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Pattern Synthesis using Modified Differential Evolution Algorithm

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Abstract:

The main aim of this work is to generate the synthesis of flat-top pattern from an array of isotropic radiating elements using modified differential evolution based on harmony search algorithm as an optimization method. In-order to synthesize a desired sector pattern main beam with sidelobes, element amplitude excitations and phase excitations are optimally determined. The sector pattern is then wrapped and perturbed to the desired shaped beam by deterministic perturbations. It is found that the new algorithm provides a great improvement in main lobe shaping control and sidelobe levels which is present in the obtained radiation pattern. The proposed method is most reliable, accurate and best optimization technique. The optimized numerical simulation results show that the method improves the performance of the algorithm significantly, in terms of both convergence rate and exploration ability.

Key words:

Antenna arrays, Pattern synthesis, Desired Shaped beam, Beam-former, Modified Differential Evolution Algorithm, Flat-top or sector beam pattern, sidelobe level (SLL).

1. Introduction:

The array pattern synthesis with shaped beam patterns has many applications in communications, phased array radar, satellite, military and radars. When the array antenna is designed to generate flat-top beams over required angular sectors, the gain is found to be reduced. Flat-top pattern is a kind of shaped beam pattern. To provide wide angular coverage flat-top or sector beams are used (R. S. Elliot, 2003 and C. A. Balanis, 1997). These sector beams are also used for surface search from ship-borne antennas and air search from ground-based antennas.

Many antenna array synthesis techniques for shaped patterns have been developed in the past years, and most of them deal only with uniformly spaced arrays. In recent years these techniques have been carried out for both equally spaced linear arrays (R.S. Elliot and J. G. Stern, 1984) and unequally spaced linear arrays. (A. Ishimaru, 1962) have provided the antenna pattern synthesis to determine desired array shaped beam radiation pattern. Later various improvements can be found to generate the desired beam shapes.

The main objective is to find an appropriate weighing vector and layout of elements to yield the desired radiation pattern. To obtain this, a set of element amplitude excitation coefficients and phase excitations are required for the desired beam shapes. The creation of shaped pattern from an equally spaced linear array has been a subject of inquiry since (P. M. woodward, 1947) addressed the problem. (David. J. Sadler, 2005) discussed a sector beam synthesis using linear and nonlinear optimization techniques. Recently (A. Chakraborty et al., 1981) have explored the beam shaping using nonlinear phase-only perturbations to obtained field-in patterns. (Hyneman et al., 1967) reported a technique for the synthesis of shaped beam with equal percentage ripple.

Many of the pattern synthesis techniques can be found in the literature for the generation of sector beam radiation pattern. (A. Chakraborty et al., 1986) developed a nonlinear phase function for the generation of $cosec^2$ patterns. In these analytical methods the original radiation pattern is somewhat close to the desired radiation pattern and it generated by standard synthesis methods. (A. Sudhakar, G. S. N. Raju et al., 2004) generated a stair-step patterns from a line source of isotropic elements. (G. S. N. Raju et al., 2003) investigated on the generation of sum and difference patterns from array antennas. (K. V. S. N. Raju et al., 2010) studied on radiation patterns of arrays using new space distributions. But these analytical techniques in synthesizing shaped beam patterns are not efficient enough to provide an optimum pattern.

More recently, evolutionary algorithms including Simulated annealing (V. Murino et al., 1996), PSO and some PSO variants (W. T. Li and X. W. Shi, 2008) are more popular and applicable to array pattern synthesis of phased arrays. (Guney. K et al., 2007) shows the use of Bees algorithm for the synthesis of Amplitude-only pattern nulling of linear antenna arrays. (A. Chatterjee, G. K. Mahanthi et al., 2012) design of fully digital controlled dualbeam concentric ring array using firefly; PSO and gravitational search algorithm are discussed.

In a similar way one of the latest evolutional computational algorithm useful for finding the global optimal solution of a given objective function, namely, the Differential Evolution Algorithm (Chuan. Lin et al., 2010) the design of spaced arrays in an array pattern synthesis is introduced. The effectiveness of DE is used to find out the optimal values of the amplitude coefficients that are obtained in the desired sector patterns. Philip Yuanping [31] presents a new pattern synthesis algorithm for arbitrary array based on adaptive array theory in pattern synthesis shaped beam.

In this paper, a Modified version of Differential Evolution strategy based on harmony search algorithm is introduced as a new optimization method (K. Guney and M. Onay, 2011) for the generation of flat-top shaped sector main beam with sidelobes in the pattern synthesis technique. For this purpose there are usually three evolutionary operations in classic DE, which are summed up to two operations in the modified DE. A new proposed method (Fenggan Zhang, Weimin Jia et al., 2013) is developed for the linear aperiodic array synthesis. By adding the parameter section strategy into the classic DE, the Modified DE combines the good local search capability of the classic DE and the great search diversity of harmony search algorithm.

The main theme of this work is to generate desired sector main beam radiation pattern in an array synthesis with the optimal element excitation amplitude and phase distributions. The effectiveness of MDE is used to find out the optimum values of amplitude and phase excitation coefficients so that it can produce an optimum desired flat-top radiation pattern. The rest of this paper is organized as follows: Section II briefly introduces Modified Differential Evolution algorithm, Section III describes the Mathematical formulation, Section IV reported the Numerical simulation results and finally the conclusion is presented in Section V.

2. Modified Differential Evolution Algorithm

2.1 Initialization:

The idea is to produce the trail vectors according to the manipulation of the target and difference vectors. If the trail vector yields a better fitness than a predetermined population member the new trail vector will be adopted into the vector base.

The classic DE search starts with randomly initiated population of N_p D-dimensional parameter vectors. Each target vector $X_{i,G}$ = [1, 2, … N_p] is a solution to the optimization problem. Where the index ‘i’ denotes the population, i= [1, 2, ... N_p] and ‘G’ denotes the subsequent generation to which the population belongs.

For each target vector $X_{i,G}$, i= 1, 2, 3…N_p, a mutant vector is generated according to

$$V_{i,G+1} = X_{i,G} + F.\left(X_{r_2,G} - X_{r_3,G}\right) \qquad (1)$$

Where the indexes r_1, r_2, r_3 Є {1, 2, 3 … N_p} are randomly selected such that $r_1 \neq r_2 \neq r_3 \neq i$, F is a real and constant factor Є [0, 2] which controls the amplification of the differential variation $\left(X_{r_2,G} - X_{r_3,G}\right)$.

There are usually three evolutionary operations in the classic DE which are summed up to two operations in the modified differential evolution.

In this problem, MDE based on harmony search called Harmony search differential evolution algorithm (HSDEA) is developed to optimize the linear aperiodic arrays with a minimum peak sidelobe level, which is inspired by the harmony search algorithm. The MDE algorithm with a new differential mutation and crossover base strategy, namely best of random, is applied to the synthesis of equally spaced antenna arrays.

2.2 Differential Mutation and Crossover:

For each individual $X_{i,G}$ in the population, a mutant vector $V_{i,G}$ is produced according to the following formula.

$$V_{i,G} = \begin{cases} X_{i,G} + \sum_{y\geq 1} F.(X_{r_2,G} - X_{r_3,G}), & \text{rand j } [0,1] \leq C_R \\ & \text{Or j= rand j } [0, 1] \\ X_{i,G} & \text{otherwise} \end{cases} \qquad (2)$$

Where the indexes r_1, r_2, r_2 ϵ {1, 2, 3 ….NP} are randomly selected such $r_1 \neq r_2 \neq r_2 \neq i$.

The element of mutant vector $V_{i,G}$ is generated by the differential mutation, whenever a randomly generated number between [0, 1] is less than or equal to the C_R value otherwise, it is equal to the corresponding element of the individual $X_{i,G}$.

F is a real and constant factors ϵ [0, 2] which controls the amplification of the differential variation ($X_{r2,G}$-$X_{r3,G}$). Different values of 'y' could lead to differential mutation strategies such as DE /rand/1/bin and DE/best/2/bin.

2.3 Selection:

The population for the next generation is selected from the individual in current population and its corresponding trail vector according to the following rule.

$$X_{i,G+1} = \begin{cases} V_{i,G+1} & \text{if} \quad f(V_{i,G}) < f(X_{i,G}) \\ X_{i,G} & \text{otherwise} \end{cases} \tag{3}$$

Where f (.) is the objective function to be minimized. It is to say that if the new vector $X_{i,G}$ produced by differential mutation and crossover operations yields a lower value of the objective function, it would replace the corresponding individual $X_{i,G}$ in the next generation.

In the modified differential evolution strategy, a new parameter H_R is introduced. The element of mutant vector $V_{i,G}$ is generated randomly in the range between [0, 1] is greater than the specified constant H_R. Otherwise, the element is produced by the classic DE. In this case, the probability that each element of mutant vector $V_{i,G}$ is produced in three ways are followed by the schematic structure as shown in the figure 1.

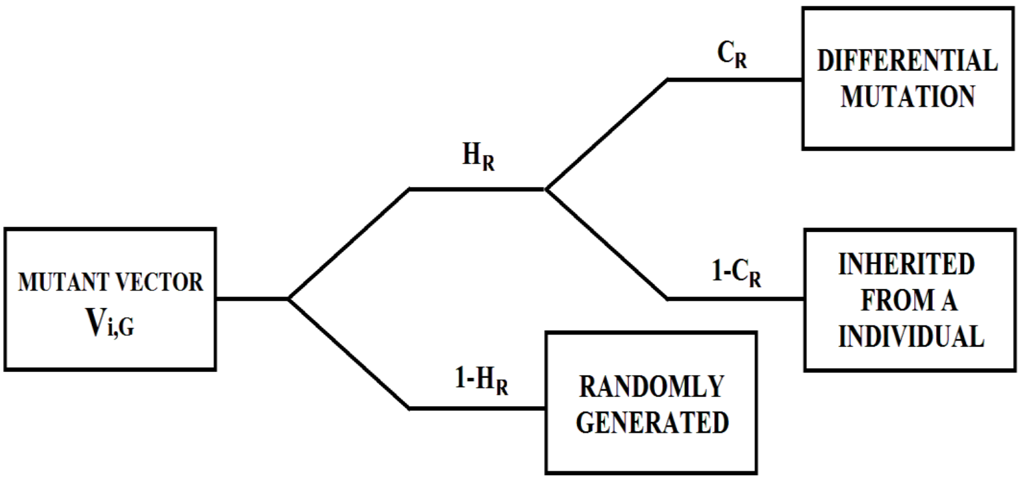

Figure 1: Probability of the new element generated using harmony search differential evolution algorithm

The new way to produce mutant elements injects the random noise into the population and improves its diversity. The changes of two key parameters C_R and F have a great influence on the algorithm performance. The parameter selection of HSDEA can be reffered at that of harmony search algorithm. Here C_R and F are updated as follows.

$$\left\{ \begin{array}{l} C_R(gn) = C_{Rmin} + (C_{Rmax} - C_{Rmin}) * \frac{gn}{Gmax} \\ \\ F(gn) = F_{max} \exp(C.gn),\ C = \ln\left(\frac{Fmin}{Fmax}\right) / G_{max} \end{array} \right\} \tag{4}$$

Where C_{Rmax} and C_{Rmin} are the maximum and minimum adjusting rate of C_R and F_{min} , F_{max} are the minimum and maximum values of F respectively.

The steps of MDE are presented as follows:

1. Initialize the model parameters and generate Np the N-dimensional parameter vectors as initial vector base
2. Select two vectors randomly from the vector base
3. Carry out the mutation and crossover operations in HSDEA and produce a new element vector
4. Execute the selection operation in DE update the vector base
5. Adjust the parameters C_R(gn) and F (gn)
6. Check the stopping criteria. If not meet, return to step 2.

To show the excellence performance of Modified Differential Evolution Algorithm the specifications have been selected from the parameter values of harmony search. These rules are governed by some parameters, i.e., a scaling weighted factor 'F' and a probability CR, which control the crossover operator. The control parameter values of minimum and maximum crossover are selected optimally which depends on the problem. The operator parameter values are HR= 0.9, F_{max}= 0.6, F_{min}= 0.1 and iterations G_{max}= 5000.

3. Mathematical formulation

3.1 Array synthesis Methodology:

A linear array having N-isotropic elements placed along Z-axis with equal inter-element spacing 'd' and a symmetric geometry of linear array as shown in figure 2.

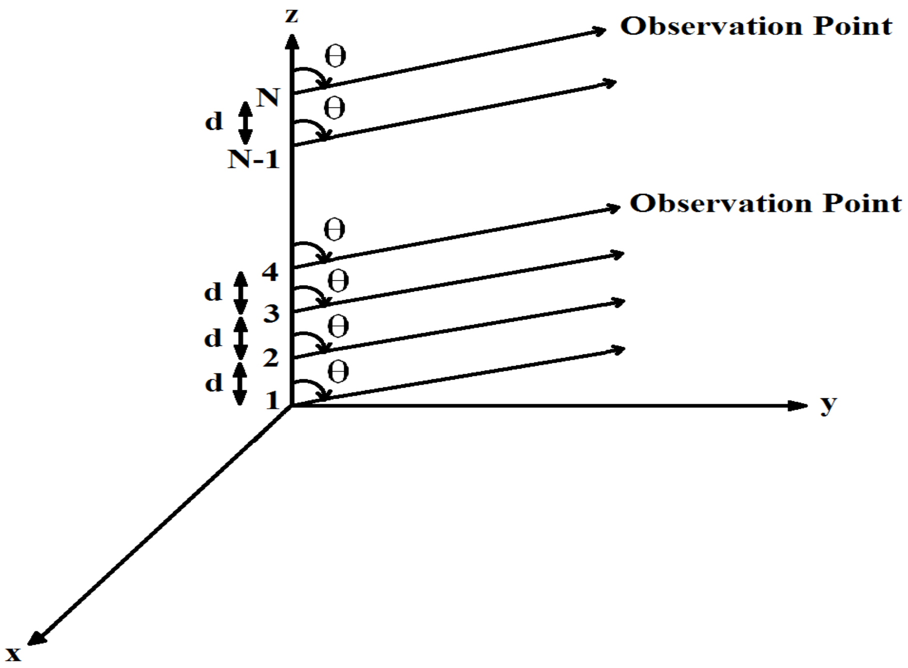

Figure 2: Schematic diagram for linear array geometry

Generally suitable values of three parameters as amplitude, phase and the inter-element spacing are considered to design an array factor for the desired radiation pattern. Here the elements are spaced at a distance'd' operated at wavelength of λ/2 in order to avoid mutual coupling and grating lobes. The array factor is designed in a normalized far field pattern and it is represented in the equation 4. Finally to generate the optimized radiation pattern amplitude and phase excitation coefficients are determined and it limits the amplitudes of elements from 0 to 1 and phase values in between –π and π.

The array factor in the azimuth plane is given as

$$\sum_{n=1}^{N} A(X_n)\, e^{j\frac{2\pi}{\lambda} n\, d\, u} \tag{5}$$

Where u = sin θ and K=2π/λ is the wave number

'λ' is the wave length in integer times of fundamental frequency

'd' is the spacing between the elements

'X_n' is the position of the n^{th} element respectively and is evaluated by $X_n = \frac{2n-N-1}{N}$

3.2 Objective function:

The goal of the optimization is to generate a desired flat-top pattern of a specified width with acceptable sidelobe level by employing non-uniform excitations to individual elements of the antenna array. The normalized amplitude in the search range [0, 1] with static phase shift in the range of [-π, π] are taken as the optimization parameters.

Therefore the objective function is given as

$$f = \min (w_1 e_1 + w_2 e_2) \tag{6}$$

Where w_1 and w_2 are the controlled weights and sum of the weights should be one that is represented as

$$\sum_{i=1}^{2} W_i = 1 \tag{7}$$

Where 'e_1' is the mean square error of the main beam region

$$e_1 = \left[\frac{1}{p} \sum_{i=1}^{p} |E_1(u_i)|^2 \right]^{1/2} \tag{8}$$

Here 'p' represents the number of sampling points in mainlobe region and $E_1(u_i)$ is the error in main beam region and it is calculated as

$$E_1(u) = \{ E(u) - F(u); \quad 0 \leq u \leq -u_0 \} \tag{9}$$

Where F (u) is desired sector pattern and E (u) is pattern obtained in the evolutionary process.

Therefore the desired sector pattern is represented by

$$F(u) = \begin{cases} 1 & ; \quad 0 \leq u \leq 0.5 \\ 0 & ; \quad \text{otherwise} \end{cases} \tag{10}$$

e_2 is the least mean square error in the sidelobe region

$$e_2 = \left[\frac{1}{Q} \sum_{i=1}^{Q} \mathrm{IE_2(u_i)I^2} \right]^{1/2} \tag{11}$$

Where 'Q' is the number of azimuth angles in the sidelobe region and E_2 (u) is the error obtained in sidelobe region and it is calculated as

$$E_2\,(u) = \left\{ E\,(u) - F\,(u); \quad u < -u_0\ \&\ u > u_0 \right\} \tag{12}$$

Where E (u) is the pattern obtained in the evolutionary process and F (u) is desired sector pattern.

4. Numerical Simulation results and discussion

Many of the pattern synthesis techniques can be found in the literature for the generation of sector beam radiation pattern. (A. Chakraborty and G. S. N. Raju et al., 1986 and 2004) had developed a nonlinear phase function for the generation of cosec^2 patterns. They generated a stair-step patterns from a line source of isotropic elements. But these analytical techniques in synthesizing shaped beam patterns are not efficient enough to provide an optimum pattern.

To show the excellent performance of the new algorithm, investigate a linear antenna array of 20 elements with equal element spacing of 0.5 λ is considered. It is desired that the normalized filed pattern from the array follow the sector function for different angular widths of min 30^0 with the range (-15 to 15) to max 90^0 with the range (-45 to 45) beam width in degrees are obtained and also the ripple level which is contained in the main beam is about less than 0.5dB and the maximum value of SLL is -25dB. For the optimized radiation pattern, the normalized amplitude coefficients and the static phase shifts of the array elements are taken as the optimizing parameters of the MDE algorithm.

As can be seen, the average convergence rate for each of the optimized radiation patterns. It should be pointed out that although the same optimality criterion is adopted in the subsection, the specific forms of the fitness functions are different, thus leading to different convergence accuracy. MDE reports an example for the behavior of the cost function versus the number of 5000 iterations that are shown in the figure 3. It is clearly seen that the required number of fitness evaluations varied from 5000 to 60,000 for different cases which takes only about a minute to hours.

Figure 4 reports the optimized radiation pattern of sector beam 30^0 within the range (-15^0 to 15^0), its corresponding amplitude distribution is given in figure 5 and phase distribution is presented in figure 6. Sector beam angular width of 60^0 within the range (-30^0 to 30^0) radiation pattern is obtained that are reported in figure 7 its corresponding amplitude and phase distributions are presented in figure 8 and figure 9. Similarly optimized radiation patterns followed for the angular widths of sector beam 90^0 (-45^0 to 45^0) are obtained by MDE algorithm that are reported in figure 10 its amplitude and phase plots are given in figure 11 and figure 12. Finally the optimized excitation levels of amplitude and phase distributions for different angular widths are computed and tabulated in the tables (1, 2 and 3).

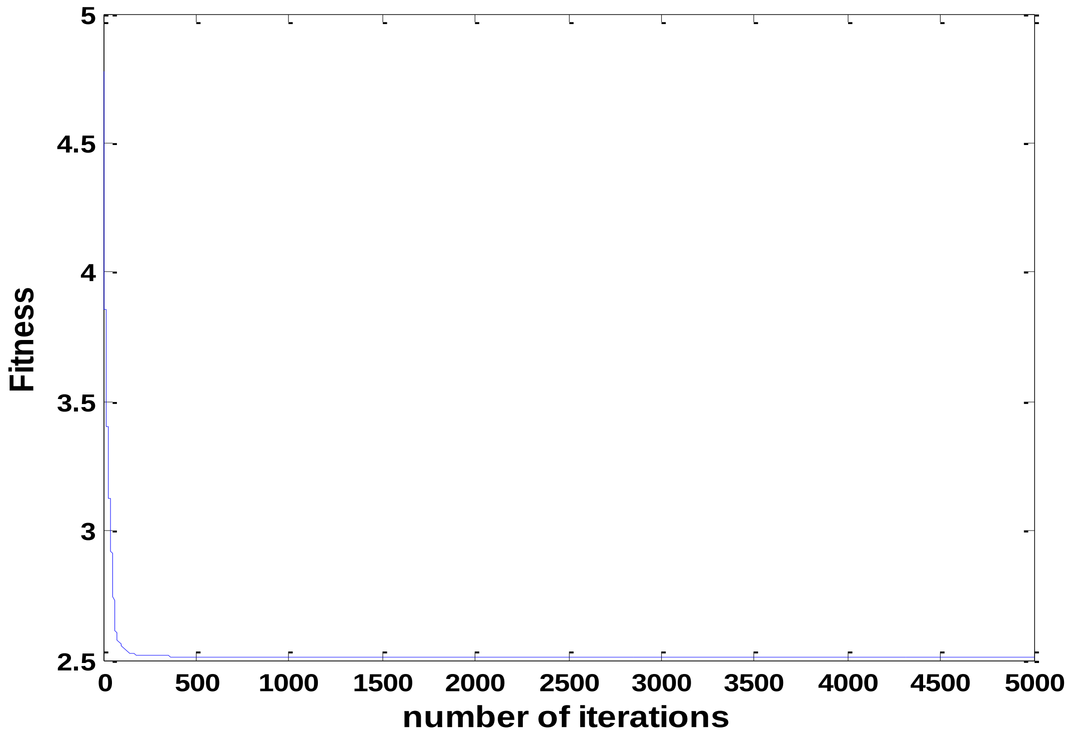

Figure 3: Behavior of fitness function versus number of iterations

Thus it shows that the MDE has ability to achieve the global minima for a non-convex problem. Hence, the optimal solutions found by this method are equivalently as good as those obtained in 20 element array. As can be seen from this simulation results, the DES family are successful in reaching the optimal value with in 100,000 number of fitness evaluations (NOFE) while the GA and PSO method failed and MDE requires the least NOFE among various methods for same accuracy.

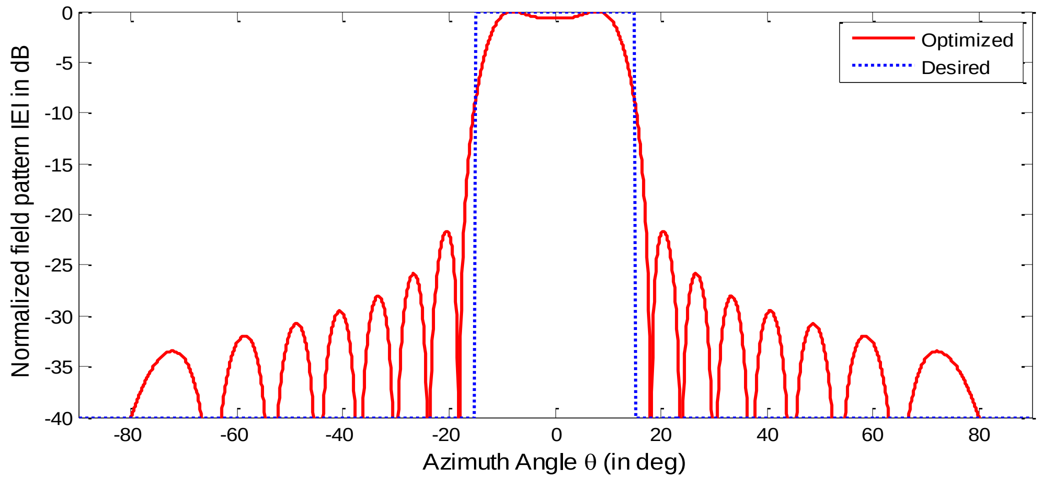

Figure 4: Radiation pattern with sector beam of 30^0(-15 to 15)

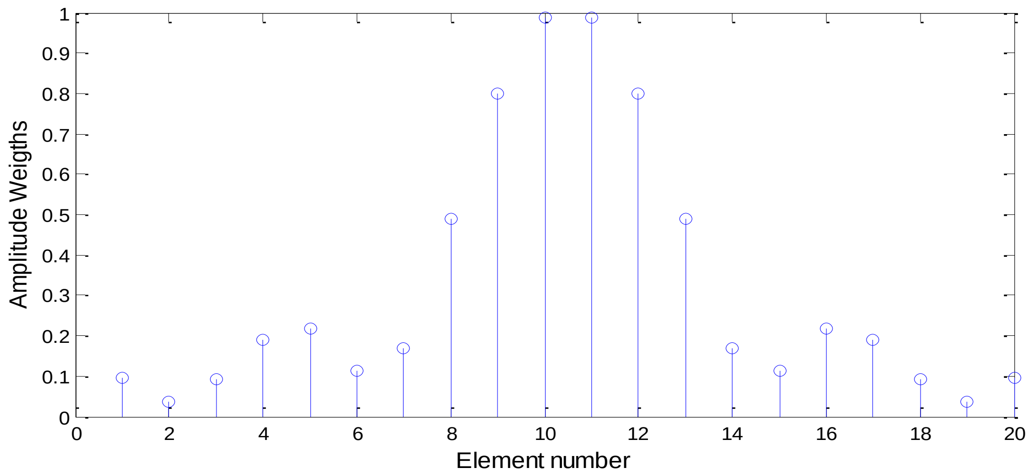

Figure 5: Amplitude distribution of sector pattern at 30^0 beam width

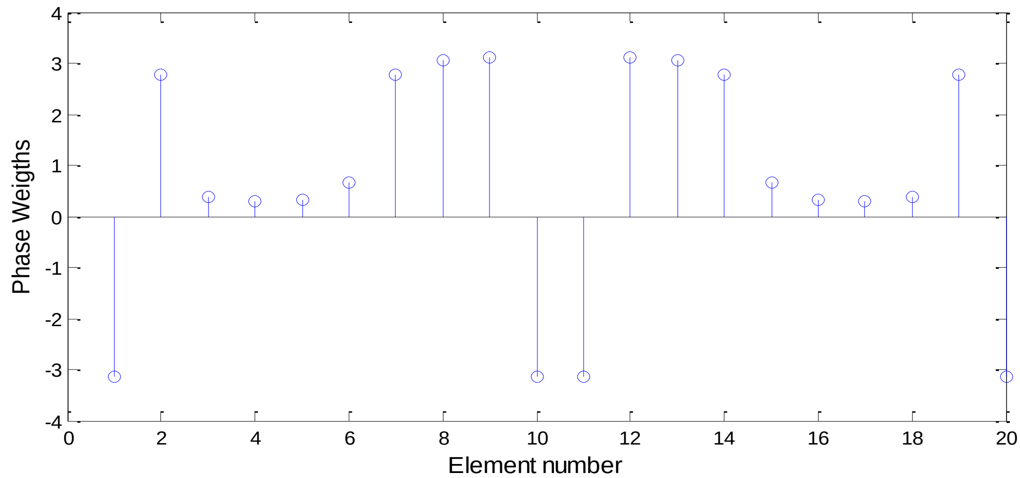

Figure 6: Phase distribution of sector pattern at 30^0 beam width

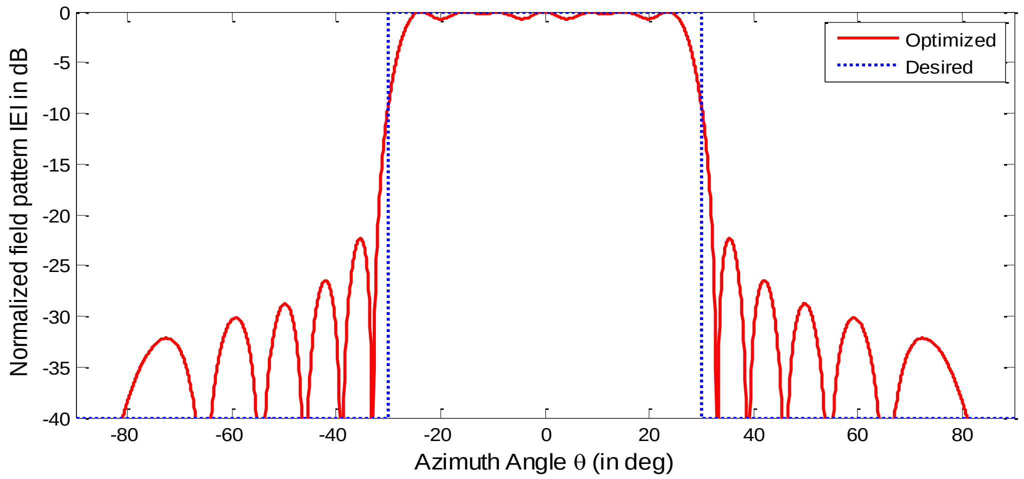

Figure 7: Radiation pattern with sector beam of 60^0(-30 to 30)

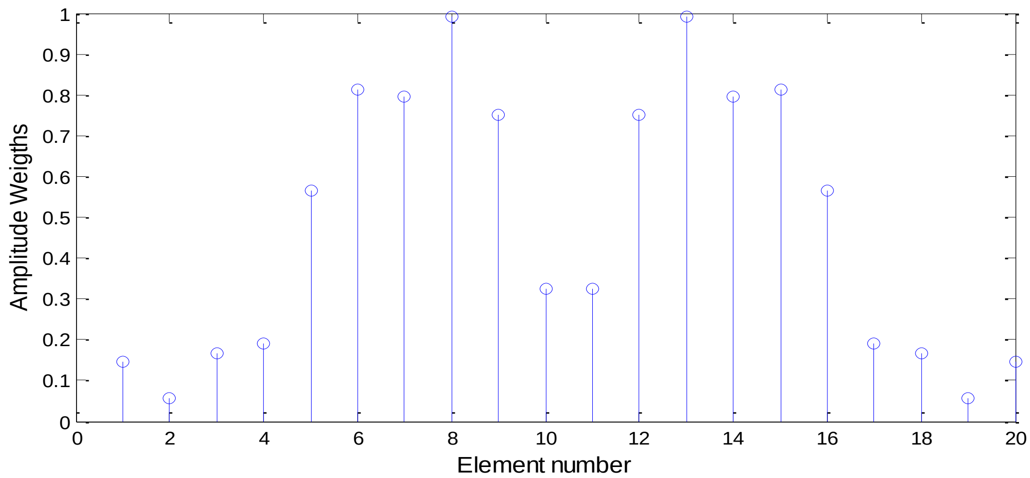

Figure 8: Amplitude distribution of sector pattern at 60^0 beam width

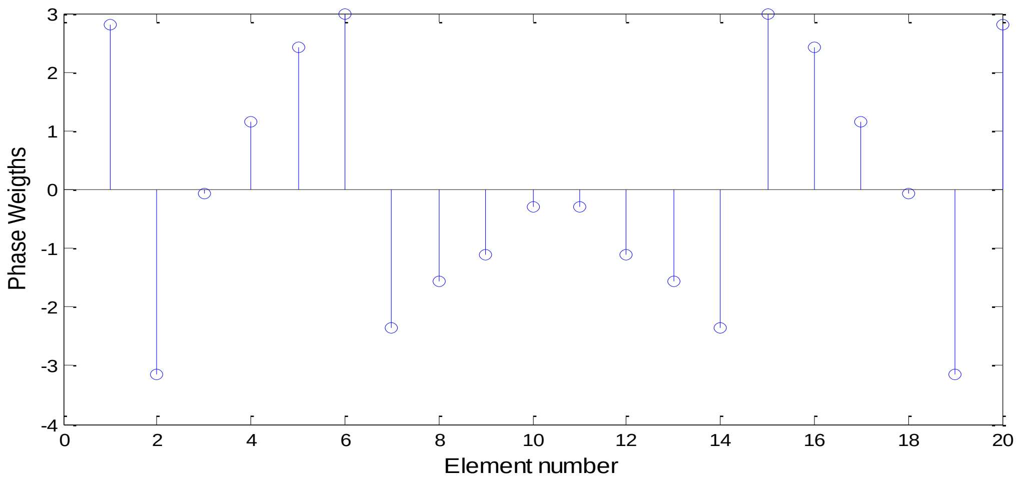

Figure 9: Phase distribution of sector pattern at 60^0 beam width

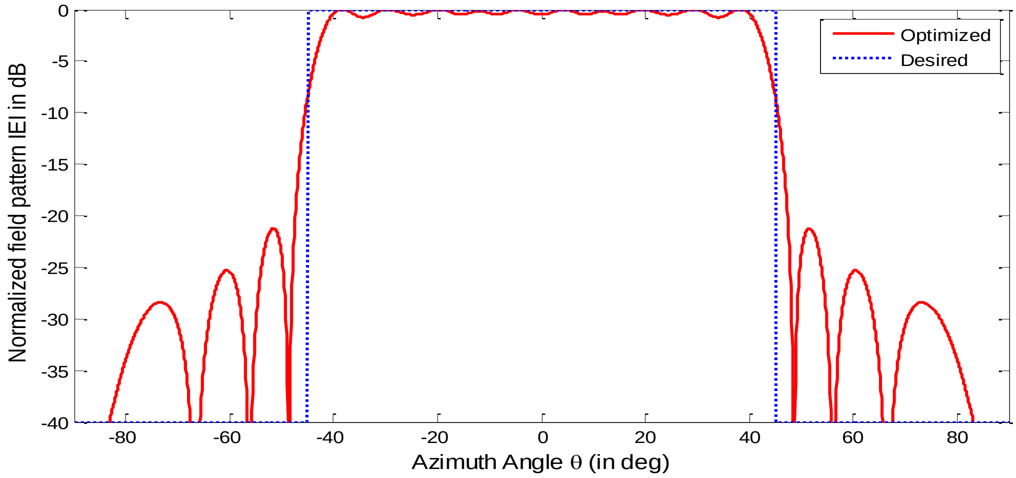

Figure 10: Radiation pattern with sector beam of 90^0(-45 to 45)

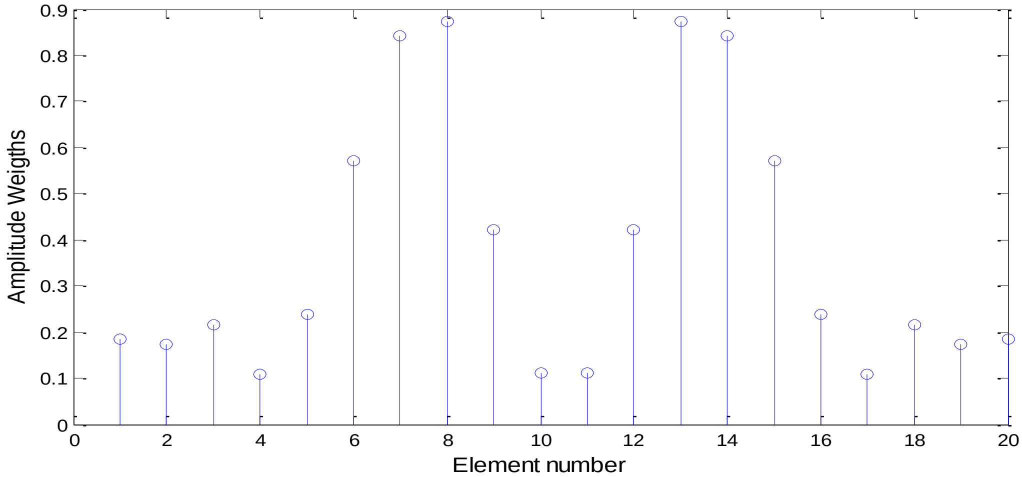

Figure 11: Amplitude distribution of sector pattern at 90^0 beam width

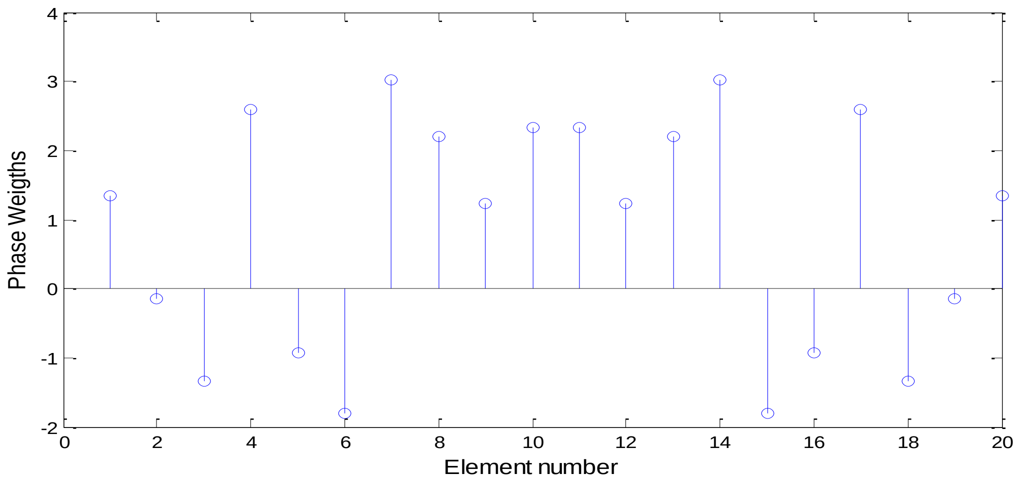

Figure 12: Phase distribution of sector pattern at 90^0 beam width

Table 1: Excitation levels of N=20 elements for width 30^0

Element Number	Element Amplitude Excitation Level	Element Phase Excitation Level	Element Number	Element Amplitude Excitation Level	Element Phase Excitation Level
1	0.095	-3.131	**11**	0.9867	-3.1388
2	0.0361	2.7761	**12**	0.7984	3.126
3	0.0913	0.3821	**13**	0.4896	3.0691
4	0.1898	0.2966	**14**	0.1685	2.7865
5	0.2161	0.3281	**15**	0.1146	0.6581
6	0.1146	0.6581	**16**	0.2161	0.3281
7	0.1685	2.7865	**17**	0.1898	0.2966
8	0.4896	3.0691	**18**	0.0913	0.3821
9	0.7984	3.126	**19**	0.0361	2.7761
10	0.9867	-3.1388	**20**	0.095	-3.131

Table 2: Excitation levels of N=20 elements for width 60^0

Element Number	Element Amplitude Excitation Level	Element Phase Excitation Level	Element Number	Element Amplitude Excitation Level	Element Phase Excitation Level
1	0.095	-3.131	**11**	0.9867	-3.1388
2	0.0361	2.7761	**12**	0.7984	3.126
3	0.0913	0.3821	**13**	0.4896	3.0691
4	0.1898	0.2966	**14**	0.1685	2.7865
5	0.2161	0.3281	**15**	0.1146	0.6581
6	0.1146	0.6581	**16**	0.2161	0.3281
7	0.1685	2.7865	**17**	0.1898	0.2966
8	0.4896	3.0691	**18**	0.0913	0.3821
9	0.7984	3.126	**19**	0.0361	2.7761
10	0.9867	-3.1388	**20**	0.095	-3.131

Table 3: Excitation levels of N=20 elements for width 90^0

Element Number	Element Amplitude Excitation Level	Element Phase Excitation Level	Element Number	Element Amplitude Excitation Level	Element Phase Excitation Level
1	0.1835	1.3417	**11**	0.1118	2.3289
2	0.1737	-0.1425	**12**	0.4229	1.2358
3	0.2169	-1.3395	**13**	0.874	2.2002
4	0.1093	2.589	**14**	0.8413	3.0171
5	0.2374	-0.9279	**15**	0.5712	-1.7959
6	0.5712	-1.7959	**16**	0.2374	-0.9279
7	0.8413	3.0171	**17**	0.1093	2.589
8	0.874	2.2002	**18**	0.2169	-1.3395
9	0.4229	1.2358	**19**	0.1737	-0.1425
10	0.1118	2.3289	**20**	0.1835	1.3417

5. Conclusion:

A modified differential algorithm based on harmony search is applied to the synthesis of spaced 20 elements, linear antenna arrays. The radiation patterns of sector shaped beams are generated using the new method. The element amplitude and phase excitation levels of different angular widths are computed for 20 element array. To show the excellent performance of Modified DE combines the good local search capability of the classic DE and the great search diversity of Harmony search algorithm. The suitable element amplitude excitations and static phase shifts are determined by the algorithm those reduce the sidelobe level of the array to satisfactorily low values. It is evident from the results using the proposed method, flat-top beams are found to have considerable ripples in the trade in region and low sidelobes in the trade off region.

A new method provides convenient mainlobe shape control and also the obtained patterns are compared to the flat-top plot in the desired region at different sector beam angle. The optimized simulation results of sector beam radiation patterns are observed and clearly shows that the superiority of MDE over the other methods in terms of finding optimum solutions for the presented problem.

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