AMSE JOURNALS –2014-Series: Modelling B; Vol. 83; N° 1; pp 67-84
Submitted July 2013; Revised April 4, 2014; Accepted May 20, 2014

Evolutional and Non-evolutional Motion of Dumbell Satellite near Resonance Oscillations

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Abstract
Evolutional and Non-evolutional motion of dumbbell satellite in elliptical orbit in central gravitational field central gravitational field of force. The gravitational field of the Earth is the main force governing the motion and magnetic field of the Earth and Oblateness of the Earth are considered to be perturbing forces, disturbing in nature. Non-linear oscillations of dumbbell satellite about the equilibrium position in the neighborhood of main resonance $\omega = 1$, under the influence of perturbing forces, which is suitable for exploiting the asymptotic methods of Bogoliubov, Krilov and Metropoloskey has been studied, considering 'e' to be a small parameter. The phase analysis has been applied to investigate the stability of the system.

AMS subject Classification 70F15
Keywords: Stability, Perturbing forces, Evolutional and Non-evolutional.

1. Introduction
This paper is devoted to the analysis of Evolutional and Non-evolutional motion of cable connected satellites system in elliptical orbit, connected by a light, flexible and inextensible cable moving in the central gravitational field of the Earth under the combined effects of the Earth magnetic field and Oblateness of the Earth in non-linear, non resonance and resonance cases. The satellites are considered to be charged material particle and the motion of the system is studied relative to their centre of mass, under the assumption that the later moves along elliptical orbit. The cable connecting the two satellites is taut and non elastic in nature such that, the system moves like a dumbbell satellite. Many space configurations of cable connected satellite system have been proposed and analysed by different authors like two satellite are connected by a rod dumbbell satellite) [5], two or more satellites are connected by a tether M.Krupa et al [8,9], Beletsky and Levin [2], Mishra and Modi [13], and spring connected satellites [31]. All these authors have mentioned numerous important applications of system and stability of relative equilibrium, if the system moves in a circular and elliptical orbit.

Beletsky and Novikova [3], studied the motion of a system of two satellite connected by a light, flexible and inextensible string in the central gravitational field of force relative to their centre of mass, which is itself assumed to more along a Keplerian elliptical orbit under the assumption that the two satellite are moving in the plane of the centre of mass. The same problem in its general form, was further investigated Singh [27,28], these works conducted the analysis of relative motion of the system for the elliptical orbit of the centre of mass in the two dimensional as well as three dimensional cases. Narayan and Singh [14,15,16], studied non-linear oscillations due to solar radiation pressure of the centre of mass of the system moves along an elliptical orbit.

The different aspects of the problem of stability of satellites in low and high altitude orbit with different perturbation forces are studied by many scientists, Sharma and Narayan [25,26], Singh et al [27,28,29], Das et al [6], and Narayan et al [14,15,16]. Special references are mentioned, Sarychev et al [23,24], studied the problem determining all equilibria of a satellite subject to gravitational and aerodynamic torque in circular orbit. All bifurcation values of the parameter corresponding to qualitative changes of stability domain are determined. Palacian; [22], studied the dynamics of a satellites orbiting are Earth like planet at low altitude orbit and perturbation is caused by inhomogeneous potential due to the Earth. Langbort [11], studied bifurcation of relative equilibria in the main problem of artificial satellite theory for a prolate body. Markeev et al [12], studied the planar oscillation of a satellite in a circular orbit. Ayub Khan et al [7], investigated chaotic motion in problem of 68dumbbell satellite.

The present paper deals with the Evolutional and Non-evolutional motion of 68dumbbell satellite in elliptical orbit under the combined effects of magnetic field of the Earth and oblateness of the Earth. The perturbing forces due to Earth magnetic field results from the interaction between space craft's residual magnetic field and the geomagnetic field. The perturbing force is arising due to magnetic moments, eddy current and hysteresis, out of these the space craft magnetic moment is usually the dominant source of disturbing effects.

2. Equation of Motion

The combined effects of the geomagnetic field and Oblateness of the Earth on the motion and stability of the satellite connected by a light, flexible and inextensible cable, under the influence of the central gravitational field of the Earth has been considered. The analysis of Evolutional and Non-evolutional motion of dumbbell satellite in elliptical orbit has been restricted to two dimensional case, we have assumed that the satellite are moving in the orbital plane of the centre of mass of the system. The motion and stability of cable connected satellite system under the effects of Earth's magnetic field, Das; et al [6], Narayan et al [17], and combined effects of Earth magnetic field and oblateness of the Earth, Narayan and Pandey [18], in elliptical and in low altitude orbit has been studied. The equation of two dimensional motion of one of the satellite under the rotating frame of reference in Nechville's co-ordinate system [21], relative to their centre of mass, which moves along equatorial orbit under the combined influence of the Earth magnetic field and Oblateness of the Earth can be represented in (2.1) :

$$x'' - 2y' - 3\rho x = \lambda_\alpha x + \frac{4Ax}{\rho} - \frac{B}{\rho}\cos\delta \quad ;$$
$$y'' - 2x' = \lambda_\alpha y - \frac{Ay}{\rho} - \frac{B\rho'}{\rho^2}\cos\delta \; . \qquad (2.1) \quad ----$$

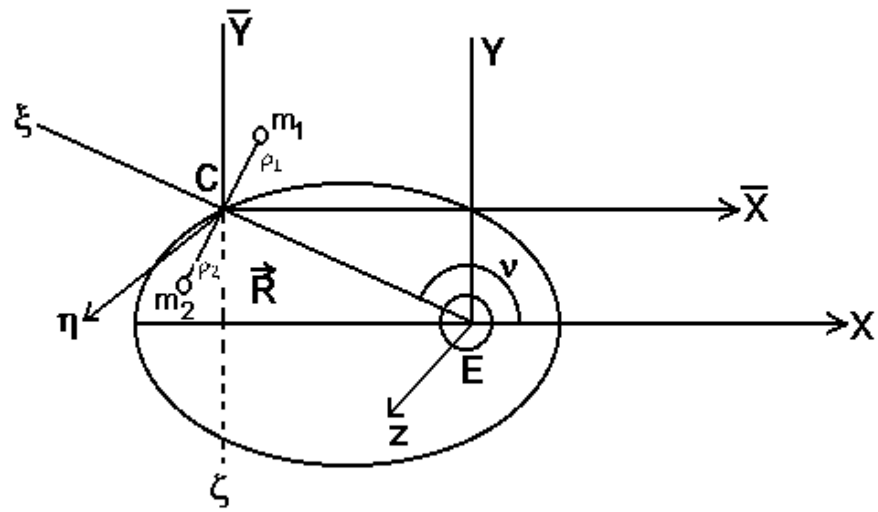

Fig. 1: Rotating Frame of Reference

Here x – axis is in the direction of position vector joining the centre of mass of the system and the attracting centre and the y – axis is along the normal to the position vector in the orbital plane of the centre of mass in the direction of the motion of the satellite m_1 where A is the Oblateness due to the Earth and B is the magnetic field of the Earth. Moreover:

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$$A = \frac{3k_2}{\rho^2};$$

$$\lambda_\alpha = \frac{p^3 \rho^4}{\mu}\left[\frac{m_1 + m_2}{m_1 m_2}\right]\lambda;$$

$$B = -\left(\frac{m_2}{m_1 + m_2}\right)\left[\frac{Q_1}{m_1} - \frac{Q_2}{m_2}\right]\frac{\mu_E}{\sqrt{\mu p}};$$ ------

(2.2)

$$\rho = \frac{R}{p} = \left(\frac{1}{1 + e\cos \mathrm{v}}\right).$$

The dipole of the Earth has its axis inclined from the polar axis of the Earth by a value of $11^0 4'$. The angle ϕ and Ω completely define the position of k_e, unit vector along the axis of magnetic dipole of the Earth.

In this case the condition for constrained is given by the in equality:

$$x^2 + y^2 \le \frac{1}{\rho^2}, \qquad \text{----- (2.3)}$$

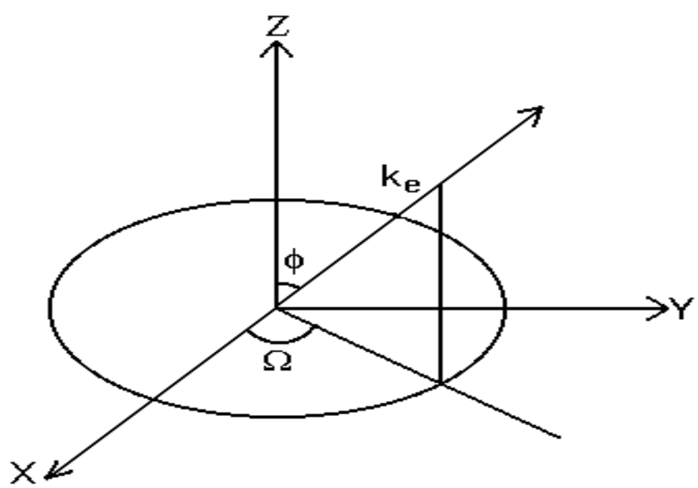

Fig.: 2 – Orientation of k_e

where λ denotes Lagrange's multiplier and μ denotes product of the gravitational constant and the mass of the Earth, where:

$Q_1 = \left[\frac{\text{Charge}\, q_i \text{ of the } i^{th} \text{ particle}}{\text{Velocity of light}}\right]$ $i = 1,2,$ on the masses m_1 and m_2, where v is the true anomaly of the centre of mass of the system in elliptical orbit.

and $$\rho = \frac{R}{p} = \left(\frac{1}{1+e\cos\mathrm{v}}\right), ;$$

where P and e are the focal parameter and the eccentricity of the orbit of the centre of mass. In equation (2.1), the prime denotes differentiation with respect to v. When the motion of the satellite m_1 of the system is determined with the help of equation (2.1), the motion of the satellite m_2 is easily determined with the help of identity.

$$m_1\vec{\rho}_1 + m_2\vec{\rho}_2 = 0, \quad ; \qquad ---(2.4)$$

where $\vec{\rho}_1$ and $\vec{\rho}_2$are the radius vectors of the satellites of masses $m_1$and m_2 respectively with respect to the centre of mass of the system.

Obviously, the actual motion of the system will be combination of three types of motion.

i) Free motion, i.e., $\lambda_\alpha = 0$.

ii) Constrained motion, i.e., $\lambda_\alpha \neq 0$.

iii) Evolutional motion (the combination of free and constrained motion).

We only interested in the constrained motion because free motion are bound to be converted into constrained motion with the lapse of time. In case of constrained motion the equality sign holds in the equation (2.3), i.e.; the particle is moving along the circle of variable radius given by:

$$x^2 + y^2 = \frac{1}{\rho^2}. \qquad ---(2.3)\text{ a.}$$

In order to discuss the non-linear planar oscillations of the system, we transform the equation (2.1), into polar form by substituting:

$$x = (1+e\cos\mathrm{v})\cos\psi, ;$$
$$y = (1+e\cos\mathrm{v})\sin\psi. \qquad ----(2.5)$$

where ψ is the angular deviation of the line joining the satellite with the stable position of equilibrium. Solving with respect to ψ and λ_α; we obtain:

$$(1+e\cos\mathrm{v})\psi'' - 2e\sin\psi' + 3\sin\psi\cdot\cos\psi + 5A(1+e\cos v)^2\sin\psi.\cos\psi$$
$$= B\cos\delta\,(1+e\cos\mathrm{v})\sin\psi - B\cos\delta\cdot\sin\mathrm{v}\cdot\cos\psi + 2e\sin\mathrm{v}. \qquad ---(2.6)$$

The equation (2.6) is the equation of motion of a dumbbell satellite in the central gravitational field of the Earth under the influence of the Earth magnetic field. The equation determining the Lagrange's multiplier is given by:

$$(1+e\cos \mathrm{v})^4(\psi'+1)^2+(1+e\cos \mathrm{v})^3(3\cos^2\psi-1)$$
$$-B\cos\delta(1+e\cos \mathrm{v})^3[\cos\psi+e\cos(\psi+\mathrm{v})-A(1+e\cos \mathrm{v})^3(4\cos^2\psi-\sin^2\psi)]=\lambda_\alpha\geq 0. \quad \text{--}$$

(2.7)

The non-linear oscillations described by (2.6), take place as long as inequality given below is satisfied.

$$(1+e\cos \mathrm{v})^4(\psi'+1)^2+(1+e\cos \mathrm{v})^3(3\cos^2\psi-1)$$
$$-B\cos\delta(1+e\cos \mathrm{v})^3[\cos\psi+e\cos(\psi+\mathrm{v})-A(1+e\cos \mathrm{v})^3(4\cos^2\psi-\sin^2\psi)]\geq 0. \quad \text{---}$$

(2.8)

where v and e are respectively true anomaly of the centre of mass of the system and the eccentricity of the orbit of the system. The prime denotes differentiation with respect to true anomaly v. The system of equation (2.1), oscillates about the stable position of equilibrium in which it lies wholly along the radius vector joining the centre of mass and the centre of force Narayan et al [18,19,20]. Substituting $2\psi=\eta$, the equation (2.1), can be expressed as follows:

$$\eta''+3\sin\eta=4e\sin \mathrm{v}+2e\eta'\sin \mathrm{v}-5A(1+e\cos v)^2\sin\eta-\eta''e\cos \mathrm{v}$$
$$+2B\cos\delta\sin\frac{\eta}{2}+2eB\cos\delta\sin(\eta-\frac{v}{2}). \quad \text{---}$$

(2.9)

Equation (2.9) describes non-linear oscillations of the system in elliptical orbit in the central gravitational field of the Earth together with the Earth magnetic field.

3. Non-linear Non Resonance Oscillations of the system about the position of equilibrium for small eccentricity.

The non-linear oscillations of the system of cable-connected satellites under the influence of above mentioned forces described by equation (2.9), will be investigated for non resonance cases.

$$\eta''+\omega^2\eta=e\left[\beta(\eta-\sin\eta)+2\eta'\sin \mathrm{v}+4\sin v-\eta''\cos \mathrm{v}+2B\cos\delta\sin\frac{\eta}{2}-5A\sin\eta\right]$$

$$+e^2\left[10A\cos v\sin\eta + 2B\cos\delta\sin(v-\frac{\eta}{2})\right].$$

$---(3.1)$

In the equation (3.1), $\omega^2 = 3$, and $\beta = \left(\dfrac{\omega^2}{e}\right)$,Moreover the non-linearity $(\eta - \sin\eta)$, will be assumed to be the order of *e*.

The system described by equation (3.1), moves under the forced vibration due to the presence of the magnetic field of the Earth. We are benefited of the smallness of the eccentricity 'e' in equation (3.1), and hence solution may be obtained by exploiting the Bogoliubov, Krilov and Mitropolskey; method [4]. For e = 0, the generating solution of zeroth order are:

$$\eta = a\cos\theta\ ;\quad \theta = \omega \mathrm{v} + \theta^{\varphi}.$$

where the amplitude '*a*' and phase θ^{φ} are constant, which can be determined by the initial conditions. The solution of equations (3.1) is obtained in the form:

$$\eta = a\cos\theta + eu_1(a,\theta,\mathrm{v}) + e^2 u_2(a,\theta,\mathrm{v}) + ... ,$$

$---(3.2)$

where the amplitude '*a*' and phase 'θ' are determined by the differential equations .

$$\frac{da}{d\mathrm{v}} = e A_1(a) + e^2 A_2(a).........,$$

$---(3.3)$

$$\frac{d\theta}{d\mathrm{v}} = \omega + eB_1(a) + e^2 B_2(a).$$

$---(3.4)$

From (3.2), we find $\dfrac{d\eta}{d\mathrm{v}}$ and $\dfrac{d^2\eta}{d\mathrm{v}^2}$ and then substituting the value of $\eta, \dfrac{d\eta}{d\mathrm{v}}$ and $\dfrac{d^2\eta}{d\mathrm{v}^2}$, in equation (3.1), and equating the coefficients of like powers of '*e*' we get:

$$\omega^2\frac{\partial^2 u_1}{\partial\theta^2} + 2\omega\frac{\partial^2 u_1}{\partial\theta\,\partial\mathrm{v}} + \frac{\partial^2 u_1}{\partial\mathrm{v}^2} - 2\omega A_1\cos\theta - 2\omega\, B_1\sin\theta + \omega^2 u_1 =$$

$$[\ 4\sin v + 2\eta'\sin v + \beta(\eta - \sin\eta) - \eta''\cos\mathrm{v} + 2B\cos\delta\cdot\sin\left(\frac{\eta}{2}\right) + 5A\sin\eta\ ].---$$

(3.5)

$$\omega^2 \frac{\partial^2 u_2}{\partial \theta^2} + 2\omega \frac{\partial^2 u_2}{\partial \theta\, \partial \mathrm{v}} + \frac{\partial^2 u_2}{\partial \mathrm{v}^2} + \omega^2 u_2 = \left[-5A\cos \mathrm{v} \sin\eta + 2\mathrm{B}\cos\delta\cos\mathrm{v}\sin\frac{\eta}{2} - 2\mathrm{B}\cos\delta\sin\frac{\eta}{2} \right]$$

$$+ A_1 a \sin\theta \frac{\partial B_1}{\partial a} - A_1 \cos\theta \frac{\partial A_1}{\partial a} - 2\omega B_1 \sin\theta \frac{\partial^2 u_1}{\partial \theta^2} + a\cos\theta (B_1^2 + 2\omega B_2) - 2\omega A_1 \frac{\partial^2 u_1}{\partial \mathrm{a} \partial \theta}$$

$$+ 2\sin\theta (\mathrm{A}_1 B_1 + 2\omega A_2) - 2B_1 \frac{\partial^2 u_1}{\partial \theta \partial \mathrm{v}} - 2A_1\omega \frac{\partial^2 u_1}{\partial \mathrm{a} \partial v} - \beta u_1 a\cos\theta + 2\omega A_2 \sin\theta + 2a\omega B_2 \cos\theta. \quad --$$

(3.6)

Using Fourier expansion given by

$$\sin(a\cos\theta) = 2\sum_{k=0}^{\infty} (-1)^k \cdot J_{2k+1}(a) \cdot \cos(2k+1)\theta;$$

$$\cos(a\cos\theta) = J_0(a) + 2\sum_{k=0}^{\infty} (-1)^k \cdot J_{2k}(a) \cdot \cos 2k\theta.$$

$---(3.7)$

where $J_k, k = 0\,1,2,3........$ stands for Bessel's function. Substituting these values in equation (3.5) and determining $A_1(a)$ and $B_1(a)$ in such a way as $u_1(a,\theta,\mathrm{v})$, should not contain resonance terms and hence, equating the coefficients of $\sin\theta$ and $\cos\theta$ to zero, separately, we obtain:

$$A_1(a) = 0;$$

$$B_1(a) = \left[-\frac{\beta a}{2\omega} + \frac{10AJ_1(a)}{2\omega} - \frac{\beta}{\omega} J_1(a) + \frac{B\cos\delta}{\omega} J_1(\frac{a}{2}) \right].$$

$---(3.8)$

With the help of the equation (3.8) it is not difficult to obtain $u_1(a,\theta,v)$ in the form:

$$u_1(a,\theta,\mathrm{v}) = \frac{4\sin \mathrm{v}}{(\omega^2 - 1)} - \frac{3a\cos(\mathrm{v}+\theta)}{2(2\omega+1)} - \frac{a\cos(\mathrm{v}-\theta)}{2(2\omega-1)} + \frac{B}{2k(k+1)} \sum_{k=1}^{\infty} (-1)^k J_{2k+1}(a) \cos(2k+1)\theta$$

$$- \frac{5A}{2k(k+1)} \sum_{k=1}^{\infty} (-1)^k J_{2k+1}(a) \cdot \cos(2k+1)\theta - \frac{B\cos\delta}{2k(k+1)} \sum_{k=1}^{\infty} (-1)^k J_{2k+1}\left(\frac{a}{2}\right) \cdot \cos(2k+1)\theta. \quad ---$$

(3.9)

In order to obtain the second approximation of the solution, we need to determine $A_2(a)$ and $B_2(a)$, and $u_1(a,\theta,\mathrm{v})$ as obtained in (3.8), and (3.9), in equation (3.6), and equating the coefficients of $\sin\theta$ and $\cos\theta$ to zero with a view to eliminate resonance terms from $u_2(a,\theta,\mathrm{v})$, we obtain:

$$A_2(a)=0\ ;$$

$$B_2(a)=\left[-\frac{\beta^2a^2}{4\omega^3}+\frac{20A\beta J_1(a)}{4\omega^3}-\frac{2\beta aJ_1(a)}{2\omega^2}+\frac{2Ba\cos\delta}{2\omega^3}J_1(\frac{a}{2})-\frac{20A\beta J_1(a)}{2\omega^3}-\frac{20AJ_1(a)B\cos\delta}{2\omega^3}J_1(\frac{a}{2})\right].$$ ----

(3.10)

Thus, in the first approximation, the solution is given by:

$$\eta=a\cos\theta+eu_1(a,\theta,v);$$

$---(3.11)$

where the amplitude 'a' and phase 'θ' are given by:

$$\frac{da}{d\mathrm{v}}=eA_1(a)+e^2A_2(a);$$ ----

(3.12)

$$\frac{d\theta}{d\mathrm{v}}=\omega+eB_1(a)+e^2B_2(a).$$

and in the second approximation the solution is obtained as:

$$\eta=a\cos\theta+\frac{4\sin\mathrm{v}}{\left(\omega^2-1\right)}-\frac{3a\cos(\mathrm{v}+\theta)}{2(2\omega+1)}-\frac{a\cos(\mathrm{v}-\theta)}{2(2\omega-1)}+\frac{B}{2k(k+1)}\sum_{k=1}^{\infty}(-1)^kJ_{2k+1}(a)\cos(2k+1)\theta$$

$$-\frac{5A}{2k(k+1)}\sum_{k=1}^{\infty}(-1)^kJ_{2k+1}(a)\cdot\cos(2k+1)\theta-\frac{B\cos\delta}{2k(k+1)}\sum_{k=1}^{\infty}(-1)^kJ_{2k+1}\left(\frac{a}{2}\right)\cdot\cos(2k+1)\theta.$$ ---

(3.13)

where the amplitude 'a' and phase 'θ' are given by:

$\dfrac{da}{d\mathrm{v}}=0$ *i.e.*, $a=a_0$ constant

$$\frac{d\theta}{d\mathrm{v}} = \omega + e\left[-\frac{\beta a}{2\omega} + \frac{10AJ_1(a)}{2\omega} - \frac{\beta}{\omega}J_1(a) + \frac{B\cos\delta}{\omega}J_1(\frac{a}{2})\right] +$$

$$e^2\left[-\frac{\beta^2 a^2}{4\omega^3} + \frac{20A\beta J_1(a)}{4\omega^3} - \frac{2\beta a J_1(a)}{2\omega^2} + \frac{2Ba\cos\delta}{2\omega^3}J_1(\frac{a}{2}) - \frac{20A\beta J_1(a)}{2\omega^3} - \frac{20AJ_1(a)B\cos\delta}{2\omega^3}J_1(\frac{a}{2})\right]$$

(3.14)

$\theta = \omega v + \theta^*$ where θ^* is constant.

From the above solution, we concluded that amplitude 'a' remain constant up to the order of the square of the eccentricity. The phase of the oscillations of the system in this case of non-linear, non-resonance oscillation varies with respect to true anomaly. However the variation of the phase is of the order of the square of the eccentricity, which is a small quantity. We have also arrived at the conclusion that the system has main resonance at $\omega = \pm 1$, and parametric resonances at $\omega = \pm\frac{1}{2}$, $\omega = \pm\frac{1}{4}$, for these values of ω, the solution fails as we get singularity. The parametric resonance at $\omega = \pm\frac{1}{4}$ arises due to non-linearity condition.

4. Non-linear Resonance Oscillations of dumbbell satellite system about the position of equilibrium for small eccentricity.

The non-linear oscillations of the dumbbell satellite under the influence of the above mentioned forces described by (2.3), will be investigated for the resonance case on the assumption that magnetic field parameter is of the order 'e' then, equation (2.3), can be put in the form:

$$\eta'' + \omega^2\eta = e\left[\beta(\eta - \sin\eta) + 2\eta'\sin\mathrm{v} + 4\sin\nu - \eta''\cos\mathrm{v} + 2B\cos\delta\sin\frac{\eta}{2} - 5A\sin\eta\right]$$
$$+ e^2\left[10A\cos\nu\sin\eta + 2B\cos\delta\sin(\nu - \frac{\eta}{2})\right].$$

(4.1)

where $\omega^2 = 3$, and $\beta = \left(\frac{\omega^2}{e}\right)$. More over the non-linearity term $(\eta - \sin\eta)$, will be assumed to be the order of e.

The system described by equation (4.1), moves under the forced vibration due to the presence of the magnetic field of the Earth and Oblateness of the Earth. This periodic sine force of perturbative nature as long as the period of oscillations of the system is different from the period of

sine force for which solution is obtain. As the period of sine force is always changing, it may become equal to the sine force, in that case the periodic sine force plays vital role in the oscillatory motion of the system. While examining the non resonance case, we conclude that the system experience resonance behavior at and near $\omega = 1$, and hence the non-resonance solution fails. We are benefitted of the smallness of the eccentricity 'e' in equation (3.1), and hence the solution of the differential equation may be obtained by exploiting the Bogoliubov, Krilov and Metropoloskey method [4].

We construct the asymptotic solutions of the system representing (4.1), in the most general case , which is valid at and near the main resonance $\omega = 1$, exploiting the well known Bogoliubov, Krilov and Metropoloskey, method [4]. The solution of equation (4.1), in the first approximation will be sought in the form :

$$\eta = a\cos(v+\theta); \qquad \text{-----} \quad (4.2)$$

$$\frac{da}{d\text{v}} = e\,A_1(a,\theta); \qquad \text{-----} \quad (4.3)$$

$$\frac{d\theta}{d\text{v}} = (\omega - 1) + e\,B_1(a,\theta) . \qquad \text{-----} \quad (4.4)$$

where $A_1(a,\theta)$ and $B_1(a,\theta)$ are particular solution periodic with respect to 'θ' of the system.

$$(\omega - 1)\frac{\partial A_1}{\partial \theta} - 2a\omega B_1 = \frac{1}{2\pi^2}\sum_{\sigma=-\infty}^{\sigma=+\infty} e^{-i\sigma\theta}\int_0^{2\pi}\int_0^{2\pi} f(a,\eta,\eta',;\eta'')e^{-i\sigma\theta'}\cos k\, d\text{v}\, dk \ ; \qquad \text{------} \quad (4.5)$$

$$a(\omega - 1)\frac{\partial B_1}{\partial \theta} + 2\omega A_1 = -\frac{1}{2\pi^2}\sum_{\sigma=-\infty}^{\sigma=+\infty} e^{-i\sigma\theta}\int_0^{2\pi}\int_0^{2\pi} f(a,\eta,\eta',;\eta'')e^{-i\sigma\theta'}\sin k\, d\text{v}\, dk .$$

where $\theta = k - \text{v} = \theta'$ and $f(a,\eta,\eta',\eta'')$, is the coefficient of 'e' on the right hand side of equation (3.1).

Where

$$f(a,\eta,\eta',\eta'') = e[\,\beta(\eta - \sin\eta) + 2\eta'\sin\text{v} + 4\sin\text{v} - \eta''\cos\text{v} + 2B\cos\delta\cdot\sin\frac{\eta}{2} - 5A\sin\eta\ ] .$$

Simple integration gives us:

$$(\omega-1)\frac{\partial A_1}{\partial\theta}-2a\omega B_1=\beta\big(a-2J_1(a)\big)-10AJ_1(a)+4B\cos\delta\,J_1\left(\frac{a}{2}\right)-4\cos\theta;$$

$$a(\omega-1)\frac{\partial B_1}{\partial\theta}+2\omega A_1=4\sin\theta.$$ -------

(4.6)

where $J_1(a)$ is the Bessel function of the first order

$$\sin(a\cos\theta)=2\sum_{k=0}^{\infty}(-1)^k\cdot J_{2k+1}(a)\cdot\cos(2k+1)\theta;$$ --------

(4.7)

$$\cos(a\cos\theta)=J_0(a)+2\sum_{k=0}^{\infty}(-1)^k\cdot J_{2k}(a)\cdot\cos 2k\theta\,.$$

where $J_k, k=0\,1,2,3........$ stands for Bessel's function .

The periodic solution of the system given by equations (4.6) can obtained as :

$$A_1=\left[-\frac{4\sin\theta}{(\omega+1)}\right];$$ --------

(4.8)

$$B_1=-\frac{1}{2a\omega}\left[(a-2J_1(a)-10AJ_1(a)-4B\cos\delta J_1(\frac{a}{2})\right]-\frac{4\cos\theta}{a((\omega+1)}\,.$$

where the amplitude 'a' and phase 'θ' are the given by the system of differential equations :

$$\frac{da}{d\mathrm{v}}=\frac{4e\sin\theta}{(\omega+1)};$$

$$\frac{d\theta}{d\,\mathrm{v}}=(\omega-1)-\frac{e}{2a\omega}\left[a-2J_1(a)-10AJ_1(a)+4B\cos\delta\,J_1(\frac{a}{2})\right]-\frac{4e\cos\theta}{a((\omega+1)}.$$ --------

(4.9)

The system of equation can be written as :

$$\frac{\partial a}{\partial v}=\frac{1}{a}\frac{\partial H}{\partial\theta};$$

$$\frac{\partial\theta}{\partial v}=-\frac{1}{a}\frac{\partial H}{\partial a}.$$ --------

(4.10)

where

$$H = \frac{(\omega-1)a^2}{2} - \frac{ea^2}{4\omega}\left[1-\left\{\frac{2+10A}{2}+\frac{4B\cos\delta}{4}\right\}\right] - \frac{ea^4}{8\omega}\left[\frac{2+10A}{16}-\frac{4B\cos\delta}{256}\right] - \frac{4ae\cos\theta}{(\omega+1)}.$$

---- (4.11)

Obviously, the system of the equation (4.10), has first integral of the form :

$$H = c_0' \quad ;$$

---- (4.12)

which reduces the problem to quadrature. Here c_0' , is the constant of integration. However, it is preferable to analyse the integral curves in the phase plane (a,θ). In order to plot the integral curves reducing the equation (4.11), in the form :

$$\frac{e(\omega+1)}{8\omega}\left[\frac{2+10A}{16}-\frac{4B\cos\delta}{256}\right]a^4 + \left[\frac{(\omega^2-1)}{2} - \frac{e(\omega+1)}{4\omega}\left\{1-\left(\frac{2+10A}{2}+\frac{4B\cos\delta}{4}\right)\right\}\right]a^2 + 4ea\cos\theta + C_0 = 0 \quad .$$

---- (4.13)

where $C_0 = (n+1)C_0'$.

---- (4.14)

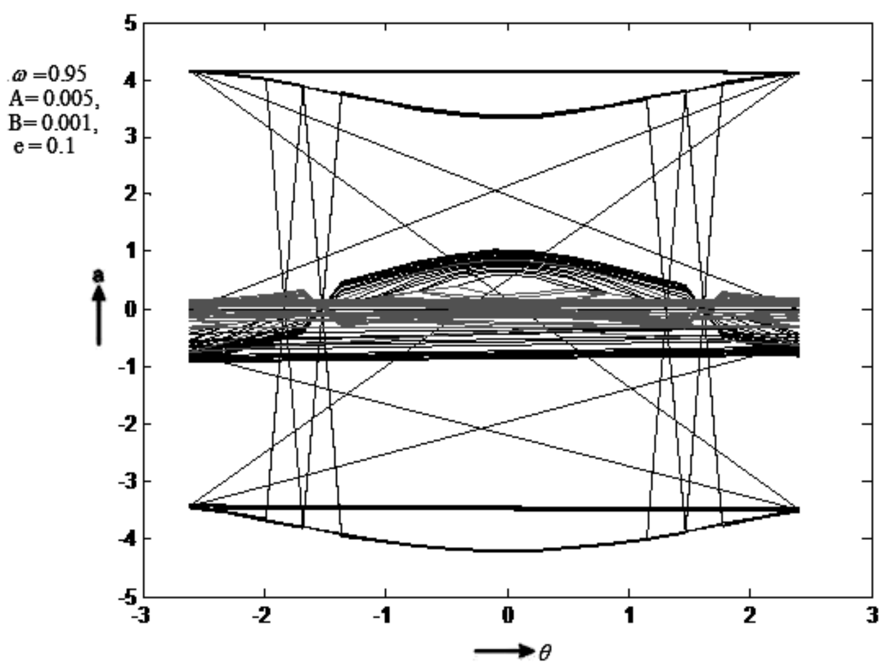

Fig. 3: Oscillation of the Dumbell satellite in elliptical orbit

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The integral curves (4.13), have been plotted in Figure (3), for ω=0.95, e =.1, A=0.005 and B=0.001. The integral curves drawn in the phase plane (a,θ), using MATLAB software 6.1 version. It is clearly indicated that there exists only one stationary regime of the amplitude and it is stable as the integral curves are closed curves.

For any other initial condition we shall obtain periodic change in the amplitude 'a' , which will be bounded. But the maximum value of 'a' in this case will always be greater than its value at the stationary regime.

Therefore, for the gravity gradient stablisation of such a space system in elliptical orbit, we are required to bring the amplitude of oscillations near the stationary regime, which gives the smallest deflection of the system from the relative equilibrium position in comparison to any other regime of oscillations.

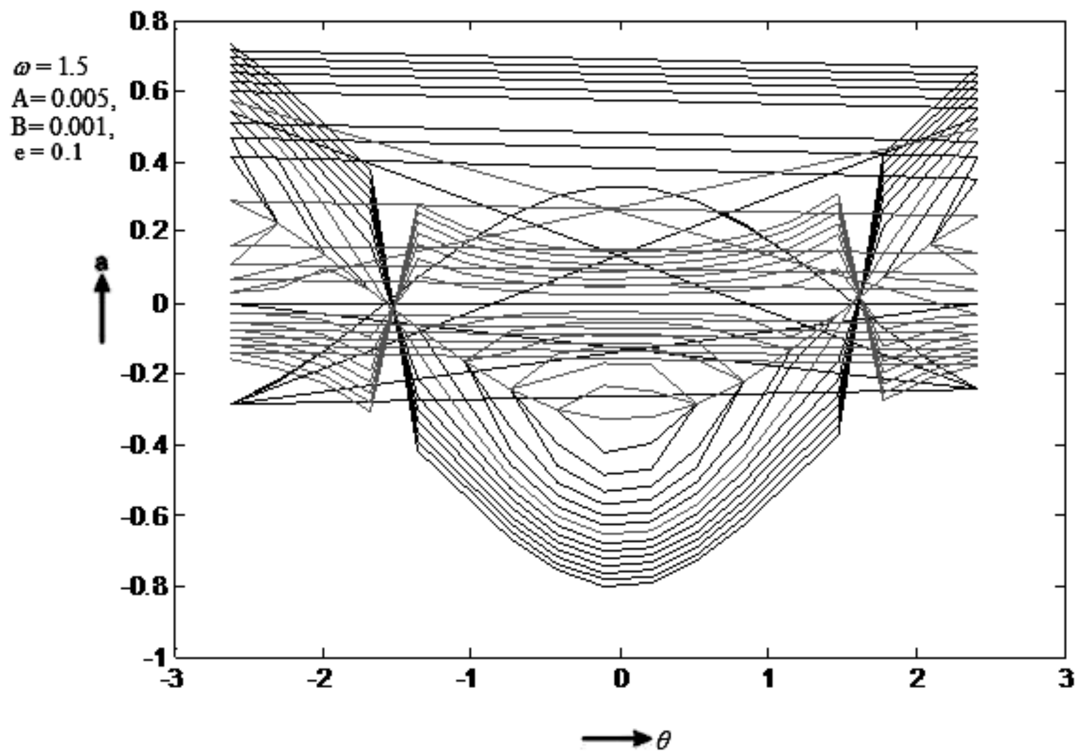

Fig. 4: Oscillations of the Dumbell satellite in elliptical orbit

In the Figure (4), the integral curves for ω= 1.5 , e =.01, A = 0.005 and B = 0.001 has been plotted and it is found that the stationary regime of the amplitude 'a' exists. In this case also there exists only one stationary regime with a slight change in its position. Minute observation of the signature (3) and (4) suggested to conclude that during the evolution of oscillations of the system as it approaches the main resonance, the stationary amplitude declines steadily with continuously changing phase.

5. Discussion

We have discussed that the combined effects of the Earth Oblateness and the magnetic field of the Earth on the evolutional and non-evolutional motion of cable connected satellites system, connected by a light, flexible and inextensible cable in the central gravitational field of the Earth for the elliptical orbit of centre of mass of the system. The satellites are considered as charge material particle. The motion of each of their relative to the centre of mass has been studied. It is assumed that centre of mass moves along Keplerian orbit around oblate Earth in elliptical orbit. It is further assumed that satellites are subjected to absolutely non elastic impacts as the cable tightened. Throughout our analysis, we assumed that the system moves like a 81dumbbell satellite. We further discussed the non-linear non-resonant and resonant oscillations of 81dumbbell satellite about the equilibrium point of the system in elliptical orbit. The equation of motion have been derived in the required form. The non-resonant oscillations of the problem has been studied with the help of Bogoliubov, Krilov and Metropoloskey, method when the eccentricity 'e' of the orbit of centre of mass has been taken as the small parameter for the solution of the system.

6. Conclusion

We arrived at the conclusion that the amplitude of oscillations of the system remains constant up to the second order of the approximation. The phase of oscillations varies with respect to true anomaly, but the rate of change is the function of the square of eccentricity of the orbit of the centre of mass. We also come to the conclusion that the system experience main resonance at $\omega = \pm 1$ and parametric resonances at $\omega = \pm \frac{1}{2}$ $, \omega = \pm \frac{1}{4}$, up to the second order of approximation. We also obtained the general solution of the non-linear oscillatory system based upon Bogoliubov, Krilov and Metropoloskey, method which is valid at near of the main resonance $\omega = 1$. The method of phase plane has been applied in order to obtain characteristic of the amplitude and the phase of the oscillatory system.

We come to the conclusion that there exists only one stationary stable regime of the amplitude for both $\omega < 1$ and $\omega > 1$.

However the stationary amplitude declines steadily, when it passes through two values of ω given by $\omega < 1$ and $\omega > 1$.

It has also been established that the system will always move like a dumbbell satellite in the phase

plane (a, θ) under consideration.

Thus, the oblateness of the Earth and magnetic field of the Earth will play important role in disturbing the attitude of the system of a dumbell satellite in elliptical orbit

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