Modelling, Measurement and Control A
Vol. 91, No. 1, March, 2018, pp. 22-31
Journal homepage: http://iieta.org/Journals/MMC/MMC_A

Evaluation of block-oriented models use for the purpose of robust controllers synthesis

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Received: 7 January 2018
Accepted: 17 April 2018

Keywords:
system identification, Hammerstein model, Wiener model, block-oriented models, robust control, linear matrix inequalities

ABSTRACT

A comparative study of the use of block-oriented and autoregressive with exogenous inputs (ARX) models in the context of robust controller synthesis is presented in this paper. Parameters uncertainties of the identified models are taken into account in the synthesis of the controllers by state feedback, which aim to assure the maximum attenuation of H_2 and H_∞ costs. The study consists in evaluating the effects of the variation in the models order and respective estimated deviation in their parameters in the synthesis of robust controllers. The models were obtained from an air heating system with nonlinear dynamics and controllers were designed by means of convex optimization procedures in the form of linear matrix inequalities. The results obtained point to the preferential use of low order Wiener or Hammerstein models, even if they have RMSE and correlation coefficient between simulation error and simulated output indexes worse than higher order models.

1. INTRODUCTION

Uncertainties are inherent to systems identification, since a series of aspects such as variations in parameters, unmodeled and/or neglected dynamics, transport delays not included in the model, changes in the break-even point (operating point), sensor noises and unforeseen disturbance inputs are preponderant factors so that the model of the process to be controlled is always an inaccurate representation of the real physical system [8]. In view of this, it is called robust control the area of science responsible for the development of techniques of analysis and design of control systems that offer guarantees of stability and/or performance against uncertainties of the model [19].

In the context of control systems, the use of convex optimization methods has made Lyapunov's approach to stability analysis and controller synthesis for uncertain systems even more popular [5]. This methodology uses linear representations of the process to be controlled, in the form of state space, and performs controller design through linear matrix inequalities, aiming to stabilize and/or meet system performance requirements throughout the domain of uncertainties.

However, Aguirre [1] states that the dynamical systems found in practice are ultimately nonlinear; and that although in some cases linear approximations are sufficient to model real plants, in many industrial applications linear models are not satisfactory and nonlinear representations should be used. Linear model structures can be used when physical system remains in the vicinity of a nominal operating point, so that linearity assumption is satisfied. However, when a wide range of operating points is involved, linear assumption may not be valid and a nonlinear model structure becomes necessary to capture dynamic behavior of the system. In addition, it is necessary to consider the energy cost that linear representation of the system can cause to the controller [17], as well as the influence of such approximation in the dimension of set of uncertainties considered, regarding the application of techniques of robust control [9].

Thus, the following question arises: if non-linear models are used in the identification of the process to be controlled, making its mathematical representation closer to actual behavior of the system, how this better representation reflects in performance indexes of the controller to be designed?

In this sense, the development of robust controllers based on Hammerstein models (Figure 1) - which consist of the interaction of linear time invariant (LTI) dynamic subsystems and static nonlinear elements, being that in this class of models static nonlinearity precedes the block linear dynamics - and Wiener models - obtained from the permutation of linear and nonlinear elements in Hammerstein model, as shown in Figure 2 - can be seen in [3-4, 11] that demonstrated in their work that the representation of non-linear processes through Hammerstein and Wiener models for application of robust control strategies can reduce the computational complexity in comparison to the implementation of conventional robust predictive controllers [18]. In this context, this paper presents a comparative study of the use of interconnected block models and autoregressive linear models with exogenous inputs (ARX) for the synthesis of robust controllers. The uncertainties in parameters of identified models are taken into account in the synthesis of the controllers - projected by means of convex optimization procedures in the form of linear matrix inequalities - by state feedback, which aim to assure the maximum attenuation of H_2 and H_∞ costs.

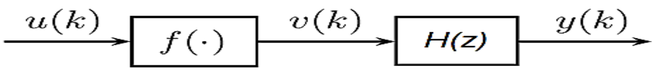

Figure 1. Hammerstein model. Source: adapted from Ribeiro and Aguirre, 2014, p.617

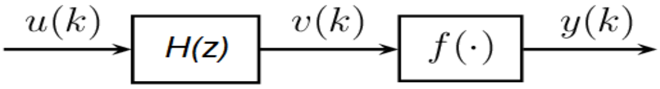

Figure 2. Wiener model. Source: adapted from Ribeiro and Aguirre, 2014, p.617

The paper is organized as follows: in the next section, some definitions and performance criteria are presented. In turn, Section 3 deals with the description of process under study and tests carried out, while Section 4 presents the methodology used. Section 5 presents the results obtained and, finally, the conclusions drawn from the research are presented in Section 6.

2. PRELIMINARIES

Consider a continuous-time system with a (sampled) control input $u_k \in \mathbb{R}^p$ and a (sampled) controlled output $y_k \in \mathbb{R}^q$, whose possibly non-linear dynamic is not known. We want to represent this system in the form:

$$\begin{aligned} x_{k+1} &= A(\beta)x_k + B(\beta)u_k \\ y_k &= C(\beta)x_k + D(\beta)u_k \end{aligned} \tag{1}$$

where $u_k \in \mathbb{R}^n$ is the state vector, and the matrices $A(\beta) \in \mathbb{R}^{nxn}$, $B(\beta) \in \mathbb{R}^{nxp}$, $C(\beta) \in \mathbb{R}^{qxn}$ e $D(\beta) \in \mathbb{R}^{qxp}$ are not precisely known, but belong to a polytopic uncertain domain $\mathcal{P}$, such that:

$$\mathcal{P} = \left\{ \begin{array}{c} (A,B,C,D)(\beta) : \ (A,B,C,D)(\beta) = \\ \sum_{i=1}^{N} (A_i, B_i, C_i, D_i)\beta_i, \beta \in \mathbb{R}^N; \\ \sum_{i=1}^{N} \beta_i = 1, \beta_i \geq 0 \end{array} \right\} \tag{2}$$

where $N = 2^l$ is the number of vertices of the polytope, where l is the number of uncertain parameters.

We investigate in this paper the design of a control law by state feedback given by:

$$u_k = Kx_k \tag{3}$$

with $K \in \mathbb{R}^{1xn}$ that robustly stabilizes the system described by (1) for all $\beta \in \mathcal{P}$, according to the following criteria.

2.1 H_2 guaranteed cost

Consider the system given by (1). The transfer function that relates external perturbation, w_k, to the output of interest, y_k, is given by:

$$G_{wy}(z) = C(zI - A)^{-1}B_w + D_w \tag{4}$$

H_2 norm of $G_{wy}(z)$ is defined as (de Oliveira et al., 2004):

$$\left\| G_{wy}(z) \right\|_2 = \frac{1}{2\pi} \int_{-\pi}^{\pi} Tr\{[G_{wy}(e^{j\omega})]^* [G_{wy}(e^{j\omega})]\} d\omega \tag{5}$$

Minimizing H_2 norm of a $G_{wz}(s)$ system effectively means minimizing the amount of energy that is transferred from exogenous input, w, to the output of interest, z, attenuating the influence of measurement noise and external disturbances on system response.

De Oliveira et al. [6] present linear matrix inequalities for computation of H_2 guaranteed cost through Lyapunov functions dependent on parameters for discrete-time uncertain systems, from which are derived the synthesis conditions used in this paper, presented below.

Lemma 2.1 [6]. If there exist symmetric positive matrices $W_i \in \mathbb{R}^{n \times n}$, $i = 1, \dots, N$, matrices $H \in \mathbb{R}^{p \times n}$, $J \in \mathbb{R}^{n \times n}$ and a symmetric positive matrix $X \in \mathbb{R}^{p \times p}$ such that:

$$\begin{bmatrix} -X + C_i H' + H C_i' & -H + C_i J' \\ -H' + J C_i' & W_i - (J + J') \end{bmatrix} \leq 0, \tag{6}$$

then:

$$C(\alpha)W(\alpha)C(\alpha)' \leq X \tag{7}$$

is verified with $W(\alpha) = W(\alpha)' > 0$ given by:

$$\begin{aligned} &W(\alpha) = \sum_{i=1}^{N} \alpha_i W_i \, ; \ \alpha_i \geq 0; \\ &i = 1, \dots, N; \ \sum_{i=1}^{N} \alpha_i = 1 \end{aligned} \tag{8}$$

Lemma 2.2 [6]. If there exist symmetric positive definite matrices $W_i \in \mathbb{R}^{n \times n}$, $i = 1, \dots, N$, matrices $Q \in \mathbb{R}^{n \times n}$ and $L \in \mathbb{R}^{n \times n}$ such that:

$$\begin{bmatrix} -W_i & A_i L + B_i Z & B_{w_i} \\ L' A_i' + Z' B_i' & W_i - (L + L') & 0 \\ B_{w_i}' & 0 & -I \end{bmatrix} \leq 0; \quad i = 1, \dots, N \tag{9}$$

then:

$$A(\alpha)W(\alpha)A(\alpha)' - W(\alpha)B(\alpha)B(\alpha)' \leq 0 \tag{10}$$

is verified with $W(\alpha) = W(\alpha)' > 0$ given by (8).

The gain of robust state feedback is given by $K = ZL^{-1}$. If conditions of Lemmas 2.1 and 2.2 are satisfied, the optimal H_2 guaranteed cost, ρ, for an uncertain system with $(A(\alpha), B(\alpha), C(\alpha)) \in \mathcal{P}$ is given by the solution of:

$$\rho = \sqrt{\min(trace(X))} \tag{11}$$

2.2 H_∞ guaranteed cost

Consider the system (1) again. H_∞ norm of the transfer function, $G_{wy}(z)$, which relates external perturbation, w_k, to the output of interest, y_k, is given by:

$$\|G_{wy}(z)\|_\infty = \sup_{\omega\in\mathbb{R}} \sigma_{max}\left[G_{wy}(e^{j\omega})\right] \quad (12)$$

According to Zhou and Doyle [18], H$_\infty$ norm is related to greater gain that can exist between exogenous inputs and the system outputs, throughout the spectrum of signals, that is, it quantifies the greater increase of energy that can occur between the inputs and outputs of a given system. Silva [13] reiterates that, for SISO uncertain systems, H$_\infty$ norm corresponds to the maximum value of Bode magnitude diagram of the set of uncertainties. Therefore, minimizing H$_\infty$ norm of $G_{wz}(s)$ means minimizing the greater gain that can exist between the disturbance input and the system output, attenuating the effect of w_k on y_k.

De Oliveira et al. [6] present in their paper LMIs that perform H$_\infty$ guaranteed cost computation through parameter dependent Lyapunov functions for discrete-time uncertain systems, from which synthesis conditions used herein are derived.

Lemma 2.3 [6]. If there exist symmetric positive definite matrices $P_i \in \mathbb{R}^{n\times n}$, $i = 1, \dots, N$, matrices $F \in \mathbb{R}^{n\times n}$ and $G \in \mathbb{R}^{n\times n}$ such that:

$$\begin{bmatrix} P_i & A_iG + B_iZ & 0 & B_{w_i} \\ G'A_i' + G'Z' & G + G' - P_i & G'C_i' & 0 \\ 0 & C_iG & I & D_i \\ B_{w_i}' & 0 & D_i' & \mu_D I \end{bmatrix} > 0; \quad (13)$$
$$i = 1, \dots, N$$

then:

$$\begin{bmatrix} \Xi & A(\alpha)'P(\alpha)B(\alpha) - \varsigma^2 C(\alpha)'D(\alpha) \\ \star & I - B(\alpha)'P(\alpha)B(\alpha) - \varsigma^2 D(\alpha)'D(\alpha) \end{bmatrix} > 0, \quad (14)$$

where $\Xi \triangleq P(\alpha) - A(\alpha)'P(\alpha)A(\alpha) - \varsigma^2 C(\alpha)'C(\alpha)$ and $\star$ represents symmetrical blocks, is verified with $P(\alpha) = P(\alpha)' > 0$ given by (8), with W replaced by P.

The gain of robust state feedback is given by $K = ZL^{-1}$. If conditions of Lemma 2.3 are satisfied, optimal H$_\infty$ guaranteed cost, ρ, for an uncertain system with $\left(A(\alpha), B(\alpha), C(\alpha)\right) \in \mathcal{P}$ is given by the solution of:

$$\varsigma = \sqrt{\min(\mu_D)} \quad (15)$$

3. SYSTEM DESCRIPTION

The modeling plant consists of an electric oven located in Signals and Systems Laboratory of Centro Federal de Educação Tecnológica de Minas Gerais (CEFET-MG) - Campus Divinópolis, designed and developed by Franco [10].

The furnace, shown in Figure 3, presents external and internal structures of dimensions 150x150x1000mm and 120x120x1000mm, respectively, both in aluminum. According to Franco [10], the space between the inner and outer walls is filled with expanded polyurethane, providing thermal insulation between the interior and exterior of the prototype, which makes it robust to abrupt variations in the temperature of external environment.

Internally, the oven is divided into eight chambers. In the first, there is an axial fan, actuator responsible for propelling the external air through the interior of the furnace, and a temperature sensor LM35, also present in the chambers 3, 5 and 7, in order to acquire the temperature in each one from them. In addition, another LM35 sensor is disposed next to the data acquisition board, in order to measure the temperature of external environment to the prototype. In turn, chambers 2, 4 and 6 have a 150W halogen bulb each, fed with a variable voltage from 0 to 220Vac, in order to heat the airflow inside the prototype. The chamber 8 is intended for exhaustion and through it the air is expelled, returning to the external environment. In order to work with a SISO system, only one input - LM35 sensor located in chamber 7 - and one halogen lamp output located in chamber 2 - will be considered in the system. The other lamps are kept off and the axial fan is driven at constant voltage equal to 80% of its nominal.

Figure 3. Prototype overview. Source: Franco, 2013, p. 8

The use of this plant is of interest due to the similarity of its dynamic behavior with that of industrial systems of great economic and environmental impact, such as furnaces found, for example, in metallurgical processes.

4. METODOLOGY

The topology detection of the purely linear models and linear plots of block-oriented models considered here is limited to defining the maximum input delays, n_u, and output, n_y. Since one of the objectives of this paper is to evaluate the relationship between the identified models complexity and the designed controllers efficiency, it will be obtained for each class of models considered, in order to compare them, low order models: one of first order ($n_u = n_y = 1$), two of second order ($n_y = 2$, $n_u = 1$ and $n_u = n_y = 2$), three of third order ($n_y = 3$, $n_u = 1$; $n_y = 3$, $n_u = 2$ and $n_u = n_y = 3$) and four of fourth order ($n_y = 4$, $n_u = 1$; $n_y = 4$, $n_u = 2$; $n_y = 4$, $n_u = 3$ e $n_u = n_y = 4$), where n_y and n_u, represent the number of output and input terms added to the model, respectively; as well as a high-order model, limiting the maximum input and output delays to 15 and using Error Reduction Rate (ERR) and Akaike Information Criterion (AIC) to identify which of the terms generated are, in fact, relevant to the system representation

After the structure detection stage, the next step in the process of identifying the plant under study is to estimate the parameters associated with each regressor in order to quantify them. This step starts with the choice of algorithm to be used. In order to avoid the polarization of the estimated parameters, the technique used in this paper is the extended least squares estimator.

Validation is the final procedure of the system identification and aims to verify if the models obtained are able to adequately represent the interest characteristics of modeled plant. Quantitative validation of the models performance is based on two indices: RMSE and correlation coefficient between simulation error and simulated output. The first one compares the predicted output of the model, or prediction, with the mean time of the system output signal, as follows:

$$RMSE = \frac{\sqrt{\sum_{k=1}^{N}[y_k - \widehat{y_k}]^2}}{\sqrt{\sum_{k=1}^{N}[y_k - \bar{y}]^2} \qquad (16)$$

where $\widehat{y_k}$ is free simulation of the model output and $\bar{y}$ is the mean value of system output signal. In turn, the performance index $J(\widehat{\boldsymbol{\theta}})$ is correlation index between simulation error and simulated output for a set of estimated parameters, given by:

$$J(\widehat{\boldsymbol{\theta}}) = \sum_{k=1}^{N} y_{i,k}\widehat{y_k} + e_k\widehat{y_k} - \widehat{y_k}^2 \qquad (17)$$

where N is the number of samples for time series used, $y_{i,k}$ is ideal portion of validation data, $\widehat{y_k}$ is free simulation of obtained model and e_k represents the additive noise or any associated uncertainty. For each order and class of models considered, the twenty models with the lowest RMSE index were selected and, among them, the one with the lowest correlation coefficient between the simulation error and the simulated output was chosen.

4.1.1 Purely linear models

Figure 4 shows RMSE and correlation coefficient between simulation error and simulated output indexes obtained by purely linear models identified. Note that the two indices are at first glance conflicting. This is due to the purpose with which each of them was used in selection of the models. Barroso [2] points out that RMSE index can lead to unreliable results if data collected have a significant noise-to-signal ratio. Correlation coefficient, although robust to noise, can provide false positive in cases of unstable models. Therefore, RMSE index was used in this paper to detect stability, that is, stable models. Among those selected, the one with the lowest correlation between simulation error and simulated output will be the one that presents parameters closer to the actual values and will therefore be the best model.

Thus, from Figure 4, by the analysis of correlation coefficient, it is inferred the model that best and worst fit the data are, respectively, the third order with two input regressors and the second order with an input regressor.

4.1.2 Hammerstein models

Figure 5 shows RMSE and correlation coefficient between simulation error and simulated output indexes obtained by Hammerstein models identified. From the observation of correlation coefficients obtained, it can be seen the model that best fits the data is the first order one. It should be noted that since the structure of low order models was defined *a priori*, without use of any structure detection tool, such models may contain spurious regressors, that is, terms that are not really necessary to compose the model, which results in a worse adjustment of the model to the data. In turn, the high correlation coefficient obtained by the fourteenth order model is explained by the fact that complex, over-adjusted models, such as the one cited, tend to model the noise present in the data, which is undesirable.

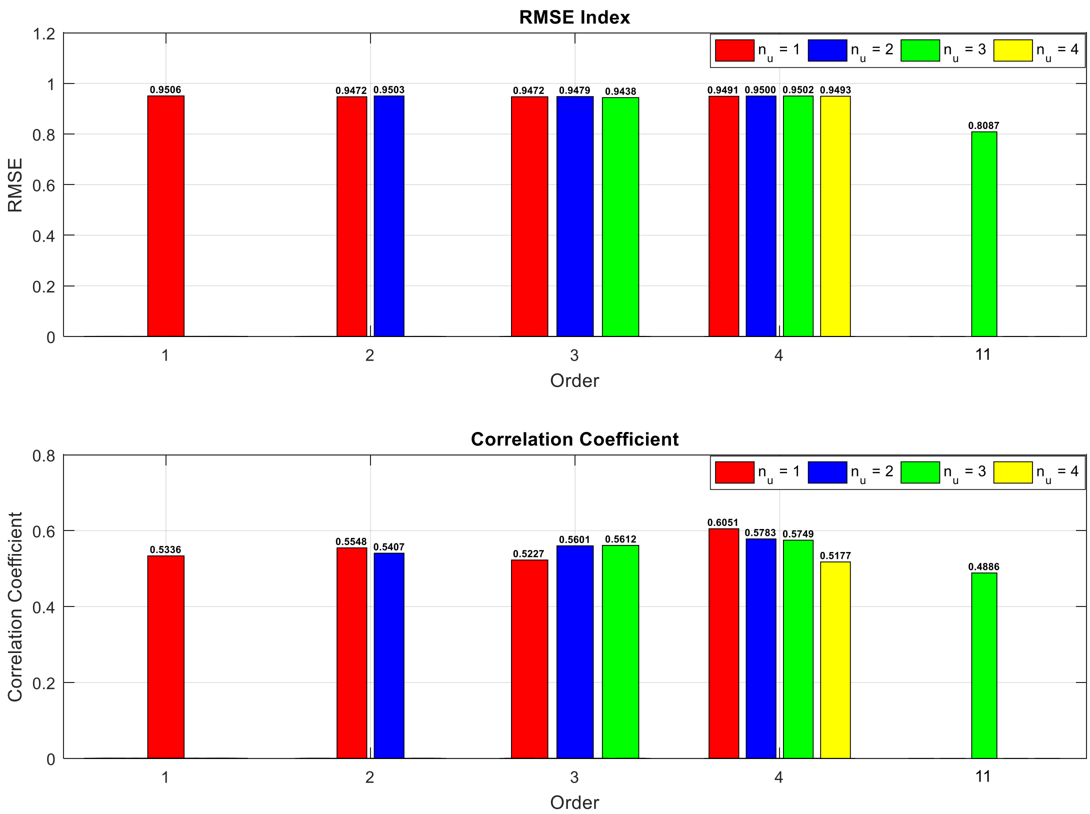

Figure 4. RMSE and correlation coefficient indexes obtained by purely linear models identified

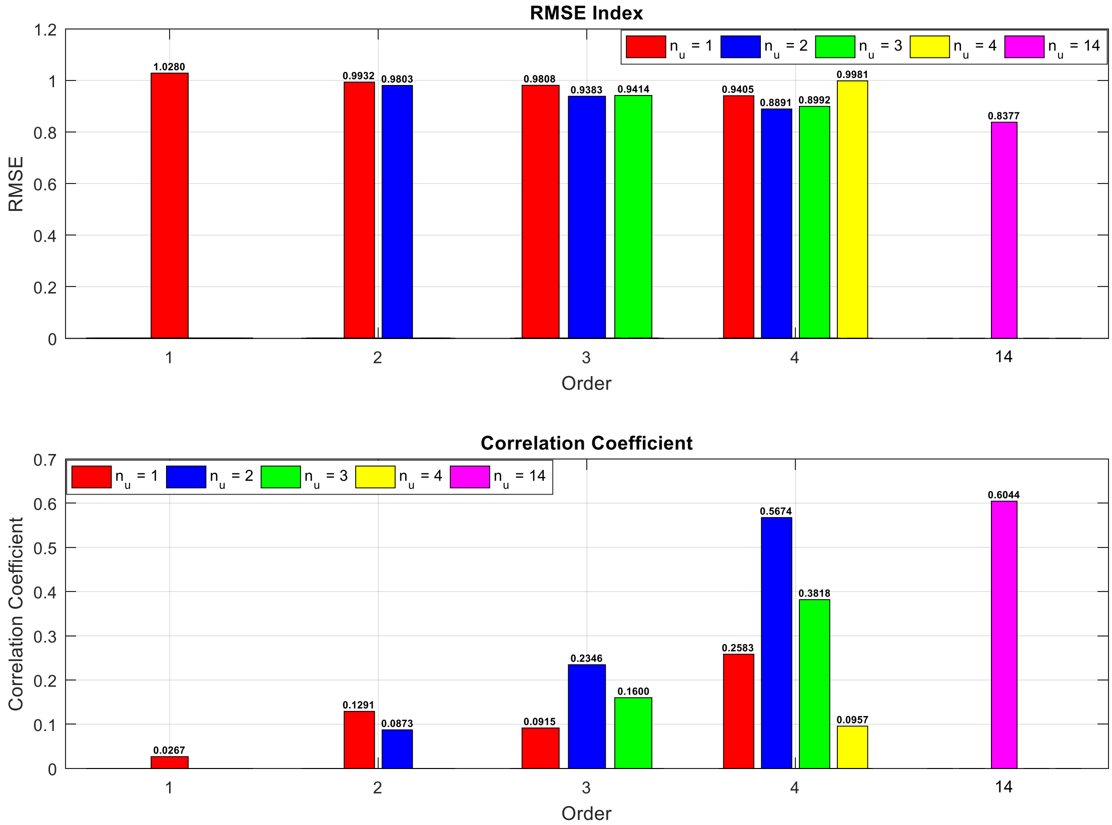

Figure 5. RMSE and correlation coefficient indexes obtained by Hammerstein models identified

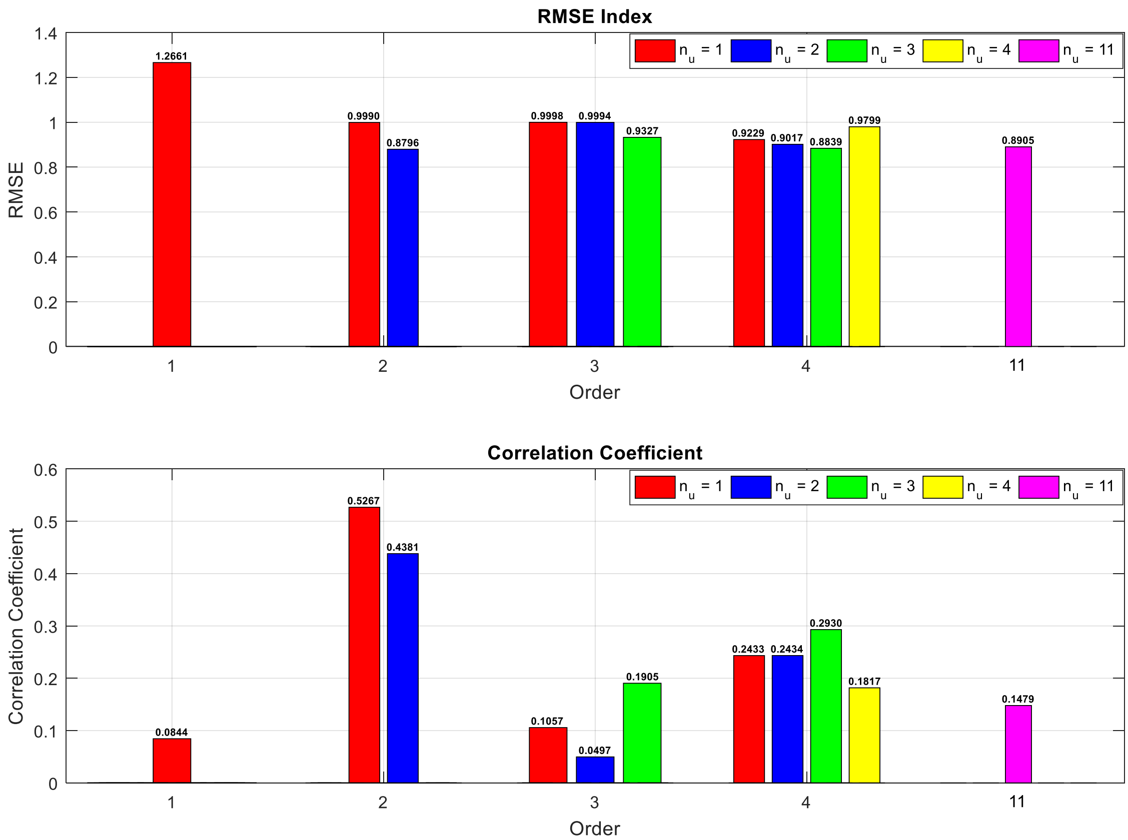

Figure 6. RMSE and correlation coefficient indexes obtained by Wiener models identified

4.1.3 Wiener models

Figure 6 shows RMSE and correlation coefficient between simulation error and simulated output indexes obtained by identified Wiener models. From the observation of correlation coefficients obtained, it can be seen the models that best and worst fit the data are, respectively, the third order with two input regressors and the second order with an input regressor.

4.1.4 Comparative analysis of identified models

Based on correlation coefficients between simulation error and simulated output obtained, it is inferred that, in general, Hammerstein and Wiener models represented the plant under study better than purely linear models. This result was expected, since the system in question presents non-linear characteristics, among which we highlight the presence of distinct heating and cooling time constants, evidencing bilinear behavior, as well as a highly nonlinear heat transfer mechanism by thermal radiation that takes place inside the oven.

However, Figure 7 shows that block-oriented models ability to represent the system more accurately when compared to purely linear models is not translated into smaller standard deviations of parameters estimated for those models and, consequently, in reducing the domain of uncertainties considered in the design of controllers. In this paper, the uncertainty corresponding to each parameter determined in the system identification is measured from the standard deviation associated with it, that is, it is considered that the value of each uncertain parameter of model a_i is restricted to the range $\left[\underline{a_i}, \overline{a_i}\right]$, where $\underline{a_i}$ is the minimum value assumed by the respective parameter, given by $\underline{a_i} = a_{iN} - \sigma_i$; a_{iN} represents the estimated nominal value of the parameter and σ_i is the standard deviation associated with it , while $\overline{a_i}$ is the maximum value assumed by a_i, given by $\overline{a_i} = a_{iN} + \sigma_i$. The sum of the uncertainties considered for identified models is carried out from pre-established limits of their parameters, taking the difference between the upper and lower constraints for each estimated parameter. Note that, in general, estimated parameters for Hammerstein and Wiener models are as uncertain as those of purely linear models, except for the fourth order linear model with two input regressors, which, possibly because they contain spurious terms, presented estimated parameters with standard deviations far superior to the others.

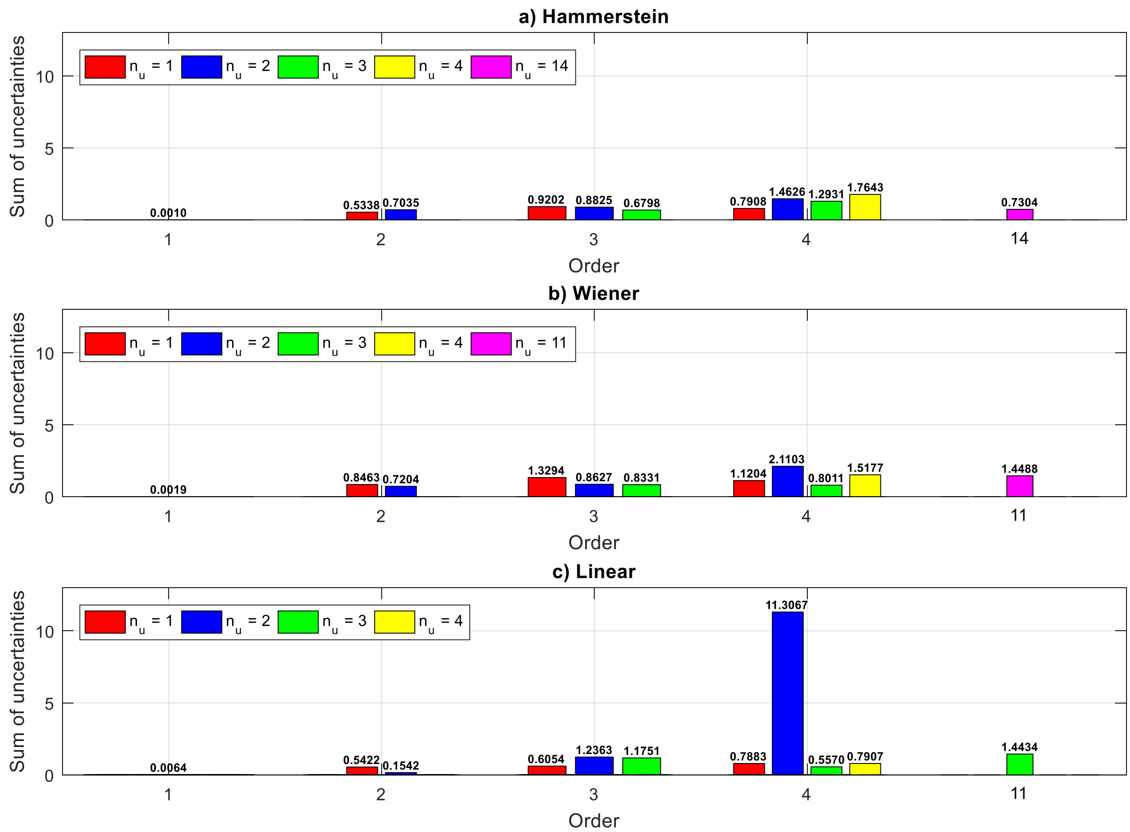

Figure 7. Sum of the uncertainties of the identified models

5. RESULTS AND DISCUSSIONS

5.1 H2 cost

Figure 8 illustrates H_2 costs obtained by controllers designed based on Hammerstein, Wiener and linear models identified in controllable canonical form. It can be seen that H_2 costs obtained by the controllers designed based on block-oriented models, especially in Wiener models, are considerably smaller than those obtained by purely linear models. In addition, it can be observed that second-order models with an input regressor were those that obtained the lowest H_2 cost and that, in general, the more input regressors added to the model, the higher the H_2 cost of the compensated system. Finally, it is important to note that the infeasibility index in the controllable form was low, occurring only in the high order models and in the fourth order linear model with two input terms.

Figure 9 shows H_2 costs obtained by the controllers designed based on the Hammerstein, Wiener and purely linear models identified in observable canonical form. The occurrence of infeasibility increases substantially in observable form, especially as the order and the number of regressors added to the model increase. In addition, comparing Figures 8 and 9, it can be observed that in cases where LMI is feasible, H_2 costs obtained in the controllable and observable forms are very close or even equal. It is also observed that in the observable form the occurrence of infeasibility is lower in purely linear models than in block-oriented models.

Finally, Figure 10 shows the sum of the uncertainties and H_2 costs obtained by Wiener models identified in controllable canonical form. It can be seen that the H_2 cost is dependent on the set of uncertainties considered; note that the upward trend of H_2 cost in fourth-order models is broken by the model with two input regressors, which presents a sum of uncertainties considerably larger than the others.

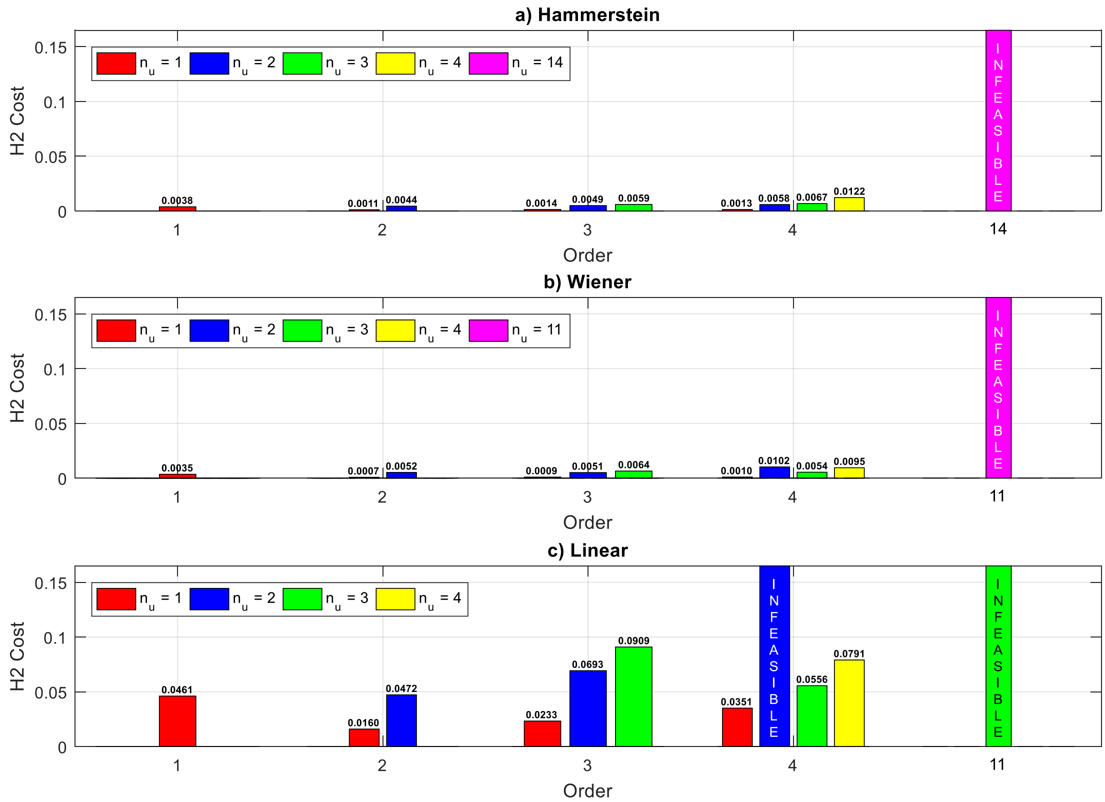

Figure 8. H_2 costs obtained by controllers designed based on Hammerstein, Wiener and purely linear models identified in controllable canonical form

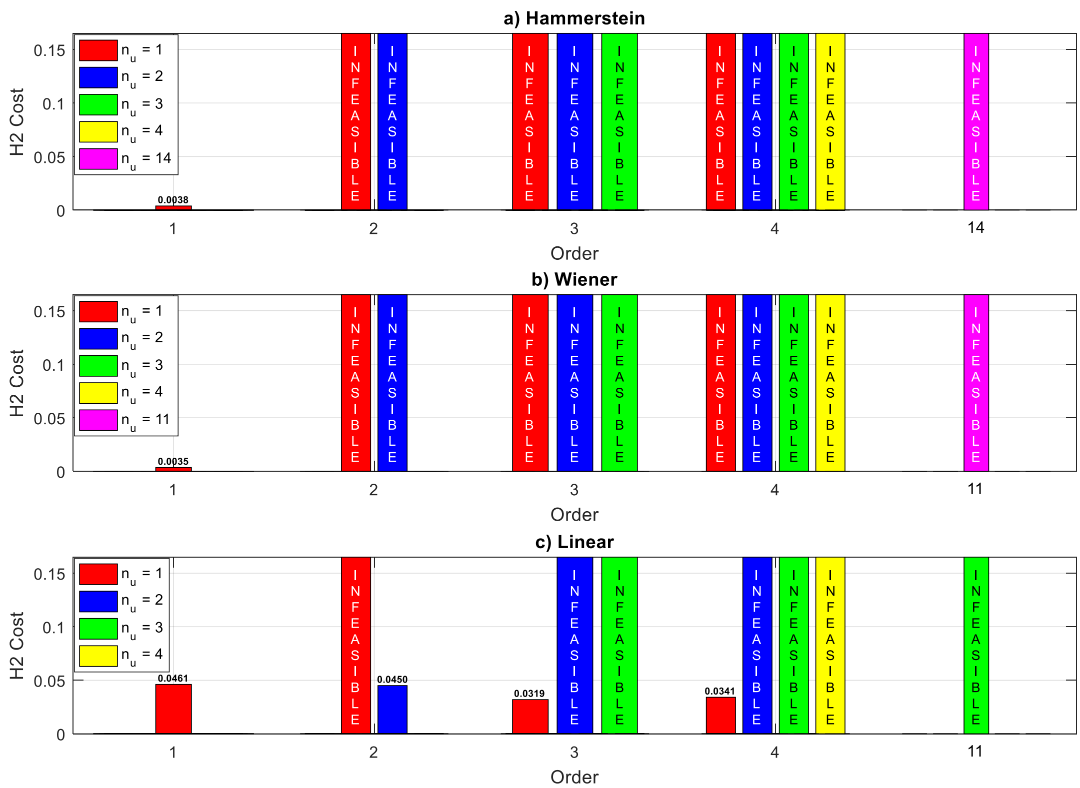

Figure 9. H_2 costs obtained by controllers designed based on Hammerstein, Wiener and purely linear models identified in observable canonical form

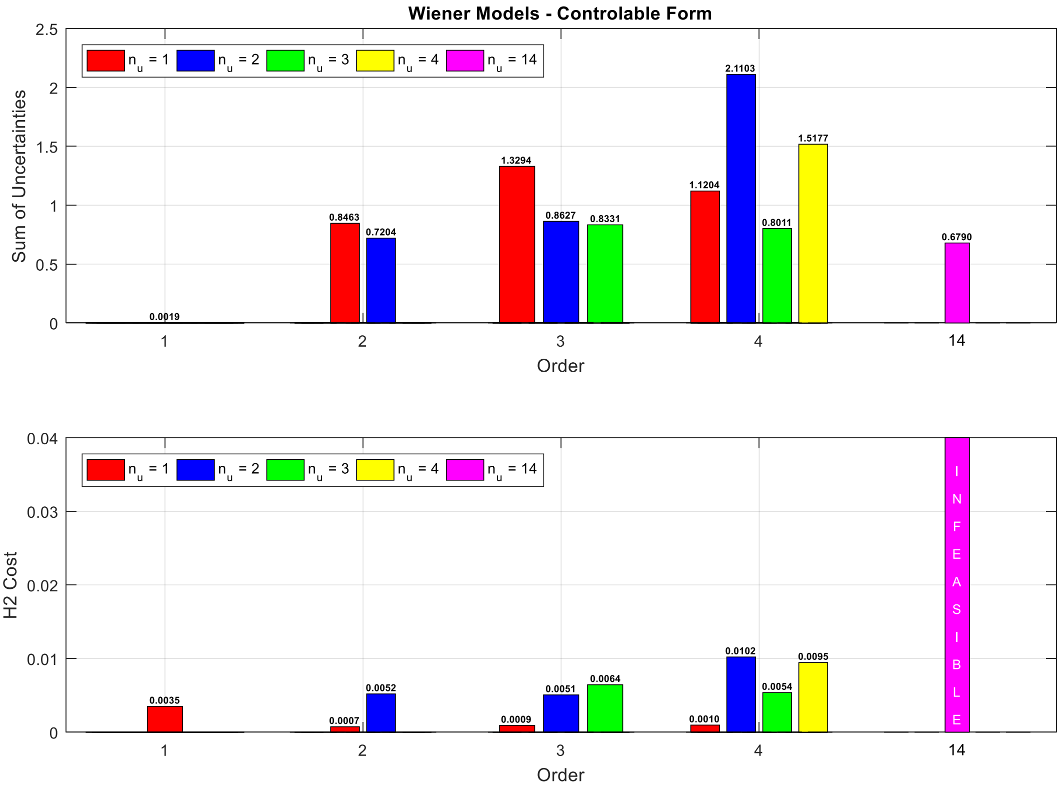

Figure 10. Influence of the set of uncertainties considered in H_2 guaranteed cost calculation

H_2 cost is therefore dependent on four factors, namely, the type of model used in the system identification: block-oriented models, especially Wiener models, presented a lower H_2 cost when compared to purely linear models; order: second order models were those that presented lower H_2 cost, corroborating the principle of parsimony, which states that between two or more candidate models effective in relation to the representation of the system dynamics, one must choose the model with the lowest number of independent parameters (Söderström and Stoica, 1989); number of input regressors added to the model: in general, the more input terms the higher H_2 cost obtained; and the sum of uncertainties: more uncertain systems tend to present higher H_2 cost. Furthermore, in this case, the controllable canonical form was more adequate than the observable form, since the infeasibility index was considerably lower in the first one.

5.2 H_∞ cost

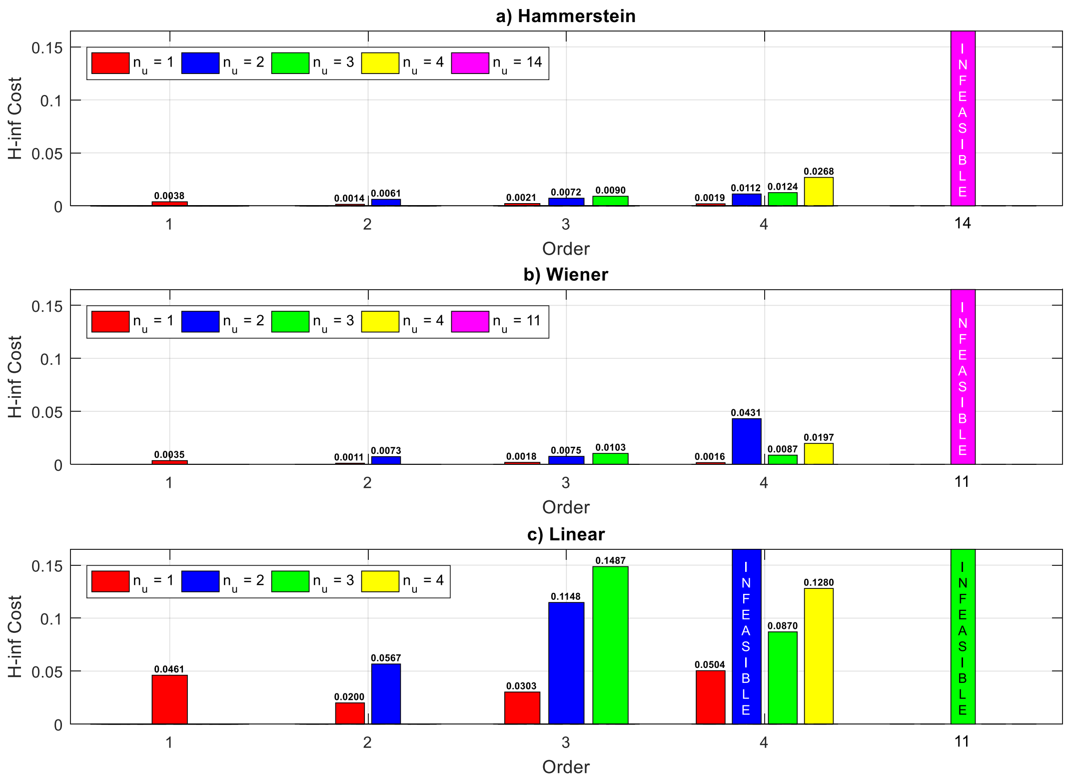

Figure 11. H_∞ costs obtained by controllers designed based on Hammerstein, Wiener and purely linear models identified in controllable canonical form

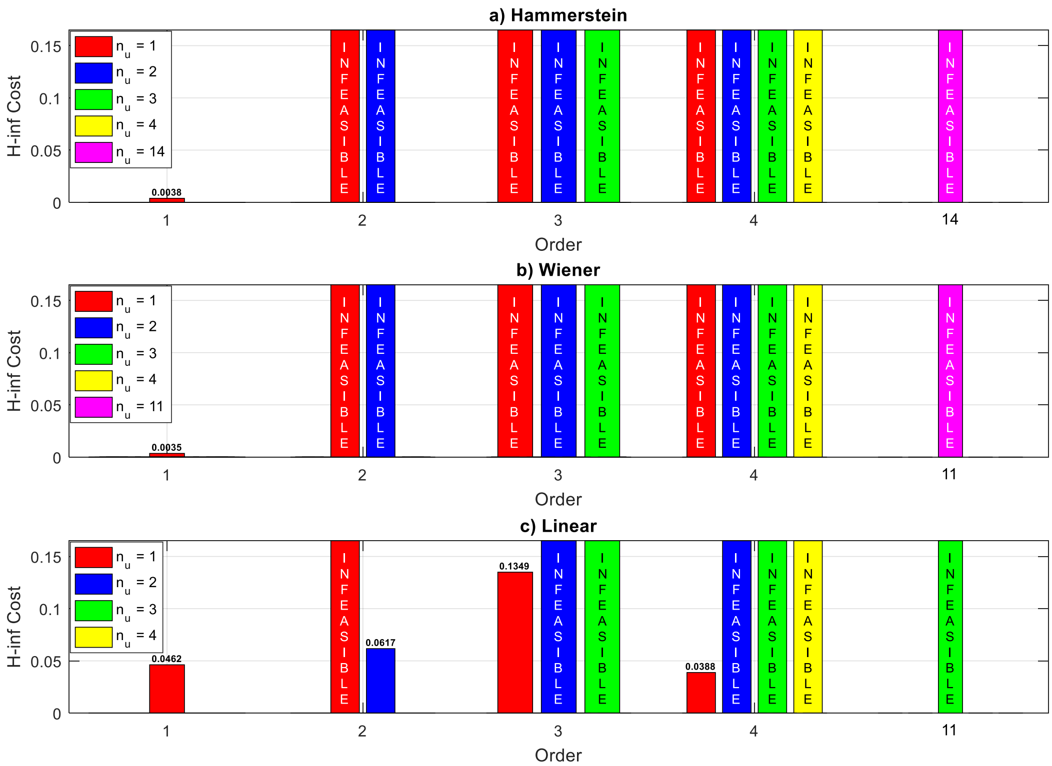

Figure 12. H∞ costs obtained by controllers designed based on Hammerstein, Wiener and purely linear models identified in observable canonical form

Figure 11 illustrates H∞ costs obtained by controllers designed based on Hammerstein, Wiener and purely linear models identified in controllable canonical form. Note that the characteristics of H∞ cost, even with respect to values, are very similar to those of H_2 cost: H∞ costs obtained by the controllers designed based on the block-oriented models, especially in Wiener models, are significantly lower than those obtained by purely linear models; the second-order models with an input regressor were the ones that obtained the lowest H∞ cost and, generally, the more input regressors added to the model, the higher H∞ cost of compensated system.

Notice now Figure 12, which shows H∞ costs obtained by controllers designed based on Hammerstein, Wiener and purely linear models identified in observable canonical form. Again, similar to H_2 cost, the occurrence of infeasibility – which in controllable form was low, occurring only in high order models and in the fourth order linear model with two input terms – increases considerably in observable form, especially as the order and the number of regressors added to the model increase.

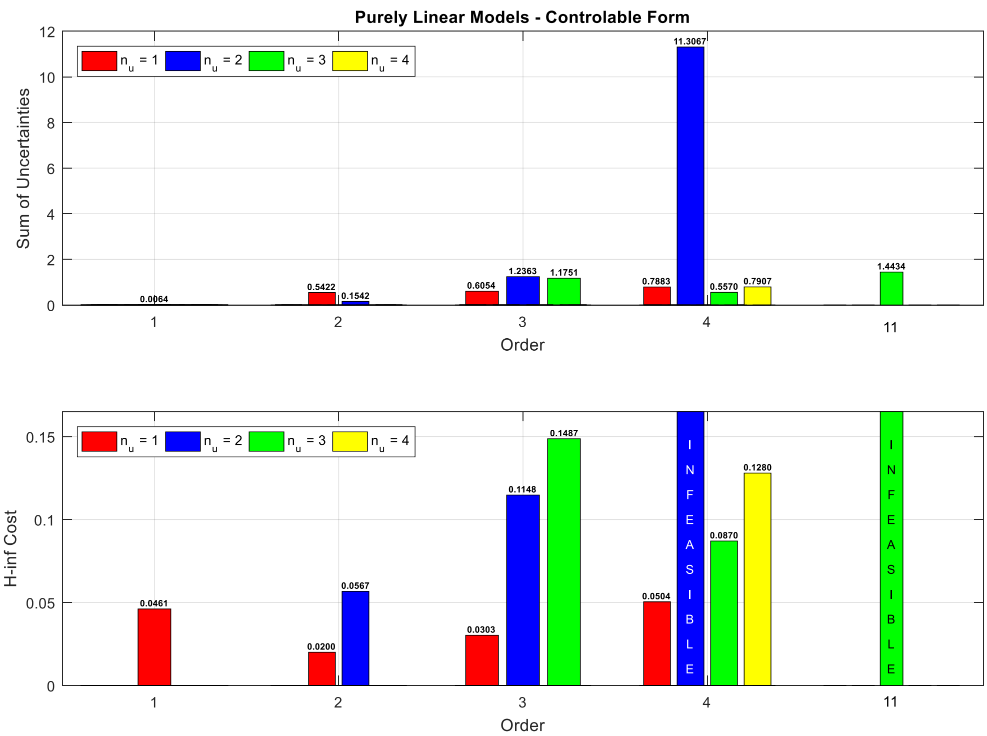

Figure 13. Influence of the set of uncertainties considered in H∞ guaranteed cost calculation

Finally, Figure 13 shows the sum of the uncertainties and H_∞ costs obtained by purely linear models identified in controllable canonical form. Here we also observe the influence of the set of uncertainties considered in H_∞ guaranteed cost statement; note that the impossibility of designing a controller capable of guaranteeing the stability of the system represented by the purely linear fourth-order model with two input terms with the lowest guaranteed H_∞ cost is due to the size of the domain of uncertainties considered, substantially larger than the others.

Thus, it is possible to state that H_∞ cost, as well as H_2 cost, depends on the type of model used in the system identification, since block-oriented models, especially Wiener models, presented lower H_∞ cost when compared to purely linear models; of the order, since the models of second order were the ones that presented lower H_∞ cost; of the number of input terms added to the model, since, in general, the more input regressors the greater H_∞ cost obtained; and the sum of the uncertainties: more uncertain systems tend to present a higher H_∞ cost, and even if the domain of uncertainties considered is too large, the LMI is infeasible, as in the case of the purely linear fourth-order model with two terms of input. In addition, also for computation of e minimal H_∞ cost, controllable canonical form was more adequate than the observable one, since the occurrence of infeasibility was considerably greater in this form than in that one.

6. CONCLUSION

In this paper, it was verified that the performance of block-oriented models, especially Wiener model, was much superior to the purely linear models, with respect to the H_2 and H_∞ costs. It was also observed that the use of Hammerstein and Wiener models in the identification of systems for the design of robust controllers can achieve better performance if the models used have low orders, even if this represents larger identification errors.

For the system in question, controllable canonical form was more adequate than observable form, since the occurrence of infeasibility was considerably lower in the first one for all LMIs considered here.

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