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Modelling and Simulation of Electric Power Transmission Line Voltage

Georgios Leonidopoulos

Electrical Engineering Dept.,
Piraeus Technological University (former Piraeus Institute of Technology),
Thevon 250, Aegaleo, Greece 12244. (E-mail: geoleonido@yahoo.gr)
(Communication address : Kilkis 11, Kalamata, Greece 24100.)

Abstract

In this paper, a well-known mathematical model of electric power transmission line is considered. From this model, a known mathematical expression has been developed which describes how the line voltage varies with the distance taking as starting point the end of the line.

We use this model and the data of a typical electric transmission line to calculate how the line voltage varies. The results are also graphed in order to have an optical view of how the line voltage behaves. Finally, the results are compared to those expected from the theoretical analysis of the model and the relative conclusions are drawn.

Keywords

Voltage, modelling, electric power transmission line, line voltage, voltage travelling wave, voltage refracted wave, electric power system, mathematical model, computer simulation

List of Symbols

R = long-wise omhic resistance of power transmission line (under sinusoidal voltage) per unit length of line (Ω/km)

L = long-wise inductance of power transmission line (under sinusoidal voltage) per unit length of line (H/km)

C = transversal capacitance of power transmission line (under sinusoidal voltage) per unit length of line (F/km)

G = transversal conductance of power transmission line (under sinusoidal voltage) per unit length of line (S/km)

l = length of power transmission line (km)

z = R+jωL = long-wise complex impedance of power transmission line per unit length of line (Ω/km)

y = G+jωC = transversal complex conductance of power transmission line per unit length of line (S/km)

Z = z.l = total long-wise complex impedance of power transmission line (Ω)

Y = y.l = total transversal complex conductance of power transmission line (S)

$\mathbf{V_S}$ = complex line to earth voltage at the beginning of power transmission line, Sending voltage (V)

$\mathbf{V_R}$ = complex line to earth voltage at the end of power transmission line, Receiving voltage (V)

$\mathbf{I_S}$ = complex phase current at the beginning of power transmission line, Sending current (A)

$\mathbf{I_R}$ = complex phase current at the end of power transmission line, Receiving current (A)

$\gamma = \sqrt{zy}$ = α+jβ = transmission co-efficient of power transmission line (km^{-1})

α = reduction co-efficient of power transmission line (neper/km)

β = phase co-efficient of power transmission line (rad/km)

$\mathbf{z_C} = \sqrt{\frac{z}{y}}$ = characteristic impedance of power transmission line (Ω)

$\mathbf{e^{j\varphi}}$ = cosφ +jsinφ = Euler's equation

$\lambda = \frac{2\pi}{\beta}$ = wave length of power transmission line (km)

υ = wave transmission velocity of power transmission line (km/sec)

τ = wave travelling time in order to cover the length of power transmission line (sec)

Δ = electric phase (angle) of power transmission line (rad)

$\mathbf{\frac{\Delta}{l}}$ = electric phase (angle) of power transmission line per unit length of line (rad/km)

$\mathbf{V_{trav}(x)}$ = voltage travelling wave as a function of distance x (V)

$\mathbf{V_{refr}(x)}$ = voltage refracted wave as a function of distance x (V)

V(x) = voltage along the electric power transmission line as a function of distance x (V)

φ(x) = electric phase(angle) of respective complex quantity as function of distance x (°)

1. Introduction

Most people think of the voltage as an element that when they put it on, it is applied immediately. They cannot imagine that the voltage is a wave (an electromagnetic wave) that travels and refracts with almost the speed of light. This understanding is due to the length of line and the inability that people have to perceive the very small time intervals (psecs, μsecs, msecs depending on the line length) that the wave needs to cover these distances.

In this paper, the length under consideration is that of an electric power transmission line of an electric power system [1-23], a length of some hundred kilometers. The equivalent electric circuit under steady state conditions is drawn and the respective differential equations are extracted from it using as independent variable the distance x from either the rears of the line. Solving the differential equations, the mathematical expression describing the voltage along the line is obtained in section 2. The above mathematical model already exists in the literature and can easily be found [1-14].

It is shown that the voltage at any point x of the line is the algebraic summation of the voltage travelling and refracted wave at that point. The proof that the above voltages are the travelling and refracted wave respectively is the mathematical expressions themselves. They are the mathematical expressions of a travelling and refracted wave respectively. The fore-mentioned can also be traced in the literature [1-14]. The information given by the above solution is analysed in further detail in section 3.

As far as I know and search in the literature, I could not find calculation and graphical representation of the voltage along an electric power transmission line. Thus, in this paper, the above mathematical expression is tried on a typical electric power transmission line and the results are presented in section 4. Furthermore, in section 5, the above results are graphed in order to have an optical image of how the voltage along the line behaves. Finally, in section 6 a discussion is developed, the results are compared to the theoretical analysis of section 3 and in section 7 the relative conclusions are drawn.

2. Development of the mathematical expression of line voltage

In figure 1, the electric equivalent representation of power transmission line under steady state conditions and using divided elements has been drawn.

where zdx = the infinitesimal long-wise complex impedance of dx

ydx = the infinitesimal transversal complex conductance of dx

From the infinitesimal element dx, the following equations are drawn :

1st law of Kirchhoff : $[I(x)+dI(x)] = I(x) + dI(x)$

2nd law of Kirchhoff : $[V(x)+dV(x)] = V(x) + dV(x)$

Voltage drop on element zdx : $dV(x) = [I(x)+dI(x)]\, zdx =$

$$\cong I(x)\, zdx \rightarrow \frac{dV(x)}{dx} = I(x)\, z \qquad (1)$$

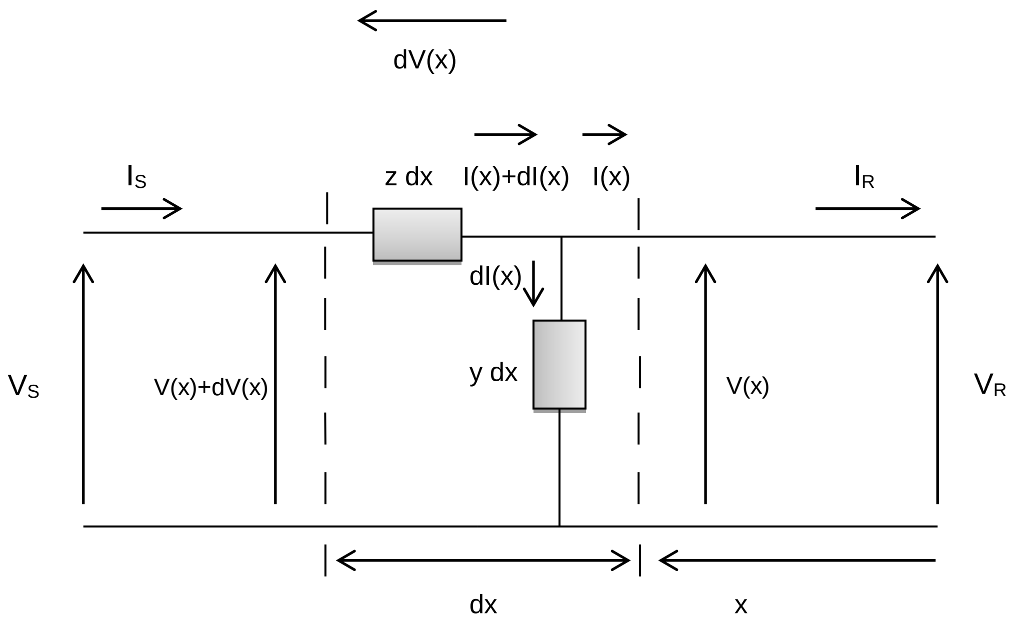


Figure 1. Electric equivalent representation of electric power transmission line

Voltage drop on element ydx : $dI(x) = V(x)\, ydx \rightarrow \frac{dI(x)}{dx} = V(x)\, y \qquad (2)$

Differentiating eqn 1 and replacing it into eqn 2, we get :

$$\frac{d^2 V(x)}{dx^2} = yz\, V(x) \qquad (3)$$

Differentiating eqn 2 and replacing it into eqn 1, we also get :

$$\frac{d^2 I(x)}{dx^2} = yz\, I(x) \qquad (4)$$

From equations (3) and (4), V(x) and I(x) are described by the same differential equations. The above implies that V(x) and I(x) are described by similar mathematical functions.

We take as initial conditions :

$$V(x=0) = V_R \tag{5}$$

$$\text{and} \quad I(x=0) = I_R \tag{6}$$

i.e. we take as x=0 the end of power transmission line

Then from equations (3), (4), (5) and (6), we extract the following mathematical expressions of line voltage :

$$V(x) = \frac{V_R + I_R\, z_C}{2} e^{\gamma x} + \frac{V_R - I_R\, z_C}{2} e^{-\gamma x} \tag{7}$$

$$V_{trav}(x) = \frac{V_R + I_R\, z_C}{2} e^{\gamma x} \tag{8}$$

$$V_{refr}(x) = \frac{V_R - I_R\, z_C}{2} e^{-\gamma x} \tag{9}$$

3. Theoretical analysis of the mathematical expression of line voltage

Equation (7) can also be written in hyperbolic form :

$$V(x) = V_R \cosh(\gamma x) + I_R.z_C \sinh(\gamma x) \tag{10}$$

The term $\cosh(\gamma x)$ can be written as :

$$\cosh(\gamma x)=\cosh[(\alpha+j\beta)x]=\cosh(\alpha x+j\beta x)=\cosh(\alpha x).\cos(\beta x)+j\sinh(\alpha x).\sin(\beta x)$$

The term $\sinh(\gamma x)$ can be written as :

$$\sinh(\gamma x)=\sinh[(\alpha+j\beta)x]=\sinh(\alpha x+j\beta x) =\sinh(\alpha x).\cos(\beta x)+j\cosh(\alpha x).\sin(\beta x)$$

The above equations (8) and (9) are nothing else but the mathematical expressions of a wave.

On one hand, the terms (V_R+$I_R z_C$) and (V_R-$I_R z_C$) are constant complex numbers since V_R, I_R and z_C are constant complex numbers. That implies that they have a constant magnitude and a constant phase.

On the other hand, the terms $e^{\gamma x}$ and $e^{-\gamma x}$ vary with distance x from the end of power transmission line.

The term $e^{\gamma x}$ can be written as $e^{(\alpha+j\beta)x} = e^{\alpha x}\, e^{j\beta x} = e^{\alpha x}[\cos(\beta x) + j \sin(\beta x)]$

The values of α and β are real positive numbers for a typical real power transmission line. This will be understood from the following analysis.

The $e^{\alpha x}$ is the magnitude of the above term while the $e^{j\beta x}$ is the phase (angle) of the above term.

The term $e^{\alpha x}$ increases as x increases i.e. the magnitude of voltage travelling wave increases as we approach the beginning of line. In other words, the magnitude (intensity) of voltage travelling wave diminishes as the wave travels from the beginning of line (where the voltage is applied and the voltage travelling wave starts) to the end of line as one expects in real world (the intensity of signal diminishes as it moves away from source).

The term βx similarly increases as x increases. With similar as above reasoning, the term βx i.e. the phase of voltage travelling wave diminishes as the wave travels from the beginning of line and moves to the end of line.

Similarly, the term $e^{-\gamma x}$ can be written as $e^{-(\alpha+j\beta)x} = e^{-\alpha x} e^{-j\beta x} = e^{-\alpha x}[\cos(-\beta x) + j \sin(-\beta x)]$

With similar as above reasoning, the term $e^{-\alpha x}$ decreases as x increases. In other words, the magnitude (intensity) of voltage refracted wave decreases as the wave moves from the end of line towards the beginning of line as one expects. It is really the part of voltage travelling wave that arrives at the end of line and refracts travelling to the opposite direction of line.

Additionally, the term $-\beta x$ decreases as x increases i.e. the phase (angle) of voltage refracted wave decreases as the wave moves from the end towards the beginning of line.

Since from equation (7), the line voltage at point x is the algebraic summation of voltage travelling and refracted wave at the same point x and also taking into consideration the results of the above reasoning and depending on line parameters and the type of load at the end of line, we can state that in general the voltage magnitude and phase decrease from the beginning to the end of line. This implies having in mind the above that either or both the voltage magnitude and phase can also increase from the beginning to the end of line.

4. Calculation of line voltage

We consider a typical electric power transmission line with the following parameters :

$R = 0.107\ \Omega/km$ $\quad$ $L = 1.362\ mH/km$

$G = 0\ S/km$ $\quad$ $C = 0.0085\ \mu F/km$

$f = 50\ Hz$ $\quad$ $l = 360\ km$

$V_R = 115470 \angle 0^\circ\ kV$ $\quad$ $I_R = 360.844 \angle 0^\circ\ A$

Then using the list of symbols and the analysis of section 2, we can calculate the other parameters of the above line :

$z_C = 406.41 \angle -7.02^\circ\ \Omega$

$\gamma = 1.085x10^{-3} \angle 82.98^\circ\ km^{-1} = (0.1326x10^{-3} + j\ 1.07687x10^{-3})\ km^{-1}$

$\alpha = 0.1326x10^{-3}$ neper/km $\quad$ $\beta = 1.07687x10^{-3}$ rad/km

$\lambda = 5834.674$ km

$\upsilon = 291733.696$ km/sec $\quad$ $\tau = 1.234$ msecs

$\Delta = 22.212^\circ$ $\quad$ $\Delta/l = 0.0617^\circ/km$

Using equations (7) or (10) and taking step Δx=10 km, we obtain the results given in table 1.

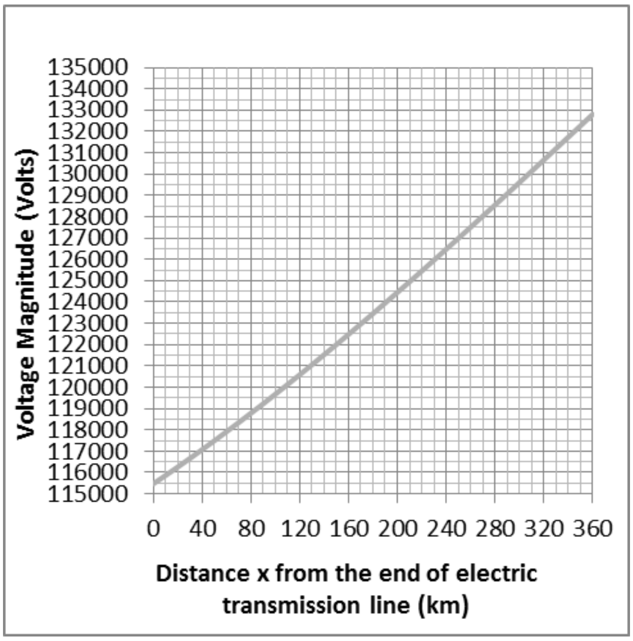
α/α	x (km)	V(x) (Volts)	φV(x) (°)	Natural logarithm (ln) of V(x)
1	0	115470.0	0	11.65677
2	10	115859.9	0.764372	11.66014
3	20	116256.9	1.525202	11.66356
4	30	116661.2	2.282437	11.66703
5	40	117072.3	3.036030	11.67055
6	50	117490.2	3.785936	11.67411
7	60	117914.8	4.532115	11.67772
8	70	118345.7	5.274531	11.68137
9	80	118783.0	6.013151	11.68505
10	90	119226.2	6.747948	11.68878
11	100	119675.4	7.478896	11.69254
12	110	120130.3	8.205973	11.69633
13	120	120590.6	8.929164	11.70016
14	130	121056.3	9.648452	11.70401
15	140	121527.2	10.36383	11.70789
16	150	122003.0	11.07528	11.71180
17	160	122483.6	11.78282	11.71573
18	170	122968.7	12.48642	11.71969
19	180	123458.2	13.18610	11.72366
20	190	123952.0	13.88186	11.72765

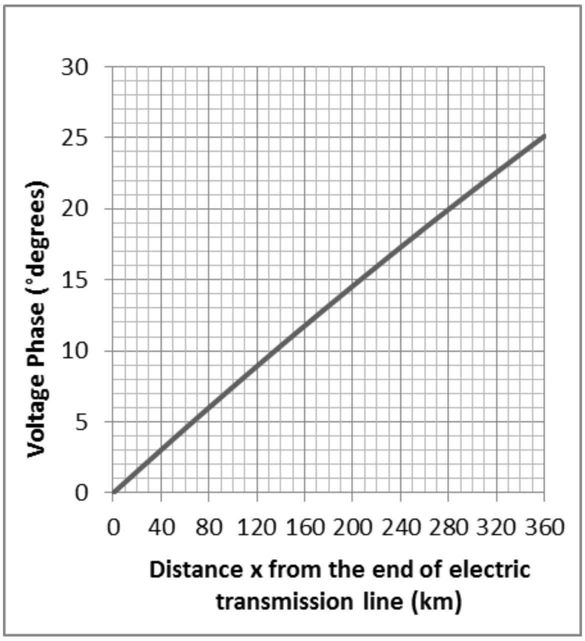
α/α	x (km)	V(x) (Volts)	φV(x) (°)	Natural logarithm (ln) of V(x)
21	200	124449.7	14.57371	11.73166
22	210	124951.3	15.26165	11.73568
23	220	125456.5	15.94571	11.73971
24	230	125965.2	16.62589	11.74376
25	240	126477.1	17.30221	11.74782
26	250	126992.1	17.97469	11.75188
27	260	127510.0	18.64335	11.75595
28	270	128030.6	19.30822	11.76002
29	280	128553.6	19.96933	11.76410
30	290	129079.1	20.62669	11.76818
31	300	129606.6	21.28035	11.77226
32	310	130136.2	21.93032	11.77634
33	320	130667.5	22.57665	11.78041
34	330	131200.4	23.21936	11.78448
35	340	131734.7	23.85850	11.78855
36	350	132270.3	24.49410	11.79260
37	360	132807.0	25.12620	11.79665

Table 1. Calculation results of voltage along the electric power transmission line

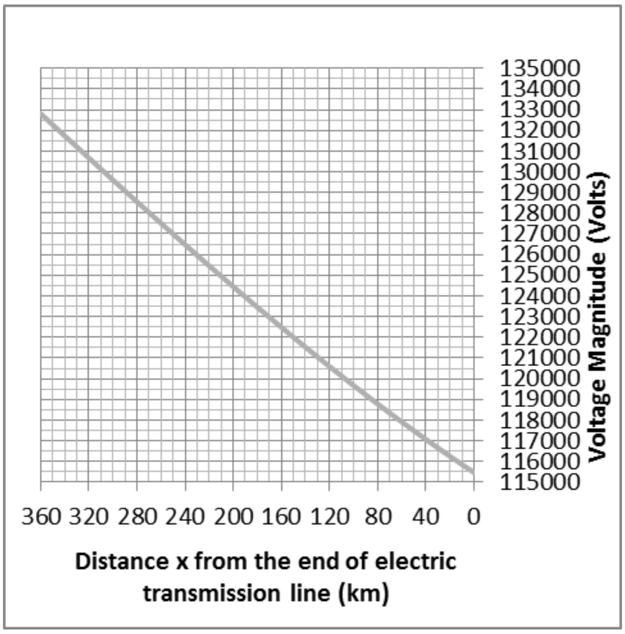
5. Graphical presentation of line voltage

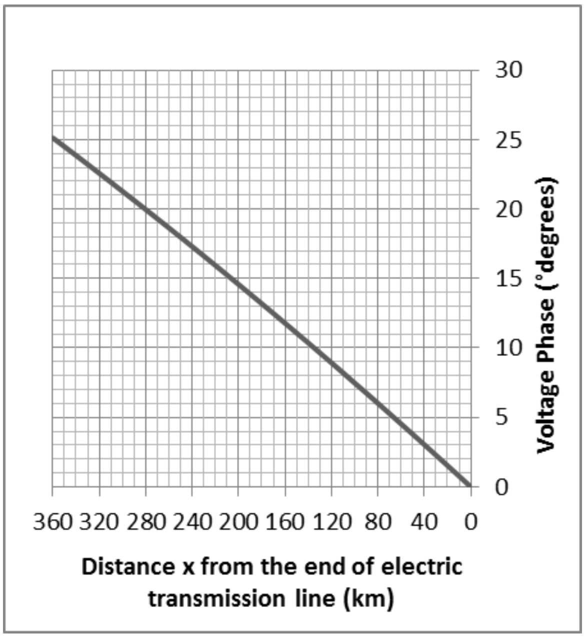
The graphical presentations of results obtained in table 1 are given in graphs 1 to 3.





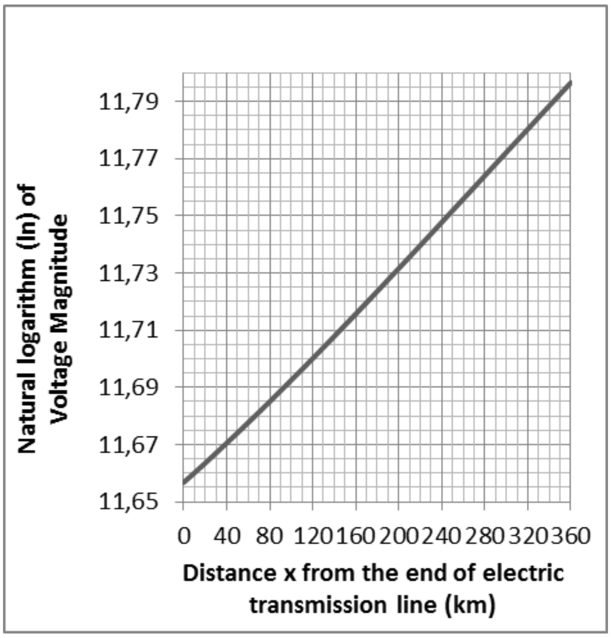
Graph 1. Voltage magnitude (left graph) and voltage phase (angle) (right graph) from the end towards the beginning of line. (direction right to left of electric power transmission line of figure 1)

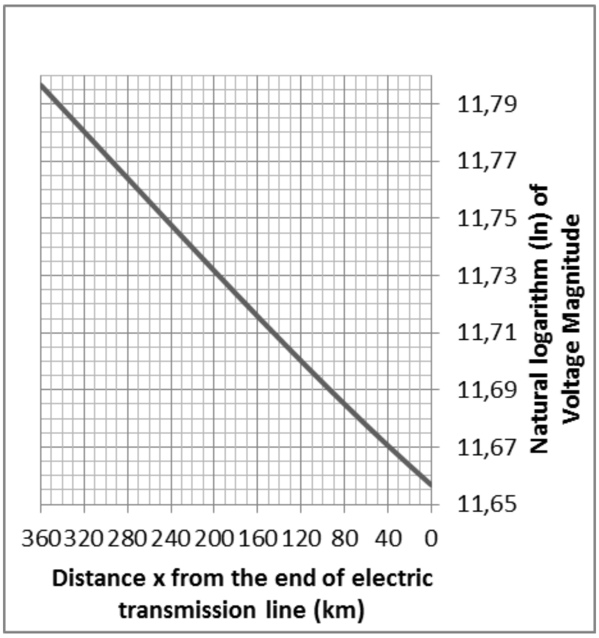




Graph 2. Voltage magnitude (left graph) and voltage phase (angle) (right graph) from the beginning towards the end of line (direction opposite to that of

graph 1 ie. left to right of electric power transmission line of figure 1)





Graph 3. Natural logarithm (ln) of Voltage magnitude from the beginning towards the end of line (left graph) and vice versa (right graph)

In graph 1 appear the line voltage magnitude and line voltage phase and how they change across the line as we move from the end to the beginning of the line ie. as we move from the right rear of the equivalent line circuit of figure 1 towards the left rear of the same circuit.

In graph 2 appear the above, ie. the line voltage magnitude and line voltage phase and how they change across the line this time as we move from the beginning of the line(left rear of line equivalent circuit of figure 1) towards the end of the line(right rear of the equivalent line circuit of figure 1).

The curves of graphs 1 and 2 may appear common but they are not. Some of them may look straight lines or almost straight lines but they are not. The above quantities have an exponential behaviour as someone can verify from equation 7. Their graphical representations depend on the values of their exponential constant factors (α and β). If their values are small and as variable x increases, the values αx and βx do not change enough in order their exponential behaviour to appear on the graphs. This is the reason they seem to be straight or almost straight lines.

Therefore, when on one hand the natural logarithm (ln) of the line voltage magnitude is graphed as a function of x (see results in table 1 and the respective graph 3) is a straight or almost a straight line indicating the exponential character of line voltage magnitude against variable x. On the other hand, the line voltage phase is given by the expression βx ie. is a linear function of x and thus when is graphed against the variable x is a straight or almost a straight line indicating the above linearity that also appears in graphs 1 and 2.

6. Discussion

From the theoretical analysis in section 3, the following can be stated :

1) the magnitude (intensity) and the phase of voltage travelling wave diminish as the wave travels from the beginning of line to the end of line

2) the magnitude (intensity) and the phase of voltage refracted wave decrease as the wave moves from the end of line towards the beginning of line

3) in general the voltage magnitude and phase decrease from the beginning to the end of line. This implies having in mind the analysis in section 3 that either or both the voltage magnitude and phase can also increase from the beginning to the end of line.

Studying the results presented in table 1 and their graphs 1 to 2 of sections 4 and 5 respectively, we can summarise the following :

1) the voltage magnitude (intensity) decreases as we move from the beginning towards the end of line

2) the voltage phase (angle) decreases as we move from the beginning towards the end of line

Regarding now the information that is drawn from the graphs 1 or 2 is discussed in the following paragraphs.

Looking at graph 2 as it is stated above, the line voltage magnitude decreases as one moves from the left rear of the line where the power source is towards the right rear of the line where the load is. This observation implies that both line and load present an ohmic-inductive behaviour. In other words, we have a reactive power flow from the source to line and load. Regarding the load is pure ohmic as one can see in section 4 from the data of the typical power line given and on which the results and graphs of sections 4 and 5 are based on. Thus the above statement is right.

The line from the data given in section 4 has an ohmic (R) as well as an inductive (L) long-wise elements plus a capacitive (C) transversal element. The above statement that the line presents an ohmic-inductive behaviour that was drawn from graph 2 means that the capacitive element of the line does not produce enough reactive power to cover the needs of the inductive long-wise element of the line and thus the source comes to cover the rest reactive power needed. This is the information drawn from the line voltage magnitude graph as a function of x.

Looking again at graph 2, we can see that the line voltage phase also decreases as one moves from the left rear of the line towards the right rear of the line. The above observation implies and cannot be otherwise that we have an active power flow from the left rear of the line where the power source is towards the right rear of the line where the load is in order to cover the needs in active power of both the ohmic element of the line and load.

7. Conclusions

Thus, from the discussion developed in sections 5 and 6, the calculations results show that line voltage magnitude and phase decrease as we move from the beginning towards the end of line which verifies the analysis developed in section 3 of the paper that in general the voltage magnitude and phase decrease from the beginning to the end of line.

For better understanding, analysis and study of electric power transmission line voltage, we propose to study it using cartesian co-ordinates. This will be the subject of a future paper.

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