AMSE JOURNALS-AMSE IFRATH Publication –2014-Series: Modelling C; Vol. 75

Special Handicap; pp 106-118

Submitted Sept. 2014; Accepted Oct. 31, 2014

Evaluation of a Prototype of a Multimodal Interface to Work with Mathematics

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Abstract

Our work concerns the non visual access to mathematics. Following an analysis of the solving processes of algebraic equations followed by students with and without visual impairment, we proposed a number of supporting functions taking into account the pedagogical context. Aiming to validate our proposal, we developed a prototype of a multimodal interface to facilitate the input of mathematical contents, the solving process, and the communication between users with and without visual impairment. The interface provides different synchronised modalities: visual, audio and braille. We conducted user tests with blind students from high school and their math teachers. The results suggest that the prototype facilitates input and manipulation of contents, and allows direct communication between the student and the teacher. We conducted a second type of tests with teachers of mathematics who have experience working with visually impaired students, with the aim to verify the suitability of the interface as a supporting tool in the learning and teaching process of algebra in an integrated environment. In this article we focus on the latter tests.

Key words: Non visual interaction, multimodal interfaces, mathematics.

1. Introduction

Students with visual impairment who learn mathematics in an integrated environment face problems which go beyond the difficulty of the subject itself. [1] suggests that the main problem is due to the non visual modalities of access to mathematical contents, which are linear and do not facilitate the direct grasping of the structure, while the visual representations used by the sighted students are bidimensional and facilitate the semantic comprehension of an expression. On the other hand, this fundamental difference between the representations used by the blind and the sighted makes it difficult to directly communicate contents between them, for example, between a student with visual impairment integrated in a mainstream school and his or her mathematics teacher. Written documents, exercises, homework and exams are usually transcribed by the transcription center in charge of providing support for the visually impaired student. However, in such a case, the communication of written contents is not straightforward and therefore it hinders the learning process, both on the side of the student and of the teacher. In France, students may benefit from the aid of an assistant for school activities *(Auxiliaire de vie scolaire, AVS)* for a personalised follow-through; however, in most cases the assistants do not have the specific competences in mathematics. On the other hand, the students must follow the verbal explanation of the teacher, who is, in most cases, used to make references which demand the visual sense. In fact, the role of the teacher is a critical factor in the accomplishment of a student of mathematics, sighted or non sighted. The teacher must create learning situations which draw the attention of the student, and guide them towards the objectives of the subject [2]. While the teacher is responsible for the didactic methods to employ, the use of technical aids seems to be of critical importance for the success of students with visual impairment in an integrated environment.

The aim of this work is to propose some supporting features to facilitate input and manipulation of algebraic contents which are needed in problem solving, allowing direct communication of contents between sighted and non sighted persons, each one using the mathematical notation familiar to them. In order to do so, we have conducted a preliminary study, aiming to analyse the strategy for solving algebraic equations followed by sighted and non sighted students, and to observe in particular their intentions while manipulating the expressions. From this analysis we have proposed a collection of assisting features which could be useful in the learning and teaching process, that is to say, to facilitate the comprehension of exercises without making the calculations that correspond to the student. In order to validate our proposal, we have developed a prototype of interface which allows input and manipulation of contents, as well as communication between sighted and non sighted users. This prototype uses a synchronised multimodal output: visual, audio and braille. The prototype was evaluated using two different protocols which complement each other:

1. The first protocol consisted in a practical evaluation with students from high school and teachers of mathematics [3]. This evaluation was focused on the effectiveness of the input method, the aids for comprehension of complex structures, and the functions for the resolution of algebraic equations, as well as the communication

between student and teacher. The results were very positive: students were able to write expressions dictated by the teacher, and to solve the given problems.

2. The second protocol involved teachers of mathematics who have experience working with visually impaired students. The aim of this evaluation was to verify the validity of the interface as a support in the learning and teaching process in an integrated environment.

This article focuses in the second type of evaluation, and presents the results and recommendations from teachers.

2. State of the art

The problem of representation and communication of mathematical contents between sighted and non sighted persons has been the subject of several projects. The NAT Braille project [4] allows the transcription of documents to braille, using different sources, in particular MathML. Aiming to facilitate access to higher studies of mathematics and science, different projects offer the transcription of documents from LaTeX to braille or to audio. One of this projects is LABRADOOR [5], which allows transcription from LaTeX into the german math braille code Marburg ; the project LaTeX-access [6] allows transcription into the Nemeth code, and the PSLM *(Programme Spécialisé de Lecture Mathématique à l'usage des non-voyants)* project produces an audio output in french [7]. The Math Genie [8] offers a synchronised visual and audio output. Another type of project focuses on the study of the audio output features to facilitate comprehension: the use of prosody and lexical cues [9], non verbal sonification [10], or the use of different audio rendering rules depending on the complexity of expressions [11][12], amongst others.

These resources represent an efficient support for visually impaired users by allowing to produce braille documents or by improving comprehension of expressions. However, supporting access and comprehension is not enough, but the current challenge consists in proposing features to facilitate doing mathematics, that is to say, manipulating contents and making calculations, enabling at the same time an efficient communication of contents using the representations which are familiar to the sighted and to the non sighted. There exist some projects which focus on these possibilities: ChattyInfty [13], a part of the Infty project, is a software which allows the edition of contents, using a synchronised visual and audio output. SensoMath [14] allows the input of contents from both the computer and the braille keyboard, and the visualisation of contents in black and in braille. The MAWEN [1] prototypes offer a synchronised view in black and in braille, as well as the use of assistants for a number of manipulations [15][16]; however, these assistants were partially implemented, and in some cases they make automatic actions, which is not recommended in a software to support learning.

The LAMBDA editor [17] uses a linear representation of expressions using a proprietary braille code which, in consequence, can be used only in this editor. LAMBDA allows the visualisation of the mathematical notation using LAMBDA characters, a special set of graphical symbols to represent the braille characters. This linear representation is not the usual notation used by sighted users, and requires an extra effort for grasping the structure of expressions. The bidimensional representation is also possible, provided that the linear expression is syntactically correct, but this representation is not interactive. LAMBDA offers some “compensation” functions for reading and editing expressions, such as partial visualisation of expressions according to their complexity. LAMBDA's braille output uses a 8-dot mathematical code, with a dictionary of symbols for each country; these symbols are inspired on the ones used in the national braille codes, aiming to facilitate their learning.

All these projects facilitate the production and communication of contents; however, most of them are prototypes or they have been only partially implemented. Currently, only LAMBDA and InftyReader/ChattyInfty are frequently used. While these software are fully functional regarding representation and exploration of contents, the functions to support doing mathematics are still limited. In addition to this, we consider that it is important to offer users the possibility to edit and visualising contents using their preferred modality.

2.1 Our approach for the development of the supporting functions

Our proposal of supporting functions was based on the analysis of the solving process of algebraic problems, carried out by teachers and students with and without sight. We have also taken into account the teaching process of mathematical concepts as explained by mathematics teachers. These functions are not easy to implement, since they must not execute any automatic procedure in the place of the student, but they must facilitate the comprehension of concepts involved in the calculations. On the other hand, these functions are necessary to compensate the limitations of the non visual modalities in relation to the visual modalities, whose bidimensional arrangement facilitates the grasping of the syntactical structure of the expression. The proposed functions aim to compensate these possibilities, which are common for sighted people but represent a disadvantage for the non sighted due to the modality of representation. Taking into account this analysis, as well as the recommendations of teachers of mathematics, we have developed a prototype of multimodal interface intended to facilitate the work on the classroom [18], on which we have implemented these supporting functions by using a user centered approach. Another key aspect is facilitating the communication of contents between sighted and non sighted people. In order to achieve this purpose, we use different synchronised views of the same content, based on the visual, audio and tactile modalities. When an expression is modified, the different views are updated dynamically. The users of the different modalities have the possibility to produce contents that will be displayed in the other modalities.

A first version of the prototype was evaluated with students with visual impairment in integration, and with teachers of mathematics. This evaluation allowed us to analyse the interactions between sighted and non sighted users and the production of mathematical contents, to refine the proposed functions and eliminate the unsuitable ones, and to propose new functions intended to facilitate doing mathematics. From this evaluation we developed and evaluated a second version of the prototype, whose results are presented in this article.

3. Characteristics of the interface

The second version of the prototype includes the edition of algebraic contents, containing powers, fractions and square roots, using a synchronised output in black, audio and braille. Equations can be entered from a computer keyboard or a braille keyboard, using the 6-dot french mathematical braille code. The equations in black are visualised according to the classical bidimensional representation. The expressions are presented in the conventional way for sighted and non sighted users, and the linear input in black is also possible. The braille output follows the rules of the standard format: for example, in french braille the terms x^2 and $x\hat{}2$ produce the same braille output. The delimiters of begin and end of block are visually represented in order to maintain a correspondence with braille and audio. The internal representation of expressions is a MathML DOM, which is transformed dynamically as terms are typed. The validation of structures is also done on the fly, so that the input will not produce a MathML syntax error.

Active reading of expressions is possible using the left and right arrow keys. The user can control the degree of visualisation according to the levels of the syntactical tree of the expression, which can represent an advantage while reading long expressions. In practice, this notion of granularity means that terms or sub-expressions can be folded and represented by a type of block that we call "semantic", because they are named after the type of element composing it, such as: "Fraction", "Product", "Member". For example, the fraction $\frac{x(x+1)+2x}{\sqrt{x^2}}$ can be presented in the forms: $\frac{x(x+1)+2x}{Denominator}$, $\frac{Product+2x}{Denominator}$, or *Fraction*. Regarding the style of the audio output using the screen reader, we have opted by reading without interpretation, except for the blocks, which semantic name is read. That is to say that the expression $\frac{x+1}{x-1}$ will be read as "*begin parenthesis, x plus one, end parenthesis, begin parenthesis...*" without indicating that it is a product, leaving the reader the interpretation task. On the contrary, if this expression is folded, it will be replaced by the name of the tag that we have associated to it in the MathML tree: *Product*.

We have implemented a number of auxiliary functions for supporting solving, which can be used differently depending on the task being executed. As stated before, these are not functions for automatic solving, but for alleviating the limitations of the non visual modalities. Our prototype includes a free search of fragments of the expression, as well as a search for common terms, with the aim of facilitating simplification in long expressions; we include as well a function for marking and unmarking terms for facilitating manipulation and follow-through of certain tasks. For example, marks can be used for indicating terms which have been processed, as an equivalent to cancelling a term on paper. It can also be used for selecting contiguous or non contiguous terms, independently of the cursor position. Presently, non contiguous selection is not available in any other software. The marked terms, as well as the search results, are indicated in red on the visual interface, and in audio the reading of the terms include a special sound to distinguish them from the non marked terms; the visualisation of the marked terms has not been implemented in braille yet. We have included shortcuts to "jump" between marked terms, so users can also use marks to indicate a term which they can access more directly. Marked terms can be copied or read aloud with a single shortcut. Contents produced in the prototype can be saved on a MathML file in order to use them afterwards, for example for importing them in OpenOffice or in NAT Braille.

The interface was developed on the Mozilla platform, with Python for the back-end. We use the NVDA screen reader with the corresponding scripts in order to personalise the audio and braille modules on Windows according to the needs of the prototype. We tested the prototype on Windows XP, Vista, 7 and 8, and on Mac OS X Snow Leopard (with limited audio output). Figure 1 presents the interface on Mac OS X Snow Leopard, displaying an example of expression in the bidimensional and linear visual modalities.

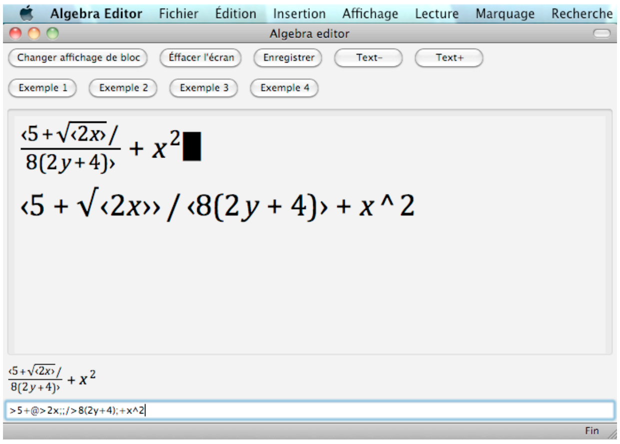

Fig.1. Example of expression in the bidimensional and linear visual modalities.

4. Evaluation with teachers of mathematics

The protocol that we followed to evaluate the second prototype was based on a cognitive walkthrough, where the evaluators were teachers of mathematics with experience teaching students with visual impairment. As potential users themselves, they can validate the usefulness of the interface regarding the needs for teaching and learning algebra.

Four teachers of mathematics participated in this evaluation: one non sighted, one visually impaired and two with normal sight. They are all familiar with the mathematical code, and three of them participated in the evaluation of LAMBDA and use it regularly with their non sighted students. The evaluation consisted of three parts:

1) *Edition and visualisation of contents*. We demonstrated the methods for writing exponents, fractions and roots, using the computer keyboard and the braille keyboard. Teachers observed the presentation of the elements in black, in braille and in audio. They analysed and discussed the appropriateness of the visual delimiters of block. We demonstrated also the option for exporting contents to a MathML file, followed by its importation into OpenOffice.
2) *Visualisation by syntactic levels*. Teachers analysed this type of visualisation on the interface and on the braille display, using different granularity levels.
3) *Solving aids.* We had a particular interest in knowing the opinion of the teachers regarding the implementation of the auxiliary functions, particularly the search for common terms, since this type of search allows finding terms that the students is supposed to find.

Every demonstration was followed by a discussion on the advantages and disadvantages of every aspect of the prototype, and the recommendations to improve it. We used a PC with the Windows XP operating system, NVDA for speech synthesis, and a Papenmeier Braillex Trio for the braille input and output. The visually impaired teacher used a Macintosh with OS X Snow Leopard, and the system zoom.

5. Results and discussion

The results of the evaluation were very positive. We present them below according to the stages presented previously.

5.1 Edition and visualisation of contents

The protocol for writing and reading exponents, fractions and square roots was considered comprehensible. Participants commented that the prototype allows to read, copy and analyse easily the elements that they have written. The use of the keyboard to input expressions in bidimensional form was considered an advantage over other modalities using the mouse. A teacher commented *"It's similar to MathType, with the exception that on MathType you need to click on things... that's the problem: blind students cannot use it because you need to click, you need to look for the symbols*". The automatic insertion of blocks for fractions and square roots was considered as acceptable, and it was not perceived as an automatic aid.

The commands for copying and pasting help prevent the problems when copying elements manually. These problems are very frequent when a student works with braille; for example, in a distribution of factors with multiple terms. A teacher explains: "*It is almost sure that students are going to make errors, because not only they need to do a lot of calculations, but there will also be a lot of terms to copy to the next line. With the copy and paste functions at least they can be sure they're copying the same thing*".

Teachers greatly appreciated the immediate visualisation of contents after the input of terms, which facilitates communication between sighted and non sighted students without a mediator. One of the teachers with visual impairment comments: "*I think it's very important to have a direct link between the teacher and the student. The problem of having an assistant (AVS) in the classroom is that the teacher does not talk to the student but to the assistant, who often doesn't have training in sciences. In my opinion, the presence of assistants works only for some handicaps*".

The audio output was considered convenient because it is rigorous and does not provide interpretation. Teachers also suggested that it would be useful to have different levels of verbosity, so that students can personalize the way of reading according to their preference. However, using audio alone will be tiresome for the user; on the other hand, in the classroom blind students will be using most probably the braille display, since the speakers or headphones will be of little help when the student is trying to listen to the teacher or peers.

Amongst the suggested improvements, we must consider the insertion of spaces in order to organise visually expressions. Spaces can be very useful to organise variables in columns while working on the solution of a system of equations by the reduction method.

Teachers found very useful the possibility to use a file produced with this prototype and to import it in OpenOffice, since they can re-use it and modify it, but they suggested that it must also work on the other way, that is to say, to be able to import into the prototype documents produced in OpenOffice or other applications which use MathML for representing mathematical equations. “*It's important; teachers can accept to use something new if it's compatible with something that they already use*”.

5.2 Visualisation by syntactic levels

Teachers considered useful the granular visualisation by blocks of syntactic levels as aid to give a general view of the expression. They consider that the names “Fraction” and “Root” are useful to represent the presence of these elements as blocks. They suggested an additional function to represent the width of an expression using spaces in a similar way as LAMBDA, except that in LAMBDA the degree of visualisation affects the whole expression. The implementation of this function could facilitate the reading strategy of a complex structure. For example, the block “Numerator” could contain the expression $8(2y + 4) - 3y + 6$, or a simple expression such as $x + y$. By displaying spaces in the place of the contents of the block “Numerator”, it would be very simple to grasp on the braille display that the numerator “   ” is larger and probably more complex than numerator “   ”. The fact of recognising by simple inspection the length of a numerator or denominator could influence the solving strategy.

It is suggested that for students with blindness it is more useful to communicate the length than the complexity of an expression. According to one of the teachers: “*If you're dictating an expression to me and you see that there's a fraction next, you may try to tell me if it's big or not; for example: ‘the numerator is 2x and then there's a long denominator’. This additional information could be useful, because if I see that the denominator is short I could think that there's nothing much inside, but if it's long I might suppose that there are complex things inside and I can decide whether I want to display the next level*”. On the other hand, the complexity of an expression is difficult to determine by the person doing the dictation, and it demands an extra effort of interpretation to the one who must interpret it. As explained by one teacher: "*If you tell me, 'inside the numerator there are three terms of complexity level 2, and two simple terms', that's going to make me think a lot: level 2? it could be a fraction, a square root, or maybe something else? On the contrary, if you just tell me the length, I can have useful and explicit information without making too much effort*".

5.3 Solving aids

Teachers found that the two types of search could be useful for supporting solving. The search for common terms could be useful in the simplification stage after a distribution, in order to know the location of a certain exponent, for example x^2. The free search could be more useful in the distribution of multiple terms, since the resulting terms will be numerous.

Teachers expressed that it is not the same thing to facilitate the tasks to the student than to do them in their place. For example, it is useful to facilitate the search for the cubic terms, but it would not be advisable to select them and copy them automatically; it is the student who must decide which terms can be simplified after obtaining the search results. Therefore, the search function does not solve anything, it only facilitates the location of the terms indicated by the student, terms which are more difficult to detect in the non visual modalities. In addition to this, participants recommended to implement the search of denominators, since the first thing you do with an equation with fractions is to process denominators: "*it would be very useful to be able to go directly to the first denominator, then pass directly to the next one without going back and forth and without having to read everything*". In a similar way, when adding two fractions, students need to know what they have as denominators; if the prototype could detect the denominators that could make solving easier. The search of exponents would also be useful in order to know the degree of the equation. The idea of browsing directly the search results was greatly appreciated. Even the access on the braille display could be improved if we could indicate simultaneously the search results, so that they can be easily identified by touch. In the current version of the prototype it is possible to use a braille display to visualise and type contents, but the functions to allow more complex interactions, such as the simultaneous communication of the search results and the selection of multiple terms are yet to be implemented.

Conclusion

We have presented here the results of the evaluation of the proposed supporting functions to solve algebraic equations, by means of an interface to facilitate edition and communication of contents between users with and without visual impairment. The results suggest that a software including the functions that we have proposed and implemented in our prototype could support the teaching and learning process in an integrated school environment, and could bring meaningful assistance to teachers of mathematics having blind students in their classroom. It is important to mention that the usefulness of such interface, as it is with all tools, depends on the didactic context in which it is used.

In order to facilitate communication between people with and without blindness, it is of crucial importance to provide the means for them to edit and visualise contents in their preferred way. On the other hand, we found out that the interoperability of documents is an important concern of teachers. Teachers appreciated the features offered by our prototype and made recommendations for its improvement. Even though for the moment we work on a standalone version of the prototype, it would be advisable to implement a plug-in version so it can be used as an extension of a text editor, in order to enlarge its possibilities.

An application including the features of our prototype could be useful to assist in some degree the teaching and learning process in an integrated environment of high school level. On the other hand, it is important to take into account higher levels of mathematics education. Presently, the standard for working with documents of complex mathematics is LaTeX. However, the LaTeX code is too verbose for its treatment in the non visual modalities, either braille or audio. As mentioned previously, there exist software to convert LaTeX into audio or into braille mathematical codes. One of our future works is the implementation of the proposed features to the treatment of the LaTeX code, in combination with audio and the mathematical braille codes.

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