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Chemical Reaction Effect on MHD Jeffery Fluid over a Stretching Sheet with Heat Generation/Absorption

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Abstract

The combined effect of heat and mass transfer in Jeffrey fluid over a stretching sheet subject to transverse magnetic field in presence of heat source/sink has been studied in this paper. The surface temperature and concentration are assumed to be very similar to the power law form. The method of solution involves similarity transformation. The coupled non-linear partial differential equations representing momentum and concentration and non homogeneous heat equation are reduced into set of non-linear ordinary differential equations. The transformed equations are solved by applying Kummer's function. The novelty of the present study is to discus the effect of Lorentz force on the veleocity and temperature profile. It is interesting to note that the velocity increases with an increase of Deborah number. The effect of pertinent parameters characterizing the flow has been presented through the graph.

Keywords: MHD flow /Visco-elastic /Mass and Heat transfer/ Porous medium/ Chemical Reaction/ Kummer's function.

1. Introduction

Interest in the boundary layer flows of non-Newtonian fluids has increased due to the applications in science and engineering including thermal oil recovery, food and slurry transportation, polymer and food processing etc. A variety of non-Newtonian fluid models have been proposed in the literature keeping in view of their several rheological features. In these fluids, the constitutive relationships between stress and rate of strain are much complicated in comparison to the Navier-Stokes equations. There is one subclass of non-Newtonian fluids known as Jeffrey fluid [1−3] which has been attracted by many of the researchers in view of its simplicity. This fluid model is capable of describing the characteristics of relaxation and retardation times.

MHD flow with heat and mass transfer has been a subject of interest of many researchers because of its varied application in science and technology. Such phenomenon are observed in buoyancy induced motions in the atmosphere, in water bodies, quasi –solid bodies such as earth, etc. In natural processes and industrial applications many transportation processes exist where transfer of heat and mass takes place simultaneously as a result of thermal diffusion and diffusion of chemical species.

Literature survey indicates that interest in the flows over a stretched surface has grown during the past few decades. Sakiadis[4] performed the first study for the flow induced by a moving surface. The flow generated by a linear stretched sheet is examined by Crane[5]. Chamkha [6] studied the MHD flow of uniformly stretched vertical permeable surface in the presence of heat generation/absorption and a chemical reaction. The effect of temperature –dependent viscosity on mixed convection flow from vertical plate is investigated by several authors (A. Hossain, S. Munir [7] and A. A. Mustafa [8]). Ishak *et al* [9] investigated theoretically the unsteady mixed convection boundary layer flow and heat transfer due to a stretching vertical surface in a quiescent viscous and incompressible fluid. Mahapatra and Gupta ([10],[11]) considered the stagnation flow on a stretching sheet. Sammer [12] investigated the heat and mass transfer over an accelerating surface with heat source in presence of magnetic field. Wang [13] studied the stagnation flow towards a shrinking sheet.

Hayat et al. [14] applied an analytic technique, namely homotopy analysis method (HAM), to study the steady mixed convection in two dimensional stagnation flows of a second grade fluid around a heated surface with the wall temperature varying linearly with the distance from the stagnation point. An investigation has been conducted by Arnold et al. [15] on the visco-elastic fluid flow and heat transfer characteristics over a stretching sheet taking into account the effects of frictional heating and internal heat generation/absorption. Bataller [16] investigated the effect of thermal radiation on heat transfer in a boundary layer visco-elastic second order fluid over a stretching sheet with internal heat source/sink. Recently, we explore the flow of a Jeffery fluid [17-18] over a stretched sheet subject to power law temperature in the presence of heat source/sink.

2. Formulation of the problem

Let us consider for a particular type of visco-elastic fluid called Jeffrey fluid given by Nadeem and Akbar [19]:

$$T = -pI + \frac{\mu}{1+\lambda}\left(R_1 + \lambda_1\left(\frac{\partial R_1}{\partial t} + V.\nabla\right)R_1\right), \tag{1}$$

where T ,the Cauchy stress tensor, p , the pressure μ , the viscosity, λ and λ_1 are the material parameters of Jeffrey fluid and R_1 is the Rivlin-Ericken tensor [20]defined by

$R_1 = \nabla v + (\nabla v)^t$

The steady two dimensional flow of an electrically conducting visco-elastic fluid like Jeffrey fluid over a stretching sheet in presence of heat source/sink has been considered. The effect of chemical reaction has also been considered in this paper. The sheet in $xz-$ plane is stretched in the $x-$ direction such that the velocity component in the $x-$ direction varies linearly along it. The goverening boundary layer equations are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\upsilon}{1+\lambda}\left(\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0{}^2 u}{\rho} + \frac{k_0}{\rho}\left(u\frac{\partial}{\partial x}\left(\frac{\partial^2 u}{\partial y^2}\right) + v\frac{\partial^3 u}{\partial y^3} + \frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x \partial y} - \frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2}\right)\right) \tag{2}$$

$$\rho C_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = K\frac{\partial^2 T}{\partial y^2} + Q(T - T_\infty) \tag{3}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2} - k_c^*(C - C_\infty) \tag{4}$$

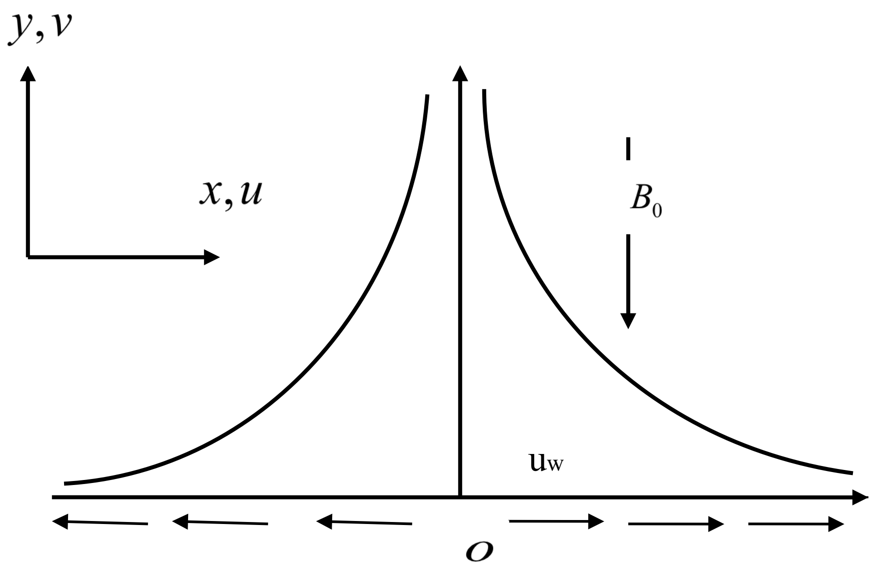

Fig.1 Physical model and co-ordinate system

The corresponding boundary conditions are:

$$\left.\begin{array}{l} u = U(x) = cx,\ v = 0, T = T_w = T_\infty + A_1\left(\dfrac{x}{l}\right)^m, C = C_w = C_\infty + A_2\left(\dfrac{x}{l}\right)^m, \quad at \quad y = 0 \\ u \to \infty,\ T \to T_\infty,\ C \to C_\infty,\ as \quad y \to \infty \end{array}\right\} \quad (5)$$

3. Solution of the flow field

Equations (1) is satisfied if we chose a dimensionless stream function

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x}\ . \quad (6)$$

Introducing the similarity transformations

$$\eta = y\sqrt{\frac{c}{\upsilon}}, \quad \psi(x, y) = x\sqrt{\upsilon c}\, f(\eta) \quad (7)$$

and substituting in (2), we get

$$f''' + (1+\lambda)\left(ff'' - f'^2\right) - R_c\left\{ff'''' - f''^2\right\} - Mf' = 0, \quad (8)$$

where f is the dimensionless stream function and η is the similarity variable. $R_c = k_0 c / \mu$, the Deborah number and $M = \sigma B_0^{\ 2} / \rho c$, the magnetic parameter.

The corresponding boundary conditions are:

$$f(0) = 0,\ f'(0) = 1,\ f'(\infty) = 0, f''(\infty) = 0. \quad (9)$$

The exact solution (8) with boundary conditions (9) is obtained as follows

$$f(\eta)=\frac{1-e^{-\alpha\eta}}{\alpha},\ \alpha>0 \tag{10}$$

where $\alpha=\sqrt{\dfrac{1+\lambda+M}{1+R_c}}$ (11)

4. Heat Transfer Analysis

Introducing non-dimensional quantities $\theta(\eta)=\dfrac{T-T_\infty}{T_w-T_\infty}$, $\mathrm{P_r}=\rho C_p/K$, $\gamma=\dfrac{Qv}{\rho C_p}$ and using equation (7) the equation (3) becomes

$$\theta''+P_r f\theta'+P_r(\gamma-mf')\theta=0 \tag{12}$$

With the boundary conditions

$$\theta(0)=1,\theta(\infty)=0. \tag{13}$$

Introducing the variable $\xi=\dfrac{P_r e^{-\alpha\eta}}{\alpha^2}$ the equation (12) transformed to

$$\xi\frac{d^2\theta}{d\xi^2}+(1-\frac{P_r}{\alpha^2}+\xi)\frac{d\theta}{d\xi}-\left(m-\frac{\gamma}{\alpha^2\xi}\right)\theta=0 \tag{14}$$

with the boundary conditions

$$\theta\left(\xi=\frac{P_r}{\alpha^2}\right)=1,\ \theta(\xi=0)=0 \tag{15}$$

Using confluent hypergeometric function (Kummer's function) we get,

$$\theta(\xi)=\left(\frac{\alpha^2\xi}{\mathrm{P_r}}\right)^{a+b}\frac{{}_1F_1\left(a+b-m,1+2a,-\xi\right)}{{}_1F_1\left(a+b-m,1+2a,-\mathrm{P_r}/\alpha^2\right)} \tag{16}$$

where, $a=\mathrm{P_r}/2\alpha^2, b=\sqrt{\left(\mathrm{P_r}\right)^2-4\alpha^2\gamma}/2\alpha^2$ and ${}_1F_1(\alpha_1,\alpha_2;x)$ is the Kummer's function defined by ${}_1F_1(\alpha_1,\alpha_2;x)=1+\sum_{n=1}^{\infty}\dfrac{\left(\alpha_1\right)_n}{\left(\alpha_2\right)_n}\dfrac{x^n}{n!},\ \alpha_2=-1,-2.....$.

Here $\left(\alpha_1\right)_n$ and $\left(\alpha_2\right)_n$ are the Pochhammer's symbols defined as

$$\left(\alpha_1\right)_n=\alpha_1(\alpha_1+1)(\alpha_1+2)............(\alpha_1+n-1)$$
$$\left(\alpha_2\right)_n=\alpha_2(\alpha_2+1)(\alpha_2+2)............(\alpha_2+n-1)$$

(17)

In terms of variable η equation (16) can be written as

$$\theta(\eta)=\frac{{}_1F_1(a+b-m,2a+1,-P_r/\alpha^2e^{-\alpha\eta})}{{}_1F_1(a+b-m,2a+1,-P_r/\alpha^2)}e^{-\alpha(a+b)\eta} \quad (18)$$

5. Mass Transfer Analysis

Introducing the similarity variable $\varphi(\eta)=\dfrac{C-C_\infty}{C_p-C_\infty}$, and using (7), in equation (4) we get,

$$\varphi''+S_c f\varphi'-S_c(mf'-k_c)\varphi=0 \quad (19)$$

With the boundary conditions

$$\begin{aligned}&\varphi'=1 \quad at \quad \eta=0\\&\varphi\to 0 \quad at \quad \eta\to\infty\end{aligned} \quad (20)$$

Again introducing a new variable $\zeta=-\dfrac{S_c}{\alpha^2}e^{-\alpha\eta}$, the equation (27) becomes

$$\zeta\frac{d^2\varphi}{d^2\zeta}+\left[1-\frac{S_c}{\alpha^2}+\zeta\right]\frac{d\varphi}{d\zeta}-\left(m-\frac{k_c}{\alpha^2\zeta}\right)\varphi=0$$

(21)

The corresponding boundary conditions are

$$\varphi(\zeta=0)=0,\varphi'(\zeta=\frac{S_c}{\alpha^2})=1 \quad (22)$$

The exact solution of equation (21) using the boundary condition (22) is

$$\varphi(\eta)=\frac{{}_1F_1(S_c/\alpha^2-m,2S_c/\alpha^2+1,-S_c/\alpha^2e^{-\alpha\eta})}{{}_1F_1(S_c/\alpha^2-m,2S_c/\alpha^2+1,-S_c/\alpha^2)}e^{-(S_c/\alpha)\eta} \quad (23)$$

6. Results and Discussion

The present study considers the flow of a visco-elastic incompressible electrically conducting fluid flow past a stretching sheet in the presence of magnetic field, uniform heat source/sink, surface fluid suction/injection and chemical reaction. The aim of the following discussion is to bring out the effect of plate temperature and chemical reaction on the flow phenomena. The heat generation/absorption contributes significantly for non-isothermal heat transfer case. In this section, the influence of emerging parameters on the velocity, temperature and concentration fields are studied.

Fig. 2 reveals the effect of visco-elastic parameter on longitudinal velocity profile for $\lambda=1.0$ in both absence/presence of magnetic parameter. Velocity field is asymptotic in nature. In both case i.e. absence of magnetic parameter ($M=0$) and the presence of magnetic parameter ($M = 1.0$) the velocity field and boundary layer thickness increases with increase in the value of R_c .

The effect of λ i.e. the ratio of relaxation and retardation time parameter on the longitudinal velocity profile in both the medium is illustrated in Fig. 3 for a Jeffery fluid with $R_c = 1$. it is seen that λ reduces the boundary layer thickness causes a decrease in velocity in both absence / presence of magnetic field.

The effect of visco-elastic parameter R_c on the transverse velocity profile is shown in Fig. 4 for $\lambda = 1.0$. It is clear that $R_c = 0$, the viscous fluid, $R_c > 0$ stands for Jeffery fluid.. As compared to the viscous fluid ($R_c = 0$), the velocity decreases with increase in magnetic parameter. The similar trends of the velocity field is also remarked in case of Jeffery fluid, $R_c > 0$.

The effect of λ i.e. the ratio of relaxation and retardation time parameter on the transverse velocity profile in both the medium is illustrated in Fig. 5 for a Jeffery fluid with $R_c = 1$. It is observed that $\lambda = 0$ reduces the boundary layer thickness as M increases and also causes a decrease in velocity in both absence / presence of magnetic field for $\lambda > 0$.

Fig.6 exhibits the effect of Magnetic parameter on the temperature profile with the fixed values of the parameters as $\mathrm{P_r} = 2.0, \gamma = 0.1, r = 2.0, \lambda = 1.0, R_c = 1.0$. A remarkable increase in the profile is observed as the magnetic parameter increases gradually. The thermal boundary layer is asymptotic in nature. In the absence of magnetic parameter ($M = 0$), the thermal boundary layer lower down the temperature profile. Hence it is conclude that Lorentz force accelerate the temperature profile at all points.

Fig.7 exhibit the variation of R_c on temperature profile keeping $P_r = 2, M = 2, \gamma = 0.1, r = 2, \lambda = 1$ as fixed. It is observed that an decrease in temperature due to the presence of elastic elements may be attributed to the fact that when a visco-elastic fluid is in flow, a certain amount of energy is depicted from the material as strain energy. In comparison to the viscous fluid ($R_c = 0$) and in case of Jeffery fluid, $R_c > 0$, the temperature decreases with increase in magnetic parameter. From Fig. 7 it is seen that the thermal boundary layer is asymptotic in nature.

The effect of λ i.e. the ratio of relaxation and retardation time parameter on the temperature profile is illustrated in Fig. 8 for a Jeffery fluid with $R_c = 1$, $\mathrm{P_r} = 2.0, \gamma = 0.1,\ r = 2.0$ and $M = 2.0$. It is seen that decreasing value of λ reduces the thermal boundary layer thickness causes a decrease in temperature in both presence of magnetic field.

The variation of Prandtl number on the temperature profile is shown in Fig. 9. with the fixed values of $\gamma = 0.1, r = 2.0, R_c = 1.0, \lambda = 1.0$ and $M = 2.0$ .The temperature field decreases with the increasing value of P_r. It is obvious that an increase in the values of P_r reduces the thermal diffusivity therefore, the thermal boundary layer thickness is decreasing in nature.

The influence of heat generation / absorption parameter on the dimensionless temperature profile can be seen in Fig. 10. With the fixed values of the other parameter it is noticed that a gradual increase in heat source parameter increases the thermal boundary layer thickness whereas the reverse trend occurs in presence of sink. Physically it reveals the fact that the thermal boundary layer leads to a higher temperature field with increase in heat source / sink.

The surface temperature parameter r has a significant role in the temperature profile is shown in Fig.11. Keeping the parameter $P_r = 0.71, \gamma = 0.1, R_c = 1.0, \lambda = 1.0$ and $M = 2.0$ as fixed it is observed that the thermal boundary layer decreases as the surface temperature parameter increases.

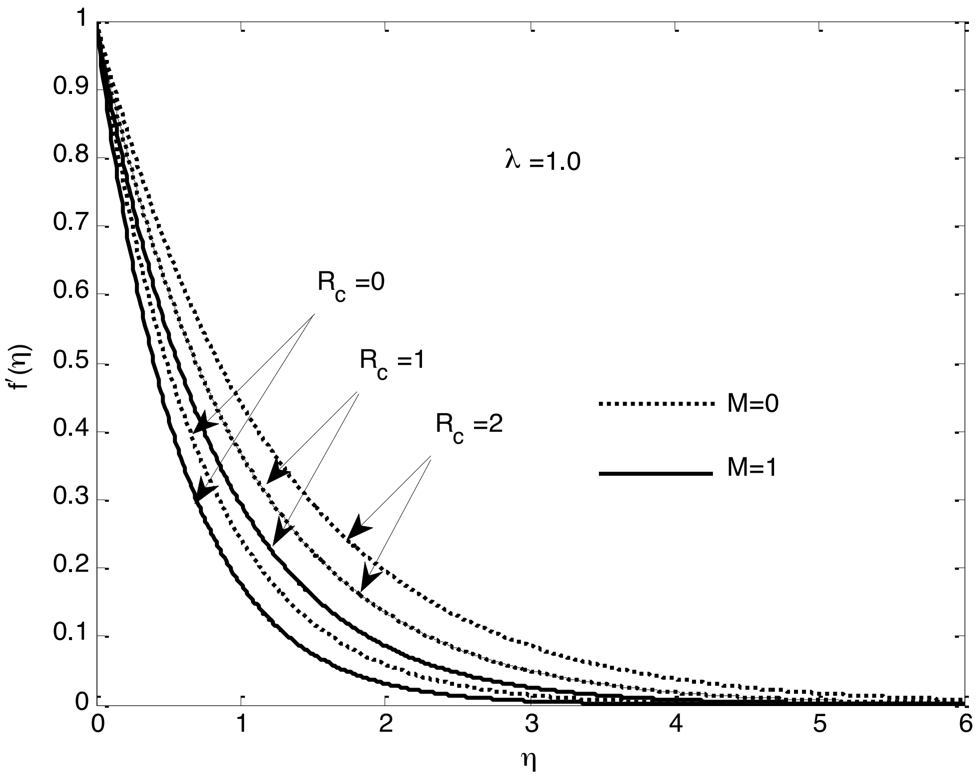

Fig.2 Effect of R_c and M on longitudinal velocity

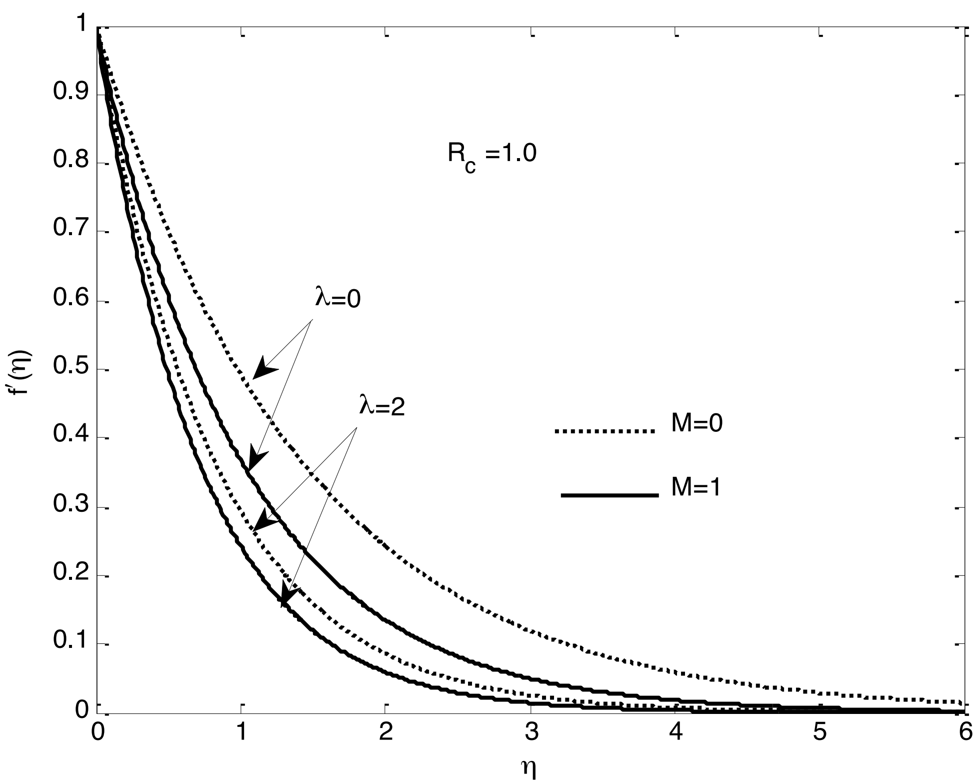

Fig.3 Effect of λ on longitudinal velocity

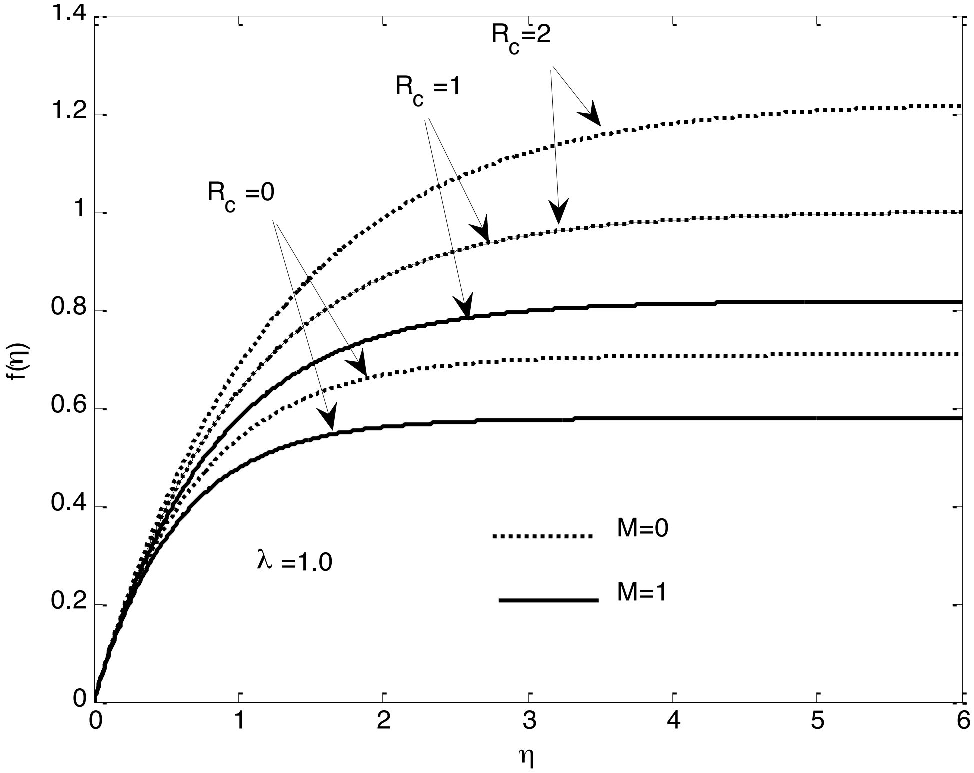

Fig.4 Effect of R_c and M on transverse velocity

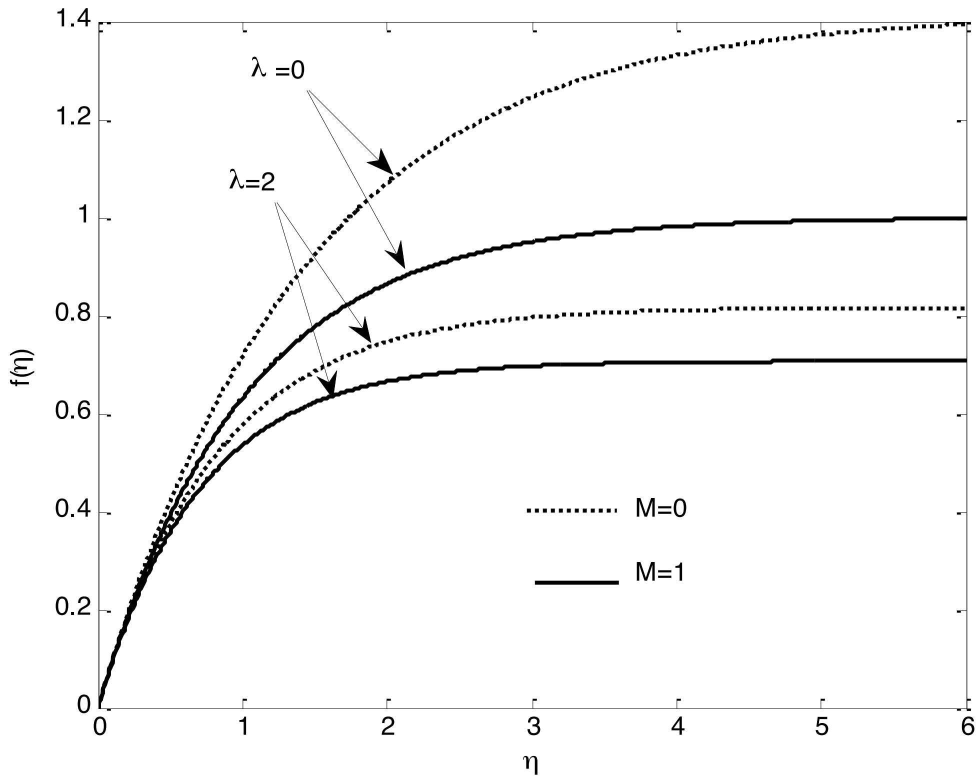

Fig.5 Effect of λ on transverse velocity

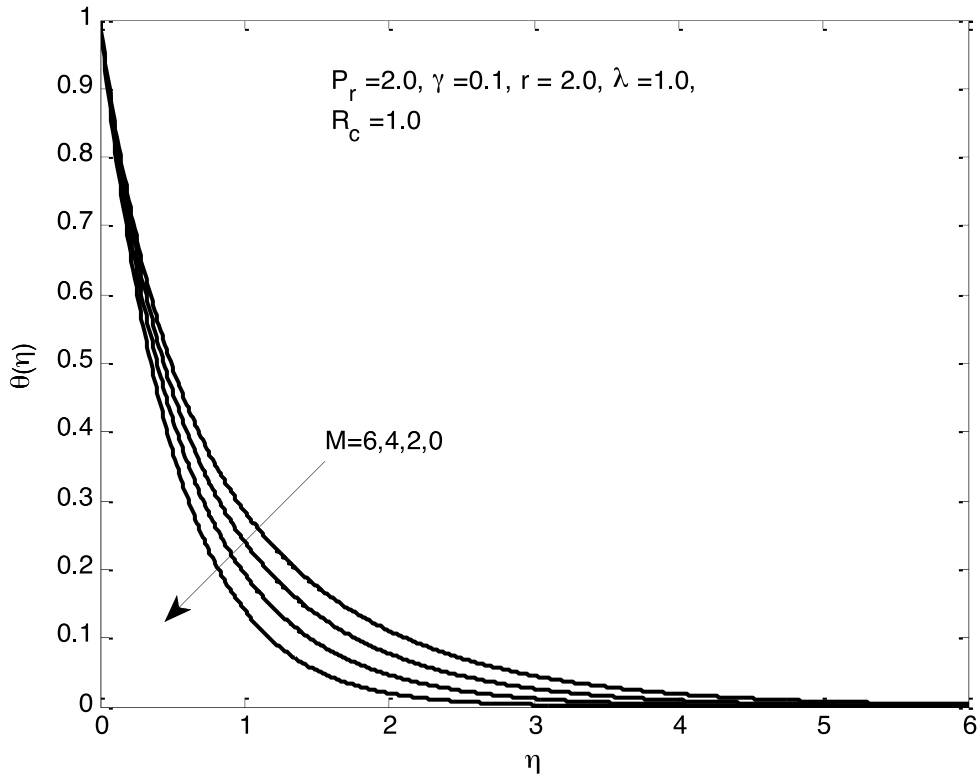

Fig. 6 Effect of M on temperature profile

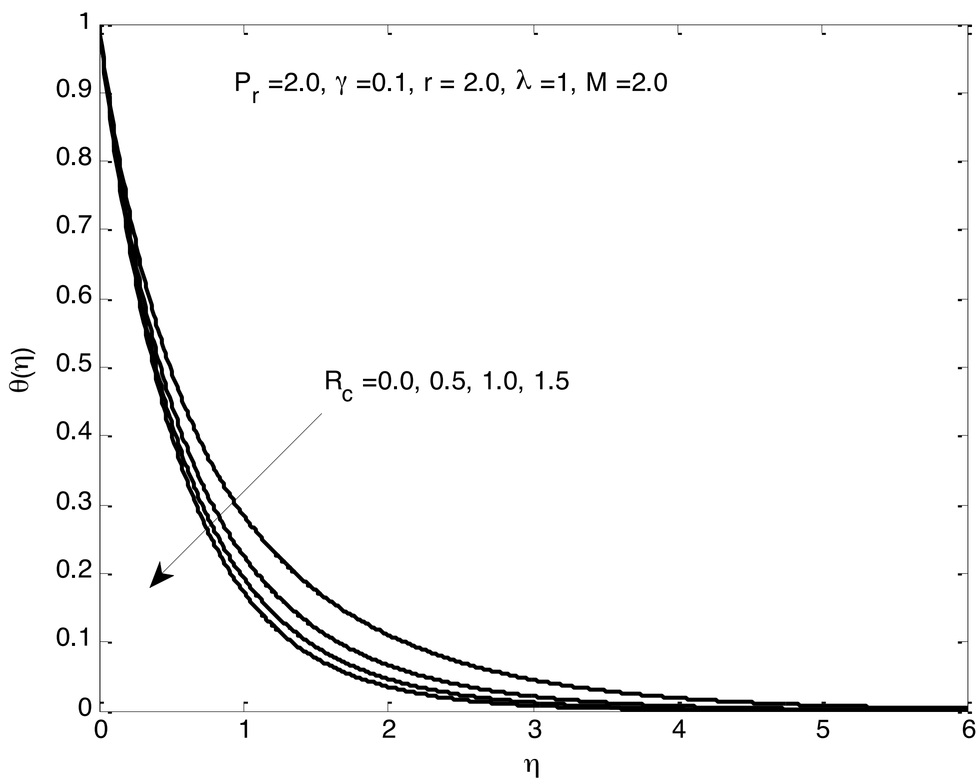

Fig. 7 Effect of R_c on temperature profile

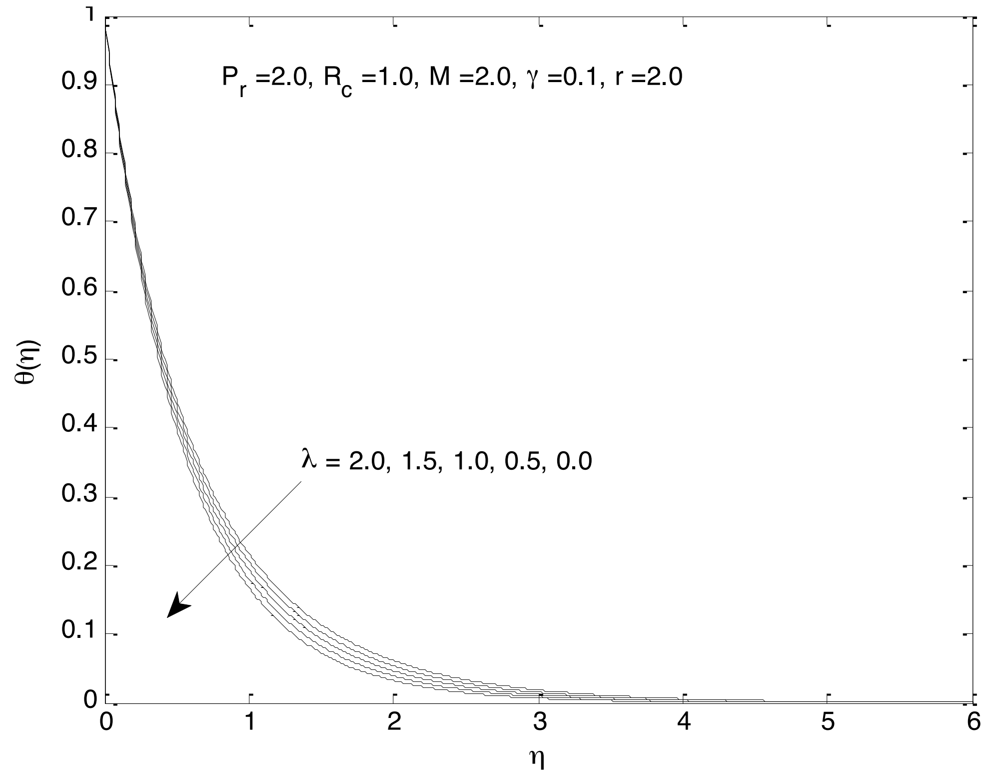

Fig. 8 Effect of λ on temperature profile

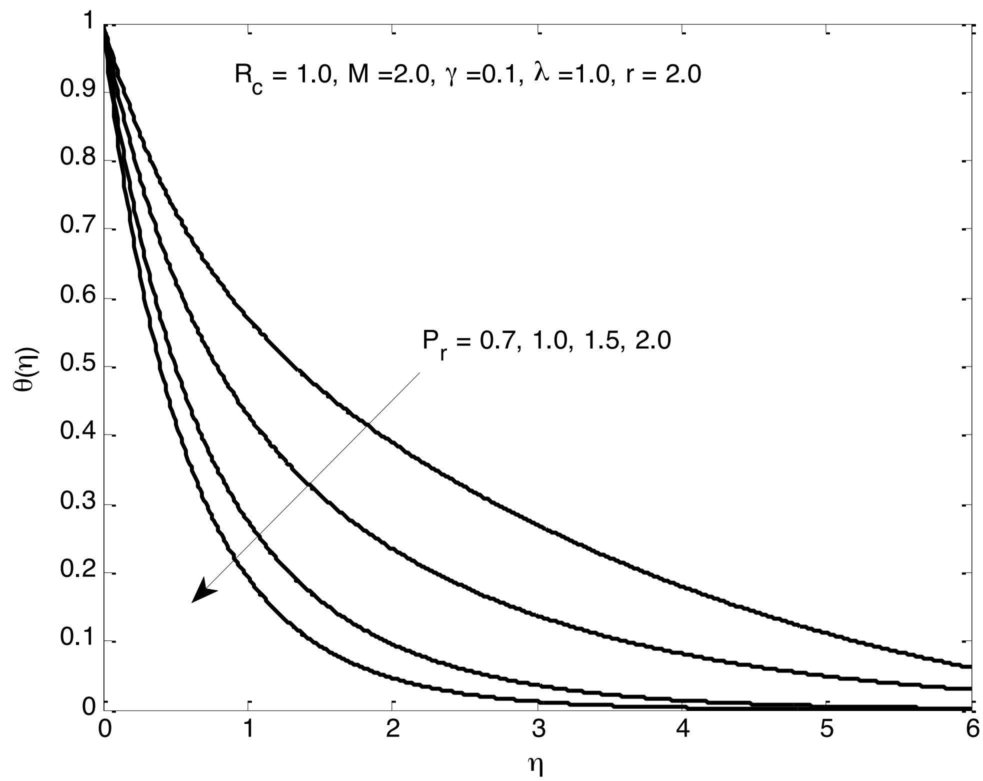

Fig. 9 Effect of P_r on temperature profile

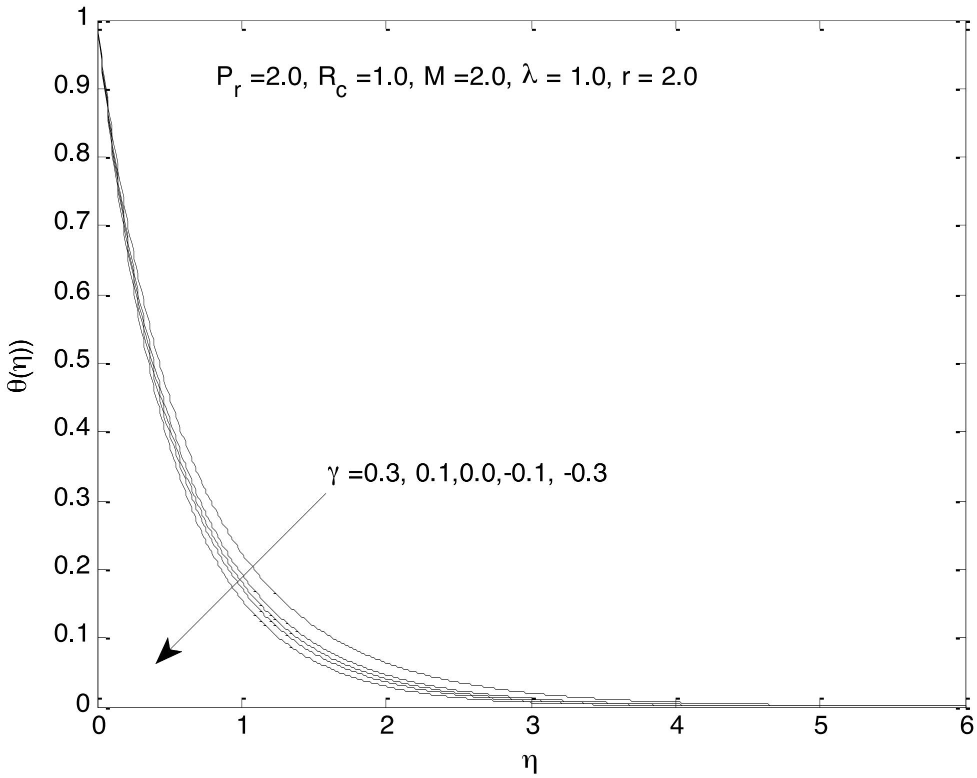

Fig. 10 Effect of γ on temperature profile

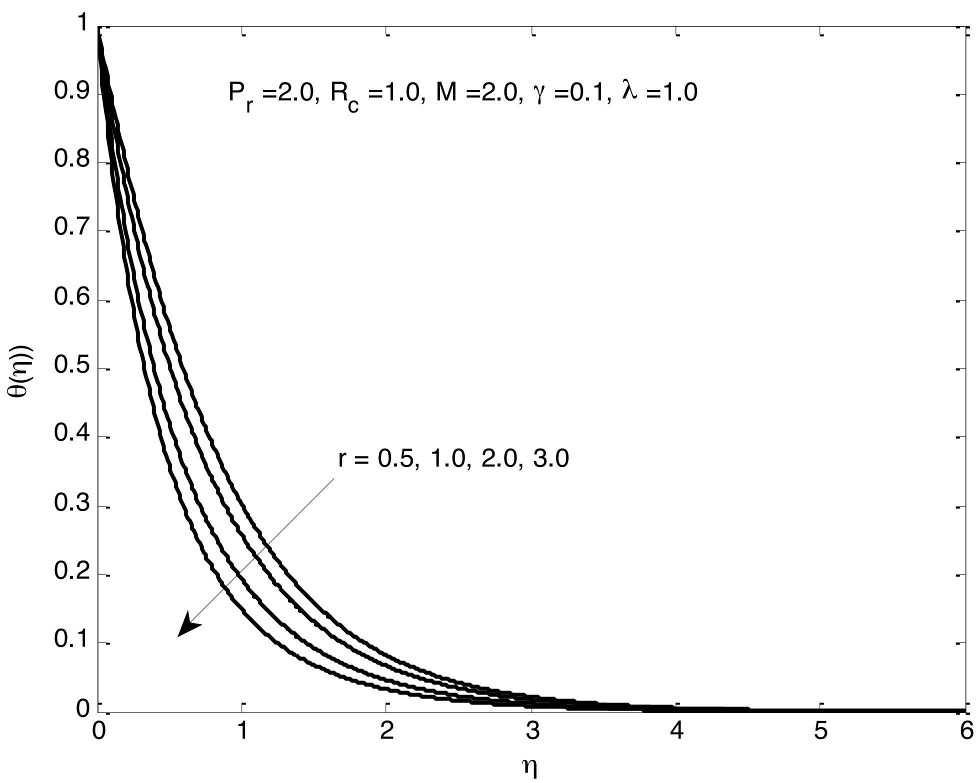

Fig. 11 Effect of r on temperature profile

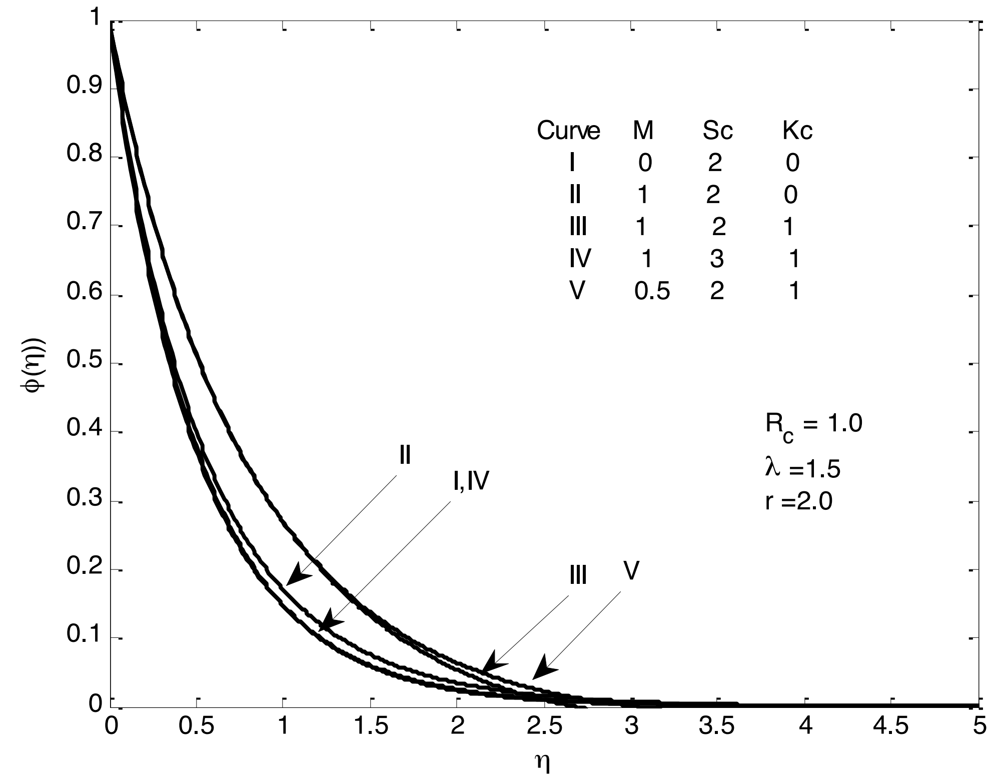

Fig. 12 Influence of S_c on Concentration profile

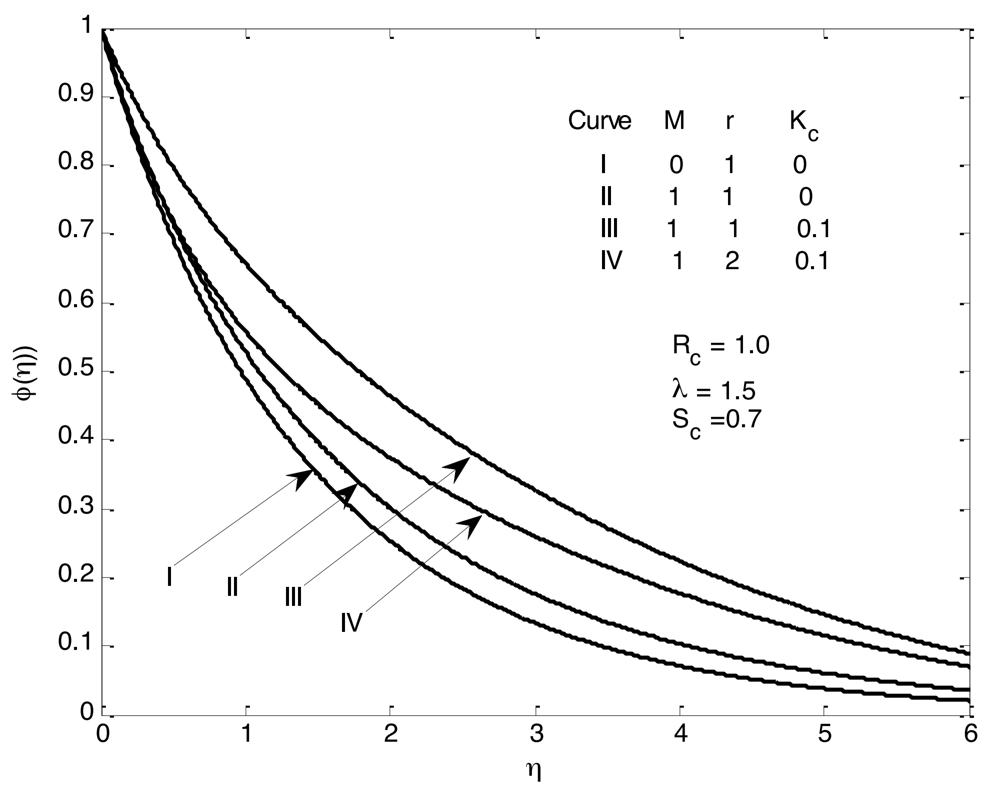

Fig.13 Influence of r on concentration profile

Fig.12 exhibits the influence of Schimdt number on the concentration profile in the absence / presence of magnetic parameter and the chemical reaction parameter with fixed values of $R_c = 1.0, \lambda = 1.5, r = 2.0$. It is observed that the absence of both M and K_c lower down the concentration profile. The increasing value of M and K_c increases the concentration level at all points but the reverse effect is observed with the increase of S_c.

The surface temperature parameter r has a significant role on the concentration profile is shown in Fig.13. It is noticed that in the absence of M, K_c and the presence of surface temperature parameter with fixed values of $R_c = 1.0, \lambda = 1.5, S_c = 2.0$ lower down the concentration profile. The reverse effect is shown as the surface temperature parameter increases. Therefore, the higher value of surface temperature parameter enhance the profile at all points with the increasing value of magnetic parameter and presence of chemical reaction parameter.

7. Conclusion

- Presence of magnetic parameter the velocity field and boundary layer thickness increases with increase in the value of visco-elastic parameter.
- Further, absence of magnetic field leads to transitory motion of the fluid.
- Presence of magnetic field also leads to increase the temperature at all points but the presence of elasticity reduce it.
- The thinning of thermal boundary layer thickness is due to slow rate thermal diffusion in presence of magnetic field.
- Magnetic parameter in case of heavier species enhance the concentration level in the presence of chemical reaction.
- But for higher value of magnetic enhance the concentration level.

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