_0 \to p_\infty,\ \theta_0 \to 0,\ \phi_0 \to 0 \tag{25}$$

The solutions of these equations (22) to (24)

$$u_0 = 1 + I_2 e^{-R_1 y} + I_3 e^{-A_0 y} - I_4 e^{-A_1 y} \tag{26}$$

$$\theta_0 = e^{-A_0 y} \tag{27}$$

$$\phi_0 = (1 + I_1) e^{-A_1 y} - I_1 e^{-A_0 y} \tag{28}$$

with $v_0 = -1,\ w_0 = 0$ and $p_0 = p_\infty$ (29)

where

$$A_0 = \frac{R_e P_r + \sqrt{R_e^2 P_r^2 - 4\left(SR_e P_r - R\right)}}{2},\ A_1 = \frac{R_e S_c + \sqrt{R_e^2 S_c^2 + 4R_e S_c K_c}}{2}$$

$$R_1 = \frac{R_e}{2} + \sqrt{\frac{R_e^2}{4} + M^2 + \frac{1}{K_p}},\quad I_1 = \frac{S_0 S_c}{A_0^2 - R_e S_c A_0 - R_e S_c K_c}$$

$I_2 = G_r A_2 + G_c (1 + I_1) A_3 - G_c I_1 A_2 - 1$, $I_3 = G_c A_2 I_1 - G_r A_2$, $I_4 = G_c (1 + I_1) A_3$

$$A_2 = \frac{R_e^2}{A_0^2 - A_0 R_e - \left(M^2 + \frac{1}{K_p}\right)},\ A_3 = \frac{R_e^2}{A_1^2 - A_1 R_e - \left(M^2 + \frac{1}{K_p}\right)}$$

When $\in \neq 0$, substituting equation (20) and non-dimensional equation

$$K(z) = \frac{K_p}{(1 + \in \cos \pi z)} \tag{30}$$

periodic permeability into the equations (13) to (18) and comparing the coefficients of identical power of $\in$, neglecting $\in^2$, $\in^3$ etc., we get the following set of equations.

$$\frac{\partial v_1}{\partial y} + \frac{\partial w_1}{\partial z} = 0, \tag{31}$$

$$v_1 \frac{\partial u_0}{\partial y} - \frac{\partial u_1}{\partial y} = G_r R_e \theta_1 + G_c R_e \phi_1 + \frac{1}{R_e}\left(\frac{\partial^2 u_1}{\partial y^2} + \frac{\partial^2 u_1}{\partial z^2}\right)$$

$$- \frac{(u_0 - 1)\cos \pi z + u_1}{R_e K_p} - \frac{M^2 u_1}{R_e}, \tag{32}$$

$$-\frac{\partial v_1}{\partial y} = -\frac{\partial p_1}{\partial y} + \frac{1}{R_e}\left(\frac{\partial^2 v_1}{\partial y^2} + \frac{\partial^2 v_1}{\partial z^2}\right) - \frac{(v_1 - \cos \pi z)}{R_e K_p}, \tag{33}$$

$$-\frac{\partial w_1}{\partial y} = -\frac{\partial p_1}{\partial z} + \frac{1}{R_e}\left(\frac{\partial^2 w_1}{\partial y^2} + \frac{\partial^2 w_1}{\partial z^2}\right) - \frac{w_1}{R_e K_p} - \frac{M^2 w_1}{R_e}, \tag{34}$$

$$v_1 \frac{\partial \theta_0}{\partial y} - \frac{\partial \theta_1}{\partial y} = \frac{1}{R_e P_r}\left(\frac{\partial^2 \theta_1}{\partial y^2} + \frac{\partial^2 \theta_1}{\partial z^2}\right) + \left(S - \frac{R}{P_r R_e}\right)\theta_1, \tag{35}$$

$$v_1 \frac{\partial \phi_0}{\partial y} - \frac{\partial \phi_1}{\partial y} = \frac{1}{R_e S_c}\left(\frac{\partial^2 \phi_1}{\partial y^2} + \frac{\partial^2 \phi_1}{\partial z^2}\right) - K_c \phi_1 + \frac{S_0}{R_e}\frac{\partial^2 \theta_1}{\partial y^2} \quad , \tag{36}$$

The corresponding boundary conditions are

$$y = 0,\ u_1 = 0,\ v_1 = 0,\ w_1 = 0,\ \theta_1 = 0,\ \phi_1 = 0$$

$$y \to \infty,\ u_1 \to 0,\ w_1 \to 0,\ p_1 \to 0,\ \theta_1 \to 0,\ \phi_1 \to 0 \tag{37}$$

Equations (31) to (36) are the linear partial differential equations which describe free convective three-dimensional flow.

For solution, we first consider (31), (33) and (34) being independent of the main flow and the temperature field we assume v_1, w_1 and p_1 in the following forms:

$$v_1(y, z) = v_{11}(y) \cos \pi z, \tag{38}$$

$$w_1(y, z) = -\frac{1}{\pi} v_{11}'(y) \sin \pi z, \tag{39}$$

$$p_1(y, z) = p_{11}(y) \cos \pi z, \tag{40}$$

where the prime in $v'_{11}(y)$ denotes the differentiation with respect to y. Expression for $v_1(y, z)$ and $w_1(y, z)$ have been chosen so that the equation of continuity (31) is satisfied. Substituting the expressions (38) - (40) into (33) - (34) we get

$$v''_{11} + R_e v'_{11} - \left(\pi^2 + \frac{1}{K_p}\right) v_{11} = R_e p'_{11} - \frac{1}{K_p} \tag{41}$$

$$v'''_{11} + R_e v''_{11} - \left(\pi^2 + M^2 + \frac{1}{K_p}\right) v'_{11} = R_e \pi^2 p_{11} \tag{42}$$

with the boundary conditions

$$\left.\begin{array}{l} y = 0;\ v_{11} = 0,\ v'_{11} = 0 \\ y \to \infty;\ v_{11} = 0,\ v'_{11} = 0,\ p_{11} = 0 \end{array}\right\} \tag{43}$$

On solving equations (41) and (42) under the boundary conditions (43), we get

$$v_1(y, z) = \frac{1}{(\pi - R_2)(\pi^2 K_p + 1)}\left(R_2 e^{-\pi y} - \pi e^{-R_2 y} + \pi - R_2\right)\cos \pi z \tag{44}$$

$$w_1(y, z) = \frac{R_2}{(\pi - R_2)(\pi^2 K_p + 1)}\left(e^{-\pi y} - e^{-R_2 y}\right)\sin \pi z \tag{45}$$

$$p_1(y, z) = \frac{R_2\left(R_e \pi + \frac{1}{K_p}\right)}{R_e \pi (\pi - R_2)(\pi^2 K_p + 1)} e^{-\pi y} \cos \pi z \tag{46}$$

where $R_2 = \frac{R_e}{2} + \sqrt{\frac{R_e^2}{4} + \pi^2 + \frac{1}{K_p}}$

For the main flow, solutions of u_1, θ_1 and ϕ_1 are assumed as

$$u_1(y, z) = u_{11}(y) \cos \pi z \tag{47}$$

$$\theta_1(y, z) = \theta_{11}(y) \cos \pi z \tag{48}$$

$$\phi_1(y, z) = \phi_{11}(y) \cos \pi z \tag{49}$$

Substitution of equations (47), (48) and (49) into equations (32), (35) and (36) gives

$$u''_{11} + R_e u'_{11} - \left(\pi^2 + M^2 + \frac{1}{K_p}\right) u_{11} = R_e v_{11} u'_0 - G_r R_e^2 \theta_{11} - G_c R_e^2 \phi_{11} + \frac{u_0 - 1}{K_p} \tag{50}$$

$$\theta''_{11} + R_e P_r \theta'_{11} + \left(S R_e P_r - \pi^2 - R\right)\theta_{11} = v_{11} \theta'_0 R_e P_r \tag{51}$$

$$\phi_{11}'' + R_e S_c \phi_{11}' - \left(R_e S_c K_c + \pi^2\right)\phi_{11} = R_e S_c v_{11}\phi_0' - S_0 S_c \theta_{11}'' \tag{52}$$

and the corresponding boundary conditions are

$$\left.\begin{aligned} &y = 0,\ u_{11} = 0,\ \theta_{11} = 0,\ \phi_{11} = 0 \\ &y \to \infty,\ u_{11} \to 0,\ \theta_{11} \to 0,\ \phi_{11} \to 0 \end{aligned}\right\} \tag{53}$$

Solving equations (50) to (52) with boundary conditions (53) and using equation (47) to (49) we get

$$u_1(y,z) = \Bigg[\frac{R_e}{\left(\pi - R_2\right)\left(\pi^2 K_p + 1\right)}\Big\{I_{49}e^{-R_5 y} - I_{21}e^{-(\pi + R_1)y}$$

$$+ I_{22}e^{-(R_2 + R_1)y} - I_{23}e^{-R_1 y} - I_{24}e^{-(\pi + A_0)y} + I_{25}e^{-(R_2 + A_0)y}$$

$$- I_{26}e^{-A_0 y} + I_{27}e^{-(\pi + A_1)y} - I_{28}e^{-(R_2 + A_1)y} + I_{29}e^{-A_1 y}\Big\}$$

$$+ \frac{G_r R_e^3 P_r}{\left(\pi - R_2\right)\left(\pi^2 K_p + 1\right)}\Big\{I_{50}e^{-R_5 y} - I_{30}e^{-R_3 y} - I_{31}e^{-(R_2 + A_0)y} + I_{32}e^{-(\pi + A_0)y} + I_{33}e^{-A_0 y}\Big\}$$

$$+ \frac{G_c R_e^3 S_c}{\left(\pi - R_2\right)\left(\pi^2 K_p + 1\right)}\Big\{I_{51}e^{-R_5 y} - I_{34}e^{-R_4 y} + I_{35}e^{-(\pi + A_1)y} - I_{36}e^{-(R_2 + A_1)y} + I_{37}e^{-A_1 y}$$

$$- I_{40}e^{-A_0 y} - I_{38}e^{-(\pi + A_0)y} + I_{39}e^{-(R_2 + A_1)y}\Big\}$$

$$+ \frac{G_c R_e^3 S_c S_0 P_r}{\left(\pi - R_2\right)\left(\pi^2 K_p + 1\right)}\Big\{I_{52}e^{-R_5 y} + I_{41}e^{-R_4 y} - I_{42}e^{-R_3 y} - I_{43}e^{-(R_2 + A_0)y} + I_{44}e^{-(\pi + A_0)y} + I_{45}e^{-A_0 y}\Big\}$$

$$+ \left(I_{53}e^{-R_5 y} + I_{46}e^{-R_1 y} + I_{47}e^{-A_0 y} - I_{48}e^{-A_1 y}\right)\Bigg]\cos \pi z \tag{54}$$

$$\theta_1(y,z) = \frac{R_e P_r}{\left(\pi - R_2\right)\left(\pi^2 K_p + 1\right)}\left[I_5 e^{-R_3 y} + I_6 e^{-(R_2 + A_0)y} - I_7 e^{-(\pi + A_0)y} - I_8 e^{-A_0 y}\right]\cos \pi z \tag{55}$$

$$\phi_1\left(y,z\right) = \frac{R_e S_c}{\left(\pi - R_2\right)\left(\pi^2 K_p + 1\right)}\Big[\Big\{I_9 e^{-R_4 y} - I_{11}e^{-(\pi + A_1)y} + I_{12}e^{-(R_2 + A_1)y} - I_{13}e^{-A_1 y}$$

$$+ I_{14}e^{-(\pi + A_0)y} - I_{15}e^{-(R_2 + A_0)y} + I_{16}e^{-A_0 y}\Big\}$$

$$+ S_0 P_r\Big\{I_{10}e^{-R_4 y} - I_{17}e^{-R_3 y} - I_{18}e^{-(R_2 + A_0)y} + I_{19}e^{-(\pi + A_0)y} + I_{20}e^{-A_0 y}\Big\}\Big]\cos \pi z \tag{56}$$

where R_2 to R_5, A_4 to A_{16} and I_5 to I_{53} are given in the appendix.

Skin friction

The expression for the skin friction in the x^*- direction in the non-dimensional form is given by

$$\tau = \frac{\tau^*}{\rho UV} = \frac{\upsilon}{VL}\left(\frac{\partial u}{\partial y}\right)_{y=0} = \frac{1}{R_e}\left[\frac{du_0}{dy} + \in \frac{du_{11}}{dy}\cos\pi z\right]_{y=0}$$

$$= \frac{1}{R_e}\Big[\ \left(I_4 A_1 - R_1 I_2 - I_3 A_0\right)$$

$$+ \in \left\{ \frac{R_e}{(\pi - R_2)(\pi^2 K_p + 1)}\big((\pi + R_1) I_{21} - R_5 I_{49} - (R_2 + R_1) I_{22} + R_1 I_{23} + (\pi + A_0) I_{24}\right.$$

$$- (R_2 + A_0) I_{25} + A_0 I_{26} - I_{27}(p + A_1) + (R_2 + A_1) I_{28} - A_1 I_{29}\big)$$

$$+ \frac{G_r R_e^3 P_r}{(\pi - R_2)(\pi^2 K_p + 1)}(R_3 I_{30} - R_5 I_{50} + (R_2 + A_0) I_{31} - (\pi + A_0) I_{32} - A_0 I_{33})$$

$$+ \frac{G_c R_e^3 S_c}{(\pi - R_2)(\pi^2 K_p + 1)}(-R_5 I_{51} + R_4 I_{34} - (\pi + A_1) I_{35} + A_0 I_{40} + (\pi + A_0) I_{38}$$

$$- (R_2 + A_0) I_{39} + I_{36}(R_2 + A_1) - A_1 I_{37})$$

$$- \frac{G_c R_e^3 S_c S_0 P_r}{(\pi - R_2)(\pi^2 K_p + 1)}(- R_5 I_{52} - R_4 I_{41} + R_3 I_{42} + (R_2 + A_0) I_{43} - (\pi + A_0) I_{44} - I_{45} A_0)$$

$$\left. + (- R_5 I_{53} - R_1 I_{46} - A_0 I_{47} + A_1 I_{48}) \right\} \cos \pi z \Big]$$

Nusselt Number

The expression for the rate of heat transfer in terms of Nusselt number is given by

$$\mathrm{Nu} = \frac{-q^*}{\rho V C_p \left(T_w^* - T_\infty^*\right)} = \frac{k}{\rho V C_p L}\left(\frac{\partial \theta}{\partial y}\right)_{y=0} = \frac{1}{R_e P_r}\left[\frac{\partial \theta_0}{\partial y} + \in \frac{\partial \theta_{11}}{dy}\cos\pi z\right]_{y=0}$$

$$= -\frac{A_0}{R_e P_r} + \frac{\in}{(\pi - R_2)(\pi^2 K_p + 1)}\{ -R_3 I_5 - (R_2 + A_0) I_6 + (\pi + A_0) I_7 + A_0 I_8\} \cos\pi z$$

Sherwood Number

The expression for the rate of mass transfer in terms of Sherwood number is given by

$$Sh = \frac{-q_1^* D}{V(C_w^* - C_\infty^*)} = \frac{D}{VL}\left(\frac{\partial \phi}{\partial y}\right)_{y=0} = \frac{1}{R_e S_c}\left[\frac{d\phi_0}{dy} + \in \frac{d\phi_{11}}{dy}\cos \pi z\right]_{y=0}$$

$$= \frac{\left\{-A_1\left(1+I_1\right)+A_0 I_1\right\}}{R_e S_c} + \frac{\in}{\left(\pi - R_2\right)\left(\pi^2 K_p + 1\right)}[\{-R_4 I_9 + (\pi + A_1) I_{11} - (R_2 + A_1)$$

$$I_{12} + A_1 I_{13} - (\pi + A_0) I_{14} + (R_2 + A_0) I_{15} - A_0 I_{16}\} + S_0 P_r \{-R_4 I_{10} + R_3 I_{17}$$

$$+ (R_2 + A_0) I_{18} - (\pi + A_0) I_{19} - A_0 I_{20}\}] \cos\pi z$$

3. Results and discussion

In order to get the physical insight into the problem, the effects of various flow, heat and mass transfer parameters in a free convection flow has been discussed. The velocity, temperature and concentration profiles are shown in Figs. 2(a), 2(b), 3 and 4. Moreover, values of the skin friction, Nusselt number and Sherwood number are presented with the help of tables 1, 2 and 3.

In order to make the result and discussion meaningful the computation has been carried out in the presence of diffusing chemically reacting species such as hydrogen, helium, ammonia etc. in the air/water medium on a cooled surface (G_r>0). The main objective of the discussion is to bring out the effect of radiation and thermal diffusion on the flow characteristics in the presence of periodic permeability, heat source and chemical reaction.

It is interesting to observe from Fig.2(a) that sudden rise in the velocity is marked in the layers near the plate, then the velocity profile attains a constant value in the free stream layer. Further, it is seen that the velocity profiles attains its maximum when G_c = 10. Thus, it maybe concluded that due to buoyancy effect caused by the concentration difference which contributes to the maximum rise of the velocity profile. Again, it is observed that for the heavier species with S_c = 0.6 (water vapour) in the presence of exothermic chemical reaction (K_c > 0), the velocity profile attains the minimum (curve VII) at all the points of the flow domain.

It is also revealed that increase in magnetic parameter (M), Schemidt number (S_c), Prandtl number (P_r), chemical reaction (K_c) lead to decrease the velocity at all points but increase in buoyancy parameters (G_r, G_c), permeability parameter (K_p), enhance the velocity at all points.

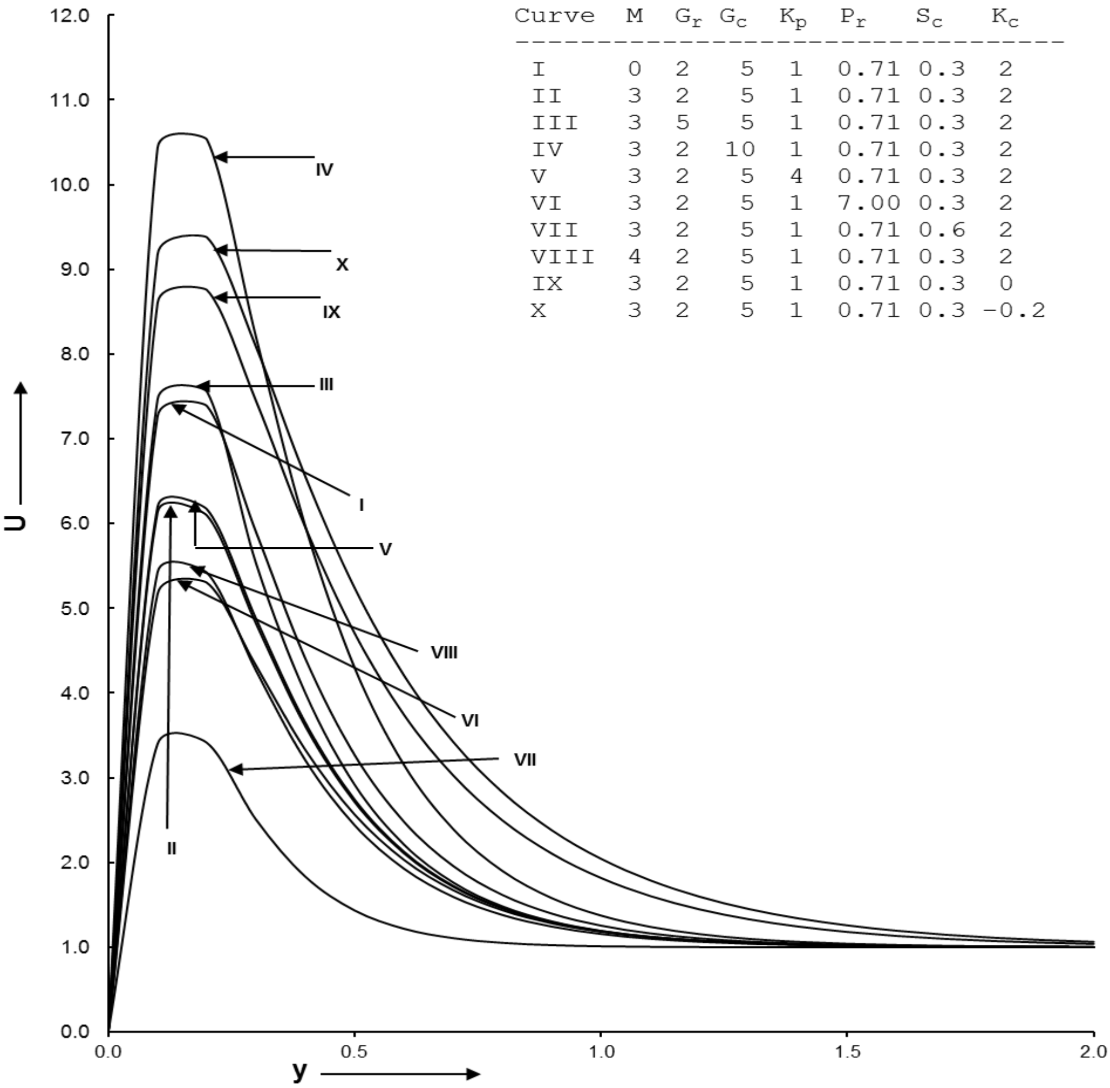
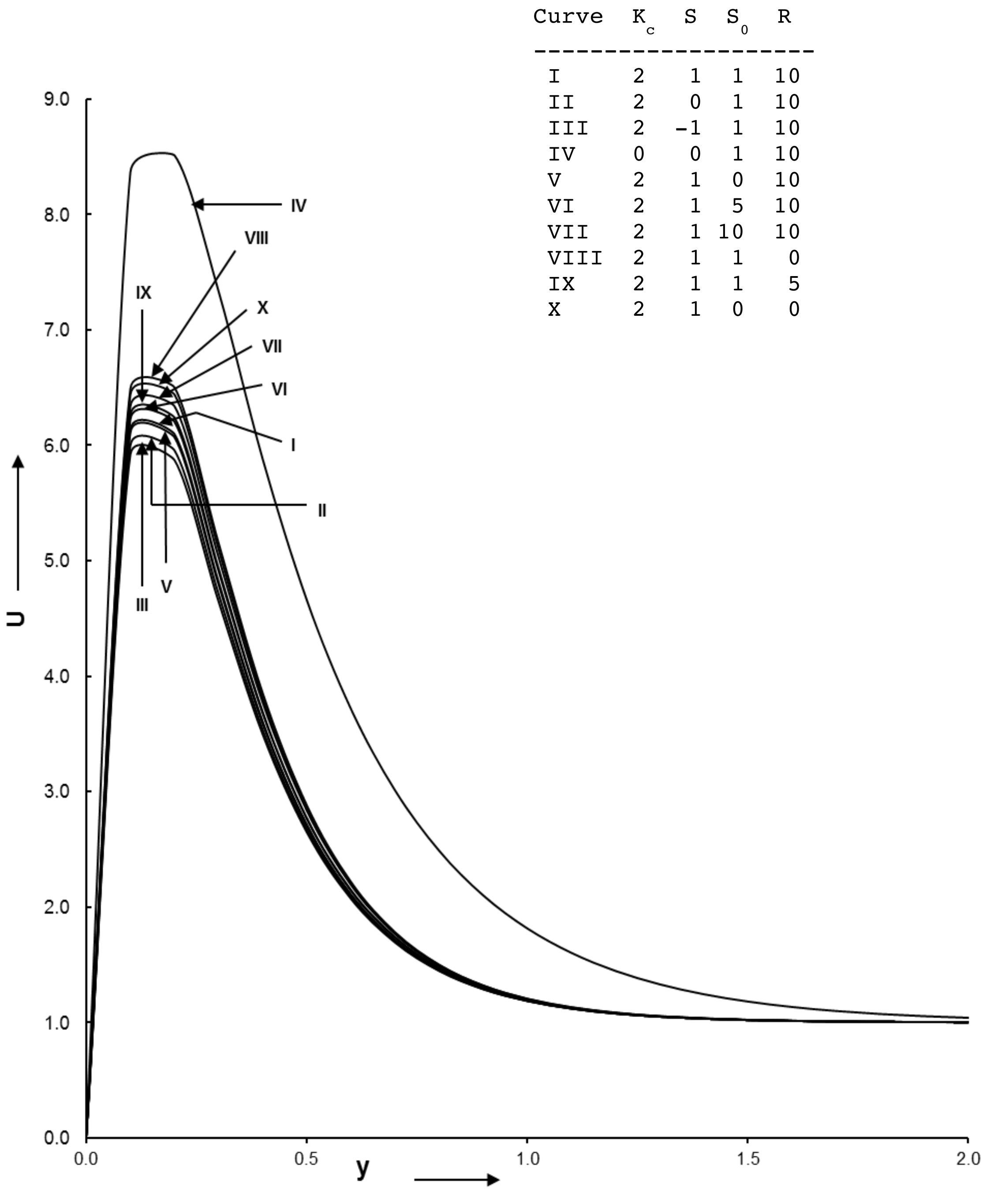


Fig- 2(a) Effect of M, G_r, G_c, Kp, P_r, S_c and K_c on Velocity profile when S = 1, S_0 = 1, R = 10, R_e = 10, $\in$ = 0.1, z = 0

In the absence of magnetic field (M = 0), the velocity increases at all points. Thus, it may be concluded that Lorentz force ($M \neq 0$) opposes the motion of the fluid particle. The similar observation is also made in the absence of chemical reaction ($K_c = 0$) as well as in the presence of endothermic reaction ($K_c < 0$). Further, it is interesting to note that presence of porous matrix reduces the flow when $K_p = 1$ (Curve II).

From Fig.2(b) it is evident that an increase in radiation parameter (R) leads to decrease the velocity at all points but increase in heat source parameter (S) and Soret number (S_0) enhance the velocity at all points. It is observed that velocity increases for S = 0 and S>0. Thus, the presence of sink (S<0) absorbs the heat energy, therby decreasing the velocity.



Curve	K_c	S	S_0	R
I	2	1	1	10
II	2	0	1	10
III	2	-1	1	10
IV	0	0	1	10
V	2	1	0	10
VI	2	1	5	10
VII	2	1	10	10
VIII	2	1	1	0
IX	2	1	1	5
X	2	1	0	0

Fig- 2(b) Effect of K_c, S, S_0 and R on Velocity profile when M = 3, $G_r = 2$, $G_c = 5$, $K_p = 1$, $P_r = 0.71$, $S_c = 0.3$, $R_e = 10$, $\in = 0.1$, z = 0

Fig. 3 exhibits the temperature profiles for different values of Prandtl number (P_r) and Radiation parameter (R). Sudden fall is marked in case of water (P_r = 7.0) and gradual fall in case of air. An increase in P_r and R leads to decrease the temperature at all points.

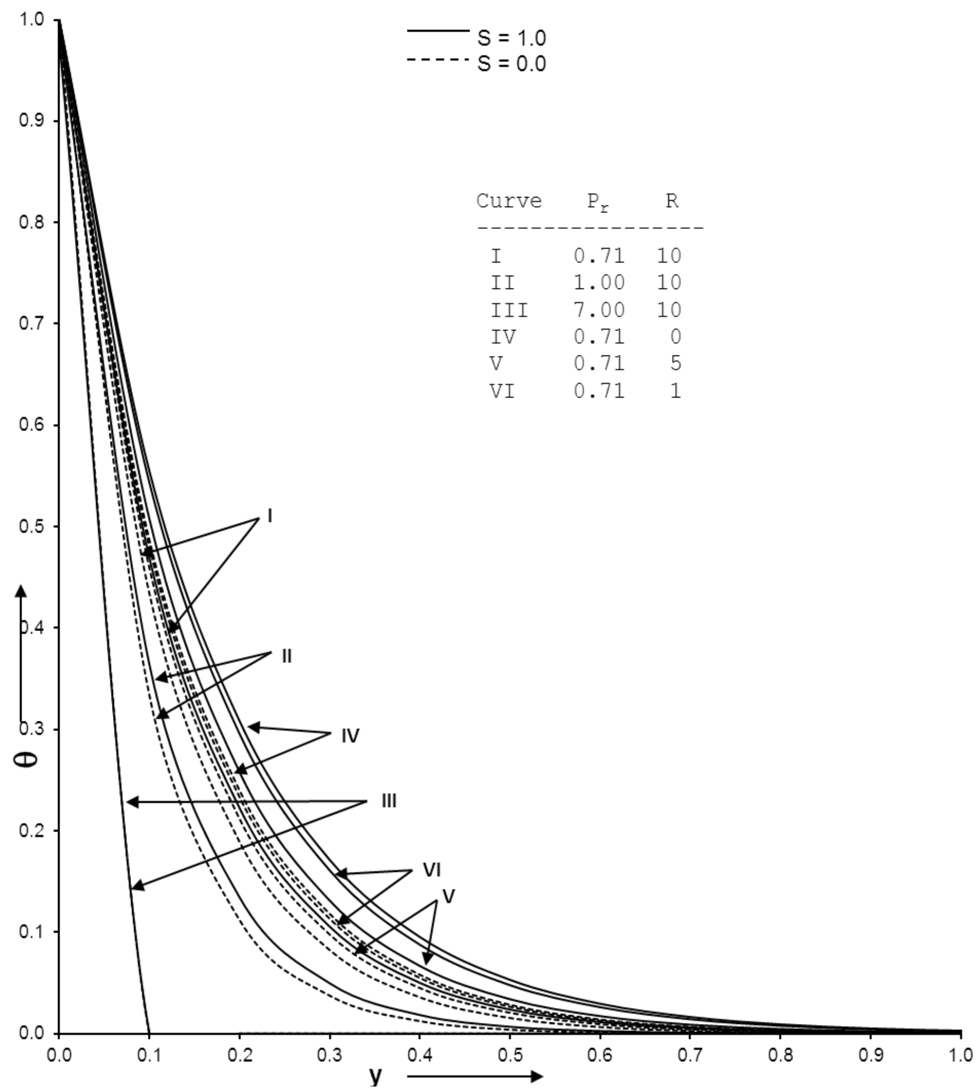


Fig.3. Effect of P_r and R on temperature profile when $R_e = 10$, $K_p = 1$, $G_r = 2$, $G_c = 5$, $K_c = 2$, $\in = 0.1$, $z = 0$

Further, it is noted that presence of heat source ($S \neq 0$), enhances the temperature.

Fig. 4 shows that the decrease in concentration is marked due to rise in K_c and S_c i.e., for heavier species with high rate of chemical reaction the concentration falls rapidly at all the layer. Computation is carried out for hydrogen (S_c =0.22), Helium (S_c =0.30), water vapour (S_c = 0.60) and ammonia (S_c = 0.78) as diffusing species in air. Thus the fall of concentration is marked due to chemical reaction and presence of heavier species with low diffusibility.

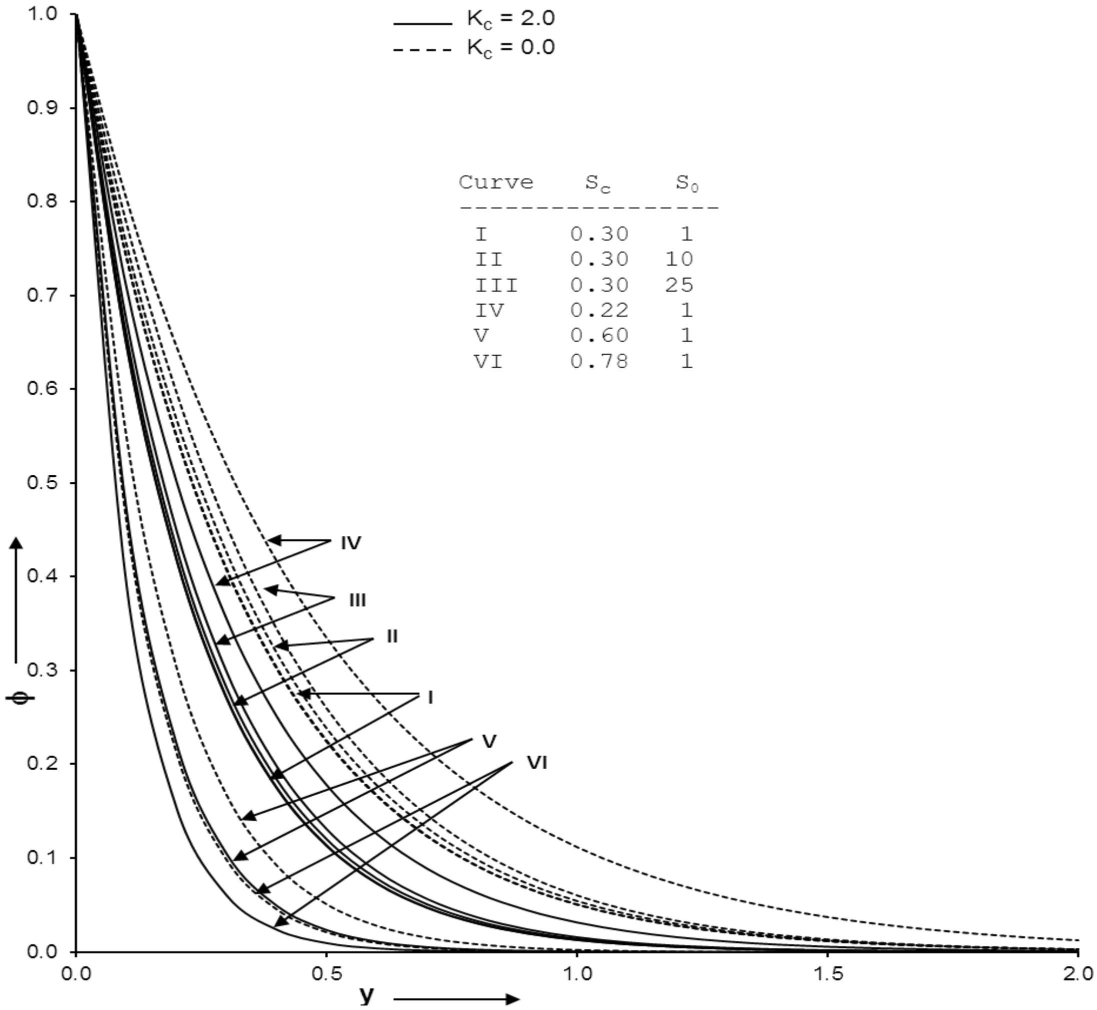


Fig.4. Effect of S_c and S_0 on concentration profile when $K_p = 1$, $G_c = 5$, $G_r = 2$, $\in = 0.1$, $z = 0$, $R_e = 10$, $R = 10$ and $S = 1$

It is also revealed that increase in the Soret number lead to increase the concentration at all points.

Table 1 shows that the skin friction increases due to higher values of heat source parameters and other parameters such as Grashoff number, modified Grashoff number, Soret number and permeability parameter. Chemical reaction, Schmidt number, reduce the skin friction. Hence the buoyancy effect is responsible to enhance the shearing stress at the plate so that fluid particle experience greater resistance due to heat and mass diffusion and in all other cases, opposite effect is observed. It is to noted that all the entries of skin friction are positive.

Table - 1 Value of the Skin friction (τ) are entered for different values of Gr, Gc, M, Kp, Kc, S, S0, R and Sc when Pr = 0.71, $\in$ = 0.1, z = 0, Re = 10

M	**G_r**	**G_c**	**K_p**	**S_c**	**K_c**	**S**	**S_0**	**R**	**τ**
0.0	2.0	5.0	1.0	0.30	0.0	1.0	1.0	10.0	19.920440
3.0	2.0	5.0	1.0	0.30	0.0	1.0	1.0	10.0	16.331250
3.0	5.0	5.0	1.0	0.30	0.0	1.0	1.0	10.0	19.914040
3.0	2.0	10.0	1.0	0.30	0.0	1.0	1.0	10.0	29.192680
4.0	2.0	5.0	1.0	0.30	0.0	1.0	1.0	10.0	14.606140
3.0	2.0	5.0	4.0	0.30	0.0	1.0	1.0	10.0	16.529010
3.0	2.0	5.0	1.0	0.30	2.0	1.0	1.0	10.0	12.989980
3.0	2.0	5.0	1.0	0.30	2.0	0.0	1.0	10.0	12.739150
3.0	2.0	5.0	1.0	0.22	2.0	1.0	1.0	10.0	14.932860
3.0	2.0	5.0	1.0	0.60	2.0	1.0	1.0	10.0	8.955756
3.0	2.0	5.0	1.0	0.78	2.0	1.0	1.0	10.0	7.911846
3.0	2.0	5.0	1.0	0.30	2.0	-1.0	1.0	10.0	12.616140
3.0	2.0	5.0	1.0	0.30	2.0	1.0	0.0	10.0	12.949710
3.0	2.0	5.0	1.0	0.30	2.0	1.0	10.0	10.0	13.326640
3.0	2.0	5.0	1.0	0.30	2.0	1.0	1.0	0.0	13.726600
3.0	2.0	5.0	1.0	0.30	2.0	1.0	1.0	5.0	13.210390
3.0	2.0	5.0	1.0	0.30	2.0	1.0	0.0	0.0	13.686460

In case of Nusselt number all the entries are negative (Table 2). It is observed that for higher values of heat source, Nusselt number (N_u) increases but fluid with higher Prandtl number (P_r) in the presence of sink (S < 0) and source (S > 0) , the Nusselt number (N_u) increases. In case of Radiation parameter (R) the reverse effect is observed. The negative value of N_u indicates that rate of heat transfer decrease in such cases.

Table - 2 Nusselt number (Nu) at z = 0, Kp = 1, Re = 10, $\in$ = 0.1

R	**P_r \ S**	**-1.0**	**0.0**	**1.0**
10.00	0.71	-1.266099	-1.170690	-1.045222
10.00	7.00	-1.017682	-1.003552	-0.907843
0.00	0.71	-1.125678	-0.990167	-0.830343
5.00	0.71	-1.187806	-1.090931	-0.955553

The rate of mass transfer at the surface increases as the Soret number increases but the reverse effect is observed in case of radiation parameter and chemical reaction (Table 3). Further for heavier species, Sherwood number decreases in the presence of endothermic reaction and for exothermic reaction the reverse effect is observed.

Table - 3 Sherwood number (S_h) at z = 0, Kp = 1, Re = 10, $\in$ = 0.1, P_r = 0.71

S_c	K_c	S_0	R	S	S_h
0.30	-0.2	0.0	0.0	0.0	-0.928020
0.30	0.0	0.0	0.0	0.0	-0.998527
0.30	2.0	0.0	0.0	0.0	-1.486124
0.30	2.0	1.0	0.0	0.0	-1.449747
0.30	2.0	1.0	10.0	0.0	-1.445250
0.30	2.0	1.0	10.0	1.0	-1.449935
0.30	2.0	10.0	10.0	1.0	-1.391921
0.30	2.0	1.0	10.0	-1.0	-1.447711
0.22	2.0	1.0	10.0	1.0	-1.568924
0.60	2.0	1.0	10.0	1.0	-1.256831
0.78	2.0	1.0	10.0	1.0	-1.204932
0.30	2.0	1.0	5.0	1.0	-1.444629
0.30	0.0	1.0	10.0	1.0	-0.990421
0.30	-0.2	1.0	10.0	1.0	-0.918182
0.22	0.0	1.0	10.0	1.0	-0.990409
0.22	-0.2	1.0	10.0	1.0	-0.888872
0.60	0.0	1.0	10.0	1.0	-0.990924
0.60	-0.2	1.0	10.0	1.0	-0.956016
0.78	0.0	1.0	10.0	1.0	-0.991265
0.78	-0.2	1.0	10.0	1.0	-0.964583

4. Conclusion

1. The Lorentz force and endothermic reaction oppose the fluid motion producing a thinner boundary layer.
2. The presence of porous matrix with periodic permeability reduces the flow and it is further reduced for higher values of S_c i.e., for heavier species.

   As bounding surface contribute substantially to flow stability within boundary layer it is appropriate to highlight the followings.

3. Rate of mass transfer at the surface increases as the Soret number increases and in the presence of endothermic reaction ($K_c < 0$) but the reverse effect is observed in case of radiation parameter and exothermic reaction ($K_c > 0$).

4. Rate of heat transfer at the surface decreases due to presence of sink ($S< 0$) and increases with source ($S > 0$) as well as for low conductive fluid.

5. The reduction of shearing stress at the plate (skin friction) is desirable as it contribute to flow stability. It is interesting to note in the mathematical model exhibiting the flow characteristics that the skin friction reduces in the presence of magnetic field, radiative surface and heavier species initiating destructive reaction whereas it increases in case of other parameters such as buoyancy forces, resistance due to porous matrix, Soret number and heat source.

Therefore, it is suggested that implementation of the theoretical results in practice for reduction of shearing stress at the radiative surface needs right choice of fluid exhibiting the above said properties withdrawing the porous matrix and heat source in a controlled condition avoiding the Soret effect.

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6. Appendix

$$R_2 = \frac{R_e}{2} + \sqrt{\frac{R_e^2}{4} + \pi^2 + \frac{1}{K_p}}, \quad R_3 = \frac{R_e P_r}{2} + \sqrt{\frac{R_e^2 P_r^2}{4} - SR_e P_r + R + \pi^2}$$

$$R_4 = \frac{R_e S_c}{2} + \sqrt{\frac{R_e^2 S_c^2}{4} + \left(\pi^2 + K_c R_e S_c\right)}, \quad R_5 = \frac{R_e}{2} + \sqrt{\frac{R_e^2}{4} + \left(\pi^2 + M^2 + \frac{1}{K_p}\right)}$$

$$A_4 = \frac{R_2}{(\pi + A_0)^2 - R_e P_r(\pi + A_0) + \left(SR_e P_r - \pi^2 - R\right)}, \quad A_5 = \frac{\pi - R_2}{A_0^2 - R_e P_r A_0 + \left(SR_e P_r - \pi^2 - R\right)}$$

$$A_6 = \frac{\pi}{(R_2 + A_0)^2 - R_e P_r(R_2 + A_0) + \left(SR_e P_r - \pi^2 - R\right)},$$

$$A_7 = \frac{R_2}{(\pi + A_1)^2 - R_e S_c(\pi + A_1) - \left(\pi^2 + K_c R_e S_c\right)}, \quad A_8 = \frac{\pi - R_2}{A_1^2 - R_e S_c A_1 - \left(\pi^2 + K_c R_e S_c\right)}$$

$$A_9 = \frac{\pi}{(R_2 + A_1)^2 - R_e S_c(R_2 + A_1) - \left(\pi^2 + K_c R_e S_c\right)}, \quad A_{10} = \frac{R_2}{(\pi + A_0)^2 - R_e S_c(\pi + A_0) - \left(\pi^2 + K_c R_e S_c\right)},$$

$$A_{11} = \frac{\pi}{(R_2 + A_0)^2 - R_e S_c(R_2 + A_0) - \left(\pi^2 + K_c R_e S_c\right)}, \quad A_{12} = \frac{\pi - R_2}{A_0^2 - R_e S_c A_0 - \left(\pi^2 + K_c R_e S_c\right)},$$

$$A_{13} = \frac{R_3^2}{R_3^2 - R_e S_c R_3 - \left(\pi^2 + K_c R_e S_c\right)}, \quad A_{14} = \frac{(R_2 + A_0)^2}{(R_2 + A_0)^2 - R_e S_c(R_2 + A_0) - \left(\pi^2 + K_c R_e S_c\right)},$$

$$A_{15} = \frac{(\pi + A_0)^2}{(\pi + A_0)^2 - R_e S_c(\pi + A_0) - \left(\pi^2 + K_c R_e S_c\right)}, \quad A_{16} = \frac{A_0^2}{A_0^2 - R_e S_c A_0 - \left(\pi^2 + K_c R_e S_c\right)},$$

$I_5 = A_0 (A_4 + A_5 - A_6)$, $I_6 = A_0 A_6$, $I_7 = A_0 A_4$, $I_8 = A_0 A_5$

$I_9 = A_1 (1 + I_1) (A_7 - A_9 + A_8) - A_0 I_1 (A_{10} - A_{11} + A_{12})$, $I_{10} = I_5 A_{13} + I_6 A_{14} - I_7 A_{15} - I_8 A_{16}$

$I_{11} = A_1 A_7 (1 + I_1)$, $I_{12} = A_1 A_9 (1 + I_1)$, $I_{13} = A_1 A_8 (1 + I_1)$, $I_{14} = A_0 A_{10} I_1$, $I_{15} = A_0 A_{11} I_1$

$I_{16} = A_0 A_{12} I_1$, $I_{17} = I_5 A_{13}$, $I_{18} = I_6 A_{14}$, $I_{19} = I_7 A_{15}$, $I_{20} = I_8 A_{16}$

$$I_{21} = \frac{I_2 R_1 R_2}{(\pi + R_1)^2 - R_e(\pi + R_1) - \left(\pi^2 + M^2 + \frac{1}{K_p}\right)},$$

$$I_{22} = \frac{I_2 R_1 \pi}{(R_2 + R_1)^2 - R_e(R_2 + R_1) - \left(\pi^2 + M^2 + \frac{1}{K_p}\right)}, \quad I_{23} = \frac{I_2 R_1 (\pi - R_2)}{R_1^2 - R_e R_1 - \left(\pi^2 + M^2 + \frac{1}{K_p}\right)}$$

$$I_{24} = \frac{I_3 A_0 R_2}{(\pi + A_0)^2 - R_e(\pi + A_0) - \left(\pi^2 + M^2 + \frac{1}{K_p}\right)},$$

$$I_{25} = \frac{I_3 A_0 \pi}{(R_2 + A_0)^2 - R_e(R_2 + A_0) - \left(\pi^2 + M^2 + \frac{1}{K_p}\right)}, \quad I_{26} = \frac{(\pi - R_2) I_3 A_0}{A_0^2 - R_e A_0 - \left(\pi^2 + M^2 + \frac{1}{K_p}\right)}$$

$$I_{27} = \frac{I_4 A_1 R_2}{(\pi + A_1)^2 - R_e(\pi + A_1) - \left(\pi^2 + M^2 + \frac{1}{K_p}\right)},$$

$$I_{28} = \frac{I_4 A_1 \pi}{(R_2 + A_1)^2 - R_e(R_2 + A_1) - \left(\pi^2 + M^2 + \frac{1}{K_p}\right)}, \quad I_{29} = \frac{I_4 A_1 (\pi - R_2)}{R_1^2 - R_e R_1 - \left(\pi^2 + M^2 + \frac{1}{K_p}\right)}$$

$$I_{30} = \frac{I_5}{R_3^2 - R_e R_3 - \left(\pi^2 + M^2 + \frac{1}{K_p}\right)}, \quad I_{31} = \frac{I_6}{(R_2 + A_0)^2 - R_e(R_2 + A_0) - \left(\pi^2 + M^2 + \frac{1}{K_p}\right)}$$

$$I_{32} = \frac{I_7}{(\pi + A_0)^2 - R_e(\pi + A_0) - \left(\pi^2 + M^2 + \frac{1}{K_p}\right)}, \quad I_{33} = \frac{I_8}{A_0^2 - R_e A_0 - \left(\pi^2 + M^2 + \frac{1}{K_p}\right)}$$

$$I_{34} = \frac{I_9}{R_4^2 - R_e R_4 - \left(\pi^2 + M^2 + \frac{1}{K_p}\right)}, \quad I_{35} = \frac{I_{11}}{(\pi + A_1)^2 - R_e(\pi + A_1) - \left(\pi^2 + M^2 + \frac{1}{K_p}\right)},$$

$$I_{36} = \frac{I_{12}}{(R_2 + A_1)^2 - R_e(R_2 + A_1) - \left(\pi^2 + M^2 + \frac{1}{K_p}\right)}, \quad I_{37} = \frac{I_{13}}{A_1^2 - R_e A_1 - \left(\pi^2 + M^2 + \frac{1}{K_p}\right)},$$

$$I_{38} = \frac{I_{14}}{(\pi + A_0)^2 - R_e(\pi + A_0) - \left(\pi^2 + M^2 + \frac{1}{K_p}\right)},$$

$$I_{39} = \frac{I_{15}}{(R_2 + A_0)^2 - R_e(R_2 + A_0) - \left(\pi^2 + M^2 + \frac{1}{K_p}\right)}, \quad I_{40} = \frac{I_{16}}{A_0^2 - R_e A_0 - \left(\pi^2 + M^2 + \frac{1}{K_p}\right)},$$

$$I_{41} = \frac{I_{10}}{R_4^2 - R_e R_4 - \left(\pi^2 + M^2 + \frac{1}{K_p}\right)}, \; I_{42} = \frac{I_{17}}{R_3^2 - R_e R_3 - \left(\pi^2 + M^2 + \frac{1}{K_p}\right)}$$

$$I_{43} = \frac{I_{18}}{\left(R_2 + A_0\right)^2 - R_e\left(R_2 + A_0\right) - \left(\pi^2 + M^2 + \frac{1}{K_p}\right)}, I_{44} = \frac{I_{19}}{\left(\pi + A_0\right)^2 - R_e\left(\pi + A_0\right) - \left(\pi^2 + M^2 + \frac{1}{K_p}\right)}$$

$$I_{45} = \frac{I_{20}}{A_0^2 - R_e A_0 - \left(\pi^2 + M^2 + \frac{1}{K_p}\right)}, \; I_{46} = \frac{I_2}{K_p\left[R_1^2 - R_e R_1 - \left(\pi^2 + M^2 + \frac{1}{K_p}\right)\right]},$$

$$I_{47} = \frac{I_3}{K_p\left[A_0^2 - R_e A_0 - \left(\pi^2 + M^2 + \frac{1}{K_p}\right)\right]}, \; I_{48} = \frac{I_4}{K_p\left[A_1^2 - R_e A_1 - \left(\pi^2 + M^2 + \frac{1}{K_p}\right)\right]}$$

$I_{49} = I_{21} - I_{22} + I_{23} + I_{24} - I_{25} + I_{26} - I_{27} + I_{28} - I_{29}$, $I_{50} = I_{30} + I_{31} - I_{32} - I_{33}$

$I_{51} = I_{34} - I_{35} + I_{36} - I_{37} + I_{38} - I_{39} + I_{40}$, $I_{52} = I_{42} + I_{43} - I_{41} - I_{44} - I_{45}$, $I_{53} = I_{48} - I_{46} - I_{47}$