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Pattern Synthesis using Accelerated Particle Swarm Optimization

* P.A. Sunny Dayal, ** G.S.N. Raju, *** S. Mishra

*Department of Electronics and Communication Engineering

Centurion University of Technology and Management, Paralakhemundi, 761 211, India

**Department of Electronics & Communication Engineering, AU College of Engineering (A), Andhra University, Visakhapatnam, 530 003, India.

***Department of Electronics and Communication Engineering,

Centurion University of Technology and Management, Bhubaneswar, 752 050, India.

(sunnydayal@live.in; profrajugsn@gmail.com; s.mishra@cutm.ac.in)

Abstract

Several conventional techniques including standard amplitude distributions like uniform, cosine on pedestal, circular, parabolic, triangular and trapezoidal are reported in literature for pattern synthesis over the last few decades. Taylor has reported synthesis of sum pattern from a line source. None of the patterns reported by the above technique are optimal in terms of sidelobes and beamwidth. However, it is possible to optimize the pattern using state of the art algorithms. In this paper, Accelerated Particle Swarm Optimization algorithm is applied to optimize the sum patterns. The realized patterns are compared with those of Taylor. It is evident from the results that patterns obtained from Accelerated Particle Swarm Optimization are better than those of Taylor in terms of sidelobes and beamwidth.

Key words

Linear arrays, Taylor series, Accelerated Particle Swarm Optimization (APSO), Pattern synthesis, Sidelobe Level (SLL), Null to Null Beamwidth (FNBW), Amplitude Excitation.

1. Introduction

Pattern synthesis of array antennas is one of the most important problems for which attention is to be paid by antenna engineers and scientists. It is inverse process of array analysis. In analysis, the generation of radiation patterns for a given distribution and array system. On the other hand, in synthesis, pattern characteristics are specified and array is to be designed. It involves the determination of excitation amplitude, phase, spacing of elements, and the selection of radiating elements in the array. However, design methodology for the above parameters is cumbersome and complex. In view of this, focus is made to synthesize amplitude distributions of excitation while keeping all other parameters constant and pre specified depending on the array application. Although several pattern shapes exist for each application, narrow beam are chosen for the design.

Array antennas have a number of applications in all types of communication, radio transmitter, navigation, radars of all types, radiometers, electromagnetic heating, direction finding, ground mapping, remote sensing, and electromagnetic energy therapy etc.

Although, array antennas with their high directive gain and simplicity, are capable of generating well defined radiation pattern shapes useful for multiple application. They are useful as temperature sensor for measuring temperature of teleobjects or planets and stars. It is done from knowledge of signal to noise ratio.

The array antennas are capable of producing desired shapes of the far-field of electromagnetic wave like electronic circuits produce any shape of signal waveform. The pattern synthesis is applied to a continuous line source as well as discrete linear array of radiators. Continuous line source is an antenna which has a long, narrow and straight geometry. Its directivity depends on the variations in field or current strength with respect to longitudinal coordinates. That is, the currents or fields are continuous functions of longitudinal coordinate. As such, the expressions obtained for far-field of continuous line source are not applicable directly to the arrays of discrete radiators. Taylor [1] reported design of line-source antennas for narrow beamwidth and low sidelobes. This method is characterized by the desired sidelobe ratio, and the boundary of the region of uniform sidelobes $\bar{n}$, an integer. Using these two parameters, the pattern, the distribution function and other relevant data are computed. The beamwidth of the pattern is a function of the sidelobe ratio. Taylor presented the variation of beamwidth as a function of sidelobe ratio for ideal pattern characteristics, along with the distribution function. The resultant patterns, for different sidelobe ratios, the amplitude distribution of continuous line sources of different types are designed. Introducing these distributions, the radiation patterns are evaluated. The distribution so designed is found to be tapered with small pedestals towards the ends of the line source. The radiation pattern consists of one main beam and a set of sidelobes with equal height depending on $\bar{n}$ and exponentially decaying sidelobes. If $\bar{n} = 6$, there exists $(\bar{n} - 1)$ number of sidelobes of equal height [2-9].

On the other hand, Particle Swarm Optimization (PSO) which is developed by Kennedy and Eberhart in 1995 [10] is based on swarm behaviour in nature such as fish and bird schooling [11]. This optimization involves the use of swarm intelligence. It is applicable to every problem of optimization. In fact, there are more than 24 variants of Particle Swarm Optimization [12-22]. The swarm intelligence is used even in ant colony, and firefly algorithms.

The trajectories of the individual particles are adjusted in searching the space of an objective function.

Particle Swarm Optimization is characterized by the following:-

- The positional vectors in a quasi – stochastic manner form the piecewise paths.
- Particle movement has stochastic and deterministic components.
- Each particle moves towards the location of current global best $\mathbf{g}^*$ and its own best location $\mathbf{x}_i^{*(t)}$ in history. It also has tendency to move randomly.
- If the new location is better than the previous one, it is taken as the best i.
- The current best for all particles n is considered in all iterations.
- Iterations are undertaken till the global best is obtained.

It is evident that in Particle Swarm Optimization, current global best and individual best are taken into account. In order to improve the algorithm, an attempt is made to accelerate the convergence of the algorithm. In this direction, algorithm is developed using only the global best. The resultant algorithm is known as Accelerated Particle Swarm Optimization (APSO). It is developed by Xin – She Yang in 2008 [22].

2. Formulation

2.1 Taylor's method of array design

Taylor [1] considers the line source concept and defined the space factor using Fourier – Transform relationship. The expression from radiation pattern is given by

$$E(u) = \cosh\pi A \frac{\sin u}{u} \prod_{n=1}^{\bar{n}-1} \left[\frac{1 - \dfrac{u^2}{\sigma^2\pi^2\left[A^2 + \left(\bar{n} - \frac{1}{2}\right)^2\right]}}{\left(1 - \dfrac{u^2}{(\pi n)^2}\right)} \right] \quad (1)$$

Here

$\bar{n} =$ an integer and it divides sidelobe structure into uniform and non-uniform. It gives the number of sidelobes of equal level.

$\bar{n} = 6$ means $(\bar{n} - 1)$ sidelobes of equal height exists.

$u = \frac{2L}{\lambda}\sin\theta$

$2L =$ length of line source

$\theta =$ angle of observer. It is measured from broadside, A is defined in such way that sidelobe ratio is obtained from $\cos\pi A$.

$$\sigma = \frac{\bar{n}}{\sqrt{A^2 + \left(\bar{n} - \frac{1}{2}\right)^2}} \quad (2)$$

The amplitude distribution is obtained from the desired radiation pattern from the following relation,

$$E(u) = \int_{-1}^{1} A(x) e^{jux} dx \quad (3)$$

Applying the steps given by Taylor, we get the relation for $A(x)$. That is,

$$A(x) = a_0 + \sum_{n=1}^{\infty} 2a_n \cos n\pi x = E(0) + 2\sum_{n=1}^{\infty} E(n\pi)\cos n\pi x \quad (4)$$

Here $E(n\pi) = 0$ for $n \geq \bar{n}$.

2.2 Accelerated Particle Swarm Optimization

The Particle Swarm Optimization depends on the current global best $\mathbf{g}^*$ and also the individual best $\mathbf{x}_i^{*(t)}$. However, Accelerated Particle Swarm Optimization depends on only the global best $\mathbf{g}^*$ [22-30].

The velocity vector is given by the following formula [22]

$$\mathbf{v}_i^{t+1} = \mathbf{v}_i^t + a(r - 0.5) + b(\mathbf{g}^* - \mathbf{x}_i^{*(t)}) \tag{5}$$

Here $r =$ random variable. It varies from 0 to 1.

If $a(r - 0.5)$ is replaced by r_t , Eq. (5) becomes

$$\mathbf{v}_i^{t+1} = \mathbf{v}_i^t + ar_t + b(\mathbf{g}^* - \mathbf{x}_i^{*(t)}) \tag{6}$$

r_t can be selected from Gaussian distribution. The subsequent position is given by

$$\mathbf{x}_i^{t+1} = \mathbf{v}_i^t + \mathbf{v}_i^{t+1} \tag{7}$$

that is,

$$\mathbf{x}_i^{t+1} = (1 - b)\mathbf{x}_i^t + ar_t + b\mathbf{g}^* \tag{8}$$

As reported by Xin-She Yang et.al. [25]

$$\begin{aligned} a &\approx 0.1 \sim 0.4, \\ b &\approx 0.1 \sim 0.7 \end{aligned} \tag{9}$$

The initial values can be $a \approx 0.2$ and $b \approx 0.5$. Both a and b are independent of $\mathbf{x}_i$ and also search domain.

Compared to Particle Swarm Optimization, Accelerated Particle Swarm Optimization has global convergence property. As a result, it is possible to reduce number of iterations. As such, we can have

$$a = a_o e^{-\gamma t} = a_o \gamma^t \text{, for } 0 < \gamma < 1 \tag{10}$$

Here, $\begin{aligned} & a \approx 0.5 \sim 1.0, \\ & and\, \gamma \approx 0.9 \sim 0.97. \end{aligned}$

2.3 Array Design using Accelerated Particle Swarm Optimization

A typical uniform linear array is shown in Fig.1.

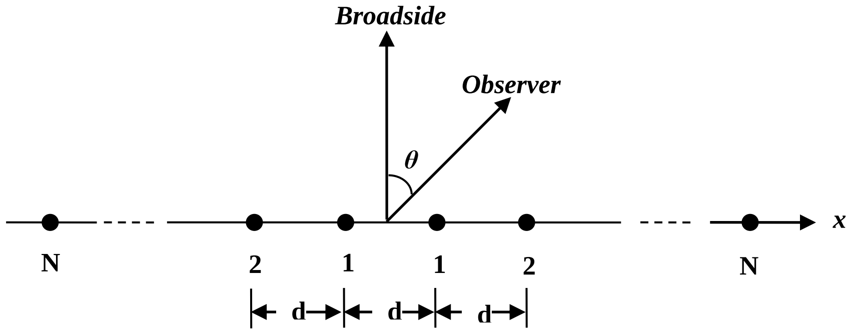

Fig.1. Geometry of Linear Array with equal spacing.

Considering a linear array of N isotropic antennas, antenna elements are equally spaced at distance d apart from each other along the x axis. The free space far-field pattern $E(u)$ is given by [9].

$$E(u) = 2\sum_{n=1}^{N} A(n)\cos[k(n-0.5)du] \tag{11}$$

Here,

$k =$ wave number $= 2\pi/\lambda$

$\lambda =$ wave length

$\theta =$ angle of observer

$u = \sin\theta$

$A(n) =$ excitation of the nth element on either side of the array, array being symmetric

$d =$ element spacing

Normalized radiation in dB is given by:

$$E(u) = 20\log_{10}\left[\frac{|E(u)|}{|E(u)|_{\max}}\right] \tag{12}$$

In the design of array, amplitude distribution is considered to be optimized keeping phase and space parameters constant, for a specified sidelobe level, $A(n)$ is computed for $d = \frac{\lambda}{2}$ and excitation phase = 0.

$$FitnessFunction = w_1(FSLL_o - FSLL_d) + w_2(LSLL_o - LSLL_d) \tag{13}$$

$$for\, w_1 = w_2 = 0.5 \quad and \quad -1 \le u \le 1,\; u \ne 0$$

Here

$FSLL_o =$ First Sidelobe level obtained

$FSLL_d =$ First Sidelobe level desired = -40dB

$LSLL_o =$ Last Sidelobe level obtained

$LSLL_d =$ Last Sidelobe level desired = -55dB

w_1 and w_2 are weighting factors which decide the relative preference given to each term in Eq. (13) and should be chosen such that $\sum_{i=1}^{2} w_i = 1$.

3. Results and Discussion

From the Eq.(1) to Eq.(4), $A(n)$ is computed using Taylor's method. At the same time from Eq.(5) to Eq.(13), it is also evaluated using Accelerated Particle Swarm Optimization. The results on $A(n)$ are compared in Figs. 2, 4, 6, 8, & 10. Introducing $A(n)$ for $E(u)$, patterns are computed and they are presented in Tables.1-5 and in Figs.2-11.

The results of both small and large are presented. The results on Sidelobe level and Beamwidth are also presented in Tables.6-9.

From results presented in Tables.1-5 it is evident that the amplitude distribution is more tapered with Accelerated Particle Swarm Optimization than that of Taylor.

The realized patterns presented in Figs.3,5,7,9,11 reveal that the patterns have low sidelobe levels obtained with Accelerated Particle Swarm Optimization than those of Taylor.

TABLE.1. – Optimized element amplitude weights for $N = 20$

n **Element Number**	$A(n)$ **Taylor method with** $\bar{n} = 6$	$A(n)$ **Accelerated Particle Swarm Optimization**
1 & 20	0.1147	0.0515
2 & 19	0.1681	0.1176
3 & 18	0.2620	0.2039
4 & 17	0.3801	0.3185
5 & 16	0.5108	0.4525
6 & 15	0.6449	0.5968
7 & 14	0.7718	0.7376
8 & 13	0.8798	0.8604
9 & 12	0.9585	0.9514
10 & 11	1.0000	1.0000

TABLE.2. – Optimized element amplitude weights for $N = 40$

n **Element Number**	$A(n)$ **Taylor method with** $\bar{n} = 6$	$A(n)$ **Accelerated Particle Swarm Optimization**	n **Element Number**	$A(n)$ **Taylor method with** $\bar{n} = 6$	$A(n)$ **Accelerated Particle Swarm Optimization**
1 & 40	0.1090	0.0656	11 & 30	0.6091	0.5858
2 & 39	0.1229	0.1008	12 & 29	0.6750	0.6549
3 & 38	0.1497	0.1239	13 & 28	0.7385	0.7222
4 & 37	0.1877	0.1648	14 & 27	0.7979	0.7858
5 & 36	0.2347	0.2090	15 & 26	0.8518	0.8405
6 & 35	0.2886	0.2622	16 & 25	0.8991	0.8911
7 & 34	0.3476	0.3198	17 & 24	0.9383	0.9327
8 & 33	0.4103	0.3824	18 & 23	0.9687	0.9661
9 & 32	0.4755	0.4487	19 & 22	0.9895	0.9871
10 & 31	0.5422	0.5173	20 & 21	1.0000	1.0000

TABLE 3 – Optimized element amplitude weights for $N = 60$

n **Element Number**	$A(n)$ **Taylor method with** $\bar{n} = 6$	$A(n)$ **Accelerated Particle Swarm Optimization**	n **Element Number**	$A(n)$ **Taylor method with** $\bar{n} = 6$	$A(n)$ **Accelerated Particle Swarm Optimization**
1 & 60	0.1079	0.0848	16 & 45	0.5976	0.5927
2 & 59	0.1142	0.1134	17 & 44	0.6418	0.6329
3 & 58	0.1264	0.1154	18 & 43	0.6853	0.6798
4 & 57	0.1443	0.1324	19 & 42	0.7276	0.7218
5 & 56	0.1673	0.1574	20 & 41	0.7682	0.7631
6 & 55	0.1948	0.1829	21 & 40	0.8067	0.8053
7 & 54	0.2262	0.2200	22 & 39	0.8427	0.8379
8 & 53	0.2607	0.2521	23 & 38	0.8757	0.8732
9 & 52	0.2979	0.2879	24 & 37	0.9055	0.9053
10 & 51	0.3373	0.3272	25 & 36	0.9317	0.9320
11 & 50	0.3783	0.3691	26 & 35	0.9540	0.9526
12 & 49	0.4208	0.4116	27 & 34	0.9722	0.9723
13 & 48	0.4642	0.4547	28 & 33	0.9860	0.9873
14 & 47	0.5084	0.4989	29 & 32	0.9953	0.9976
15 & 46	0.5530	0.5435	30 & 31	1.0000	1.0000

TABLE 4 – Optimized element amplitude weights for $N = 80$

n **Element Number**	$A(n)$ **Taylor method with** $\bar{n} = 6$	$A(n)$ **Accelerated Particle Swarm Optimization**	n **Element Number**	$A(n)$ **Taylor method with** $\bar{n} = 6$	$A(n)$ **Accelerated Particle Swarm Optimization**
1 & 80	0.1076	0.0891	21 & 60	0.5919	0.5865
2 & 79	0.1111	0.1079	22 & 59	0.6251	0.6243
3 & 78	0.1181	0.1167	23 & 58	0.6581	0.6484
4 & 77	0.1283	0.1282	24 & 57	0.6905	0.6827
5 & 76	0.1418	0.1355	25 & 56	0.7222	0.7197
6 & 75	0.1581	0.1526	26 & 55	0.7530	0.7492
7 & 74	0.1771	0.1690	27 & 54	0.7827	0.7692
8 & 73	0.1985	0.1910	28 & 53	0.8111	0.8013
9 & 72	0.2220	0.2182	29 & 52	0.8381	0.8398
10 & 71	0.2474	0.2419	30 & 51	0.8635	0.8657
11 & 70	0.2743	0.2723	31 & 50	0.8871	0.8862
12 & 69	0.3027	0.2895	32 & 49	0.9088	0.8977
13 & 68	0.3322	0.3216	33 & 48	0.9284	0.9175
14 & 67	0.3627	0.3706	34 & 47	0.9459	0.9384
15 & 66	0.3940	0.3984	35 & 46	0.9611	0.9553
16 & 65	0.4260	0.4106	36 & 45	0.9739	0.9773
17 & 64	0.4586	0.4416	37 & 44	0.9843	0.9893
18 & 63	0.4916	0.4863	38 & 43	0.9921	0.9847
19 & 62	0.5249	0.5236	39 & 42	0.9974	0.9837
20 & 61	0.5584	0.5513	40 & 41	1.0000	1.0000

TABLE 5 – Optimized element amplitude weights for $N = 100$

n **Element Number**	$A(n)$ **Taylor method with** $\bar{n} = 6$	$A(n)$ **Accelerated Particle Swarm Optimization**	n **Element Number**	$A(n)$ **Taylor method with** $\bar{n} = 6$	$A(n)$ **Accelerated Particle Swarm Optimization**
1 & 100	0.1074	0.0950	26 & 75	0.5884	0.5941
2 & 99	0.1097	0.1169	27 & 74	0.6151	0.6164
3 & 98	0.1141	0.1217	28 & 73	0.6416	0.6330
4 & 97	0.1208	0.1235	29 & 72	0.6678	0.6700
5 & 96	0.1295	0.1375	30 & 71	0.6936	0.6872
6 & 95	0.1403	0.1299	31 & 70	0.7190	0.7050
7 & 94	0.1529	0.1492	32 & 69	0.7438	0.7496
8 & 93	0.1673	0.1575	33 & 68	0.7679	0.7683
9 & 92	0.1833	0.1821	34 & 67	0.7913	0.7905
10 & 91	0.2007	0.2088	35 & 66	0.8138	0.8169
11 & 90	0.2195	0.2162	36 & 65	0.8354	0.8258
12 & 89	0.2396	0.2360	37 & 64	0.8559	0.8473
13 & 88	0.2606	0.2633	38 & 63	0.8754	0.8719
14 & 87	0.2827	0.2804	39 & 62	0.8937	0.9147
15 & 86	0.3055	0.2951	40 & 61	0.9107	0.9140
16 & 85	0.3291	0.3337	41 & 60	0.9264	0.8892
17 & 84	0.3534	0.3595	42 & 59	0.9408	0.9368
18 & 83	0.3782	0.3814	43 & 58	0.9537	0.9611
19 & 82	0.4035	0.4059	44 & 57	0.9651	0.9735
20 & 81	0.4292	0.4173	45 & 56	0.9749	0.9787
21 & 80	0.4553	0.4450	46 & 55	0.9832	0.9659
22 & 79	0.4816	0.4844	47 & 54	0.9899	0.9855
23 & 78	0.5082	0.5089	48 & 53	0.9949	0.9897
24 & 77	0.5349	0.5319	49 & 52	0.9983	1.0000
25 & 76	0.5617	0.5607	50 & 51	1.0000	0.9987

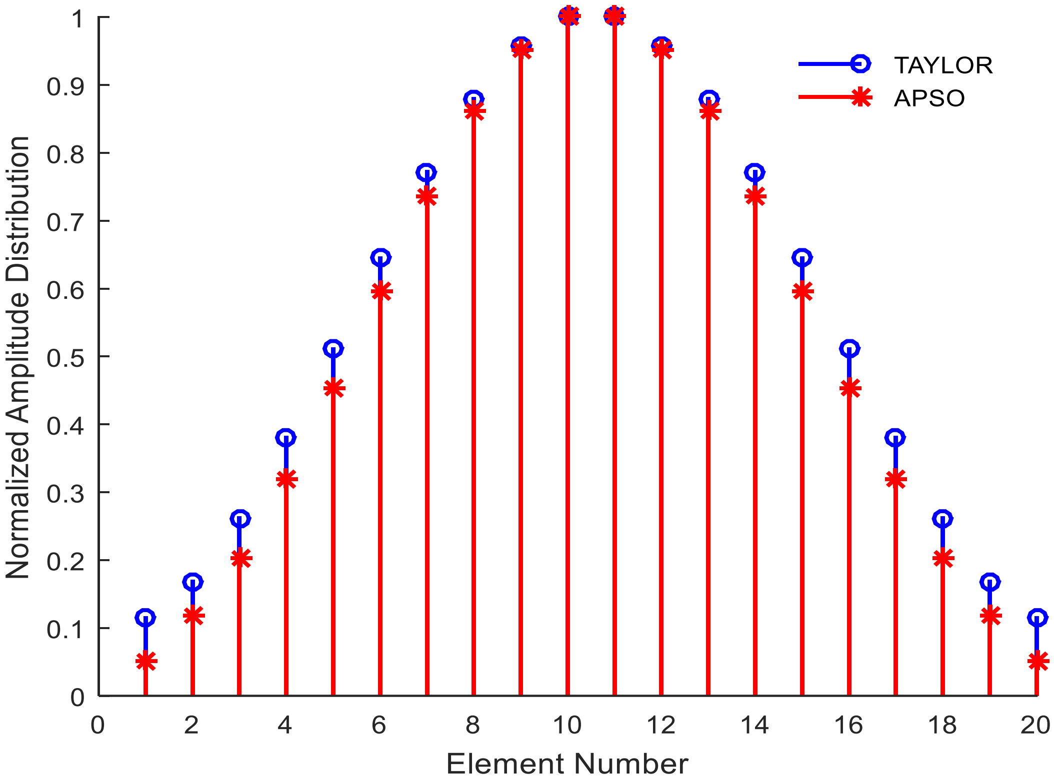

Fig.2. Element amplitude weights obtained by Taylor method with $\bar{n} = 6$ and APSO method for $N = 20$

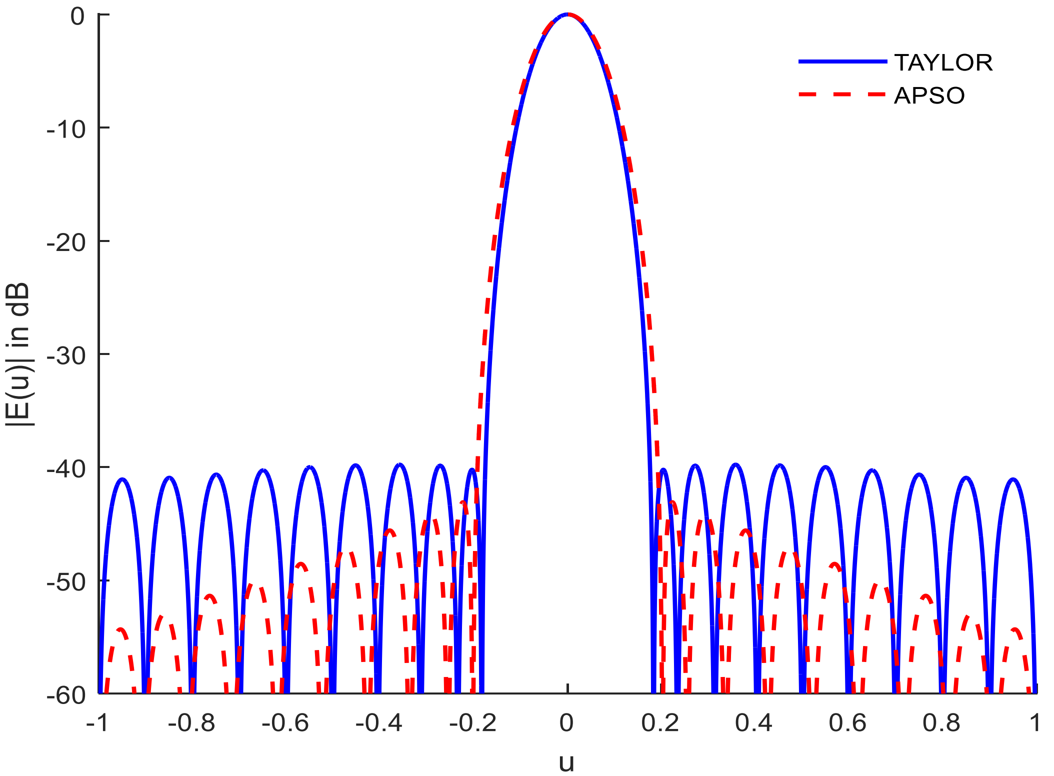

Fig.3. Optimized Sum Pattern by Taylor method with $\bar{n} = 6$ and APSO method for $N = 20$

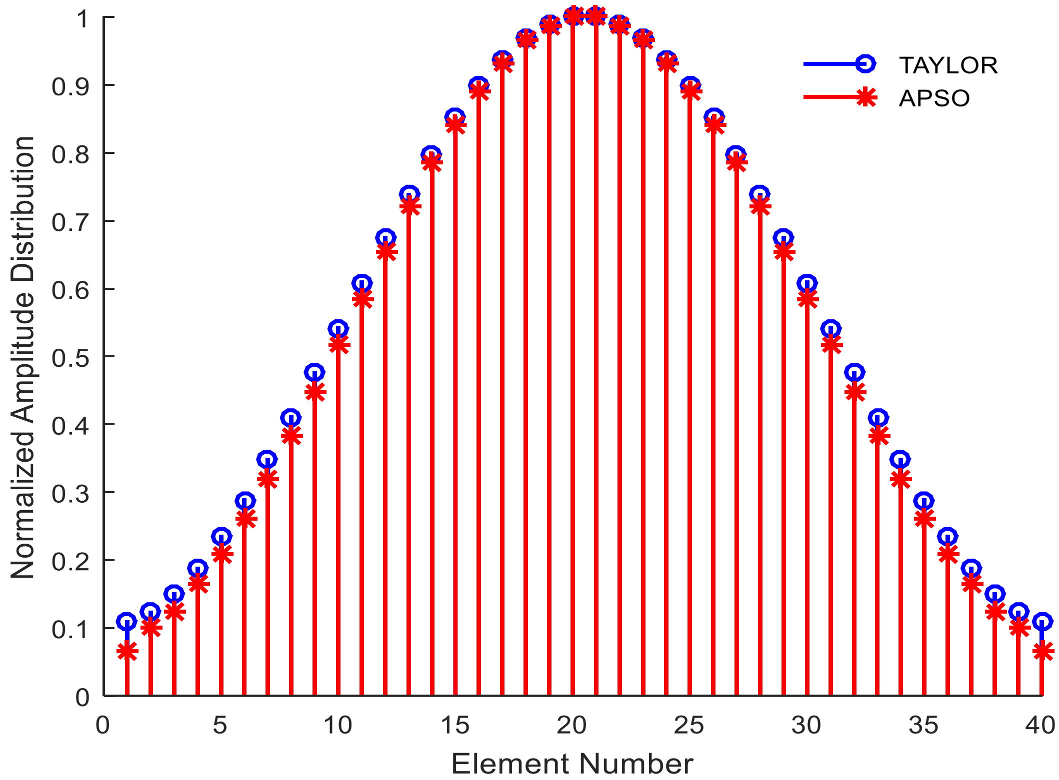

Fig. 4. Element amplitude weights obtained by Taylor method with $\bar{n} = 6$ and APSO method for $N = 40$

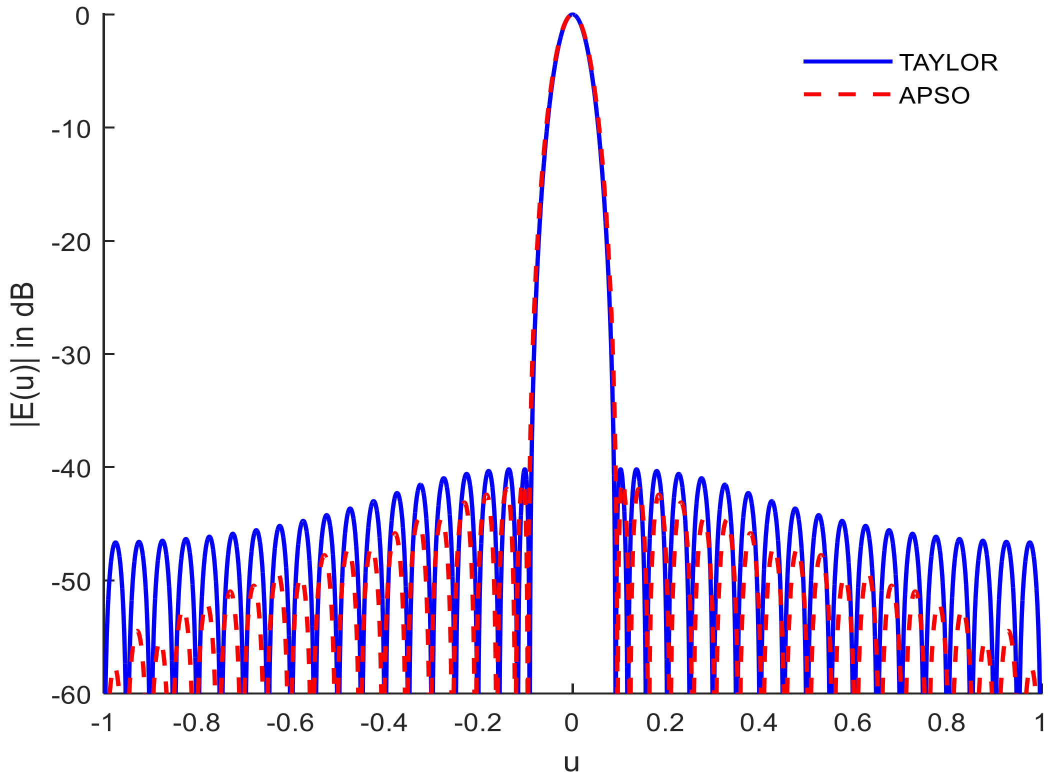

Fig. 5. Optimized Sum Pattern by Taylor method with $\bar{n} = 6$ and APSO method for $N = 40$

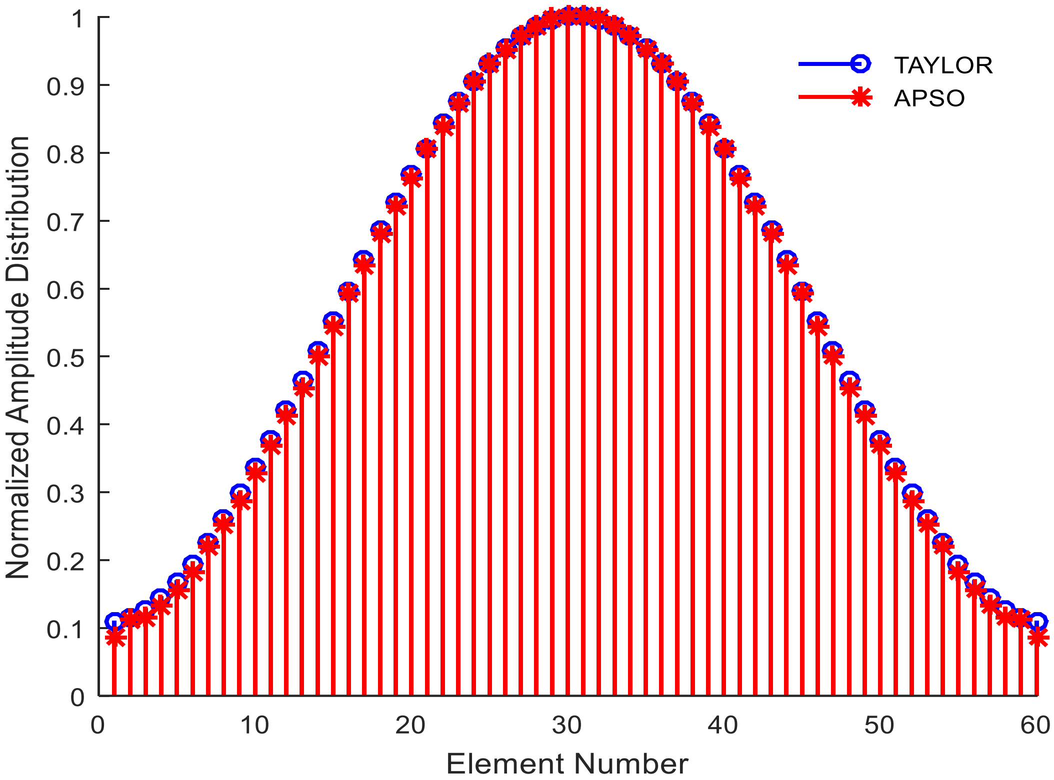

Fig. 6. Element amplitude weights obtained by Taylor method with $\bar{n}=6$ and APSO method for $N=60$

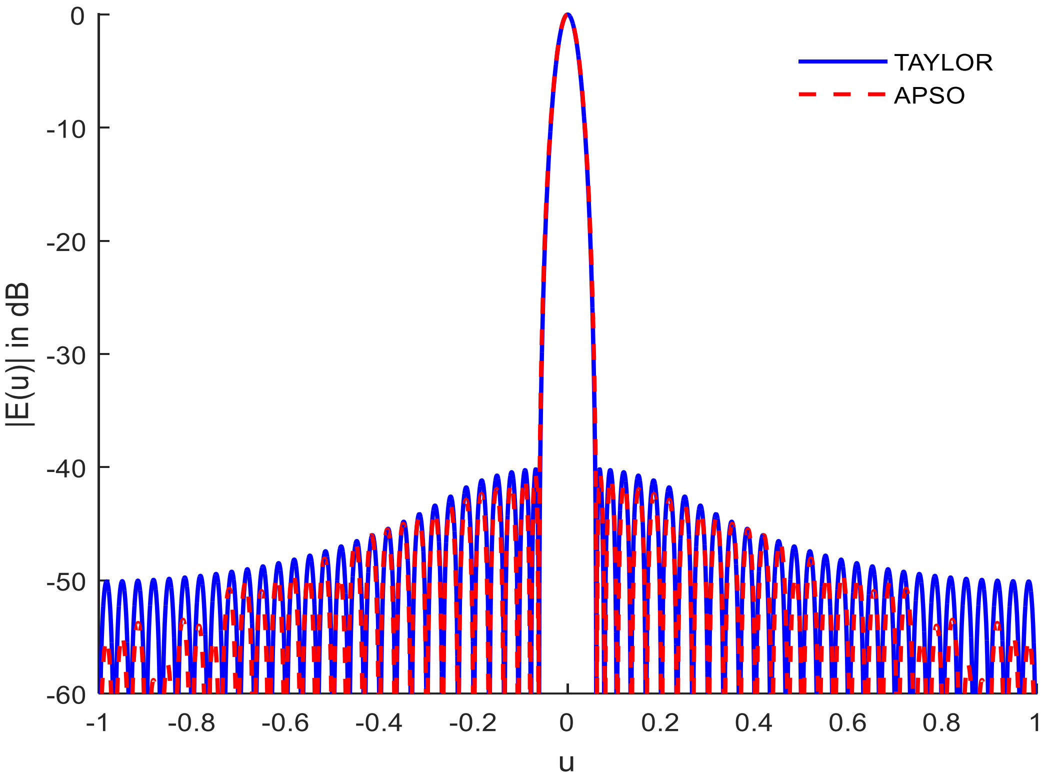

Fig. 7. Optimized Sum Pattern by Taylor method with $\bar{n}=6$ and APSO method for $N=60$

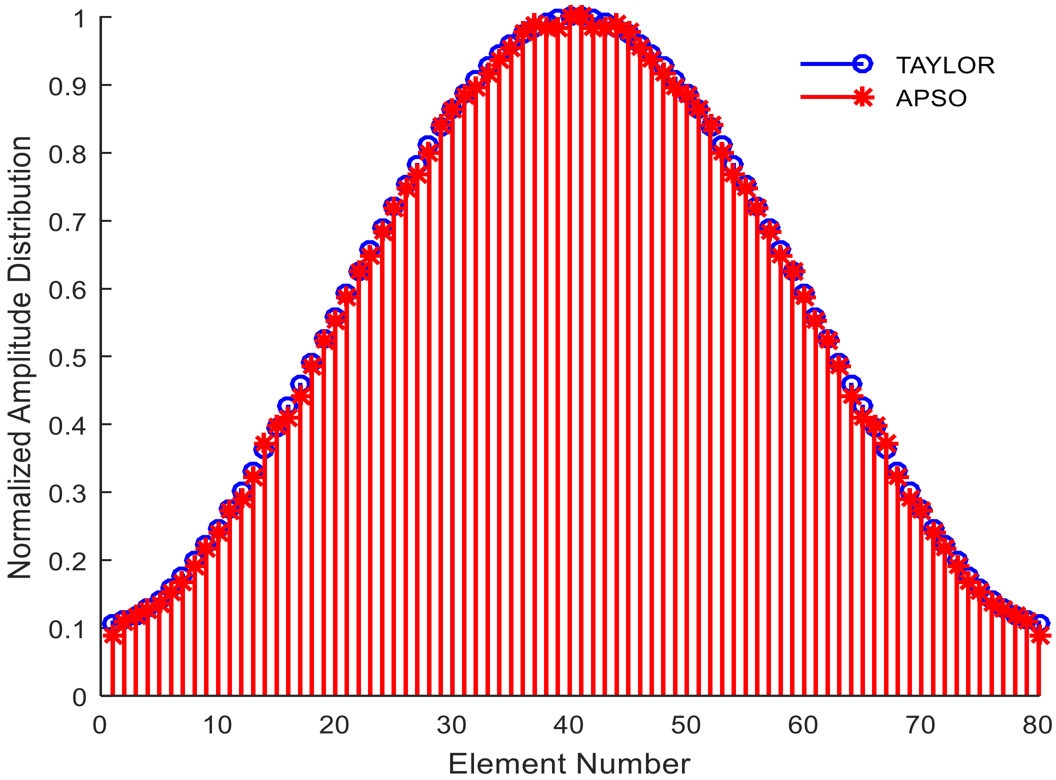

Fig. 8. Element amplitude weights obtained by Taylor method with $\bar{n}=6$ and APSO method for $N=80$

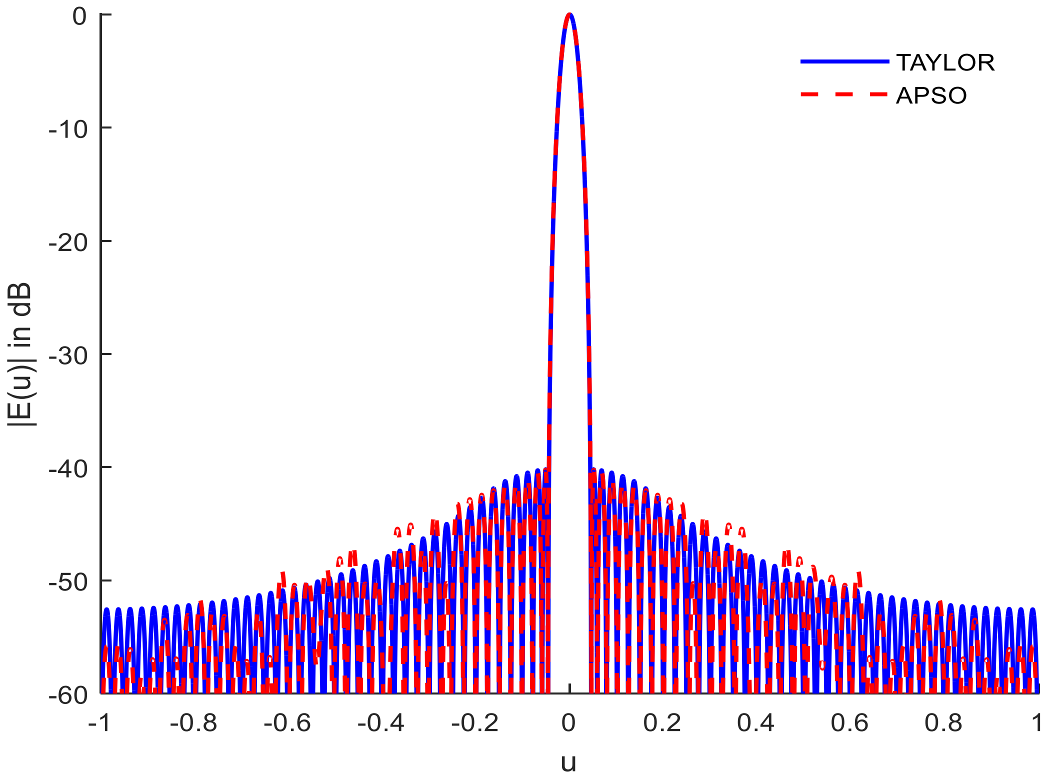

Fig. 9. Optimized Sum Pattern by Taylor method with $\bar{n}=6$ and APSO method for $N=80$

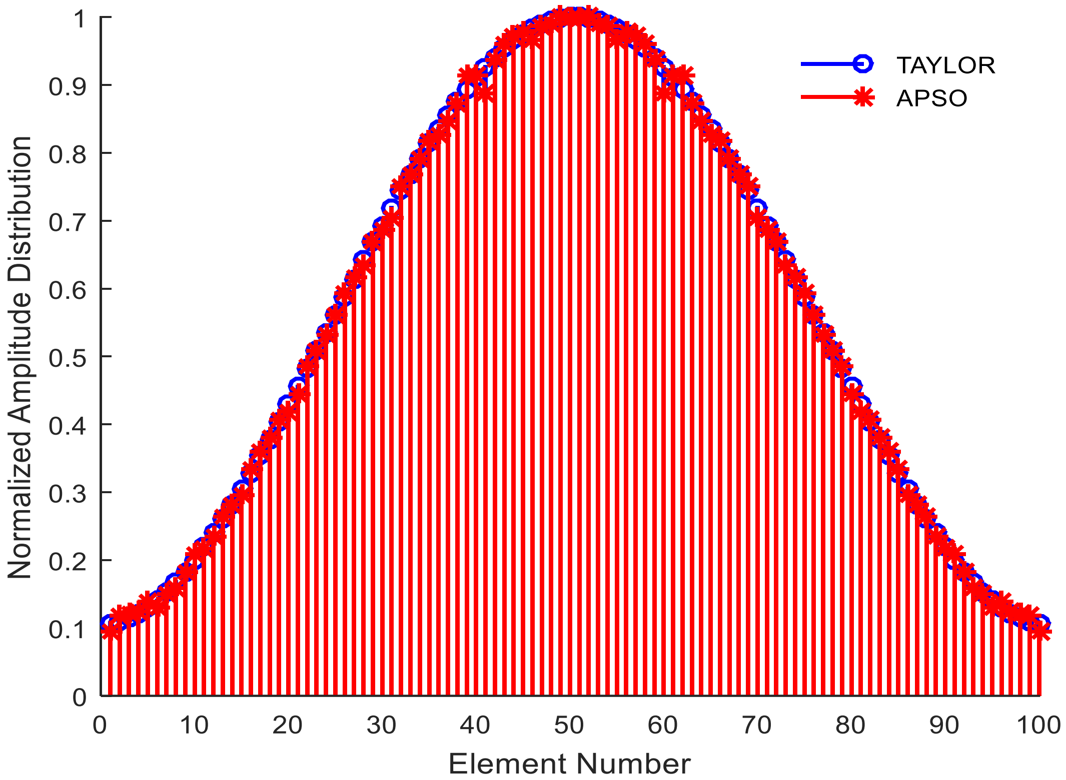

Fig. 10. Element amplitude weights obtained by Taylor method with $\bar{n}=6$ and APSO method for $N=100$

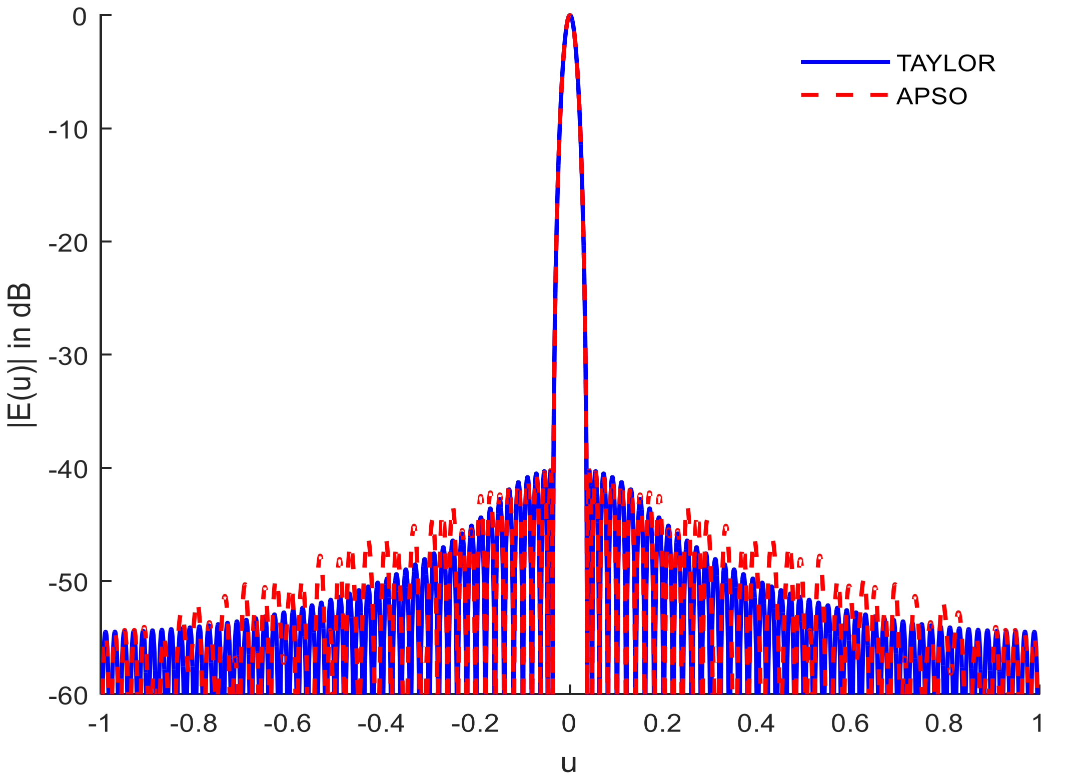

Fig.11. Optimized Sum Pattern by Taylor method with $\bar{n}=6$ and APSO method for $N=100$

TABLE.6. First Null Beamwidth for Optimized Sum Pattern

N **Number of Elements**	$FNBW(\deg)$ **Taylor method with** $\bar{n} = 6$	$FNBW(\deg)$ **Accelerated Particle Swarm Optimization**
20	20.89	23.02
40	10.44	11.03
60	6.96	7.07
80	5.21	5.24
100	4.18	4.18

TABLE.7. First Side Lobe Level for Optimized Sum Pattern

N **Number of Elements**	$First\,SLL(dB)$ **Taylor method with** $\bar{n} = 6$	$First\,SLL(dB)$ **Accelerated Particle Swarm Optimization**
20	-40.22	-43.09
40	-40.18	-41.20
60	-40.17	-40.61
80	-40.17	-40.21
100	-40.17	-40.17

TABLE.8. Last Side Lobe Level for Optimized Sum Pattern

N **Number of Elements**	$Last\ SLL(dB)$ **Taylor method with $\bar{n}=6$**	$Last\ SLL(dB)$ **Accelerated Particle Swarm Optimization**
20	-41.08	-55.32
40	-46.65	-57.65
60	-50.09	-55.25
80	-52.56	-55.83
100	-54.48	-55.84

Conclusion

It is evident from the results on $A(n)$, the resultant amplitude distribution is tapered in both the cases. But the patterns are entirely different. In Taylor's method, the patterns have $(\bar{n}-1)$ number of sidelobes of equal height, equal sidelobes and the rest have exponential decay. But in Accelerated Particle Swarm Optimization, sidelobes are tapered. Taylor's method is basically applicable to line sources. But it modified for discrete arrays by sampling at the location of the elements. Quantized levels are obtained. In the case of Accelerated Particle Swarm Optimization, discrete amplitude levels are directly determined and hence it is simple and realistic.

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