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Numerical Computation of Mixed Convection Past a Heated Vertical Plate within a Saturated Porous Medium with Variable Permeability

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Abstract

The main goal of this paper is to study the effects of variable permeability and the fluid suction/injection on the mixed convection flow along a heated vertical plate embedded in saturated porous medium by Newtonian fluid with an internal heat generation. The surface temperature is assumed to vary as a power of the axial coordinate measured from the leading edge of the plate and subjected to an applied lateral mass flux. Several similarity solutions were obtained in terms of temperature parameter, but only three physical cases were studied: isothermal plate, uniform surface heat flux and uniform lateral mass flux at the plate. The non-linear equations of the similarity analysis with boundary layer conditions have been solved numerically using a fifth-order Runge-Kutta scheme coupled with the shooting iteration technique. Also, the effect of the governing physical parameters on velocity and temperature, shear stress and Nusselt number profiles have been computed and studied with help of graphs.

Key words: Mixed convection, porous medium, suction/injection, variable permeability.

Nomenclature

A	wall temperature parameter	V_w	lateral mass flux
a	equivalent thermal diffusivity ($m^2 s^{-1}$)	v	velocity component in y direction ($m\,s^{-1}$)
B	suction/injection velocity parameter	x	coordinate along the plate (m)

C_p	specific heat of fluid ($J\,Kg^{-1}\,K^{-1}$)
f	dimensionless stream function
f_w	suction/injection parameter
Gr_x	modified local Grashof number
g	gravitational acceleration ($m\,s^{-2}$)
$K(y)$	permeability of porous medium (m^2)
$K(\eta)$	non-dimensional permeability
KB	Keller-Box method
K_t	thermal conductivity ($W\,m^{-1}\,K^{-1}$)
Nu_x	local Nusselt number
Q_w	local heat flux at the plate surface (W)
Ra_x	modified local Rayleigh number
RE	relative error
Re_x	local Reynolds number
RK_4	fourth order Runge-Kutta method
RK_5	fifth order Runge-Kutta method
T	fluid temperature (K)
u	velocity component in x direction ($m\,s^{-1}$)
y	coordinate normal to the plate (m)
β	thermal expansion coefficient (K)
γ	mixed convection parameter
ε	permeability parameter
η	similarity variable
θ	dimensionless temperature
λ	temperature exponent
ν	kinematic viscosity ($m^2\,s^{-1}$)
ρ	fluid density ($Kg\;m^{-3}$)
ψ	stream function
w	wall plate condition
∞	infinity plate condition
$'$	derivative with respect to η

1. Introduction

We call a porous medium, a solid part, including voids called pores which can communicate between them and containing one or more fluid phases being able to flow and, eventually, to exchange the energy and/or the matter between them and/or the solid part. There are numerous examples of natural or artificial porous media in everyday life : soil, porous rock, ceramics, textiles, leather and many others. A porous medium is characterized mainly by two related macroscopic quantities which are the porosity and permeability.

The appearance of temperature gradients within a porous medium leads to the existence of differences in density of the saturating fluid. Such differences may, under certain conditions, by action of gravity, be an influence on the velocity distribution of fluid in the medium. We then say that there is free convection of thermal origin, forced convection when the saturating fluid is driven by external mechanical force (pump, fan, ...) and mixed convection corresponds to the coupling of the two previous phenomena.

The study of mixed convection in a saturated porous medium has attracted the attention of many workers due to its immense practical importance as well as a number of engineering and geophysical applications such as, in petroleum reservoirs, geothermal reservoirs, industrial and agricultural water distribution, drying of porous solids and cooling processes of nuclear reactors are just a few. Excellent reviews on this topic can be found in the books by Ingham and Pop (2005, [1]), Vafai (2005, [2]), Nield and Bejan (2006, [3]) and Vadasz (2008, [4]), and in the review paper by Magyari et al. (2005, [5]). The problem of free convection heat transfer from a vertical plate embedded in a fluid saturated porous medium is studied by Cheng and Minkowycz (1977, [6]). The mixed convection boundary layer flow on an impermeable vertical surface embedded in a saturated porous medium has been treated by Pal and Shivakumara (2006, [7]). The mixed convection boundary layer flow over a vertical permeable plate in a porous medium has been investigated by Nazar and Pop (2006, [8]).

The majority of the above studies consider the permeability of the medium as constant. However, porosity measurements by Schwartz and Smith (1953, [9]), Tierney and al. (1958, [10]) and Benenati and Brosilow (1962, [11]) show that the porosity is not constant but varies from the wall to the interior due to which permeability also varies. Mohammadein and El-Shaer (2004, [12]), Singh (2012, [13]), Ibrahim and Hassanien (2000, [14]), have integrated the variable permeability to study the flow past through a porous medium and they show that the variation of permeability has greater influence on velocity and heat transfer. Vafai (1984, [15]) and Vafai et al. (1985, [16]), have investigated the variable permeability effect on forced convection flow. Chandrasekhara and Namboudiri (1985, [17]) studied the effect of variable permeability on combined free and forced convection about inclined surfaces in porous media using a similarity solution approach. The permeability effect on free convective boundary layer flow induced by a vertical flat plate embedded in a porous medium has been investigated by EL-Kabeir and Rashad (2006, [18]). The mixed convection from a continuously moving vertical surface with suction or injection has been treated by Ali and Al-Yousef (1998, [19]).

Thus, the aim of the present paper is to investigate the effects of variable permeability due to packing of particles of the porous medium and the fluid suction/injection on the mixed convection boundary layer flow adjacent to a heated vertical plate in a saturated porous medium taking into account an internal heat generation.

2. Mathematical modeling and similarity analysis

A vertical flat plate embedded in saturated porous medium by Newtonian fluid with applied lateral mass flux in the direction normal to the plate proportional to $x^{(\lambda-1)/2}$ quantity is considered as shown in Figure 1.

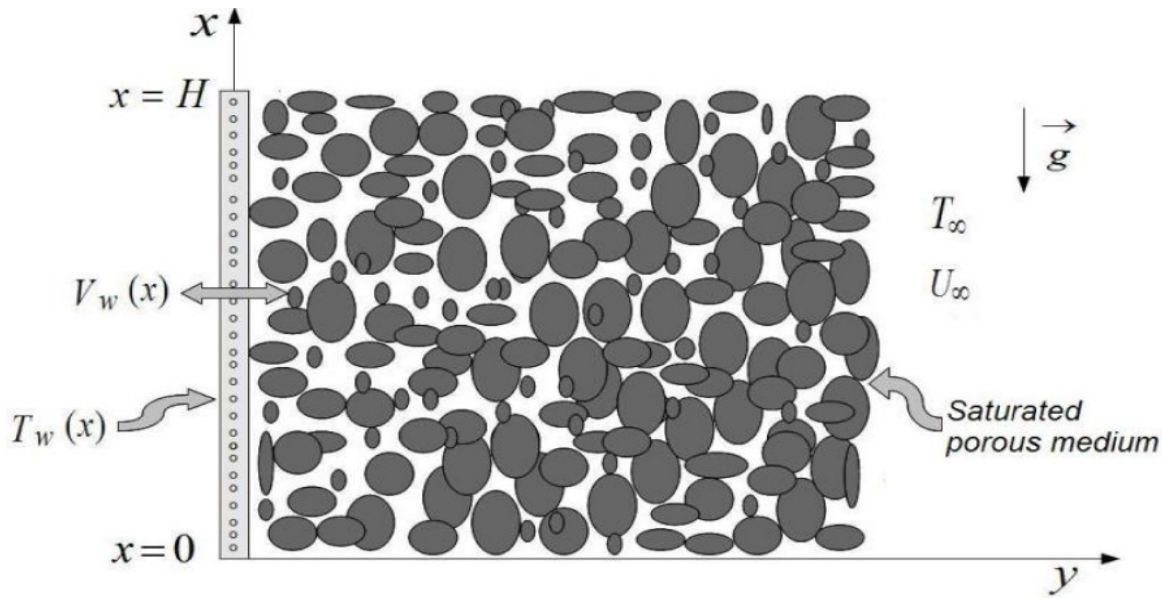

Fig.1. Vertical heated plate in a saturated porous medium.

The temperature distribution of the plate has been assumed as $T_w(x) = T_\infty + A\,x^\lambda$, where x is the distance measured along the vertical plate and λ is the constant temperature exponent. T_∞ is the temperature away from the plate assumed constant and A is a positive constant. The Cartesian coordinates x and y are measured, respectively, along and perpendicular to the plate. The flow is supposed two-dimensional, steady and laminar for an incompressible fluid. The convective fluid and the porous medium are in local thermodynamic equilibrium anywhere and no dissipation of energy by viscosity. The permeability of the porous medium is assumed to be variable and the density of the medium is considered according to temperature. Radiation heat transfer is considered negligible with respect to other modes of heat transfer. In this paper, we work in the case of the porous medium with the small filtration velocities (Reynolds number is less than 1) and the small permeability of the medium (practically of the order of 10^{-8}). For this, we can effectively neglect the inertia term in front of the viscous term. Therefore the stationary Navier Stokes equations have for limit the Darcy equation after development.

By considering the assumptions mentioned above and the Boussinesq approximation:

$$\rho = \rho_\infty (1 - \beta (T - T_\infty)) \tag{1}$$

The governing equations for this model are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2}$$

$$u = U_\infty + \frac{g\beta K(y)}{\nu} (T - T_\infty) \tag{3}$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = a \frac{\partial^2 T}{\partial y^2} + \frac{\varphi}{\rho C_p} \tag{4}$$

The boundary conditions associated with Eqs. (2) - (4) are:

$$y = 0 \quad x \geq 0 \quad , \quad v = V_w(x) \quad T = T_w(x) \tag{5}$$

$$y \to \infty \quad x \geq 0 \quad , \quad u = U_\infty \quad T = T_\infty \tag{6}$$

where u and v, are respectively, the velocity components along x and y axes. T is the temperature of the fluid and φ is the internal heat source. The constants ν, a, g and ρ are, respectively, kinematic viscosity, thermal diffusivity, gravitational acceleration and density. C_p and β are, respectively, the specific heat at a constant pressure and the coefficient of thermal expansion, $V_w = B\ x^{(\lambda-1)/2}$ is the lateral mass flux, where B is a constant.

Following Ress and Pop (2000, [20]), it is assumed here that the permeability, $K(y)$ of the porous medium varies as:

$$K(y) = K_\infty + (K_w - K_\infty)\, e^{-y/L} \tag{7}$$

where K_w is the permeability at the wall, K_∞ is the permeability of the ambient, and L is a constant.

The continuity equation (2) is satisfied by the stream function $\psi(x, y)$ defined by:

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x} \tag{8}$$

The non-linearity of our model and the complexity of the phenomena encountered (boundary layer, instability, geometry of the porous medium, ...) make difficult its direct resolution.. The transformation of the PDE system, describing the problem studied in a simple non-linear differential equation becomes indispensable.

To transform Eqs. (3) - (4) into a set of ordinary differential equations, the following dimensionless variables used by Postelnicu et al. (2000, [21]) are introduced:

$$\eta = \frac{y}{x} Ra_x^{1/2}, \quad \psi(x,y) = a\, Ra_x^{1/2} f(\eta), \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}$$
$$Ra_x = \frac{g \beta K_\infty (T_w - T_\infty) x}{a \nu}, \quad \varphi = \rho\, C_p \frac{a (T_w - T_\infty) Ra_x}{x^2} e^{-\eta} \tag{9}$$

where Ra_x Ra_x is the Rayleigh number, f and θ are the dimensionless similarity functions.

From Eq. (7), we choose $L = x\, Ra^{-1/2}$ such that the non-dimensional permeability is purely function of η, and given by :

$$K(\eta) = K_\infty \left(1 + (\varepsilon - 1)\, e^{-\eta} \right) \tag{10}$$

where $\varepsilon = \frac{K_w}{K_\infty}$ being the permeability parameter.
When $\varepsilon = 1$, which corresponds to a uniform permeability, and when are greater than 1 corresponds to a non-uniform permeability cases.

Eqs. (8) - (9) transform Eq. (3) and Eq. (4) into

$$f'(\eta) = \gamma + \frac{K(\eta)}{K_\infty} \theta(\eta) \tag{11}$$

$$\theta''(\eta) - \lambda f'(\eta)\, \theta'(\eta) + \frac{\lambda + 1}{2} f(\eta)\, \theta'(\eta) + e^{-\eta} = 0 \tag{12}$$

The boundary conditions (5) and (6) transform into

$$\eta = 0 \qquad f(0) = f_w, \quad f'(0) = \gamma + \varepsilon, \quad \theta(0) = 1 \tag{13}$$

$$\eta \to \infty \qquad f'(\infty) = \gamma, \quad \theta(\infty) = 0 \tag{14}$$

By injecting the Eq. (11) in the Eq. (12), we get the following non-linear differential equation:

$$f'''(\eta)+3M(\eta)f''(\eta)+(\lambda\gamma+N(\eta))f'(\eta)-\lambda f'(\eta)^2+(\frac{\lambda+1}{2})\times$$
$$(f(\eta)f''(\eta)+M(\eta)f(\eta)f'(\eta)-\gamma M(\eta)f(\eta))+\frac{e^{-\eta}}{1-M(\eta)}- \quad (15)$$
$$\gamma N(\eta)=0$$

and the boundary conditions Eqs. (13) – (14) are reduced to Eqs. (16) – (17)

$$\eta=0 \quad f(0)=f_w, \quad f'(0)=\gamma+\varepsilon \quad (16)$$

$$\eta\to\infty \quad f'(\infty)=\gamma \quad (17)$$

where $f_w = -\frac{2B}{\lambda+1}\left(\frac{\nu}{a\,g\,\beta K_\infty A}\right)^{1/2}$ is the suction/injection parameter, $\gamma=\frac{Re_x}{Gr_x}$ is called the mixed convection parameter, where $Re_x=\frac{U_\infty x}{\nu}$ and $Gr_x=\frac{g\beta K_\infty (T_w-T_\infty)\,x}{\nu^2}$.

$M(\eta)$ and $N(\eta)$ are the similarity functions such that:

$$M(\eta)=1-\frac{K_\infty}{K(\eta)}$$
$$N(\eta)=1+2\frac{K_\infty{}^2}{K(\eta)^2}-3\frac{K_\infty}{K(\eta)} \quad (18)$$

The local heat flux at the plate surface is given by:

$$Q_w=-K_t\left(\frac{\partial T}{\partial y}\right)_{y=0}$$
$$=-K_t A^{3/2} S^{1/2} x^{(3\lambda-1)/2}\,\theta'(0) \quad (19)$$

where $S=\frac{g\beta K_\infty}{a\,\nu}$, $\theta'(0)$ is the gradient of the temperature at the plate surface and K_t is the thermal conductivity.

The local Nusselt number is defined as:

$$Nu_x=-\theta'(0)\,Ra_x^{1/2} \quad (20)$$

Expression for shear stress τ can be developed from the similarity

solution: $\tau=\frac{a\,\mu\,Ra_x^{3/2}}{x^2}f''(\eta)$

(21)

where $f''(\eta)$ describes the dimensionless shear stress distribution in the boundary layer area and the particular value at $\eta=0$ represents the dimensionless shear stress on the plate surface.

3.Numerical solution method

The third order ODE Eq. (15) governed by the boundary conditions Eqs. (16) - (17), is non-linear. However it is still difficult to solve it analytically. For this, we can routinely rewrite it as a system of three first order ODEs by posing:

$$g_1 = f(\eta), \qquad g_2 = f'(\eta), \qquad g_3 = f''(\eta) \tag{22}$$

So the non-linear differential Eq. (15) is equivalent to the system of differential equations of the first order Eq. (23) coupled with the appropriate boundary conditions Eq. (24):

$$\begin{cases} g_1' = g_2 \\ g_2' = g_3 \\ g_3' = -3\,M(\eta)\,g_3 - (\lambda\,\gamma + N(\eta))\,g_2 + \lambda\,g_2^2 - \\ (\dfrac{\lambda+1}{2})\,(g_1\,g_3 + M(\eta)\,g_1\,g_2 - \gamma\,M(\eta)\,g_1) - \\ \dfrac{e^{-\eta}}{1-M(\eta)} + \gamma\,N(\eta) \end{cases} \tag{23}$$

$$\begin{cases} \eta = 0 \qquad g_1 = f_w, \qquad g_2 = \gamma + \varepsilon \\ \eta \to \infty \qquad\qquad g_2 = \gamma \end{cases} \tag{24}$$

The system of differential equations Eq. (23) subject to the boundary conditions Eq. (24) has been solved numerically by the fifth-order Runge-Kutta scheme associated with the shooting iteration technique.

Since we have the initial conditions on g_1 and g_2, it would be natural to seek the condition on g_3 at $\eta = 0$ ($g_3(0)$). For given values of λ the value of $g_3(0)$ is estimated and the differential equations of the system Eq. (23) were integrated until the boundary condition at infinity, $g_2(\eta)$ decay exponentially to γ. If the boundary condition at infinity is not satisfied, then the numerical routine uses the calculate correction to the estimated value of $g_3(0)$. This process is repeated iteratively until exponentially decaying solution in g_2 is obtained. A step size of $\Delta\eta = 0.0001$ was found to be sufficient to give results that converge to within an error of 10^{-6} in nearly all cases. The value of η_∞ was chosen as large as possible.

4. Numerical results and discussions

Similarity solutions are obtained for distinct values of the temperature exponent λ and suction/injection parameter f_w. However, only three physical solutions were selected for $\lambda = 0, 1/3$ and 1 which corresponds to isothermal plate, uniform surface heat flux and linear temperature distribution with uniform lateral mass flux at the plate, respectively. The calculations were performed for different cases of the suction/ injection parameter f_w which corresponds to the suction ($f_w > 0$), injection ($f_w < 0$) and impermeable plate ($f_w = 0$). We notice that the importance of heat transfer regime relative to the other is characterized by the mixed convection parameter γ such that: $\gamma = 0$ corresponds to the free convection, $\gamma = 1$ corresponds to the mixed convection and $\gamma > 1$ corresponds to the forced convection.

The comparisons are made with Postelnicu et al. (2000, [21]) and Cortell (2010, [22]) results in terms of $\theta'(0)$ with various values of suction/injection parameter f_w and temperature exponent λ, are presented in Table 1. Although, all the relative errors from the previous results are less than 1% for $\lambda = 1/3$. The present method yields results which are generally in good agreement with those of Postelnicu et al (2000, [21]) and Cortell (2010, [22]). However, it could be said here with a positive note that the method used in this current model converges rapidly and accurately when compared to the method adopted by the previous work.

$-f''(0) = -\theta'(0) = Nu_x \ Ra_x^{-1/2}$							
λ	$f(0)$	$f'(0)$	[21] (KB)	[22] (RK4)	Present results (RK5)	RE / [21]	RE / [22]
$1/3$	-1.0	1	-0.0662	-0.066178	-0.066291	0.13 %	0.17 %
	-0.6	1	-0.0094	-0.009357	-0.009433	0.35 %	0.80 %
	0.6	1	0.2869	0.286887	0.286969	0.024 %	0.028 %
	1.0	1	0.4289	0.428891	0.428903	0.00069 %	0.0027 %

Tab.1. Comparison of the temperature gradient at the plate surface with previously published results for $\lambda = 1/3$, $f_w \neq 0$ and $\gamma = 0$ when $\varepsilon = 1$.

Figure 2 shows the temperature profiles in the boundary layer area for an isothermal and impermeable plate drowned vertically in a saturated porous medium with a variable K. It is seen, from this figure, that the increase of the parameter γ reduces the temperature profiles in the boundary layer area. This may be due to the intensity of the flow velocity and the high permeability close to the plate which promote the fast cooling around it.

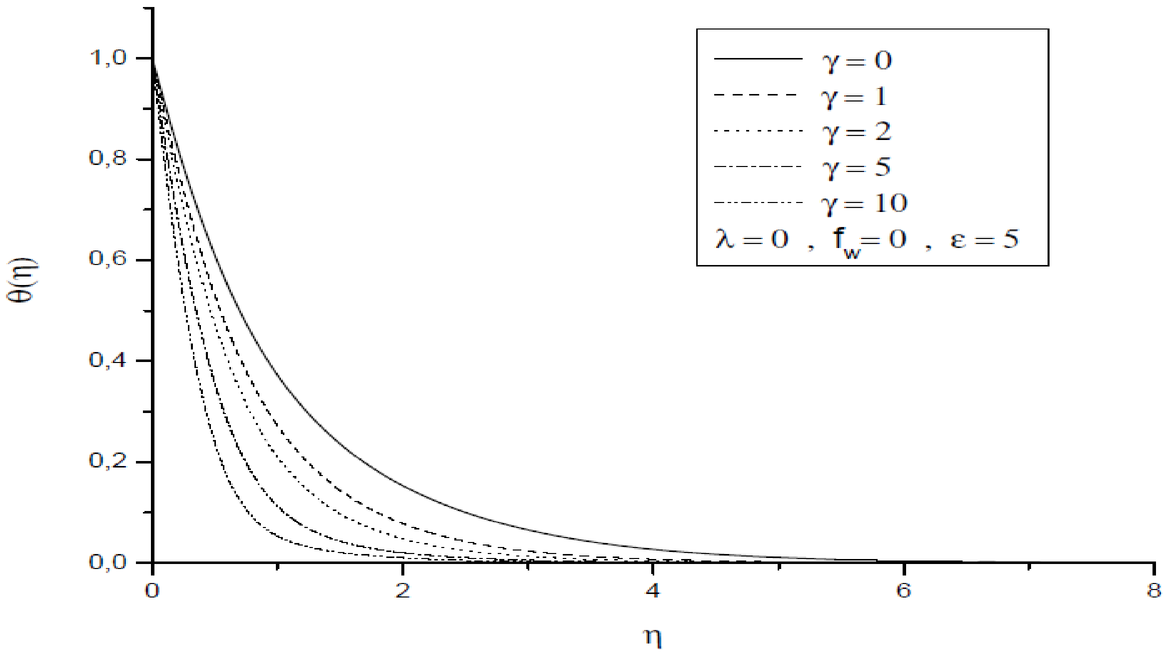

Fig.2. Dimensionless temperature profiles at $\lambda = 0$, $f_w = 0$ and $\varepsilon = 5$ for selected values of γ.

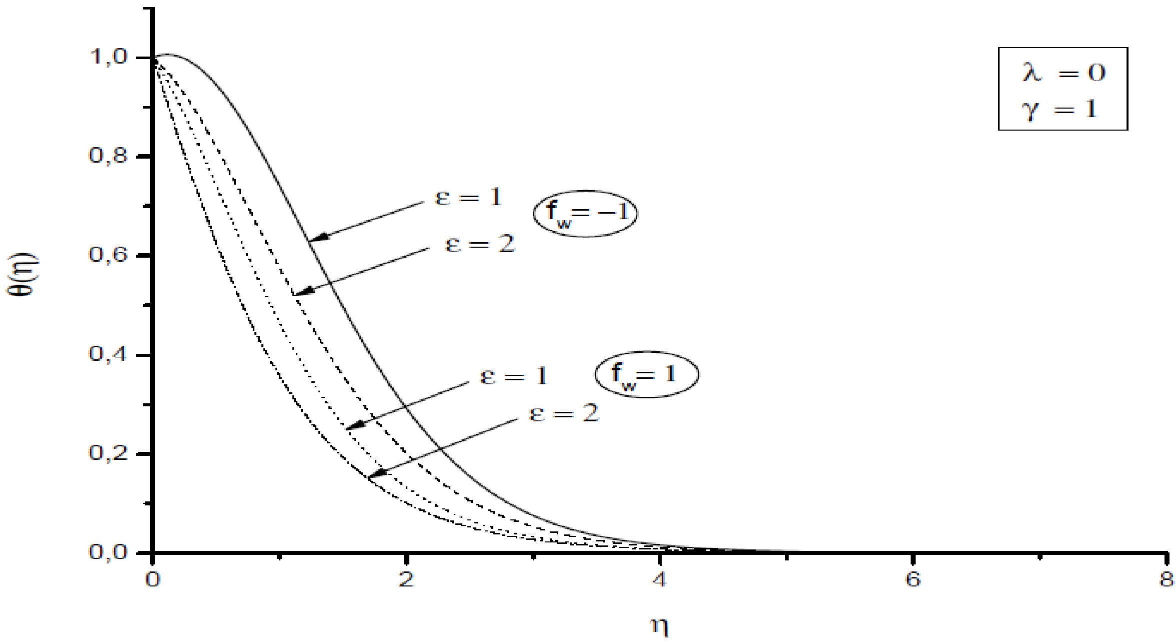

Fig.3. Dimensionless temperature profiles for $\lambda = 0$, $\gamma = 1$ and $f_w \neq 0$ at $\varepsilon = 1$ and $\varepsilon = 2$.

In Figures 3, 4 and 5 we present for $\gamma = 1$, the temperature profiles for a vertical permeable plate embedded in a saturated porous medium with a uniform ($\varepsilon = 1$) and a non-uniform ($\varepsilon = 2$)

permeability and for $\lambda = 0$, $\lambda = 1/3$ and $\lambda = 1$ which corresponds physically to isothermal plate, uniform surface heat flux and uniform lateral mass flow at the plate, respectively. It is clear that the fluid suction through the plate generates cooling in the boundary layer area for the two values of ε.

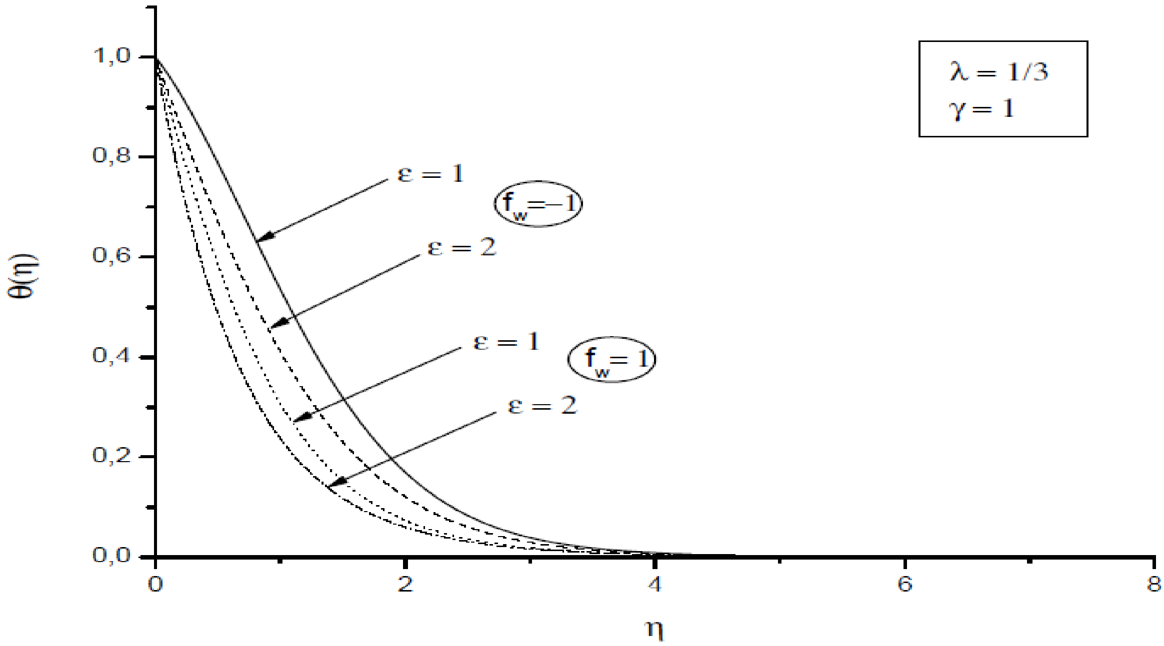

Fig.4. Dimensionless temperature profiles for $\lambda = 1/3$, $\gamma = 1$ and $f_w \neq 0$ at $\varepsilon = 1$ and $\varepsilon = 2$.

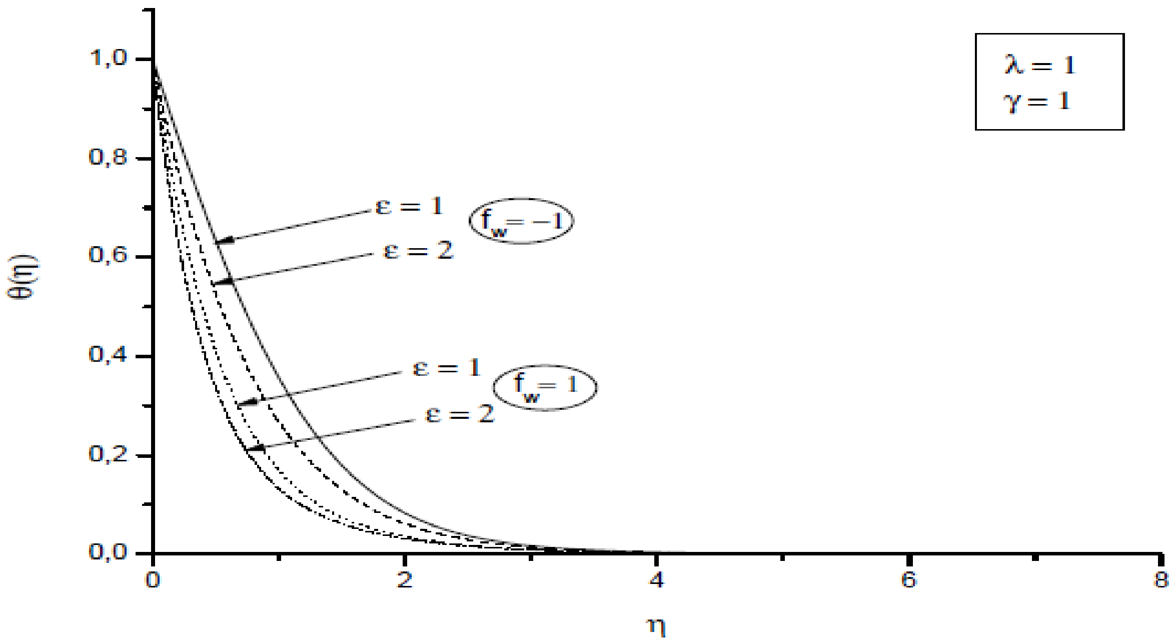

Fig.5. Dimensionless temperature profiles for $\lambda = 1$, $\gamma = 1$ and $f_w \neq 0$ at $\varepsilon = 1$ and $\varepsilon = 2$.

Furthermore, we notice that for the two values of f_w (suction/injection), the non-uniformity of the porous medium permeability contributes to the rapid decrease of the temperature adjacent to the plate which is normal since the permeability of the porous medium near the plate is more important than away from the latter.

The displayed Figure 6 shows the dimensionless velocity profiles in the boundary layer area for an isothermal and impermeable plate with non-uniform K, for three heat transfer regimes ($\gamma = 0$, $\gamma = 1$, $\gamma > 1$). We notice that the velocity profiles are amplified and stabilized quickly in passing from the free convection ($\gamma = 0$) to forced convection ($\gamma > 1$). This can be justified by the positive effect of the imposed flow velocity away from the plate and the high permeability near the plate.

Figures 7, 8 and 9 show for $\gamma = 1$, the dimensionless velocity distributions in the boundary layer area with a uniform ($\varepsilon = 1$) and a non-uniform ($\varepsilon = 2$) permeability and for three physical values of $\lambda = 0, 1/3$ and 1. Here, we also find that the fluid suction at the plate leads to the reduction of the velocity profiles. On the other hand, it is clear that the non-uniformity of the porous medium permeability in the fluid suction and injection cases permit to accelerate the velocity flow near the plate and this is quite logical to fact that the media porous near the plate is more permeable than it away from the later. This is consistent with the observations made in the results for the dimensionless temperature discussed above.

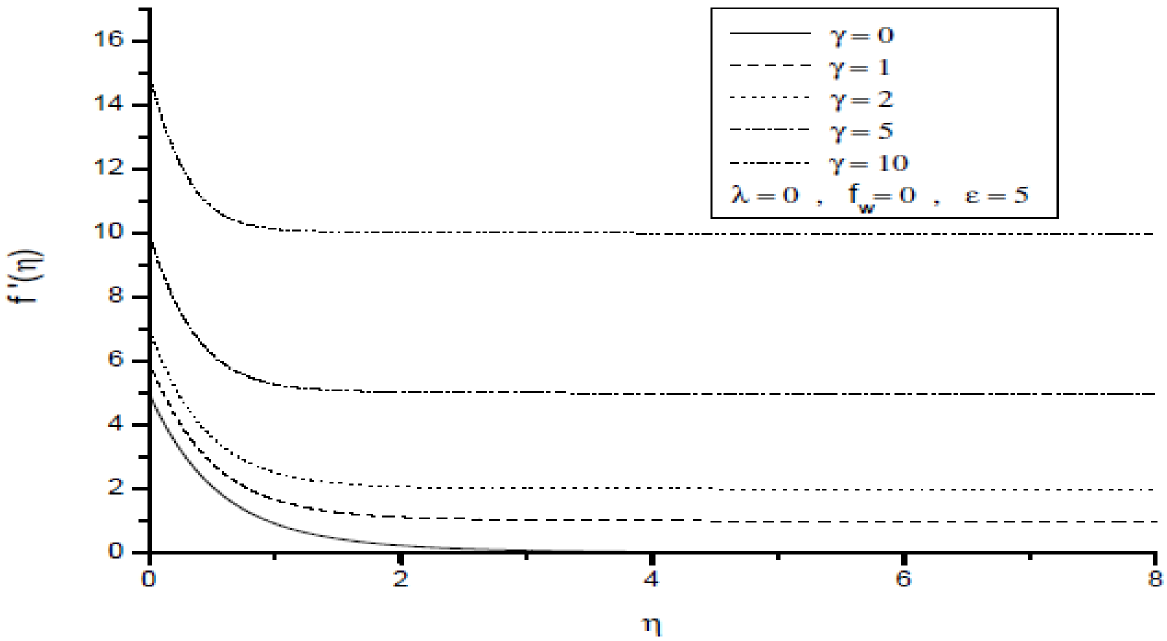

Fig.6. Dimensionless velocity profiles at $\lambda = 0$, $f_w = 0$ and $\varepsilon = 5$ for selected values of γ.

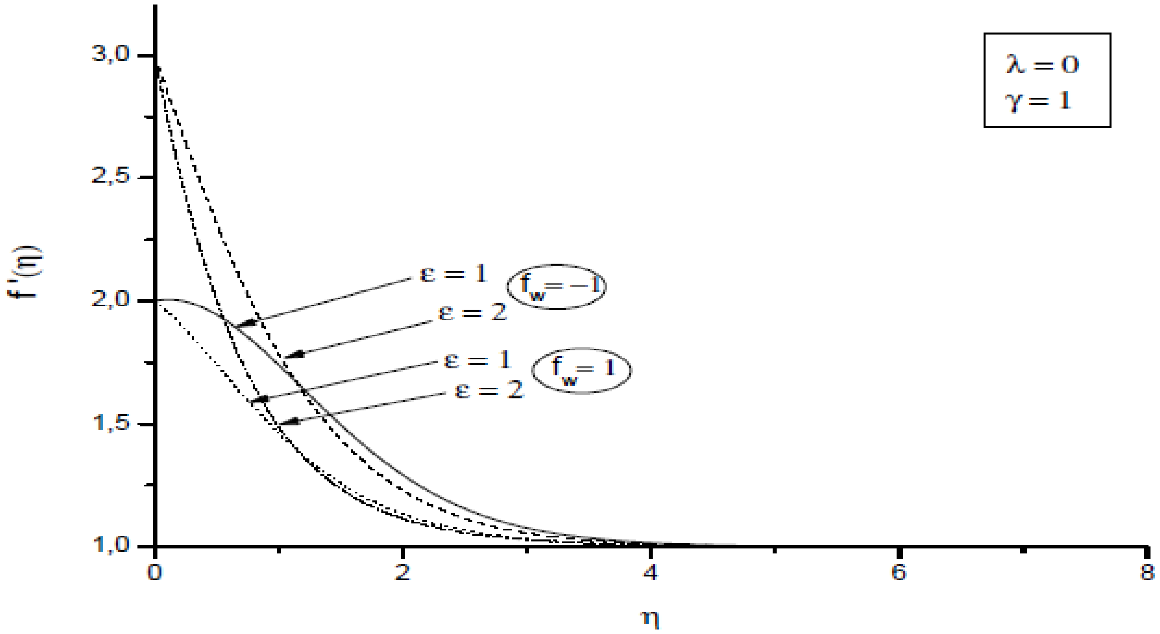

Fig.7. Dimensionless velocity profiles for $\lambda = 0$, $\gamma = 1$ and $f_w \neq 0$ at $\varepsilon = 1$ and $\varepsilon = 2$.

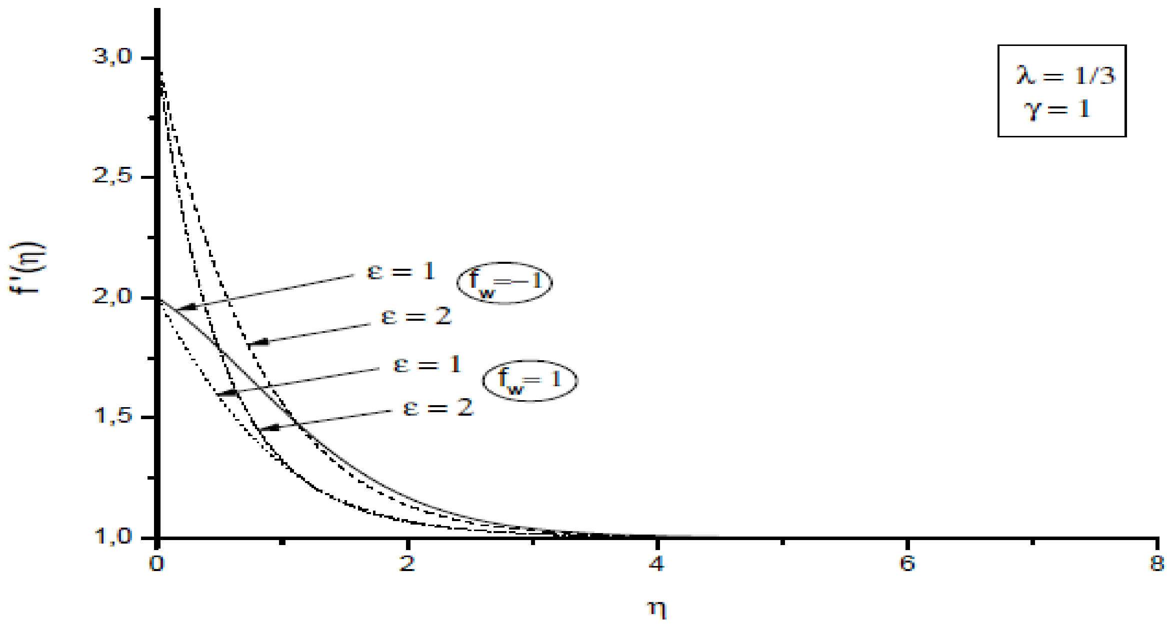

Fig.8. Dimensionless velocity profiles for $\lambda = 1/3$, $\gamma = 1$ and $f_w \neq 0$ at $\varepsilon = 1$ and $\varepsilon = 2$.

Figure 10 describes for $\gamma = 1$, the dimensionless shear stress profiles around the isothermal and impermeable plate for various values of the parameter ε. It is remarkable that the frictional forces become important near the plate where the porous medium is more permeable. This can be physically justified by the effect of the flow velocity which is important close to the plate and that promotes the shear stresses.

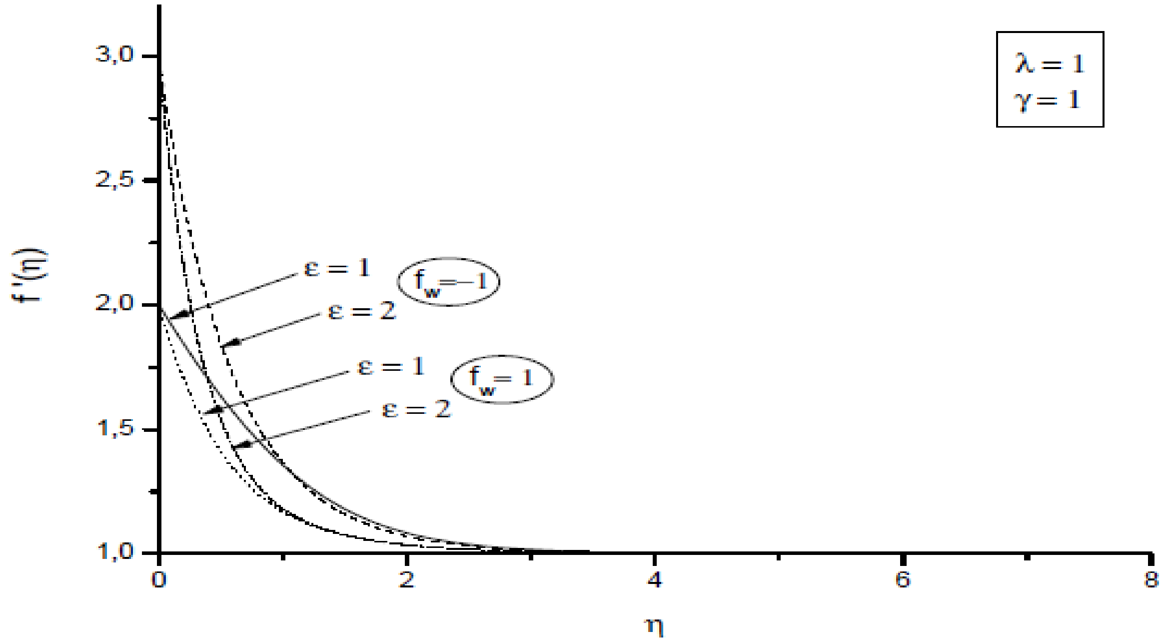

Fig.9. Dimensionless velocity profiles for $\lambda = 1$, $\gamma = 1$ and $f_w \neq 0$ at $\varepsilon = 1$ and $\varepsilon = 2$.

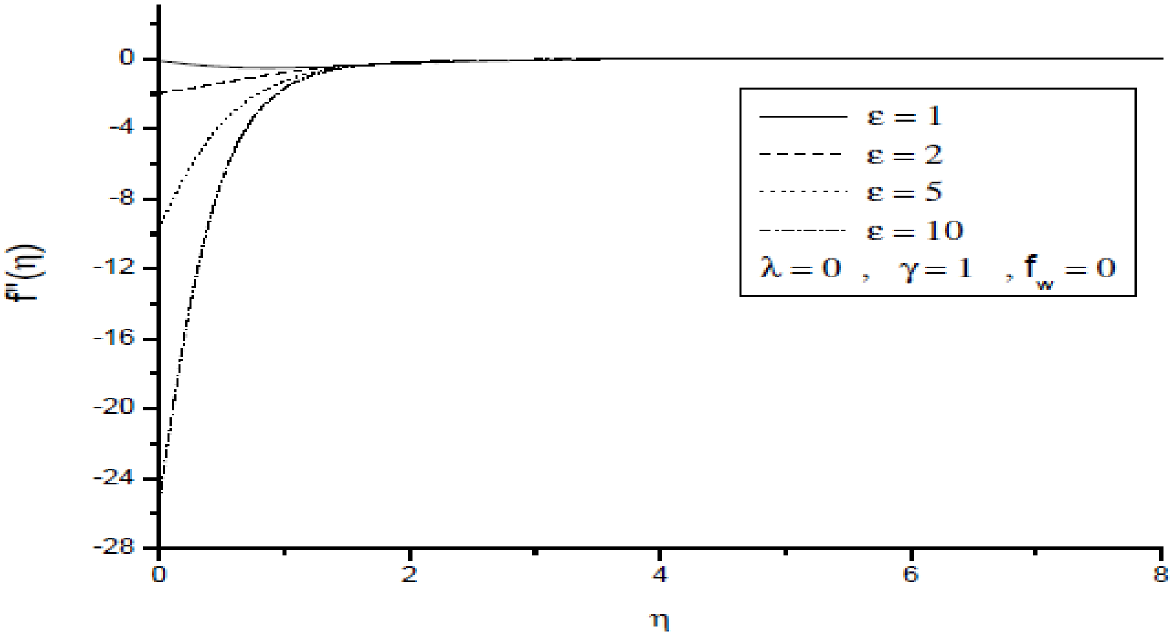

Fig.10. Dimensionless shear stress profiles for $\lambda = 0$, $\gamma = 1$ and $f_w = 0$ for various values of ε.

Figure 11 shows that for an isothermal plate, the local Nusselt number profiles according to the mixed convection parameter γ in the boundary layer area for uniform and non-uniform permeability of the porous medium in the cases of impermeable plate, fluid suction and injection. It is clearly remarkable that the non-uniformity of the porous medium permeability allows amplifying the rate of heat transfer in the boundary layer area for various values of f_w. We also note that the rate of heat transfer increases with the fluid suction around the plate. On the other hand, the heat transfer rate increases with γ in the two cases of the porous medium permeability ($\varepsilon = 1$ and $\varepsilon = 2$). It's obvious that the fact that the non-uniformity of the porous medium permeability, the fluid suction along the plate and the great values of the parameter γ, all contributes to the intensity of the flow velocity in the boundary layer area which promotes the heat transfer by convection.

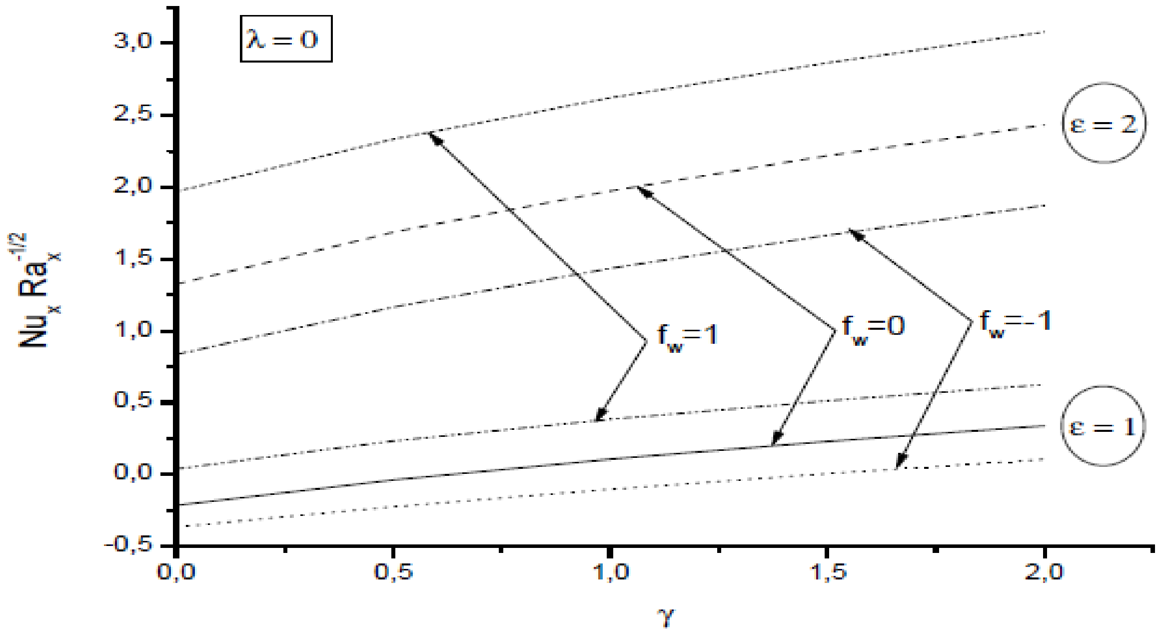

Fig.11. Nusselt number profiles according to γ at $\lambda = 0$ for selected values of f_w and ε.

5. Conclusion

The similarity equations are solved numerically by using the fifth-order Runge-Kutta scheme associated with the shooting iteration technique. The influence of the parameters ε, γ, f_w and λ on dimensionless temperature and velocity, dimensionless shear stress and local Nusselt number profiles have been examined and discussed in details. Special cases are considered for the temperature exponent λ with the lateral mass flux controlled by the suction/injection parameter f_w. From the present numerical study, we conclude that:

- The passing from the free convection regime to the forced convection, the flow velocity is amplified and the temperature field decreases in the boundary layer area respectively and they quickly stabilize in a porous medium with high permeability ($K_w > K_\infty$).

- In the mixed convection regime, the fluid suction along the plate reduces the thickness of the thermal and dynamic boundary layer.
- In the case of the mixed convection regime, the non-uniformity of the porous medium permeability ($K_w > K_\infty$) promotes the advantage, the intensity of the flow velocity and the shear stresses in the boundary layer area.
- In the variable permeability case, the rate of the heat transfer is much higher than in a uniform permeability case for different heat transfer regimes and for various values of the suction/injection parameter.

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