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Decoupled State-Feedback Controller of Three Phase Shunt Active Power Filter: Unbalanced Current Compensation

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Abstract

The non linearity of inverter model constitutes one of the big problems in control of the three phase shunt active filter. In this paper we propose a linear model and state feedback control to overcome this complexity. To drive a linear model from the non linear model we introduced a power balanced equation. The currents injected by the three phase shunt active power filter and dc voltage are controlled in the synchronous orthogonal dq frame by applying a linear control based in the decoupled state-feedback controller. The accuracy of the linear proposed model and performances of the linear control are evaluated for unbalanced current compensation. The simulation results show the effectiveness of the proposed model and the control technique.

Keywords

Linear model, State-feedback linear controller, Three phase shunt active power filter, Unbalanced current perturbation

1 Introduction

The intensive use of unbalanced equipments in industry constitutes an important and several source of power quality problems. These equipments, which are often nonlinear loads, generate harmonic currents, reactive power and single phase loads cause system unbalance. A power quality distortion is a source of low system efficiency and disturbance of other consumers.

The three phase shunt active power filter (SAPF) widely used to improve power quality on the load side (Rejil et al. (2013), Ravindra et al. (2011) and Chennai et al. (2014)).

The effectiveness of any SAPF is associated to its configuration, the model established for the system (Mendalek et al. (2003) and Karzerni (2001)), the closed loop control strategy applied, the method implemented to obtain the references current (Abaali et al. (2007, 2008)), and the modulation technique used (Chennai et al. (2014)). The SAPF state of the art is well documented; hundreds of works are reviewed in Singh et al. (1999) and Brandao et al. (1999). The nonlinearity of the model of shunt active power filter, due to the inverter nonlinearity, is the major problem of control synthesis given in Kazerani et al. (2001) and Mendalek et al. (2003).

In this paper we propose the decoupled state-feedback control method applied to the linear model to overcome the complexity of the non linearity model of three phase shunt active filter. A power balance equation and nonlinear input transformation are used to drive a linear model from nonlinear model. The current injected by the three phase shunt active power filter and dc voltage are controlled in the synchronous orthogonal dq frame by applying a linear control based in the decoupled state-feedback controller (Tnani et al. (2006)). The accuracy of the linear proposed model and performances of the linear control are evaluated for selective compensation of unbalanced current compensation using Matlab simulation.

This paper is organized as following: after the introduction and short description of general structure, in the third section a mathematical recall of the nonlinear model of SAPF is developed. The fourth section gives the linear model of SAPF. The closed loop control of SAPF is decrypted in the fifth section. In the sixth section, the simulation results are presented. Finally, these results are discussed and commented in seventh section.

2 Nonlinear model of three phase shunt active power filter

The SAPF generate and inject the compensation current at the Point of Common Connection (PCC). The injected current is equivalent to the load current perturbations. Thus, the resulting total current drawn from the ac mains is sinusoidal. The main circuit is given in Fig. 1.

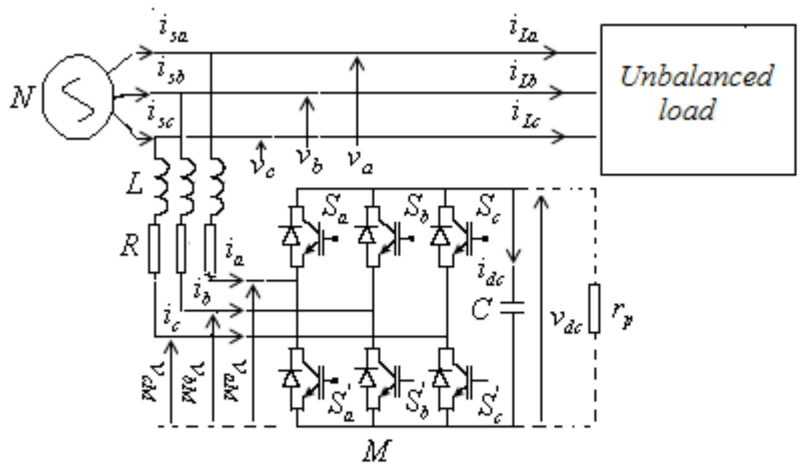


Fig. 1: General structure of the *SAPF*

The following terminology will be used:

$[v]=[v_a\ v_b\ v_c]^T$:Three phase voltage source

$[i]=[i_a\ i_b\ i_c]^T$:Output current inverter,

$[v_M]=[v_{aM}\ v_{bM}\ v_{cM}]^T$:Output voltage inverter

v_{dc} :Voltage in the dc side

i_{dc} : Current in the dc side.

2.1 Nonlinear model of SAPF in the stationary reference

The Kirchhoff's rules at the CCP of the SAPF allow one to write the following three equations corresponding to three phases three wire in the stationary "abc" frame (Mendalek et *al*. (2003):

$$\begin{cases} v_a = L\dfrac{di_a}{dt} + Ri_a + v_{aM} + v_{MN} \\ v_b = L\dfrac{di_b}{dt} + Ri_b + v_{bM} + v_{MN} \\ v_c = L\dfrac{di_c}{dt} + Ri_c + v_{cM} + v_{MN} \end{cases} \tag{1}$$

By doing the sum of the three equations, taking into account the absence of the zero-sequence in the currents into a three wire system, and assuming that the ac supply voltages are balanced, we obtain the following relation:

$$v_{MN} = -\frac{1}{3}\sum_{i=a}^{c} v_{iM} \tag{2}$$

The switching function c_k of the k^{th} leg of the converter equal to '1' if S_k is on (S'_k is off) and equal to '0' if S_k is off (S'_k is on).

Hence, one can write:

$$v_{kM} = c_k v_{dc} \tag{3}$$

From (2) and (3), the equations system (1) can be written as follows:

$$L\frac{di_k}{dt} = -Ri_k - (c_k - \frac{1}{3}\sum_{i=a}^{c} c_i)v_{dc} + v_k \tag{4}$$

The switching state function can be defined by:

$$m_k = (c_k - \frac{1}{3}\sum_{i=a}^{c} c_i) \tag{5}$$

The current and voltage in the *dc* side are relegated to the output current of the inverter by:

$$\frac{dv_{dc}}{dt} = \frac{1}{C}i_{dc} = \frac{1}{C}\sum_{i=a}^{c} c_i\, i_i \tag{6}$$

Since the zero sequence is assumed to be zero and we can verify the relationship $\sum_{i=a}^{c} m_i\, i_i = \sum_{i=a}^{c} c_i\, i_i$ we can write (6) in the reduced form given by:

$$\frac{dv_{dc}}{dt} = \frac{1}{C}(m'_a i_a + m'_b i_b) \text{ with } m'_a = 2m_a + m_b \text{ et } m'_b = m_a + 2m_b \tag{7}$$

By combining (7) and (4), the reduced nonlinear model of SAPF can be formulated in the following matrix:

$$\frac{d}{dt}\begin{bmatrix} i_a \\ i_b \\ v_{dc} \end{bmatrix} = \begin{bmatrix} -\frac{R}{L} & 0 & -\frac{m_a}{L} \\ 0 & -\frac{R}{L} & -\frac{m_b}{L} \\ \frac{1}{C}m'_a & \frac{1}{C}m'_b & 0 \end{bmatrix}\begin{bmatrix} i_a \\ i_b \\ v_{dc} \end{bmatrix} + \begin{bmatrix} \frac{v_a}{L} \\ \frac{v_b}{L} \\ 0 \end{bmatrix} \tag{8}$$

2.2 Nonlinear model of SAPF in the rotating frame *dq*

To simplify the control of the system, we transform the model (8) in the reference frame dq. dq rotates at fundamental frequency, then the fundamental quantities become constant. The *abc/dq* transformation matrix is given by:

$$C_{dq}^{abc} = \sqrt{\frac{2}{3}} \begin{bmatrix} \cos\theta & \cos(\theta - \frac{2\pi}{3}) & \cos(\theta - \frac{4\pi}{3}) \\ -\sin\theta & -\sin(\theta - \frac{2\pi}{3}) & -\sin(\theta - \frac{4\pi}{3}) \end{bmatrix} \text{ with } \theta = \omega t \text{ et } C_{abc}^{dq} = (C_{dq}^{abc})^T \quad (9)$$

The reduced form of the transformation matrix (9) is given by:

$$C_{dq}^{ab} = \sqrt{2} \begin{bmatrix} \cos(\theta - \frac{\pi}{6}) & \sin\theta \\ -\sin(\theta - \frac{\pi}{6}) & \cos\theta \end{bmatrix} \text{ and } C_{ab}^{dq} = \sqrt{\frac{2}{3}} \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin(\theta - \frac{\pi}{6}) & \cos(\theta - \frac{\pi}{6}) \end{bmatrix} \quad (10)$$

The third equation of (8) can be written in matrix form as follows:

$$\frac{dv_{dc}}{dt} = \frac{1}{C} \left[m'_{ab} \right]^T \left[i_{ab} \right] \quad (11)$$

Applying the transformation (10) to (11) yields (12). M'_d et M'_q are respectively the transformation of m'_a and m'_b in the dq reference.

$$\frac{dv_{dc}}{dt} = \frac{1}{C} (C_{dq}^{ab} \left[m'_{ab} \right])^T (C_{dq}^{ab} \left[i_{ab} \right]) = \frac{1}{C} \left[M'_{dq} \right]^T \left[I_{dq} \right] \quad (12)$$

The application of the reduced form of the transformation matrix (10) to the first two equations of the model (8) yields the following equation:

$$\frac{d}{dt} \left[I_{dq} \right] = - \begin{bmatrix} \frac{R}{L} & -\omega \\ \omega & \frac{R}{L} \end{bmatrix} \left[I_{dq} \right] - \frac{1}{L} \left[M_{dq} \right] v_{dc} + \frac{1}{L} \left[v_{dq} \right] \quad (13)$$

Finally, the dynamic non linear model of SAPF expressed in the rotating frame is given by (14). The nonlinearity of this model is due to the coupling between the state variables $\{I_d \; I_q \; v_{dc}\}$ and the input $\{M_d \; M_q\}$ (Mendalek et *al*. (2003)).

$$\frac{d}{dt} \begin{bmatrix} I_d \\ I_q \\ v_{dc} \end{bmatrix} = \begin{bmatrix} -\frac{R}{L} & \omega & -\frac{M_d}{L} \\ -\omega & -\frac{R}{L} & -\frac{M_q}{L} \\ \frac{M'_d}{C} & \frac{M'_q}{C} & 0 \end{bmatrix} \begin{bmatrix} I_d \\ I_q \\ v_{dc} \end{bmatrix} + \begin{bmatrix} \frac{v_d}{L} \\ \frac{v_q}{L} \\ 0 \end{bmatrix} \quad (14)$$

3 Linear model of SAPF

An alternative equation, based on the power balance input-output of the inverter, can be used to describe the dynamics of the dc voltage v_{dc} expressed by the third equation of nonlinear model

(14). The active power at the ac side and dc side of the inverter can be described respectively by (15) and (16).

$$P_{ac} = \frac{3}{2} v_d I_d + \frac{3}{2} v_q I_q \tag{15}$$

$$P_{dc} = v_{dc} I_{dc} = C v_{dc} \frac{d}{dt} v_{dc} \tag{16}$$

Then the power balance can be expressed by:

$$P_{ac} = P_{dc} + P_p \tag{17}$$

The power losses in the resistor *R*, due to the switching losses of the power components, is P_p. The resistance *R* is very low these power losses are almost negligible compared to the power losses of the inverter which can be modelled by a resistance r_p in parallel with the capacitor energy storage C.

According to (17), the resulting equation of dynamics of v_{dc} is given by:

$$C v_{dc} \frac{d}{dt} v_{dc} + \frac{1}{r_p} v_{dc}^2 = \frac{3}{2} v_d I_d + \frac{3}{2} v_q I_q \tag{18}$$

The equation (18) can be written as follows:

$$\frac{d}{dt}(v_{dc}^2) = \frac{3}{C} v_d I_d + \frac{3}{C} v_q I_q - \frac{2}{r_p C} v_{dc}^2 \tag{19}$$

The quantity v_{dc}^2 is taken as state variable instead of v_{dc}, (19) becomes linear. Especially since v_{dc} is unipolar, the choice of v_{dc}^2 as a state variable will not cause any problems. In the nonlinear model, both input M_d and M_q are coupled with the state variable v_{dc}. So we change the input variables M_d and M_q with the new input variables v_{dM} and v_{qM}. Finally the linear model of SAPF form standard linear state space can be written as follows:

$$\frac{d}{dt}\begin{bmatrix} I_d \\ I_q \\ v_{dc}^2 \end{bmatrix} = \begin{bmatrix} -\frac{R}{L} & \omega & 0 \\ -\omega & -\frac{R}{L} & 0 \\ \frac{3v_d}{C} & \frac{3v_q}{C} & -\frac{2}{r_p C} \end{bmatrix} \begin{bmatrix} I_d \\ I_q \\ v_{dc}^2 \end{bmatrix} + \begin{bmatrix} \frac{U_d}{L} \\ \frac{U_q}{L} \\ 0 \end{bmatrix} \tag{20}$$

where v_{dc}^2, I_d and I_q are the state variables, the input variables U_d and U_q are related to the input variables v_{dM} and v_{qM} by (21) with (v_d, v_q) are the grid voltage source in the rotating *dq* frame. The constants *R*, r_p, *L*, *C* et ω are the system parameters.

$$\begin{cases} U_d = v_d - v_{dM} \\ U_q = v_q - v_{qM} \end{cases} \quad (21)$$

4 State space presentation of SAPF model

The linear model of SAPF has two inputs and three outputs and can be expressed in matrix form (22). The known matrix $A(3x3)$, $B(3x2)$ and $C(3x3)$ are the state matrix, the control matrix and the observation matrix. The variables $x \in R^3$, $u \in R^2$ and $y \in R^3$ are respectively the vector state variable, the input vector and the output vector.

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx \end{cases} \quad (22)$$

4.1 State feedback control of SAPF

The block diagram of the closed loop system controlled by state feedback (Tnani et *al.* (2006)) is given by Fig. 2:

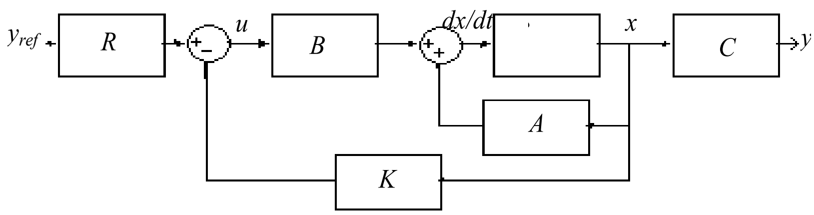


Fig 2: Closed-loop system with a controller state feedback

The control equation is given by :

$$u = -Kx + Ry_{ref} \quad (23)$$

with $K(2x3)$, $R(2x3)$ and y_{ref} are respectively the gain of state feedback, a matrix used to obtain a unity gain steady and current reference vector. The transfer function of closed loop is given by:

$$G_c(s) = G_1(s)R \text{ with } G_1(s) = C(sI - A + BK)^{-1} B \quad (24)$$

The state feedback gain matrix K is calculated by placing the poles of the closed loop transfer function $G_1(s)$ and R is the pseudo inverse of $G_1(0)$.

4.2 Pole placement of closed loop transfer function

The open loop system is third order; it has a real pole and two purely imaginary poles. The poles placement of the open loop system in the complex plane is shown in Fig. 3a.

The polynomial characteristic of the closed loop system can be written as:

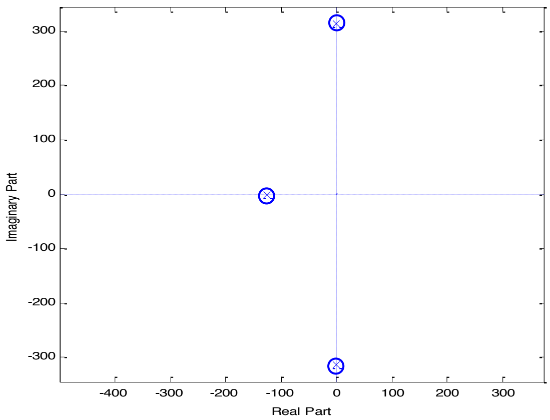
$$\det(sI - A + BK) = (s - \lambda_1)(s - \lambda_2^*)(s - \lambda_2) \tag{25}$$

λ_1 , $\lambda_2 = -\zeta\omega_n + j\omega_n\sqrt{1-\zeta^2}$ and $\lambda_2^* = -\zeta\omega_n - j\omega_n\sqrt{1-\zeta^2}$) are respectively the real pole and the two conjugated complex poles with ζ and ω_n ars the damping and the angular frequency of the system.

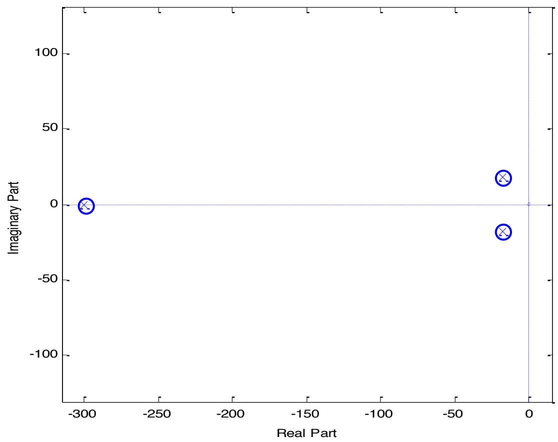
1 The real pole is placed far to the conjugated complex poles become dominant.

2 The conjugated complex poles (little amortized amortization) are reduced to a specific damping ($\zeta \approx 0{,}69 \quad \omega_n \approx \omega/10$).

The Fig. 3.b shows the poles placement of the closed loop system in the complex plane.



(a)



(b)

Fig. 3 : Poles placement of the system in complex plan (a) : open loop, (b) : closed loop

5 Simulation results

The simulation parameters are :

– Output passive filters (*L, R*)=(*3 mH, 1Ω*)

– The capacitor in the dc side (*C=500 μF*),

– Inverter power looses are modelised by r_p resistor equivalent to 5% of P_{ac}.

– Reference value of dc voltage $v_{dcref}=740V$,

– Three wires of the power network (*220V, 50Hz*).

The linear model of SAPF is evaluated using a unbalance current compensation. The current loads and current reference are given by (26)

$$\begin{cases} i_{La}(t) = \sqrt{2}I_1 \sin\omega t + \sqrt{2}I_2 \sin\omega t \\ i_{Lb}(t) = \sqrt{2}I_1 \sin(\omega t - \frac{2\pi}{3}) + \sqrt{2}I_2 \sin(\omega t + \frac{2\pi}{3}) \\ i_{Lc}(t) = \sqrt{2}I_1 \sin(\omega t + \frac{2\pi}{3}) + \sqrt{2}I_2 \sin(\omega t - \frac{2\pi}{3}) \end{cases}, \begin{cases} i_{aref}(t) = \sqrt{2}I_2 \sin\omega t \\ i_{bref}(t) = \sqrt{2}I_2 \sin(\omega t + \frac{2\pi}{3}) \\ i_{cref}(t) = \sqrt{2}I_2 \sin(\omega t - \frac{2\pi}{3}) \end{cases} \quad (26)$$

The Fig. 4, 5, 6, and 7 shows respectively the load current before compensation, the dc voltage v_{dc} superposed with its reference v_{dcref}, the injected current I_d and I_q superposed with there references I_{dref} and I_{qref}. The Fig. 8 show the grid current after compensation practically balanced. The dynamics and response times of the system are acceptable.

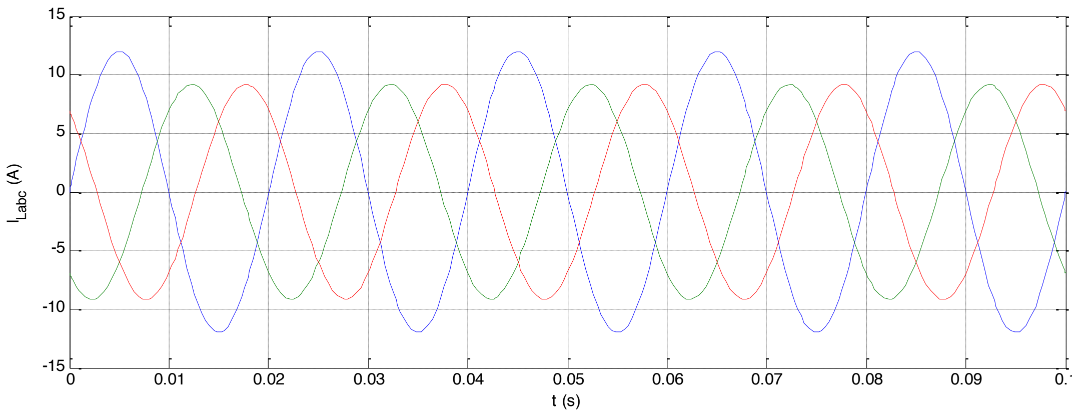


Fig. 4 Load current before compensation

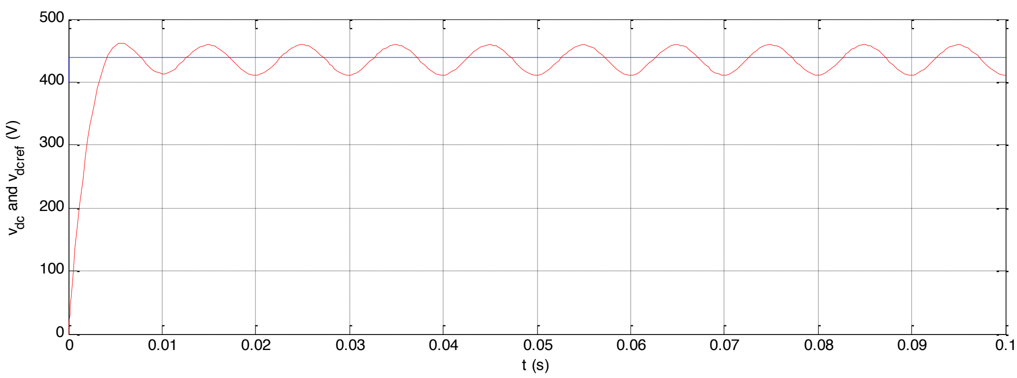


Fig. 5 : v_{dc} and v_{dcref}

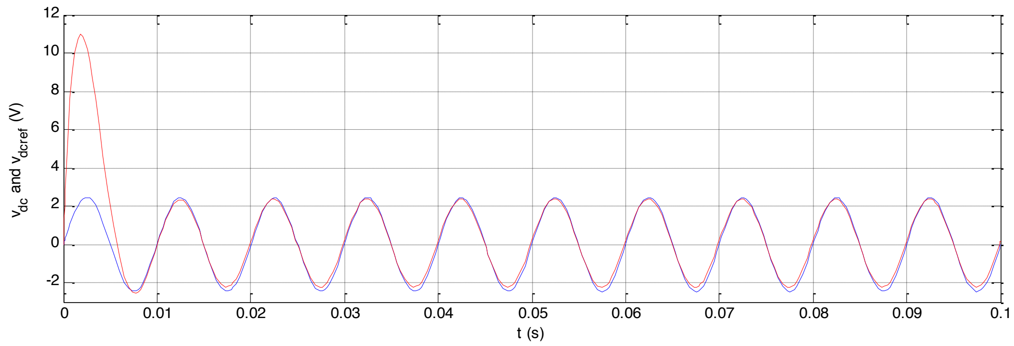


Fig. 6 : I_d and I_{dref}

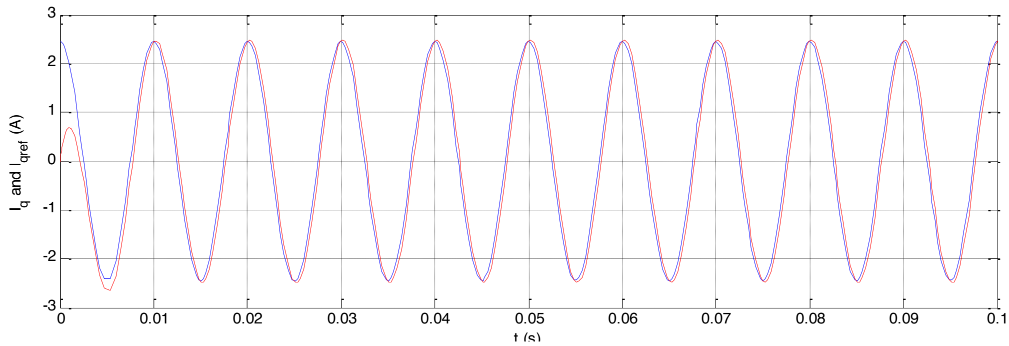


Fig. 7 : I_q and I_{qref}

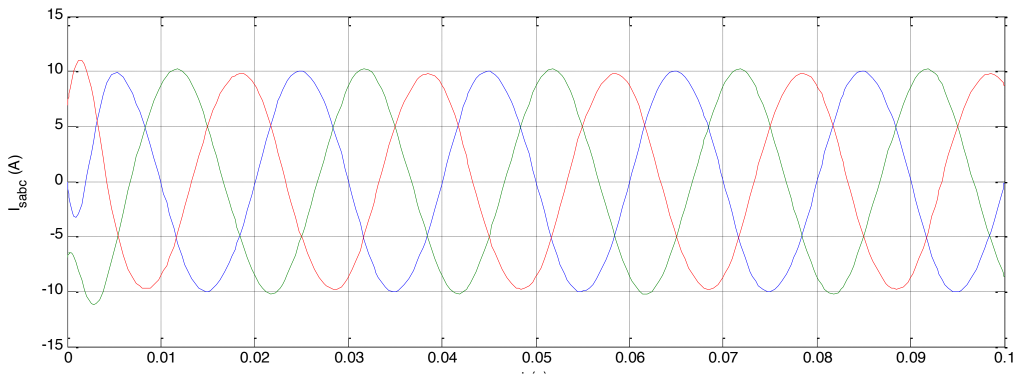


Fig. 8 : Grid current after compensation

6 Conclusion

In this paper, we have developed a nonlinear model of three-phase shunt active power filter, which we deduced a linear model and close-loop control using the state feedback controller. According the simulation results, it is shown that the linear model and applying the state-feedback control technique, the dynamics of the system is considerably improved resulting in short response times. The proposed control method is limited to compensate the low frequency current perturbation.

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