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Model Reference Adaptive Feed-Forward Control for Non-Minimum Phase System in Two-Degree-of-Freedom Framework

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Abstract

This paper proposes a design of Model Reference Adaptive Controller (MRAC) for Non-minimum phase (NMP) system in Two-degree-of-freedom (2DOF) structure. Exact reference input tracking can be possible using inverse transfer function model of the system and Feed-forward Model Reference Adaptive Control is also an effective method for command input tracking . As direct model reference adaptive control needs inversion based system, the properties of inverse dynamics of Non-minimum phase system restrict the direct application of this powerful control technique. So, the designing of tracking control system for non-minimum phase system is a challenging issue in control system engineering, especially when the inverse model based Model Reference Adaptive Control scheme is to be designed for the non-minimum phase System. To circumvent this difficulty, a novel technique of adaptive decoupled two-degree-of-freedom control has been developed to achieve the set point trajectory tracking for a class of non-minimum phase systems. In this two-degree-of-freedom structure, feed-forward and feedback control design can be done independently and satisfactory results have been observed by applied in 3rd and 4th order NMP systems in simulation environment.

Key words

Model Reference Adaptive Control (MRAC), Non-minimum Phase (NMP), State feedback Control, Two-degree-of-freedom (2DOF).

1. Introduction

Design of tracking control system is one of the essential as well as challenging tasks in the field of control system engineering. In frequency domain analysis large phase lag and initial undershoot in the time domain response is unavoidable due to right hand plane (RHP) zero of

NMP system. Another peculiar phenomena of NMP system has been found that when transfer function model and inverse transfer function model of NMP system are connected in cascaded, the output response always become oscillatory and unbounded, but obviously, it is not true for minimum phase system, here numerator denominator of transfer function model cancelled each other, and as a result exact set point tracking can be achieved. So, designing of controller for a non-minimum phase system is a challenging tasks due to its RHP zero dynamics and this unusual nature of RHP zero dynamics inspired the researcher to design an appropriate control system for NMP system. In control system engineering, the numbers of closed loop transfer function which are to be controlled indicates Degree of freedom (DOF) of the system [1]. To get a precise control performance, the 2DOF system strategy in design technique is obviously being a better way than 1DOF design technique. Using literature survey regarding 2DOF system, it has been found that, this concept of 2DOF becomes more and more reliable tool for research work. Using 2DOF controller in Ball and Beam system [2], the ball can track the square wave signal with certain given specifications. Model predictive 2DOF control structure can automatically tune the single input single output industrial process with model uncertainty [3]. As Adaptive feed-forward zero phase error tracking control in 2DOF control structure [4] for minimum as well as non-minimum phase systems shows acceptable satisfactory result ,the combination of adaptive feed-forward and feedback compensators in 2DOF structure may be an effective tool for the designing of tracking control system of NMP system .

Among all Adaptive Controller, Model Reference Adaptive Control (MRAC) is very much popular as it is a direct approach to force the uncertain system to obtain the desired performance [5]. Here is an added advantage for control designer that, they can choose the shape of the transient part of the output response, and it is only possible by reference model, which is an essential part of the MRAC design [6]. A Model Reference Adaptive Controlled Permanent Magnet Synchronous Motor has been simulated successfully, where time response shows optimum performance [7]. Both MIT rule and Lyapunov stability rule can apply for designing of Model Reference Adaptive Control and its efficiency has been observed by implementing on a Chemical Reactor [8], that the stability of the Controller is completely granted for Lyapunov stability theory. To improve the lateral stability of articulated heavy vehicles (AHVs), MRAC has been theoretically applied to active trailer steering (ATS) [9]. The transient performance of closed loop Model reference adaptive controlled fractional order nonlinear system improves in the sense of generating smooth system output [10].Proportional plus Integral (PI), Proportional plus Integral plus Derivative (PID), Phase lead or Phase lag Controller may not be able to control all the poles of the higher order system independently. To circumvent these difficulties, state feedback control with arbitrary pole placement approach can solve this problem independently under the certain condition, that the system must be completely state controllable [11]. Transient dynamics of piezo-actuated bimorph atomic force microscopy (AFM) probe is controlled by state feedback controller, where quality factor and the resonance frequency of the probe have been adjusted simultaneously [12]. An optimal state feedback design has been considered for the classical unstable plant like Inverted Pendulum by calculating the appropriate state feedback gains using flower pollination algorithm (FPA) [13]. The efficiency of MRAC and Pole placement method encourages us to design a suitable controller for inversion based Non-minimum system.

The objective of this paper is to propose a 2DOF controller for NMP system, in which inverse system model becomes compensated by Model Reference Adaptive Control as feed-forward technique and non-inverse model one has been controlled by arbitrary pole placement method as feedback counter part of the 2DOF structure. This proposed method is verified by a 3rd order and 4th order NMP system with step, ramp and Sinusoidal Signal.

This paper is organized as follows, section 2, 3 represent brief descriptions of the 2DOF, MRAC respectively. Problem formulation and the mathematical background of MRAC have been provided by the section 4 and 5. Both numerical examples and simulation results are given in section 6. The demonstration ended with discussion and concluding remarks in section 7and 8 separately.

2. **Two-degree-of-freedom System**

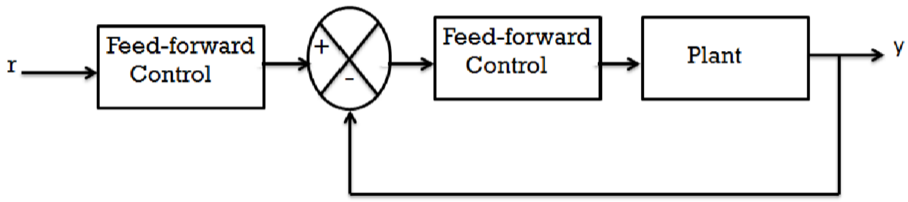

Fig1. Block Diagram of Two-degree-of-freedom Controlled System

In the above figure, the feed-forward and feedback controller are connected in such a fashion that output signal of closed loop feed-forward controlled system fed to the input of feedback controller and they are not depend on each other. Feedback controller mainly used to reject the effect of disturbance of the system whereas feed-forward controller help to track the regerence trajectories given to the system

3. **Model Reference Adaptive Control**

The model reference adaptive control system shown in the Fig. 2 consists of two control loops, inner and outer. The inner loop is an ordinary feedback loop composed of plant and controller and the outer control loop consists of the adjustment mechanism and reference model. Outer loop calculates the control parameters on the basis of error which is produced by the difference of reference and plant model outputs. In a model reference adaptive control, the mechanism for adjusting the parameters can be obtained in two ways, by using gradient method or by using stability theory.

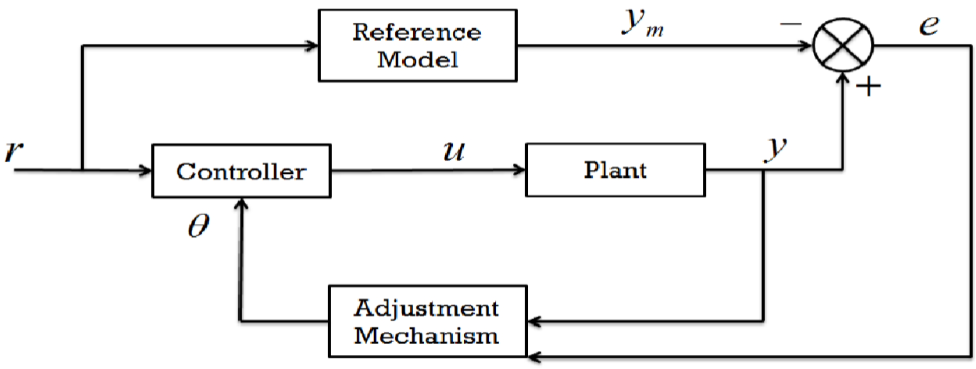

Fig2. Model Reference Adaptive Control System

4. Problem Formulation

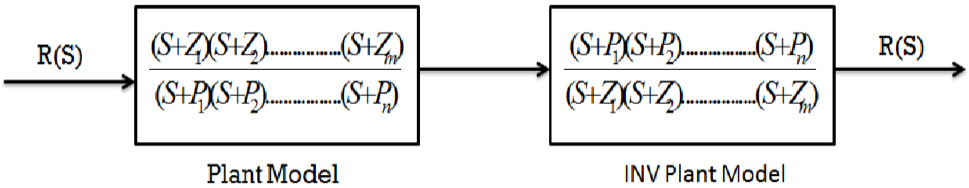

Fig3. Block Diagram of the series combination minimum phase system with inversion transfer function model

In the above block diagram [Fig3], $-z_1, -z_2, \ldots\ldots, -z_m$ are the zeros and $-p_1, -p_2, \ldots\ldots -p_n$ are the poles of the transfer function model of minimum phase system and, here, exact matching of output with reference input is possible as poles and zeros of the transfer function models are cancelled with each other, but when this concept is applied for NMP system, unbounded output response is obvious. These difficulties may be overcome by the proposed structure of Fig4.

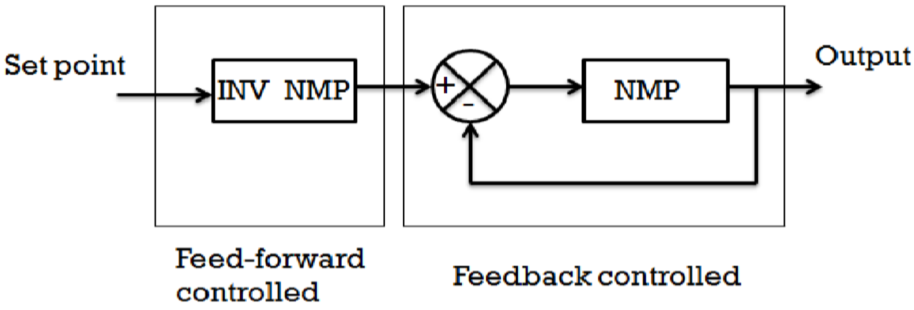

Fig4. Block Diagram of 2DOF Controlled NMP Plant

Here, feed-forward controller is used to tackle inverse model NMP system whereas feedback controller is applied for non inverse NMP system to stabilize the system in Two-degree-of-freedom structure. With the help of reference model system direct model reference adaptive control as feed-forward counterpart of this control structure shape the trajectory of the output response. One 3rd order and 4th [14] order practical non-minimum system have been taken for experimental simulation.

5.0 Derivation of the Proposed Closed Loop Model Reference Adaptive Controller Employing 2DOF Control System

Consider a n^{th} order plant model and a reference model such as

$$y^{(n)} = -a_{n-1}y^{(n-1)} - a_{n-2}y^{(n-2)}............ + bu \tag{1}$$

$$y_m{}^{(n)} = -b_{n-1}y_m{}^{(n-1)} - b_{n-2}y_m{}^{(n-2)} - .... + b_m r \tag{2}$$

Where, y, y_m are the plant model and reference model output respectively. u is the control input to the plant whereas r is the command input of the closed loop system. $a_{n-1}, a_{n-2}......$ and $b_{n-1}, b_{n-2}........$ are the co-efficient of the differential equations.

The aim of the closed loop controlled system output is to follow the reference model's output trajectory.

Let, Control input: $u = \theta_1 r - \theta_2 y^{(1)} - \theta_3 y^{(2)} - .... - \theta_n y^{(n-1)}$ (3)

Tracking error: $e = y - y_m$ (4)

$\theta_1, \theta_2, ...\theta_n$ are the control parameters

By subtracting the equation (2) from equation (1), and replacing control input in the equation we get,

$$e^{(n)} = y^{(n)} - y_m{}^{(n)} = -a_{n-1}y^{(n-1)} - a_{n-2}y^{(n-2)} - ..... + bu - (-b_{n-1}y_m{}^{(n-1)} - b_{n-2}y_m{}^{(n-2)} - .... + b_m r) \tag{5}$$

Assuming the initial values of the control parameter $\theta_1, \theta_2, \theta_3..............\theta_n$, and integrating equation (5) $(n-1)$ times with respect to t, we get,

$$e^{(1)} = -b_{n-1}e - (b\theta_n + a_{n-1} - b_{n-1})y - (b\theta_{n-1} + a_{n-2} - b_{n-2})y^2 - .... + (b\theta_1 - b_m)r \quad (6)$$

From the error dynamics it is observed that the tracking error will go to zero if $b\theta_n = b_{n-1} - a_{n-1}, b\theta_{n-1} = b_{n-2} - a_{n-2}, b\theta_{n-2} = b_{n-3} - a_{n-3}, ..., b\theta_1 = b_m$.

The parameter adjustment rule thus achieve the goal.

5.1 Proof of Closed-loop stability

In this closed loop system, adaptation mechanism will drive both the tracking error and error in estimation of control parameters to zero. Consider a quadratic function has been taken as Lyapunov function for the error dynamics,

$$v(e,\theta_1,\theta_2) = \frac{1}{2}\left(e^2 + \frac{1}{b\gamma}(b\theta_n + a_{n-1} - b_{n-1})^2 + \frac{1}{b\gamma}(b\theta_{n-1} + a_{n-2}. - b_{n-2})^2 + ... + \frac{1}{b\gamma}(b\theta_1 - b_m)^2 \right) \quad (7)$$

where $b\gamma > 0$.

This function is zero when e is zero and the controller parameters are equal to the correct values. For a valid Lyapunov function ,time derivative of Lyapunov function must be negative. The derivative is given by

$$V^{(1)} = ee^{(1)} + \frac{1}{\gamma}(b\theta_n + a_{n-1} - b_{n-1})\theta_n^{(1)} + \frac{1}{\gamma}(b\theta_{n-1} + a_{n-1} - b_{n-1})\theta_{n-1}^{(1)} + ... + \frac{1}{\gamma}(b\theta_1 - b_m)\theta^{(1)} \quad (8)$$

Substituting the value of $e^{(1)}$ from equation (6) into equation (8), we get,

$$V^{(1)} = -b_{n-1}e^2 + \frac{1}{\gamma}(b\theta_n + a_{n-1} - b_{n-1})(\theta_n^{(1)} - \gamma ye) + \frac{1}{\gamma}(b\theta_{n-1} + a_{n-2} - b_{n-2})(\theta_{n-1}^{(1)} - \gamma y^2 e) +$$
$$......... + \frac{1}{\gamma}(b\theta_1 - b_m)(\theta_1^{(1)} + \gamma re) \quad (9)$$

If the parameters are updated as $\theta_n^{(1)} = \gamma ye, \theta_{n-1}^{(1)} = \gamma y^2 e, \theta_{n-2}^{(1)} = \gamma y^3 e, ...\theta_1^{(1)} = -\gamma re$ (10)

The derivative of Lyapunov function will be negative semi definite that indicates $V(t) \geq V(0)$. This ensures that, e, θ_1, θ_2 must be bounded.

5.2. Barabalat Lemma

To ensure that tracking error goes to zero, second time derivative of Lyapunov function has been done,

$$V^{(1)} = -b_{n-1}e^2 \quad (11)$$

$$V^{(2)} = -2b_{n-1}e^{(1)} \quad (12)$$

Equation (13) substituting the value of $e^{(1)}$ in equation (5), we get,

$$V^{(2)} = -2b_{n-1}e[-b_{n-1} - (b\theta_n + a_{n-1} - b_{n-1})y - (b\theta_{n-1} + a_{n-2} - b_{n-1})y^2 - .... + (b\theta_1 - b_m)r] \quad (13)$$

$$V^{(2)} = f(e, \theta_1, \theta_2, ..., \theta_n, y, r) \quad (14)$$

Since, all the parameters are bounded and as $e = y - y_m$ is also bounded. $V^{(2)}$ is also bounded which implies $V^{(1)}$ uniformly continuous. So, using Barbalat's Lemma we conclude that tracking error converges to zero.

6. Simulation Results

In order to examine the proposed control metodology, two numerical examples have been chosen.

Example 1 $$G(S) = \frac{(S-0.526)(S^2+1.526S+3.803)}{(S+1.185)(S^2-0.185S+4.219)}.$$

In this 3rd order transfer function model two complex conjugate poles and zeros are at (0.0925±j1.7946) and (0.7630±j1.7946) respectively. A single pole at -1.185 and an RHP zero placed at 0.5260. It is inherently an unstable system as not only one zero at unstable plane, one pair of complex conjugate pole also at right hand plane of s plane.

Example 2 $$G(S) = \frac{123.853\times10^4(3.5-S)}{(S^2+6.5S+42.25)(S+45)(S+190)}$$

This model has been taken from our previous paper [14], the 4th order transfer function model is the part of a real mechanical system [15] which consists of mass, spring damper element and which produce non-minimum phase characteristics as it has RHP zero placed at 3.5. This system has poles are at -45,-190 and one pair of complex conjugate poles are at -3.35±j5.6292. Ramp, sine wave signal are being used for both controlled and uncontrolled 3rd order and 4th order inverse model connected NMP system. All the simulation experiments have been performed in MATLAB environment. Unit step signal has been used to compare the proposed 2DOF controlled NMP system with PID and State feedback controlled same NMP system.

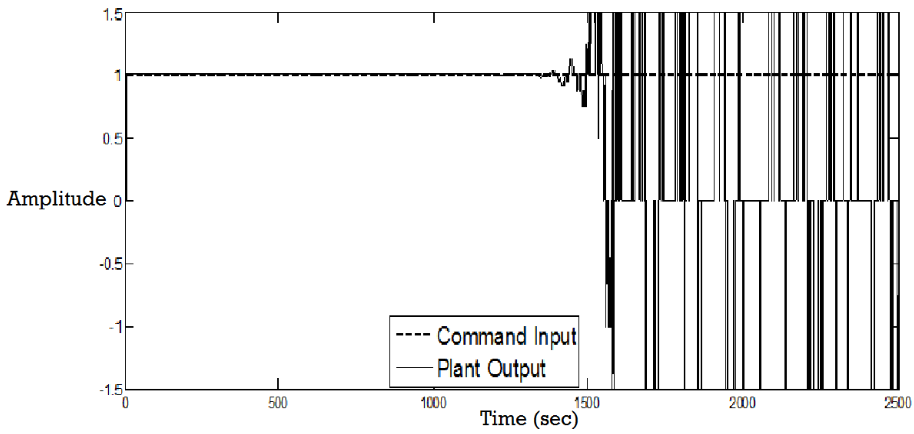

Fig5. Unit Step response of inverse model connected uncontrolled 3rd order NMP System

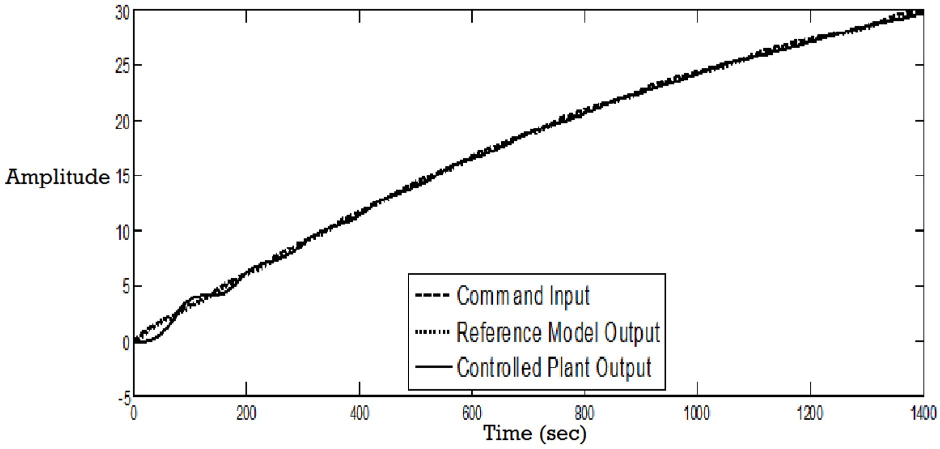

Fig6. Unit ramp response of 2DOF controlled 3rd order NMP System

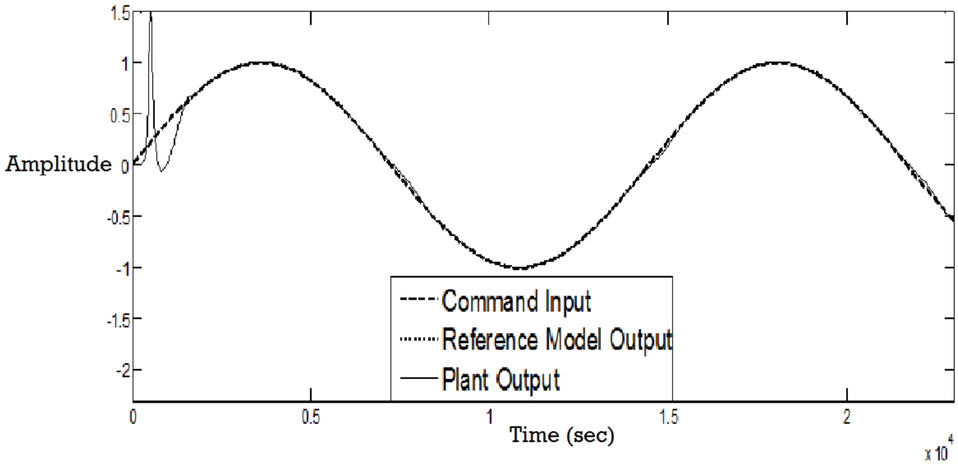

Fig7. Sine wave responseof 2DOF controlled 3rd orderNMP System

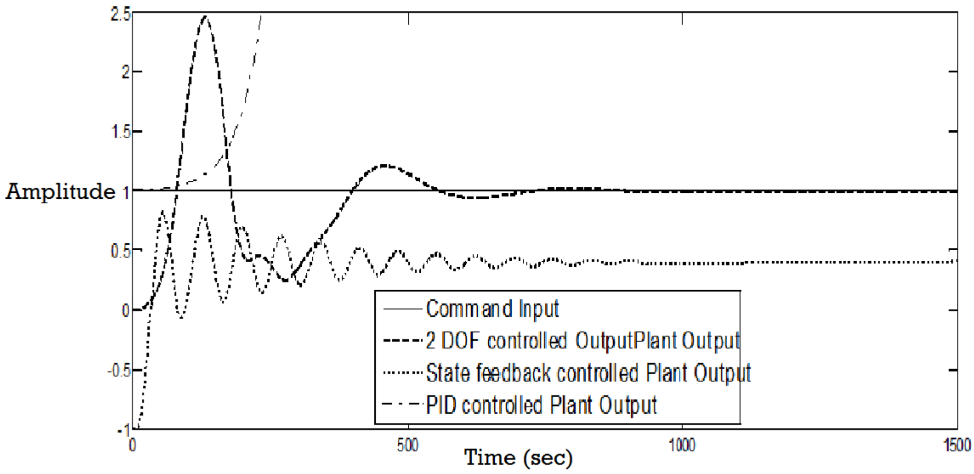

Fig8. Unit step response of State feedback , PID and 2DOFcontrolled 3rd order NMP System

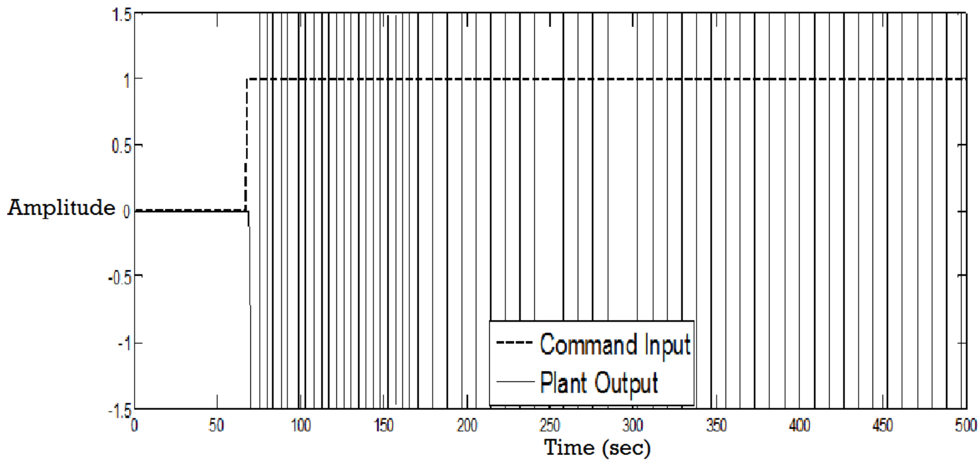

Fig9. Unit Step response of inverse model connected uncontrolled 4th order NMP System

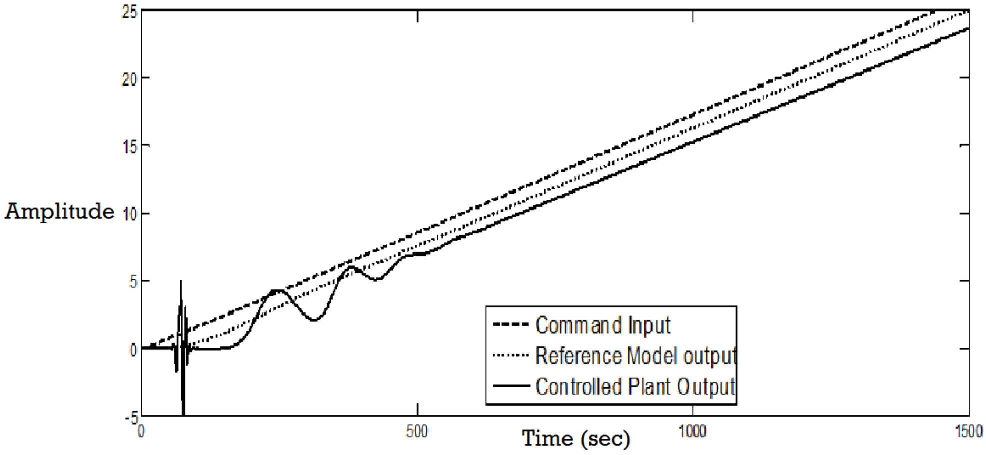

Fig10. Unit ramp response of 2DOF controlled 4th order NMP System

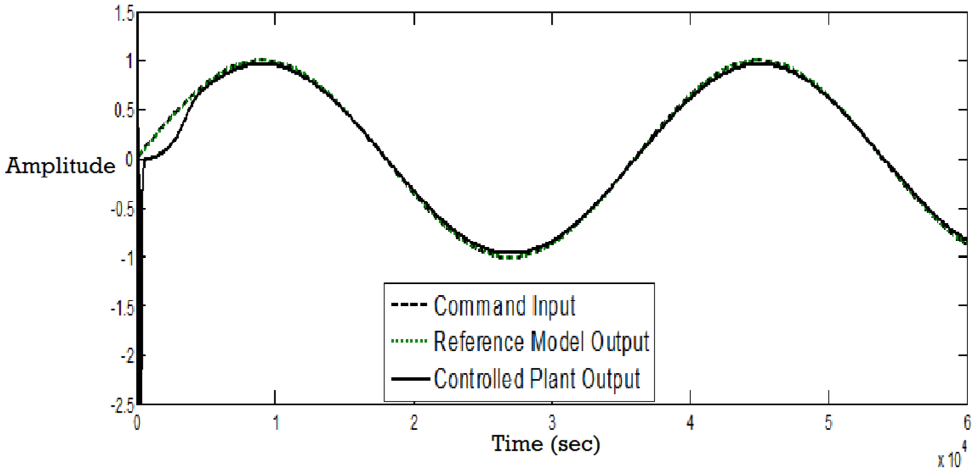

Fig11. Sine wave responseof 2DOF controlled 4th orderNMP System

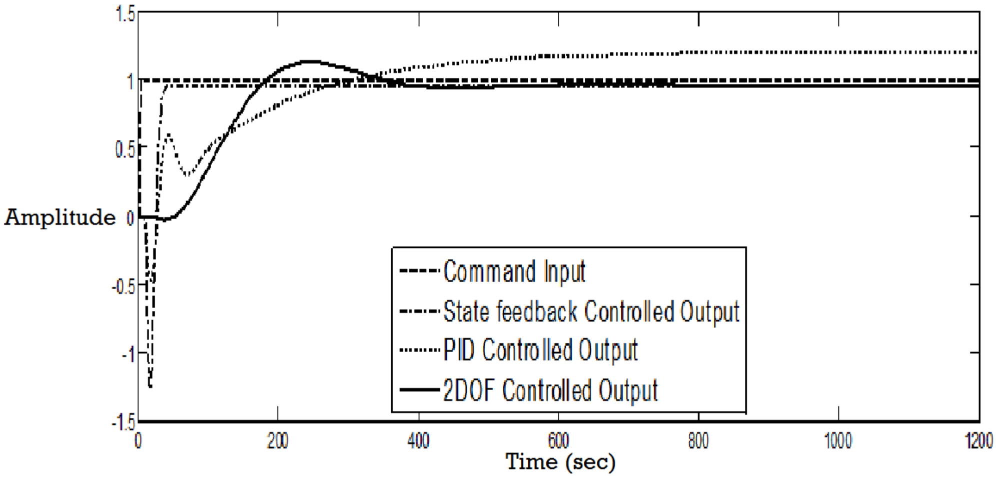

Fig12. Unit step response of State feedback , PID and 2DOFcontrolled 4th order NMP System

7. Discussion

Unlike minium phase sytem, inverse transfer function model connected Non-minimum phase system gives unstable output response [Fig5, Fig9] though their numeratot denominator polynomial cancelled each other. To overcome this unnatural phenomena, Two-degree-of-freedom controller has been applied for two different NMP systems. Two different signals like ramp and sine wave signals are used as command input to obtain satisfactory simulation result for 3rd order [Fig6, Fig7] and 4th order [Fig10, Fig11] NMP system to strengthen the proposed control methodology. To compare the 2DOF controller with PID and State feedback control system, unit step response of 3rd order and 4th order systems with the above said three controller has been superimposed and the following comparison chart is observed.

Comparison chart for Controlled 3rd and 4th order system dynamics has been shown below.

Name of the Controller	Order of The system	Initial Undershoot	Steady State Error	Rise Time
State Feedback	3rd order	1	0.4	infinity
	4th order	1.27	0.04	infinity
PID	3rd order	0	infinity	infinity
	4th order	0.47	20	6.1 Sec.
2DOF	3rd order	0	0.004	79 Sec.
	4th order	0.03	0.04	3.8 Sec

8. Conclusions

This paper presents a theoretical technique for designing a Model Reference Adaptive control (MRAC) as feed-forward compensator in Two-degree-of-Freedom (2DOF) framework to meet the trajectory tracking problem of NMP system. Feed-forward techniques need inversion of system model and as inversion of NMP system always shows unbounded output response as it's RHP zeros become RHP poles, so it is very difficult to stabilize the closed loop system. To overcome this unavoidable difficulties, here, a novel technique of 2DOF control structure has been considered, where the state feedback controller as feedback compensator placed the closed loop poles of non-inverse NMP system at suitable position of left hand plane to stable the system and the feed-forward counter part of 2DOF structure, being decoupled with feedback controller, is here to meet trajectory tracking problem of NMP system. The simulation study of this proposed 2DOF control methodology has been experimented in MATLAB environment on 3^{rd} order as well as 4^{th} order NMP system with ramp and sinusoidal command inputs and unit step response of this 2DOF controlled system along with PID and State feedback controlled system have been superimposed to get clear comparison of dynamic characteristics to establish the effectiveness of the proposed control methodology.

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