Research on Fractal Method for Soft Fault Diagnosis of Nonlinear Analog Circuits

Xinmiao Lu, Hong Zhao, Qiong Wu

The Higher Educational Key Laboratory for Measuring & Control Technology and Instrumentations of Heilongjiang Province, Harbin University of Science and Technology
Harbin, China, (lvxjmiao0611@126.com)

Abstract

The soft fault diagnosis of nonlinear analog circuits is an important guarantee of the stable and reliable operation of electronic products. In view of the low accuracy and heavy computation load of current soft fault diagnosis methods for nonlinear analog circuits, this paper presents a soft fault diagnosis method for nonlinear analog circuits based on fractal theory. Analyzing the single-fractal and generalized multi-fractal diagnosis mechanisms, and taking the fault signal as an example, the proposed method calculate the fractal dimension of the fault signal by the single-fractal box dimension and generalized multi-fractal dimension calculation method, and analyzes the influence of different frequency input signals on the features of the fault state signal through experimental simulation. It is concluded that the increasing frequency of the input signal has little effect on the fractal characteristics of the fault signal. Comparing the single-fractal and the generalized multi-fractal diagnosis method, the author discovers that the effect is better when generalized multi-fractal dimension sequence is used to diagnose the circuit fault.

Key words

Single-fractal, generalized multi-fractal, feature extraction, fault diagnosis, nonlinearity

1. Introduction

In spite of its extensive use in all aspects of our daily life, the nonlinear analog circuits are facing a major obstacle against its development, i.e. the outdated fault diagnosis. The traditional diagnosis method often handles the nonlinear analog circuits as linear circuits. Simple as it is, the diagnosis method has a low reliability. The nonlinear factor must be taken into account in order
to fully grasp the various situations in the circuit. In essence, the circuit fault diagnosis is pattern recognition and fault location. The key lies in identifying the fault features of the fault signal of the circuit. At present, the artificial intelligence is the major means for feature extraction and diagnosis. Typical examples are Volterra-kernel analysis, Wiener-kernel analysis, and wavelet-based feature extraction, etc.

The Volterra series model describes the features of a stable causal time-invariant nonlinear system. The variation of Volterra series usually indicates the existence of a fault [1][2]. Similar to the transfer function of the linear system, Volterra series has nothing to do with the input and output. This property can be used to diagnose fault and identify the circuit state. Curse of dimensionality poses a great challenge to Volterra series model. Since the order of Volterra series grows rapidly during the calculation process, the data dimension would also grow exponentially, which brings tremendous difficulties to the calculation [3]. If the order of Volterra series is reduced, the fault features of the nonlinear analog circuit would not be fully expressed. Therefore, the future of Volterra series model rests in accurate measurement of the soft fault state of nonlinear analog circuits and the Volterra series of the system of all orders [4]. Lin et al. [5] adopted Wiener series to solve the curse of dimensionality. They applied Gaussian white noise excitation to the nonlinear system with unknown characteristic parameters, and expanded its response by orthogonal functional series to obtain a set of functions which characterize the system features. Besides, they used the time dependent function as the estimation of the correlation functions of all orders to solve the Wiener kernel of the system, and employed the BP neural network to realize the fault diagnosis. The Wiener kernel description of the nonlinear analog circuit boasts high diagnostic accuracy. Nevertheless, the large amount of information of the Wiener kernel is not conducive to computing. There is still room for improvement in how to obtain more efficient and accurate Wiener kernel of the circuit. The most commonly used theory for analog circuit fault diagnosis is the wavelet-based feature extraction. The wavelet transform analysis retains the advantages of Fourier transform, and overcomes its shortcomings in time-dependent problems. With time-frequency characteristics, the method is very suitable for analysis of non-steady-state signals [6]. However, in the nonlinear analog circuit fault diagnosis, the wavelet transform analysis has a low diagnostic accuracy, and the wavelet analysis alone cannot solve all the problems, especially in locating fault components [7][8].

With the development of fault diagnosis technology, scholars have proposed a new diagnostic method, i.e. the fault diagnosis based on fractal theory. Mao et al. first proposed a method based on fractal features and support vector machines to diagnose the faults of analog
circuits. Taking the single-fractal grid dimension as the fault feature and using support vector machine as the feature classification of the fault diagnosis, the method has yielded fruitful results [9]. Zhou et al. proposed analog circuit fault feature extraction method based on the fractal dimension of fractional Fourier transform [10]. With this method, they mapped the original fault signal to different fractional spaces, calculated the single-fractal dimension of the fault response signal of different fractional orders, and treated the results as the fault feature. Finally, they used the neural network to carry out classification and diagnosis, obtaining a good diagnosis effect. Despite the above efforts, the research of circuit fault diagnosis based on fractal theory is mainly based on single-fractal analysis, which only reflects the irregularity of the whole signal rather than all the local features of the signal [11-15]. As a result, the researchers should focus on the improvement of the fineness of signal feature expression at present and in the near future.

In light of these problems, this paper digs deep into the fault diagnosis method based on fractal theory, analyzes mechanisms of single-fractal and multi-fractal fault diagnosis, and provides an effective method to improve the accuracy of circuit fault diagnosis.

2. Basic Principles of Fractal Theory

2.1 Single Fractal Box Dimension

The dimension of a single multi-fractal box is known as the capacity dimension or the volume dimension. It is the most widely used dimension calculation method in dimensional measurement. Featuring good accuracy and high efficiency, it is particularly suitable for simple fractal. Cover the fractal body $F$ with small boxes (length of the side $\delta$). As there are cavities and cracks on various levels within the fractal body, some of the small boxes are empty, and some cover part of the fractal body. Denote the total number of non-empty boxes as $N(F, \delta)$, and define the box dimension $D$ as:

$$D = \lim_{\delta \to 0} \frac{\ln N(F, \delta)}{-\ln \delta}$$

(1)

Plus, a set of double-logarithmic coordinates is often used. That is to say, take $\ln \delta$ as the x-axis, and $\ln N(F, \delta)$ as the y-axis. When $\delta$ changes, mark the log value of $N(F, \delta)$, the number of boxes recorded each time, on the coordinates. If these markers present a straight line on the coordinates, the slope of the line is the fractal dimension of the fractal body. In addition to the measurement of the Euclidean space, the single-fractal box dimension can also be used for data measurement.
2.2 Generalized Multi-fractal Dimension

Assuming there is a fractal set \( F \) and the measure is \( \mu \), the order of the probability distribution \( q \) is defined as:

\[
\chi_q(\delta) = \sum_i p_i^q(\delta)
\]

(2)

Where, \( \delta \) is the unit measure used to decompose \( F \), \( p_i \) is the probability distribution of the \( i \)-th unit.

The generalized measure of the \( r \)-th dimension is defined as

\[
M^r(q) = \lim_{\delta \to 0} M^r_\delta(q)
\]

(3)

And

\[
M^r_\delta(q) = \chi_q(\delta)\delta^r = \sum_i p_i^q(\delta)\delta^r
\]

(4)

Assume that the critical exponent \( \tau(q) \) is:

\[
M^r(q) = \begin{cases} 
\infty, & r < \tau(q) \\
\text{Finite Value}, & r = \tau(q) \\
0, & r > \tau(q)
\end{cases}
\]

(5)

Where, \( \tau(q) \) is the quality index.

If \( r = \tau(q) \), \( M^r(q) \) is a non-zero finite value. According to Formulas (5), it is obtained that

\[
\chi_q(\delta) \sim \delta^{-\tau(q)}.
\]

Thus

\[
\tau(q) = \lim_{\delta \to 0} \frac{\ln \chi_q(\delta)}{-\ln \delta} = \lim_{\delta \to 0} \frac{\ln \sum_i p_i^q(\delta)}{-\ln \delta}
\]

(6)

Then, introduce the generalized multi-fractal dimension \( D(q) \):
\[
D(q) = \begin{cases} 
\frac{1}{q-1} \lim_{\delta \to 0} \sum_{i} \ln p_{i}(\delta) / \ln \delta, & q \neq 1 \\
\lim_{\delta \to 0} \frac{\ln p_{i}(\delta) \ln p_{i}(\delta)}{\ln \delta}, & q = 1
\end{cases}
\]

(7)

From the above formula, it is obtained that:

\[
D(q) = \begin{cases} 
\tau(q), & q \neq 1 \\
-\tau'(q), & q = 1
\end{cases}
\]

(8)

Based on the concepts of multi-fractal spectrum \( f(\alpha) \) and singular intensity \( \alpha \), \( f(\alpha) \sim \alpha \) and \( D(q) \sim q \) are associated through Legendre transform:

\[
\begin{cases} 
f(\alpha) = q\alpha - (q-1)D(q) \\
\alpha = \frac{d}{dq} [q-1]D(q)]
\end{cases}
\]

(9)

3. Research on Mechanisms of Fault Fractal Diagnosis

3.1 Mechanism of Single-fractal Fault Diagnosis

Single-fractal fault diagnosis is a diagnostic method that takes the fractal dimension of the fault signal of a certain measure as the fault state eigenvalue. In essence, the diagnosis process is to calculate the dimension of the nonlinear analog circuit fault signal, and get a dimension point value. In the single-fractal fault diagnosis, the outputted state space of fault state signal is viewed as a large closed space, and different states in the closed space are expressed as closed subspaces. See Fig.1 for the spaces at different states in the fault space of the nonlinear analog circuit.

Fig.1. Fault State Space of Single-fractal Fault Diagnosis
From Fig. 1, assuming that the non-linear analog circuit has $N$ states, including the normal state, the state space should contain $N$ closed subspaces. In the single-fractal fault diagnosis, the different fault states are distributed in the $N$ closed subspaces within the state space, and each subspace interval contains all the values between the minimum and maximum fractal dimensions of the state. Taking fault state 1 as an example, the fractal dimension intervals in the state space are $[\bar{D}, D]$. $\bar{D}$ and $D$ are respectively the minimum and maximum fractal dimensions of fault state 1.

The division of state intervals is of critical importance in the single-fractal fault diagnosis of the nonlinear analog circuits. If the state intervals are clearly divided, it is much easier to identify the fault state. During the division of circuit fault states, the sub-spaces should not overlap each other. If dimension intersections appear between different subintervals, i.e. the range of dimensionality is overlapped, it is impossible to tell which fault state the dimensions of the overlapping region belong to. In this case, such a division of fault state subintervals cannot distinguish different states or achieve the purpose of fault identification. Assume that the situation in Fig. 2 occurs.

In the overlapping region, fault state 1 and fault state 2 share the same fractal dimension. It is impossible to determine which state is represented by the fractal dimension within the intersection. As the fractal dimension within the intersection reflects the fault features of fault state 1 and fault state 2 at the same time, it is unreasonable to distinguish the fault state according to such a division. However, non-overlapping interval division is an ideal state and is unattainable in actual practice. Overlapping is a commonplace due to the signal difference and noise in the circuit and the fluctuation of dimension in fractal theory. Therefore, in the single-
fractal fault diagnosis, the requirements on interval division should be relaxed properly and special circumstances should be handled appropriately. For the single-fractal diagnosis of fault circuits, it is only necessary to extract the fault signal, calculate the single-fractal dimension, and find out the sub-interval that the dimension value falls into. In this way, it is possible to identify the fault status indicated by the subinterval of the current circuit.

3.2 Mechanism of Generalized Multi-fractal Fault Diagnosis

The generalized multi-fractal diagnosis is an extension of the single-fractal diagnosis. The single measure is expanded to multiple measures, and the point set is replaced with the sequence. Different from the single-fractal diagnosis, the generalized multi-fractal fault diagnosis does not require the division of state intervals because it is impossible to distinguish the multi-fractal fault state based on the range of point value in single-fractal diagnosis. In generalized multi-fractal diagnosis, the generalized multi-fractal dimension sequence forms the state space with fractal value composition curves in different measures. See Fig.3.

In the generalized multi-fractal circuit fault diagnosis, the state space \( S \) of the fault state signal is regarded as a plane. In the state plane, the weight factor \( q \) serves as the x-axis, and the fractal dimension value corresponding to the weight factor serves as the y-axis. The generalized multi-fractal dimension sequence of each type of fault is presented as a curve in the state plane. The generalized multi-fractal dimension sequences are used as characteristic samples of all the fault states of the nonlinear analog circuit. Under the \( i \)-th fault state, the characteristic samples are:
\[ D_q^i = [D_0, D_1, \cdots, D_n] \]

(10)

Where, the weight factor is \( q = 0, 1, \ldots, n \).

In the state space of the generalized multi-fractal diagnosis, the weight factor \( q \) is the a-axis of the state plane, indicating the measure of the different state signals. In some cases, different state signals in the same measure share the same fractal dimension. For example, in Fig.3, when the measure is \( q^1 \), the fractal curve of state 1 intersects the fractal curve of state 3, that is, the two states share the same fractal dimension value; when the measure is \( q^2 \), the fractal curve of state 2 intersects the fractal curve of state 3, that is, the two states share the same fractal dimension value. This means the single-fractal dimension method cannot distinguish between state 1 and state 2 under the measure of \( q^1 \), or distinguish between state 2 and state 3 under the measure of \( q^2 \). After all, the single-fractal dimension method only supports measurements in the same measure. Adapted from the single-fractal dimension, the generalized multi-fractal dimension is capable of measuring the signal features in multiple measures. Thus, the generalized multi-fractal dimension has a better ability of identifying fault state.

During the circuit fault diagnosis, the fractal dimension sequence of the circuit fault state is listed in the state space in the form of curve segment. As shown in Fig.4, there are four fault states of the analog circuit. Their sample sequence curve segments are \( \overline{D_q^1} \), \( \overline{D_q^2} \), \( \overline{D_q^3} \) and \( \overline{D_q^4} \). The curve segment of the dimension sequence of the circuit fault state is set as \( \overline{D_q^i} \). Then, the generalized fractal dimension is used as the eigenvalue for state identification. In fact, the fault diagnosis is about finding out which curve segment of the state space is closest to the curve segment of the dimension sequence of the fault signal. In Fig.4, the curve segment \( \overline{D_q^4} \) of the signal is basically coincident with the curve segment of the sample sequence \( \overline{D_q^4} \). Hence, it is easy to conclude that the fault state of the non-linear analog circuit at this time is the same as the fault state at \( i = 4 \). The philosophy of generalized multi-fractal nonlinear analog circuit fault diagnosis is as follows: when the curve segment of the output signal dimension of the circuit of unknown state is close to that of a certain state sample sequence, the fault state of the circuit must be the same with that of the sample sequence.
4. Simulation and analysis of fractal dimension feature extraction

In fractal theory, the key to the extraction of signal features lies in dimension values for fractal dimension helps identify fractal features. In the fault diagnosis, therefore, the fractal dimension is used as the eigenvalue to distinguish between different states. If two signals belong to the same state under the same measure, they have a lot in common and share very similar fractal features. In other words, their fractal dimension values are close to each other. Signals in different states exhibit different fractal features, and thus different fractal dimension values. Making use of this law, the fault diagnosis method based on fractal theory can distinguish different fault states. In the following section, an AC circuit is taken as an example. The author extracts the circuit features by single-fractal dimension and multi-fractal dimension methods respectively, and analyzes the influence of different input signal frequencies on the fractal features of circuit fault state. The circuit parameters and structure are shown in Fig.5.

The input signal is taken as \( u_i = 10 \sin \left( \omega t - \frac{\pi}{4} \right) \), and the tolerance of the resistive element and the capacitive element in the circuit is set to 5\% of the nominal value. In this test, the fault
state is set as a positive/negative deviation of 20% from the nominal value. The author only gathers and analyzes the signals at the output node $u_o$, and does not take other measurable nodes into consideration. There are 2,048 sampling points. The cycle is 0.8 milliseconds. Table 1 lists the types of faults. In total, the author configures 6 types of fault. The fault state corresponding to each type is expressed as “element $i \uparrow \downarrow 20\%$”, i.e. a positive/negative deviation of 20% from the nominal value of the element. For instance, “$R_1 \uparrow 20\%$” means the type of fault that the resistance $R_1$ deviates positively from the nominal value by 20%.

The author places the circuit into different fault states with $\omega=100$, 500 and 1000 rad/s respectively, and calculates the outputted fault signal by the single-fractal box dimension method. In the experiment, 5 groups of data are taken for each state to calculate the box dimension value and the mean value. See Tables 2, 3, and 4 for details.

<table>
<thead>
<tr>
<th>State Code</th>
<th>Fault State</th>
<th>Fault Code</th>
<th>Fault State</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Normal</td>
<td>4</td>
<td>$R_2 \uparrow 20%$</td>
</tr>
<tr>
<td>2</td>
<td>$R_1 \uparrow 20%$</td>
<td>5</td>
<td>$R_2 \downarrow 20%$</td>
</tr>
<tr>
<td>3</td>
<td>$R_1 \downarrow 20%$</td>
<td>6</td>
<td>$C \downarrow 20%$</td>
</tr>
</tbody>
</table>

Table 2. Fault State Box Dimensions with $\omega=100$ rad/s

<table>
<thead>
<tr>
<th>Fault code</th>
<th>Data 1</th>
<th>Data 2</th>
<th>Data 3</th>
<th>Data 4</th>
<th>Data 5</th>
<th>Mean value</th>
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<td>1</td>
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<tr>
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<td>1.2671</td>
<td>1.2622</td>
<td>1.2654</td>
<td>1.2637</td>
<td>1.2643</td>
</tr>
<tr>
<td>3</td>
<td>1.2942</td>
<td>1.2967</td>
<td>1.2995</td>
<td>1.3001</td>
<td>1.2925</td>
<td>1.2966</td>
</tr>
<tr>
<td>4</td>
<td>1.2756</td>
<td>1.2732</td>
<td>1.2716</td>
<td>1.2769</td>
<td>1.2784</td>
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<tr>
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<td>1.2161</td>
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<td>1.2275</td>
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<td>1.2292</td>
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Table 3. Fault State Box Dimensions with $\omega=500$ rad/s

<table>
<thead>
<tr>
<th>Fault code</th>
<th>Data 1</th>
<th>Data 2</th>
<th>Data 3</th>
<th>Data 4</th>
<th>Data 5</th>
<th>Mean value</th>
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</thead>
<tbody>
<tr>
<td>1</td>
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<td>1.3017</td>
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<tr>
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Table 4. Fault State Box Dimensions with $\omega = 1000 \text{ rad/s}$

<table>
<thead>
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<th>Fault code</th>
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<th>Data 2</th>
<th>Data 3</th>
<th>Data 4</th>
<th>Data 5</th>
<th>Mean value</th>
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</table>

According to Tables 2, 3, and 4, the fractal box dimension of output signal varies with fault state and thus can be used for feature recognition. Fig.6. displays the comparison between the mean values of single-fractal box dimensions of the fault signal at different input frequencies.

Fig.6. Comparison of Fault Signal Box Dimensions with Different $\omega$ Values

As illustrated in Fig.6., when the frequency of the input signal remains constant, the mean value of fractal box dimension differ from fault state to fault state, indicating that each type of fault has its unique fractal features. As the frequency of the input signal rises, the mean value of the fractal box dimension of each fault state increases, and the increase does not blur the distinction of the fractal features of each fault state. Thus, the single-fractal box dimension is capable of extracting the features of fault state, thereby achieving the recognition of fault state.

Next, the author uses the generalized multi-fractal dimension to calculate the dimension of the circuit state, and to analyze the influence of different input signal frequencies on the fractal characteristics of the circuit. Keeping the test parameters unchanged, the author still only gathers the signals at the output node $u_0$, and does not take other measurable nodes into consideration. The circuit is put in different fault states with $\omega = 100, 500, \text{ and } 1000 \text{ rad/s}$ respectively. The
output fault signals are calculated by the generalized multi-fractal dimension. See Fig. 7, 8 and 9 for the simulation results, in which \( q = 0, 1, 2, \ldots, 5 \).

![Fig. 7. Generalized Multi-fractal Dimensions of Fault Signals with \( \omega = 100 \) rad/s](image1)

![Fig. 8. Generalized Multi-fractal Dimensions of Fault Signals with \( \omega = 500 \) rad/s](image2)
Based on the simulation results in Fig. 7, 8 and 9, the author obtains the generalized multi-fractal dimension eigenvalues of each fault state at three different frequencies. See Tables 5, 6 and 7 for the results.

Table 5. Generalized Multi-fractal Dimension Eigenvalue of Fault States with $\omega=100$ rad/s

<table>
<thead>
<tr>
<th>Fault code</th>
<th>$D_0$</th>
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<th>$D_2$</th>
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Table 6. Generalized Multi-fractal Dimension Eigenvalue of Fault States with $\omega=500$ rad/s

<table>
<thead>
<tr>
<th>Fault code</th>
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<th>$D_3$</th>
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<td>1.271</td>
<td>1.264</td>
<td>1.259</td>
</tr>
<tr>
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<td>1.332</td>
<td>1.322</td>
<td>1.311</td>
<td>1.305</td>
<td>1.296</td>
<td>1.291</td>
</tr>
<tr>
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<td>1.315</td>
<td>1.301</td>
<td>1.286</td>
<td>1.273</td>
<td>1.260</td>
<td>1.251</td>
</tr>
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<td>1.238</td>
<td>1.227</td>
<td>1.221</td>
<td>1.214</td>
<td>1.210</td>
</tr>
<tr>
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<td>1.253</td>
<td>1.242</td>
<td>1.232</td>
<td>1.227</td>
<td>1.223</td>
</tr>
</tbody>
</table>

Table 7. Generalized Multi-fractal Dimension Eigenvalue of Fault States with $\omega=1000$ rad/s

<table>
<thead>
<tr>
<th>Fault code</th>
<th>$D_0$</th>
<th>$D_1$</th>
<th>$D_2$</th>
<th>$D_3$</th>
<th>$D_4$</th>
<th>$D_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.333</td>
<td>1.321</td>
<td>1.311</td>
<td>1.302</td>
<td>1.297</td>
<td>1.293</td>
</tr>
<tr>
<td>2</td>
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<td>1.352</td>
<td>1.340</td>
<td>1.329</td>
<td>1.317</td>
<td>1.312</td>
</tr>
<tr>
<td>3</td>
<td>1.394</td>
<td>1.383</td>
<td>1.372</td>
<td>1.365</td>
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<td>1.354</td>
</tr>
<tr>
<td>4</td>
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<td>1.357</td>
<td>1.343</td>
<td>1.328</td>
<td>1.313</td>
<td>1.303</td>
</tr>
<tr>
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<td>1.272</td>
<td>1.268</td>
</tr>
<tr>
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<td>1.312</td>
<td>1.301</td>
<td>1.290</td>
<td>1.285</td>
<td>1.281</td>
</tr>
</tbody>
</table>

It can be seen from Tables 5, 6 and 7 that the output signals of different fault states have different generalized multi-fractal dimensions. Even if the dimensions intersect under some measures, the signals are distinguishable by the dimension values under other measures, thereby
supporting feature recognition. Comparing the generalized multi-fractal dimensions of fault signals at different input frequencies, it is concluded that the generalized multi-fractal dimensions of different fault states are different under the same input signal frequency. This indicates that the fractal features of a fault state are different under different measures. Even if there are overlapping dimension values under some measures, the fractal features are different under other measures. Like the single-fractal box dimension, the generalized multi-fractal dimension of each fault state also increases with the input signal frequency. However, the increase does not affect the distinction of the generalized multi-fractal features of each fault state. Thus, the generalized multi-fractal dimension is also capable of extracting the features of fault state, thereby achieving the recognition of fault state.

**Conclusion**

Due to the nonlinear and unstable features of soft fault signals in nonlinear analog circuits, it is very difficult to extract fault features using traditional signal processing methods. In view of the problem, this paper analyzes the fault diagnosis of nonlinear analog circuits respectively by single-fractal theory and generalized multi-fractal theory. To fulfill the purpose of fault diagnosis, the author analyzes the single-fractal and generalized multi-fractal diagnosis mechanisms, extracts the soft fault features of the nonlinear analog circuit, divides the dimension range of the state intervals, and identifies the interval that the dimension of the signal falls into. Besides, the author probes into the fault diagnosis method which uses the generalized multi-fractal dimension to describe the fault features of the circuit. To complete fault diagnosis, the author identifies fault state based on the similarity of dimension sequences, i.e. the closeness between the original sample sequence and the fault sequence. The experiment results demonstrate the advantages and disadvantages of the single-fractal and generalized multi-fractal fault diagnosis methods for nonlinear analog circuits. It is concluded that the generalized multi-fractal fault diagnosis has a better effect.

**References**