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Modelling of Parameter Uncertainties and Disturbance Rejection of a Real-Life System Employing L_1 Adaptive Control Scheme

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Abstract

This paper presents a novel scheme for modelling of parameter uncertainties and disturbance rejection of a real-life system utilizing newly evolved L_1 adaptive control technique. In L_1 adaptive controller use of high adaptation gain incurs fast transient performances and the resulted high frequency component is then filtered out and is applied to the system to keep good robustness. To validate L_1 adaptive controller in real-life experimentation speed control of a DC motor model is evaluated. At the onset DC motor is modelled to include parameter uncertainties present in the experimental setup. Then the DC motor with disturbances is evaluated by employing L_1 adaptive controller whose parameters are tuned by particle swarm optimisation (PSO) in an off-line manner.

Key words

Modelling, parameter uncertainties, disturbance rejection, real-life experimentation, L_1 adaptive control, particle swarm optimization, transient performance, robustness.

1. Introduction

Classical controllers are incapable of controlling systems with time varying uncertainties and time varying disturbances (Dincel et al., 2016). Adaptive controllers (Calvet, 2016) are introduced to tackle those problems. Adaptive fuzzy logic is hugely used to control systems with nonlinearities, disturbances, uncertainties (Bechkaoui et al.,2015; Boulkroune et al., 2014; Jha et al., 2013). Model reference adaptive controllers (MRAC) are used to control systems with uncertainties, disturbances, delay (Hashemipour et al., 2017; Ganesan et al., 2017). MRAC are dependent on initial conditions of the system, changes in input, incurs slow transient performance. To eliminate those problem L_1 adaptive controller comes into places (Cao et al., 2007; Choe et al., 2016). L_1 adaptive controller uses high adaptation gain to get fast transient performance and at the same time control signal is filtered out through a low pass filter to guarantee high robustness (Cao et al., 2007).

L_1 adaptive controller is used to control armed robot manipulator (Cao et al., 2007), aircraft model (Hellmundt et al., 2015). L_1 adaptive controller out-performs to deal with the systems containing nonlinearities, unknown uncertainties (Song et al., 2016), time varying disturbances. In previous literatures L_1 adaptive controller is used in simulation case study only. But it is very essential to verify any adaptive controller in real-life experimentation to see whether it is capable of quickly adapting and efficiently rejecting unknown uncertainties, time varying disturbances present in the system. Till date in only one literature L_1 adaptive controller is used to control an underwater submarine system by D. Maalouf et al (2013). They strengthen L_1 adaptive controller by a proportional-integral controller for betterment of results. Here merely L_1 adaptive controller is used to tackle a real-life system containing uncertainties and disturbances.

In this work firstly the DC motor parameters are estimated, uncertainties are modelled and then a L_1 adaptive controller is incorporated to speed control of that DC motor consists of parameter uncertainties, time varying disturbances. The parameters of L_1 adaptive controllers are so tuned by using a stochastic optimisation technique named as particle swarm optimisation (PSO) (Maiti et al., 2016) that with fast adaptation high robustness is also assured. Effect of high adaptation gain to get quick transient performance is nullified by a low pass filter into the control channel, incurs guaranteed stability. The DC motor is tested with different reference trajectories and certain load disturbances.

This paper is organised as follows. Section 2 describes the L_1 adaptive controller design for a real-life system. Its two sub-sections consists of modelling and estimation of parameter uncertainties and adaptation of L_1 adaptive controller. Section 3 lights up on a real-life experimentation. There results and discussions are given in section 4. Section 5 drawn the conclusion of the paper.

2. L_1 adaptive controller design for a real-life system

L_1 adaptive controller is very much efficient to control systems with time varying uncertainties and disturbances. A DC motor model of 25 W, 50 Volts, 3000 rpm is used here to validate the effectiveness of L_1 adaptive controller. To design L_1 adaptive controller the DC motor with parameter uncertainties and disturbances have to be modelled at first. Then L_1 adaptive controller will be designed.

2.1. Modelling of DC motor with parameter uncertainties

Input to the motor is voltage signal and speed is taken as the output. A tachometer is attached to measure the speed and then the speed is converted into equivalent voltage inside the setup. A conversion ratio of speed/voltage = 1000rmp/2V is set by the manufacturer. A flywheel is attached to the shaft and a permanent magnetic is so arranged that it can give load to the motor by attracting that flywheel. The DC motor experimental setup is given in fig. 1.

Consider the DC motor model as:

$$\ddot{\omega}+\left(\frac{R_a}{L_a}+\frac{B}{J}\right)\dot{\omega}+\left(\frac{BR_a}{JL_a}+\frac{K_bK_T}{JL_a}\right)\omega=\frac{KK_T}{JL_a}v(t) \tag{1}$$

$$y=\omega \tag{2}$$

Where the parameters of DC motors are: resistance of the armature (R_a), inductance of the armature (L_a), back emf coefficient (K_b), torque coefficient (K_T), driver circuit coefficient (K), inertia of the motor (J) and damping ratio (B). $\underline{\omega}\in\Re^2$ is the state of the motor in rad/sec, $v(t)\in\Re$ is the dc voltage input to the motor, $y\in\Re$ is the output of the motor i.e. angular speed in rad/sec.

At the time of output measurement the parameter uncertainties and disturbances are included in the DC motor model. Let the unknown uncertainties for all the parameters mentioned above are ΔR_a, ΔL_a, ΔK_b, ΔK_T, ΔK, ΔJ and ΔB respectively. Now the DC motor model of (1) with unknown uncertainties and time varying disturbances $d(t)$ become:

$$\begin{aligned}&\ddot{\omega}+\left(\frac{R_a+\Delta R_a}{L_a+\Delta L_a}+\frac{B+\Delta B}{J+\Delta J}\right)\dot{\omega}+\left(\frac{(B+\Delta B)(R_a+\Delta R_a)}{(J+\Delta J)(L_a+\Delta L_a)}+\frac{(K_b+\Delta K_b)(K_T+\Delta K_T)}{(J+\Delta J)(L_a+\Delta L_a)}\right)\omega\\&+d(t)=\left(\frac{(K+\Delta K)(K_T+\Delta K_T)}{(J+\Delta J)(L_a+\Delta L_a)}\right)v(t)\end{aligned} \tag{3}$$

After some manipulation, it can be written as:

$$\ddot{\omega}+\left(\frac{R_a}{L_a}+\frac{B}{J}+\Delta P_1\right)\dot{\omega}+\left(\frac{BR_a}{JL_a}+\frac{K_b K_T}{JL_a}+\Delta P_2\right)\omega+d(t)=\left(\frac{KK_T}{JL_a}+\Delta P_3\right)v(t) \tag{4}$$

where, $\Delta P_1=\dfrac{L_a\Delta R_a-R_a\Delta L_a}{L_a(L_a+\Delta L_a)}+\dfrac{J\Delta B-B\Delta J}{J(J+\Delta J)}$,

$$\Delta P_2=\frac{R_a\Delta B+B\Delta R_a+\Delta B\Delta R_a-BR_a(L_a\Delta J+J\Delta L_a+\Delta J\Delta L_a)}{JL_a(J+\Delta J)(L_a+\Delta L_a)}+\frac{K_T\Delta K_b+K_b\Delta K_T+\Delta K_b\Delta K_T-K_bK_T(L_a\Delta J+J\Delta L_a+\Delta J\Delta L_a)}{JL_a(J+\Delta J)(L_a+\Delta L_a)}$$ and

$\Delta P_3=\dfrac{K_T\Delta K+K\Delta K_T+\Delta K\Delta K_T-KK_T(L_a\Delta J+J\Delta L_a+\Delta J\Delta L_a)}{JL_a(J+\Delta J)(L_a+\Delta L_a)}$ are the uncertainties related to angular speed, angular acceleration and voltage input of the DC motor respectively.

Therefore the DC motor state-space model with unknown uncertainties and time varying disturbances takes a form:

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix}=\begin{bmatrix}0 & 1\\ -\left(\frac{R_a}{L_a}+\frac{B}{J}\right) & -\left(\frac{BR_a}{JL_a}+\frac{K_bK_T}{JL_a}\right)\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}+\begin{bmatrix}0\\ 1\end{bmatrix}\left(\left(\frac{KK_T}{JL_a}+\Delta P_3\right)v+\begin{bmatrix}-\Delta P_1 & -\Delta P_2\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}+d(t)\right) \tag{5}$$

Therefore, it can be clearly shown that the DC motor model have unknown constant $\omega=\left(\dfrac{KK_T}{JL_a}+\Delta P_3\right)$, uncertainties $\underline{\theta}=[\theta_1 \quad \theta_2]=[-\Delta P_1 \quad -\Delta P_2]$ and disturbance $\sigma=d(t)$. These $[\omega \ \underline{\theta} \ \sigma]$ values are bounded to some compact set given by: $[\omega \ \underline{\theta} \ \sigma]\in[\Omega \ \underline{\Theta} \ \Sigma]$.

Therefore, equation (5) can be written as:

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix}=\begin{bmatrix}0 & 1\\ -\left(\frac{R_a}{L_a}+\frac{B}{J}\right) & -\left(\frac{BR_a}{JL_a}+\frac{K_bK_T}{JL_a}\right)\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}+\begin{bmatrix}0\\ 1\end{bmatrix}\left(\omega v(t)+\underline{\theta}^T(t)\underline{x}(t)+\sigma(t)\right) \tag{6}$$

2.2. Estimation of parameter uncertainty of a DC motor

After mathematical modelling the practical plant have to be evaluated to get the parameter values including parameter uncertainties and disturbances. At first the DC motor runs in open loop condition with variable step reference input such as: $v_{oc}(t) = \begin{cases} 28.8 * 0.5u(t) & 0 < t \le 300\,\textbf{\textit{sec.}} \\ 28.8 * 0.9u(t) & 300 < t \le 600\,\textbf{\textit{sec.}} \end{cases}$ and time step $\Delta t = 0.1$sec. for 10 minutes. Speed of the DC motor in form of equivalent voltage is taken as the output. In simulation environment parameter estimation is done with the help of a well-known and efficient stochastic optimisation technique named as particle swarm optimisation (PSO) (Maiti, 2017a). Integral absolute error is taken as the objective function of PSO. The error between the two speed obtained from experimental setup and simulation model is considered when same input is given to them. PSO runs 200 iteration with 30 particle to give global best optima. After repeating the whole experiment ten times, the best result is given in fig. 2 and in table 1. In fig. 2, the blue and red line represents the open loop response of practical plant and simulation model respectively. Table 1 represents the estimated parameter values of DC motor.

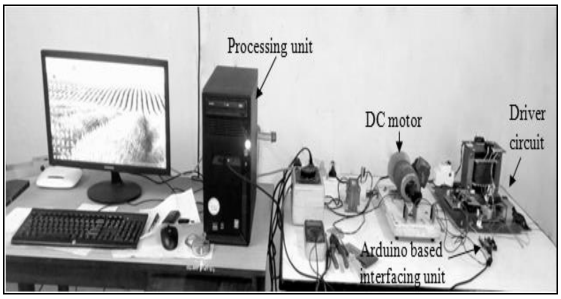

Fig. 1: DC motor experimental setup

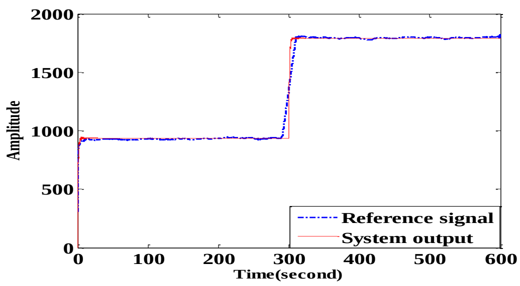

Fig. 2: Open loop system responses

Table 1: Estimated parameter values of DC motor

R_a	L_a	J	B	K	K_b	K_T
0.43401	0.00073	0.00620	0.00043	0.43025	0.00329	0.00329

From the result pictured in fig. 2, it is evident that the results obtained from simulation model and experimental setup differs due to inclusion of parameter uncertainties and disturbances in the DC motor experimental model. In this paper our objective is to predict and quickly adapt the values of uncertainties and disturbances present in the system and by eliminating them efficient and robust control effort have to be produced that the plant can track the desired trajectory properly. Therefore online estimation and adaptation of L_1 adaptive control scheme is proposed in this paper.

2.3. L_1 adaptive controller implementation

Predictor model have to be designed that the values of $[\omega \ \underline{\theta} \ \sigma]$ can be predicted accurately. Consider the predictor model as:

$$\begin{bmatrix} \dot{\hat{x}}_1 \\ \dot{\hat{x}}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\left(\frac{R_a}{L_a}+\frac{B}{J}\right) & -\left(\frac{BR_a}{JL_a}+\frac{K_bK_T}{JL_a}\right) \end{bmatrix} \begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \left(\hat{\omega}(t)v(t) + \underline{\hat{\theta}}^T(t)\underline{x}(t) + \hat{\sigma}(t) \right) \quad (7)$$

Where, $\hat{\omega}(t) = \left(\frac{KK_T}{JL_a} + \Delta\hat{P}_3 \right) \in \Re$, $\underline{\hat{\theta}}(t) = \left[-\Delta\hat{P}_1 \quad -\Delta\hat{P}_2 \right] \in \Re^n$ and $\hat{\sigma}(t) = \hat{d}(t) \in \Re$ are the estimation of unknown constant, time varying uncertainties and time varying disturbances respectively. After predicting those values adaptation law is formulated following the Lyapunov stability criteria as: $\dot{\hat{\omega}}(t) = \Gamma_1 \operatorname{Pr}oj\left(\left(\frac{KK_T}{JL_a} + \Delta\hat{P}_3 \right), -\underline{\tilde{x}}^T(t)P \begin{bmatrix} 0 \\ 1 \end{bmatrix} v(t) \right)$,

$\dot{\underline{\hat{\theta}}}(t) = \Gamma_2 \operatorname{Pr}oj\left(\left[-\Delta\hat{P}_1 \quad -\Delta\hat{P}_2 \right], -\underline{x}(t)\underline{\tilde{x}}^T(t)P \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right)$, $\dot{\hat{\sigma}}(t) = \Gamma_3 \operatorname{Pr}oj\left(\hat{d}(t), -\left(\underline{\tilde{x}}^T(t)P \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right)^T \right)$.

Where, $\underline{\tilde{x}} = \hat{x} - x$ is the state error vector. P is the solution of the algebraic Lyapunov equation of the system given by,

$$\begin{bmatrix} 0 & 1 \\ -\left(\frac{R_a}{L_a}+\frac{B}{J}\right) & -\left(\frac{BR_a}{JL_a}+\frac{K_bK_T}{JL_a}\right) \end{bmatrix}^T P + P \begin{bmatrix} 0 & 1 \\ -\left(\frac{R_a}{L_a}+\frac{B}{J}\right) & -\left(\frac{BR_a}{JL_a}+\frac{K_bK_T}{JL_a}\right) \end{bmatrix} = -Q,$$ where Q is an arbitrary symmetric matrix satisfying $Q = Q^T > 0$.

Three different adaptive gains $\underline{\Gamma} = [\Gamma_1 \, \Gamma_2 \, \Gamma_3]$, each of high value are used here to get quick adaptation of uncertainties to get fast transient performances. To remove high frequency component inclusion into the control channel a low pass filter is added with controller and the overall controlled voltage input to the system become:

$$v(s) = -kC(s)\left[\hat{\eta}(s) - k_g r(s) \right] \quad (8)$$

Where, k is the gain to the controller, k_g is pre filter gain, $\hat{\eta}(s)$ is the Laplace transform of $\hat{\eta}(t)$ given by, $\hat{\eta}(t) = \left(\frac{KK_T}{JL_a} + \Delta\hat{P}_3 \right) v(t) + \left[-\Delta\hat{P}_1 \quad -\Delta\hat{P}_2 \right]^T \underline{x}(t) + \hat{d}(t))$.

$C(s)$ is the low pass filter of form: $C(s) = \dfrac{f_c\, D(s)}{1 + f_c\, D(s)}$ with dc gain C(0)=1. f_c is the cut off frequency of the filter.

3. Real-life experimentation

The L_1 adaptive controller is very much efficient to give quick transient performance as well robustness in simulation environment by making judicious trade-off between high adaptive gain and low pass filter (Maiti, 2017b). It is of great importance to validate an adaptive controller in real-life environment. To justify theoretical findings, L_1 adaptive controller is tested on a DC motor experimental setup. The range of the parameter values getting from mathematical calculation does not imply optimal results. Selecting the proper parameters of L_1 adaptive controller in real-life environment to get good transient performance as well high robustness PSO is used. The candidate solution vector of PSO is set as: $\underline{CSV} = [\hat{\omega} \,|\, \underline{\hat{\theta}} \,|\hat{\sigma} \,|\, \underline{\Gamma} \,|\, k \,|\, k_g \,\,|\, f_c]$. PSO runs for 200 iterations with 30 particles to get optimal integral absolute error ($IAE = \sum_{i=0}^{itr} \underline{\tilde{x}}$) by making state error vector $Lim\, \underline{\tilde{x}} = 0$ as $t \to \infty$ in each iteration. The $\underline{CSV}$ can be segregated in two parts, adaptive parameters component $\underline{CSV}_{adap} = [\hat{\omega} \,|\, \underline{\hat{\theta}} \,|\hat{\sigma}]$ and non-adaptive parameters component $\underline{CSV}_{non-adap} = [\underline{\Gamma} \,|\, k \,|\, k_g \,\,|\, f_c]$. The $\underline{CSV}$ is tuned and the optimal results obtained are set. The experimental setup is run and the adaptive parameters are adapted online in each iteration to cope up with time varying uncertainties and disturbances occurs certainly or continuously.

4. Results and discussions

4.1. Case study on experimental setup

The DC motor experimental setup is run and the angular speed output consists of uncertainties and disturbances is measured. Angular speed is then converted into r.p.m following $N = 60\omega/2\pi$. Predictor is simulated by using R-K 4th order method and the two outputs are compared to produce adaptation law. From that, control law is calculated, filtered out and is given to the plant as voltage input. The DC motor is run for 10 minutes with time step Δt = 0.1 second. In each iteration the adaptive parameters are adapted online to produce efficient control effort by eliminating uncertainties and time varying disturbances continuously. Three different reference signals are used to test the proposed method. Two variable step trajectories and a step trajectory with full load disturbance is given as:

Input 1: $r(t) = \begin{cases} 1200u(t) & 0 < t \le 300\text{sec.} \\ 1800u(t) & 300 < t \le 600\text{sec.} \end{cases}$, Input 2: $r(t) = \begin{cases} 1000u(t) & 0 < t \le 300\text{sec.} \\ 1500u(t) & 300 < t \le 600\text{sec.} \end{cases}$

Input 3: $r(t) = 1000u(t) \;\; 0 < t \le 600\text{sec}$ with certain full load disturbance.

In first case all the parameters of L_1 adaptive controller is set from PSO. In our proposed method, non-adaptive parameters are set from PSO and adaptive parameters are adapted in real-life continuously. The results obtained are given in terms of integral absolute error (IAE) and control energy (CE) for 10 minutes evaluation period in fig. 3 and table 2.

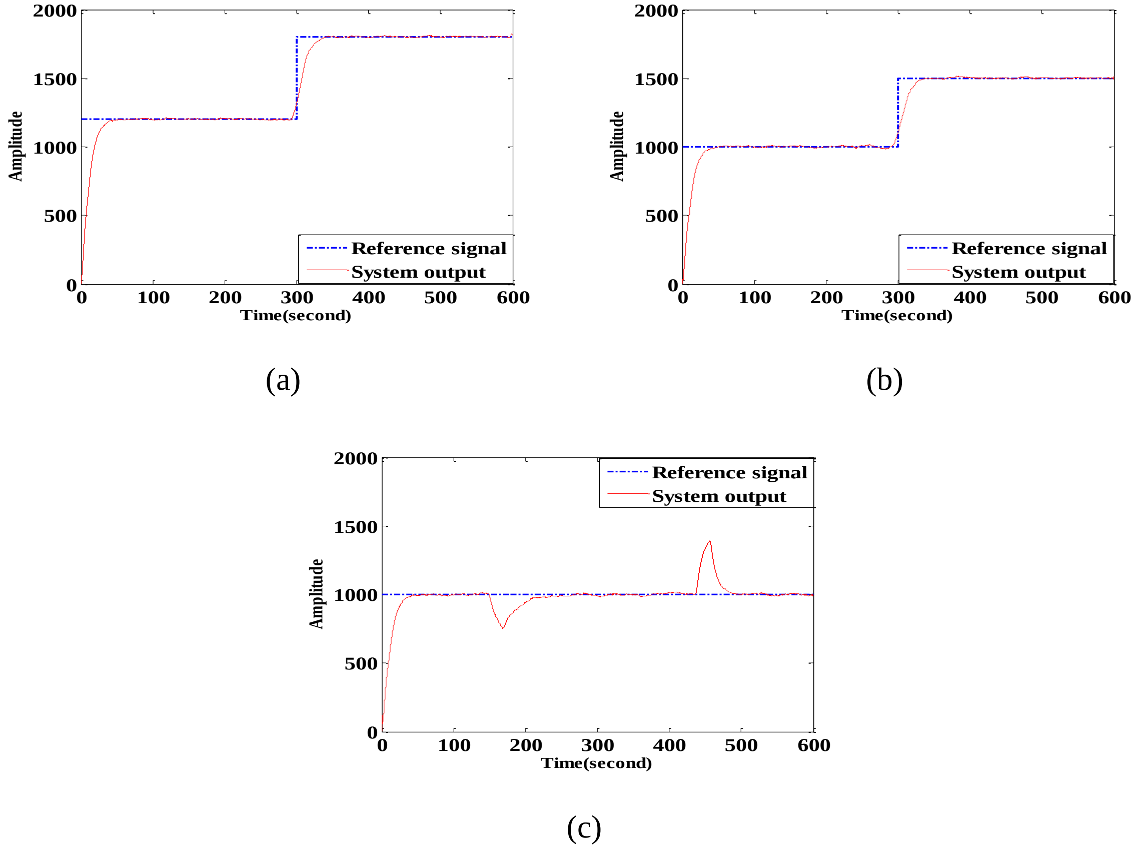

Fig 3: Reference with respect to response of DC motor experimental setup employed L_1 adaptive controller for (a) Input 1, (b) Input 2, (c) Input 3.

Table 2: Offline and online adaptation results in terms of IAE and CE

	Offline Adaptation		Online Adaptation	
	IAE	**CE**	**IAE**	**CE**
Input 1	21852.44247	268923.00484	21506.45040	271016.18497
Input 2	19216.24839	199663.63952	18793.58506	195755.34141
Input 3	32627.18739	286896.42751	30465.47159	256854.53710

4.2. Discussion

From table 2 it can be shown that the IAE and control energy required for proposed method is less than that of the other method compared here. Fig. 3 represents system response with references for three different inputs. From fig. 3 it can be clearly shown that the system response properly track the reference signal. From fig. 3(c) it is evident that after giving load disturbance certainly the speed of the motor starts to decrease but within very few time it again came back to the reference and even with load it can track the desired trajectory accurately without becoming unstable.

5. Conclusion

From those results it is evident that the proposed method can track the desired trajectory properly by eliminating uncertainties and disturbances present in the system or certainly comes to the system. It gives fast transient performance by quickly eliminating the occurrence of load disturbance. At the same time without becoming unstable during the loaded condition it guarantee robust control also. Therefore it can be concluded that the proposed online adaptation based L_1 adaptive controller can be successfully employed to control any practical system consists of uncertainties and time varying disturbances.

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