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Multi-Goal Motion Planning of Planar Robot Manipulators based on Optimal Control and Biased Randomization

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Abstract

In this paper, the motion planning problem for a planar robotic manipulator with two degrees of freedom, which is constrained to pass through a set of waypoints in the plane, is studied. In this problem, both the discrete optimal sequence of points and the continuous optimal control inputs to the system must be determined. The motion planning problem is formulated as an optimal control problem of a dynamic system with integer variables used to model the waypoints constrains. The nature of this problem, highly nonlinear and combinatorial, makes it particularly difficult to solve. The proposed method combines a meta-heuristic algorithm to determine the promising sequence of discrete points with a collocation technique to optimize the continuous trajectory of the system. Although this method does not guarantee the global optimum, it can solve instances of dozens of points in reasonable computation time.

Key words

robotics, optimal control, motion planning, meta-heuristics, biased-randomization

1. Introduction

A fundamental problem in robotics is the task planning problem. Given the models of the robot manipulator and the environment in which it operates, the problem is to generate a sequence of actions to accomplish a given task [1]. For assembly, material handling, spot welding, measuring, testing, and inspecting one wants to generate a continuous intersection-free motion of the robot manipulator that connects several given configurations of the end-effector.

In general terms, the motion planning problem is an optimal control problem of a mechanical system. Each system comprises a dynamic model to take into account [2, 3]. That is, besides geometrical feasibility, it is also important to ensure dynamical feasibility. And related to optimality, the motion planning must be executed with minimum energy consumption. Furthermore, being a dynamic system, the solution of the problem must provide the optimal scheme of accelerations and velocities during the motion.

In this work, the energy-optimal motion planning problem for planar robot manipulators with two revolute joints is studied. In addition, the end-effector of the robot manipulator is constrained to pass through a set of waypoints, whose sequence is not predefined. The presence of these waypoint constraints adds a combinatorial complexity to this optimal control problem and makes it particularly difficult to solve. We propose a multi-start approach to solve the problem that determine the promising discrete path and evaluate the continuous dynamic trajectory.

The paper is organized as follows. The robot motion planning problem studied in this paper will be introduced and its main characteristics will be stated in Section 2. Our Multi-Start approach is described in Section 3, including its correspondent flow-chart. In Section 4 the results obtained applying our approach to several instances are reported. Finally, some conclusions will be drawn in Section 5.

2. The Robot Motion Planning Problem

In this section, the main characteristics of the robotics system studied in this paper will be formalized. First, the problem will be described and the combinatorial problem for the robotics system will be stated. Then, the dynamic model of the robotics system will be introduced. Finally, the general optimal control problem and its particular reformulation for the robotics system will be described.

2.1 Problem description

The following problem has been studied: find the control inputs that steer the end-effector of a robot manipulator, from an initial position $\bar{p}_I$ to a final position $\bar{p}_F$, passing through a set of n_p waypoints $\bar{P} := \{\bar{p}_1, \dots, \bar{p}_{np}\}$, that minimize the energy consumption during the motion. Therefore, the solution of the problem must provide the optimal scheme of accelerations and velocities during the motion, as shown in Fig. 1.

Furthermore, the total time, t_F-t_I, to carry out the task is supposed to be assigned, where t_I and t_F are the initial and final times, respectively. And, the end-effector of the robot manipulator is constrained to pass once through all the n_p waypoints of $\bar{P}$ when moving between $\bar{p}_I$ and $\bar{p}_F$. It is important to point out that, since both the order in which the waypoints are visited and the corresponding passage times are not specified, they must be determined. Thus, the combinatorial problem is introduced.

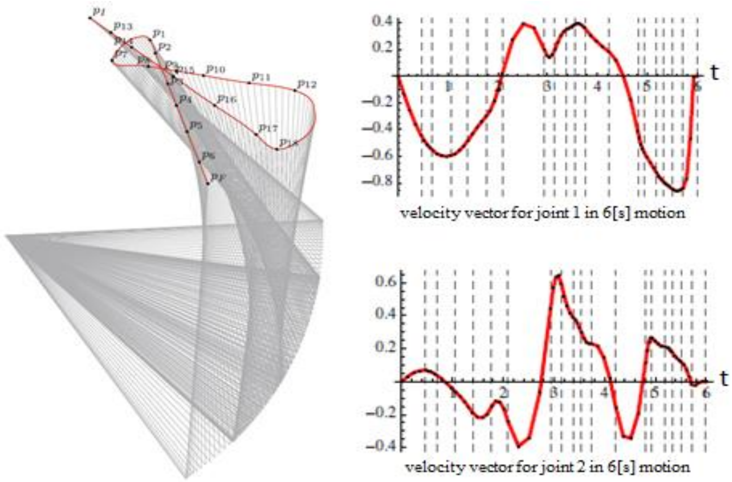

Fig. 1. Optimal trajectory and control variables within 18 waypoints.

2.2 Robot dynamic model

The *RR* robot is a paradigmatic model which appears in most robotics textbooks [4]. And a *RR* robot qualitatively corresponds to the model of the first two links of a SCARA (Selective Compliant Assembly Robot Arm) without taking into account the vertical one (see left hand side of Fig. 2). It is composed by two homogeneous links and two actuated joints moving in a horizontal plane $\{x, y\}$, as shown in right hand side of Fig. 2, where l_i is the length of link i, r_i is the distance between joint i and the mass center of link i, m_i is the mass of link i, and I_{zi} is the barycentric inertia with respect to a vertical axis z of link i, for i = 1, 2.

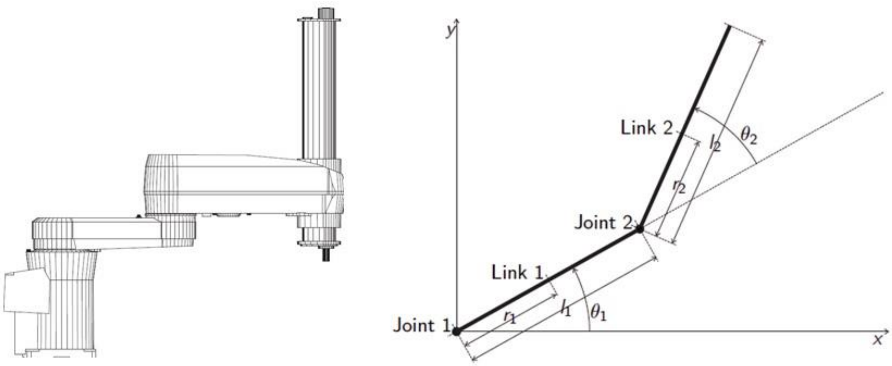

Fig. 2. A robot manipulator that moves in a horizontal plane

The dynamic model of this robotic system is represented by the second order differential equation

$$B(\theta)\ddot{\theta} + C(\theta, \dot{\theta})\dot{\theta} + F_v\dot{\theta} = \tau \;, \quad (1)$$

where the matrices $B(\theta)$, $C(\theta, \dot{\theta})$ and F_v have the following expressions:

$$B(\theta) := \begin{bmatrix} \alpha + 2\beta\cos(\theta_2) & \delta + \beta\cos(\theta_2) \\ \delta + \beta\cos(\theta_2) & \delta \end{bmatrix}, \quad (2)$$

$$B(\theta) := \begin{bmatrix} -\beta\, sin(\theta_2)\dot{\theta}_2 & -\beta\, sin(\theta_2)(\dot{\theta}_1 + \dot{\theta}_2) \\ \beta\, sin(\theta_2)\dot{\theta}_1 & 0 \end{bmatrix}, \quad (3)$$

$$B(\theta) := \begin{bmatrix} f_{v1} & 0 \\ 0 & f_{v1} \end{bmatrix}. \quad (4)$$

The first term of (1) represents the inertia joint forces, the second term represents the Coriolis and centrifugal joint forces, the third term represents the viscous friction joint forces, and τ, in the right-hand side, represents the forces applied at the joints. Since the robot manipulator moves in a horizontal plane, the gravity term does not appear in the dynamic equation (1). Vector θ = $(\theta_1, \theta_2)^T$ is the vector of configuration variables, where θ_1 is the angular position of link 1 with respect to the x axis of the reference frame $\{x, y\}$ and θ_2 is the angular position of link 2 with respect to link 1, as illustrated in Fig. 2. Vector $\dot{\theta}$ = $(\dot{\theta}_1, \dot{\theta}_2)^T$ is the vector of angular velocities, where $\dot{\theta}_1$ and $\dot{\theta}_2$ are the angular velocities at joint 1 and joint 2, respectively. Vector $\ddot{\theta}$= $(\ddot{\theta}_1, \ddot{\theta}_2)^T$ is the vector of accelerations, where $\ddot{\theta}_1$ and $\ddot{\theta}_2$ are the accelerations at joint 1 and joint 2, respectively. Vector τ = $(\tau_1, \tau_2)^T$ is the vector of control inputs of the system, where τ_1 is the torque applied by the actuator at joint 1 and τ_2 is the torque applied by the actuator at joint 2.The parameters α, β and δ in (2) and (3) have the following expressions:

$\alpha := I_{z1} + I_{z2} + m_1r_1^2 + m_2(l_1^2+r_2^2)$, $\beta := m_2l_1r_2$, $\delta := I_{z2} + m_2r_2^2$.

Parameters f_{v1} and f_{v2} in (4) are the viscous friction coefficients.

In particular, the optimization problem will find the control inputs τ_1(t) and τ_2 (t), whose dynamic model is given by (1), that minimize the robot energy during its motion.

2.3 Optimal control problem

The problem formulation of the optimal control problem and the correspondent reformulation for the problem studied in [5]. The continuous optimal control problem is formulated as follows:

$$\text{Min } J[x(t), u(t), s] := E[x(t_F), s] + \int L[x(t), u(t), s]\, dt \quad (5)$$

subject to:

$$\dot{x}(t) = f[x(t), u(t), s], \quad t \in [t_I, t_F] \quad (6)$$

$$0 = g[x(t), u(t), s], \quad t \in [t_I, t_F] \quad (7)$$

$$0 \leq c[x(t), u(t), s], \quad t \in [t_I, t_F] \quad (8)$$

$$r_{ineq}[x(t1), x(t2), \ldots, x(t_{nrineq}), s] \leq 0 \quad (9)$$

$$r_{eq}[x(t1), x(t2), \ldots, x(t_{nreq}), s] = 0 \quad (10)$$

$$x(t_I) = x_I \quad (11)$$

$$\psi[x(t_F)] = 0 \quad (12)$$

The objective functional in (1) is given in Bolza form and it is expressed as the sum of the Mayer term, which is assumed to be twice differentiable, and the Lagrange term. Variable $t \in [t_I, t_F]$ represents time, where t_I and t_F are the initial and final time, respectively. *x(t)* represents the state variables within both, differential and algebraic variables and *u(t)* represents the control functions, also referred to as control inputs, which are assumed to be measurable. The vector s contains all the time-independent variables of the problem. Equation (6) and (7) represents a Differential Algebraic Equations (DAE) system. The function *f* is assumed to be piecewise Lipschitz continuous to ensure existence and uniqueness of a solution. The system must satisfy the algebraic path constraints c in (8) and the interior point inequality and equality constraints r_{ineq} and r_{eq} in constraints (9) and (10), respectively, which are assumed to be twice differentiable. Finally, x_I in (11) represents the vector of initial conditions given at the initial time t_I and the function ψ in (12) provides the terminal conditions at the final time t_F, which is assumed to be twice differentiable.

In addition, we can introduce integer variables to obtain a multi-phase problem, and therefore, the so-called mixed integer optimal control problem appears [6]. Moreover, we can reformulate the mixed-integer optimal control problem into a mixed integer nonlinear programming problem including these transformations [7]: i) making unknown passage times through the waypoints part of the state, ii) introducing binary variables to enforce the constraint of passing once through each waypoint, and iii) applying a fifth-degree Gauss-Lobatto direct collocation method [8] to tackle the dynamic constraints. High degree interpolation polynomials allow the number of variables of the problem to be reduced for a given numerical precision. Further details about the optimal control problem reformulation are in [5].

3. Problem resolution

We observe that the continuous path is strongly affected by the discrete path, i.e. the sequence of points. In particular, we experimentally checked that main energy consumption is due to changes in velocity which occur when there is a change in the direction of the motion. Therefore, the preferable sequence of waypoints for minimizing energy consumption will comprise straight paths within smooth turns.

On the other hand, the optimal control problem described in 2.3 will became a NLP problem with non-convex feasible region as far as we fix the sequence of waypoints. And the solution for this NLP problem is achieved in IPOPT solver [9] in a short time. Therefore we can take advantage of that property and look for promising discrete paths, which fix the sequence of waypoints as well as will avoid sharp turns, and obtain its continuous trajectories (i.e. continuous paths plus velocity profile along them) by solving its correspondent NLP problem.

In order to construct sequences of waypoints which include the minimum changes of direction as possible, we will look for straight segments and then join the segments and the isolated points. The flow chart for this algorithm is depicted in Fig. 3, and basically, this approach includes the procedures that follow:

1. Search possible segments in the workspace. In this procedure we consider each edge from the workspace and compute one by one the acute angle respect to all the other waypoints. In case we obtain 180º the waypoint will be included. We find a segment when there are at least two more waypoints included in the edge.

2. Use biased randomization. Once we obtain the list of segments, we sort that list using a skewed probability distribution. A skewed distribution is used here in order to assign higher probabilities of being in the top of the list by segments composed with a larger number of waypoints. In our case, a Geometric distribution with $\alpha = 0.25$ was employed to induce this biased-randomization behavior [10]. So, we always select the segment in the top to be the discrete path solution. Next, we update the list of segments, since the waypoints from the segment selected must be removed from the other segments. Then, sort the list and pick the one in the top again. Step 2 ends when the list of segments is empty.

3. Join segments and isolated waypoints. Once we have a "good" selection of segments to be in the discrete path, we compute Euclidean distance for all possible connections, i.e. ends of the segments and isolated waypoints. And we sort the list of possible connections using a biased-randomization of a geometric distribution with $\beta = 0.25$. So, we always join using the connection in the top of the Euclidean distance list. Next, we update the list of connections, since there would

be connections that must be removed after we have joined. Then, sort the list and join using the one in the top again. Step 3 ends when all the waypoints are connected in a discrete path.

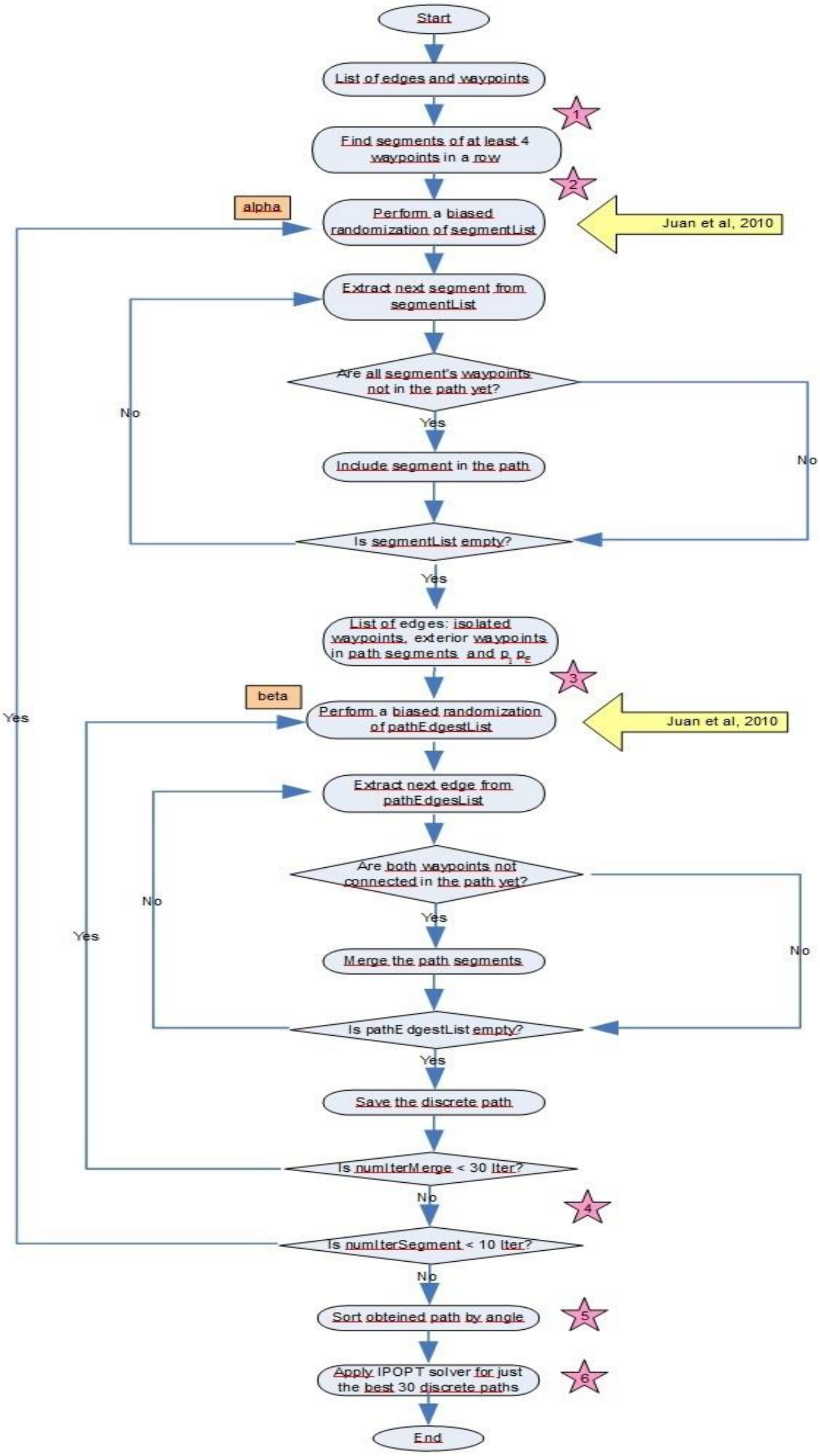

Fig. 3. Flow chart for the *RR* pseudo-optimal solution, i.e. continuous trajectory

4. Repeat step 2 and 3. After having studied the parameter analysis, the proper stopping criteria for this Multi-Start approach for *RR* problems with 12-18 waypoints, are maximum 10-15 iterations for step 2 and maximum 30 iterations for step 3. It means, for each group of selected lines in step 2, we will try 30 different connections in step 3. So, the Multi-Start approach will create 10-15 X 30 discrete paths, although in many cases we obtain a repeated one from previous iterations.

5. Sort discrete paths. We calculate the total acute angle in every distinguished discrete path. The total acute angle is the addition of each acute angle, i.e. acute angle of each 3 consecutive waypoints in the discrete path. And, we sort the discrete path from the lowest to the highest total angle computed.

6. Solution of the RR. We call IPOPT through the correspondent .nl file within the NLP problem formulation. Because the discrete path will be fixed according to the list of path, the IPOPT solver provides trajectories quickly. In *RR* problems with 12-18 waypoints, it will be enough checking the first 30 paths from the list.

4. Computational results

In this section, the results of several numerical experiments where the robot is constraint to pass through the first 12 waypoints and through all the 18 waypoints listed in Table 1, as well as, the initial point p_I and end point p_F specified in each instance.

p_1 = (0.455718, 0.660622)	p_2 = (0.472266, 0.616427)	p_3 = (0.510878, 0.513305)
p_4 = (0.538458, 0.439647)	p_5 = (0.571554, 0.351256)	p_6 = (0.610167, 0.248135)
p_7 = (0.335096, 0.591699)	p_8 = (0.450359, 0.571340)	p_9 = (0.536806, 0.556071)
p_{10} = (0.623253, 0.540801)	p_{11} = (0.767332, 0.515353)	p_{12} = (0.911410, 0.489904)
p_{13} = (0.331795, 0.685981)	p_{14} = (0.397159, 0.636696)	p_{15} = (0.527889, 0.538125)
p_{16} = (0.658618, 0.439554)	p_{17} = (0.789347, 0.340984)	p_{18} = (0.854711, 0.291699)

Table 2. Coordinates of the waypoints used in the experiments LAT

In Table 2 we report the results obtained while using the biased randomization in the *RR* problem. It has been implemented in a C++ code which generates a set of good discrete path where those with minimum total angle are evaluated in IPOPT optimization engine. For each instance we indicate the correspondent case and the number of waypoints considered. In all the experiments we use the initial time t_I = 0 [s] and the final times t_F = 4 [s] and t_F = 6 [s] for instances of 12 and 18 waypoints, respectively. Notice that instances with cases A, B and C differ in the location of the final point p_F, which is specified in each instance case.

Instance	initial point and final point sequence of waypoints from Multi-Start
Ins-12A	p_I = (1.0843, 0.459365) p_F = (0.637747, 0.174476) (p_I, p_{12}, p_{11}, p_{10}, p_9, p_8, p_7, p_1, p_2, p_3, p_4, p_5, p_6, p_F)
Ins-12B	p_I = (1.0843, 0.459365) p_F = (0.2027, 0.6151) (p_I, p_{12}, p_{11}, p_1, p_2, p_3, p_4, p_5, p_6, p_{10}, p_9, p_8, p_7, p_F)
Ins-18A	p_I = (0.2664, 0.7353) p_F = (0.637747, 0.174476) (p_I, p_{13}, p_{14}, p_{15}, p_{16}, p_{17}, p_{18}, p_{12}, p_{11}, p_{10}, p_9, p_8, p_7, p_1,p_2, p_3,p_4, p_5,p_6, p_F)
Ins-18B	p_I = (0.2664, 0.7353) p_F = (0.933149, 0.232556) (p_I, p_{13}, p_7, p_9, p_{10}, p_{11}, p_{12}, p_6, p_5, p_4, p_3, p_2, p_1, p_{14}, p_8, p_{15},p_{16}, p_{17},p_{18}, p_F)
Ins-18C	p_I = (0.2664, 0.7353) p_F = (1.0843, 0.459365) (p_I, p_{13}, p_{14}, p_{15}, p_{16}, p_{17}, p_{18}, p_6, p_5, p_4, p_3, p_2, p_1, p_7, p_8, p_9,p_{10}, p_{11},p_{12}, p_F)

Table 2. Multi-Start approach pseudo-optimal discrete paths

Finally, Fig. 4 shows the 12 and 18 waypoints configurations both in case B. Also, depicts the continuous path of minimum energy consumption in couple of instances, according to our Multi-Start approach.

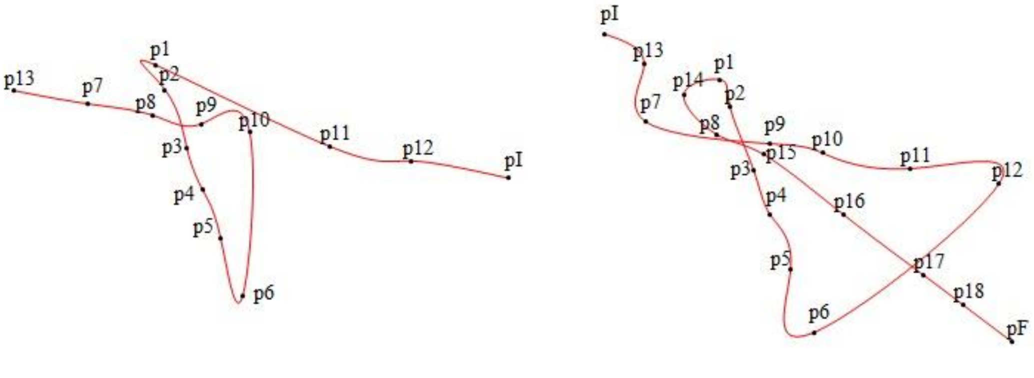

Fig. 4. Continuous path for instances Ins-12B and Ins-18B using biased randomization

Conclusion

In this paper we studied the motion planning problems for planar robot manipulator with two revolute joints whose end-effector is constrained to pass through a set of points whose sequence is not predefined. One of the main difficulties of the problem is to ensure that the sequence of discrete configurations that have to be interpolated, although geometrically feasible, do also result dynamically feasible according to physical limitations of the actuators. One of the crucial features of the resolution method is the use of a fifth-degree direct collocation method to tackle the dynamic constraints and another is the search of promising discrete paths with a Multi-Start algorithm. Therefore, we apply biased randomization to obtain a set of good discrete paths and we use the NLP problem to be displayed in IPOPT solver to obtain the dynamicity of the trajectory. And, since the discrete path is already fixed in the NLP problem and selected as good candidate, the numerical experiments in terms of computational times are much affordable than if no discrimination of sequences were preceded in the use of IPOPT solver.

Acknowledgment

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An Adaptive SA-based Simheuristic for the Stochastic Portfolio Optimization Problem

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Abstract

Combinatorial optimization has been at the heart of financial and risk management. This body of research is dominated by the mean-variance efficient frontier (MVEF) that solves the portfolio optimization problem (POP), pioneered by Harry Markowitz. The classical version of the POP minimizes risk for a given expected return on a portfolio of assets by setting the weights of those assets. Most authors deal with the variability of returns and covariances by employing expected values. In contrast, we propose a simheuristic methodology (combining the simulated annealing metaheuristic with Monte Carlo simulation), in which returns and covariances are modeled as random variables following specific probability distributions. Our methodology assumes that the best solution for a scenario with constant expected values may have poor performance in a dynamic world. A computational experiment is carried out to illustrate our approach.

Key words

Portfolio optimization, Simheuristics, Metaheuristics, Adaptive simulated annealing

1. Introduction

Investments play an essential role in our society through wealth creation, sustainable economic growth and ultimately improvements in welfare standards. In this context the portfolio optimization problem (POP) can be described as a strategy of *a)* selection of financial assets and *b)* determination of the optimal weights allocated to those assets that result in a desired portfolio return and an associated minimum level of risk. This combinatorial optimization problem (COP) constitutes a milestone of modern portfolio theory, founded by Harry Markowitz [1]. It consists of a quadratic objective function that is *a)* computed by aggregating over covariances of asset returns, and *b)* minimized subject to a desired rate of return. It is worth noting that other risk measures have been applied in the literature such as value-at-risk or semi-variance (measuring downside risk exposure). Additionally, portfolio weights must add up to one and take on non-negative values. A realistic POP introduces further constraints such as: pre-assignment, quantity and cardinality constraints. The pre-assignment constraint allows the investor to pre-select certain assets. The quantity constraint confines the weight allocated to an asset within a desired range of values. While the upper limit attempts to reduce the exposure to each asset, the lower rules out investments in negligible quantities, which may be prohibitively costly because of the transaction costs. Finally, the cardinality constraint sets a minimum and maximum number of assets that are included in a portfolio. The lower bound seeks to alleviate risk exposure by allocating wealth within a sensible subset of assets. The upper bound is dictated by the evidence that marginal benefits of diversification starts to decrease after the number of assets already selected hits a certain threshold [2]. Moreover, portfolios with a large number of assets are more costly in terms of complexity, managerial effort and transaction costs. These constraints make the problem NP-hard [3]. As a consequence, metaheuristics are commonly employed.

A textbook version of the POP builds on the empirically unsupported assumption that expected returns and covariances are constant – a key limitation in a large and growing body of research. Our main contribution is to address this limitation by modeling expected returns and covariances as random variables. The resulting problem is referred to as the *stochastic* POP (SPOP). We propose a simheuristic approach [4] (i.e., combination between metaheuristics and simulation) to address the realistic SPOP. This approach integrates Monte Carlo simulation (MCS) into an adaptive simulated annealing (SA) metaheuristic [5]. Basically, while the metaheuristic searches for promising solutions, simulation techniques are applied to assess their quality.

2. Description of the Problem

Let assume there is a set $A = \{a_1, a_2, \dots, a_n\}$ of n assets, where each asset a_i ($\forall i \in \{1,2,\dots,n\}$) has associated an expected return r_i. The covariance between two assets a_i and a_j ($\forall i,j \in \{1,2,\dots,n\}$) is represented by σ_{ij}. A solution is described by the vector $X = (x_1, x_2, \dots, x_n)$, where each element x_i

$(0 \leq x_i \leq 1)$ is the fraction of wealth invested in asset a_i. The aim of the POP is to minimize the portfolio risk while obtaining an expected return greater or equal to an investor-given threshold R. A rich version considers pre-assignment, quantity, and cardinality constraints. Pre-assignment constraints imply that assets pre-selected by the investor have to be included in the solution before any other assets are considered. The parameter p_i indicates whether an asset has been pre-selected ($p_i = 1$) or not ($p_i = 0$). The quantity constraint specifies a lower and an upper bound for the weights x_i, ε_i and δ_i $(0 \leq \varepsilon_i \leq \delta_i \leq 1)$, respectively. The cardinality constraint defines the minimum and maximum number of assets, k_{min} and k_{max} $(1 \leq k_{min} \leq k_{max} \leq n)$.

The stochastic version of the problem differs in the modeling of asset returns and covariances. Whereas they are represented by expected values in the deterministic version, random variables are simulated in the stochastic version.

The realistic POP can be modeled as follows:

$$\min f(X) = \sum_{i=1}^{n} \sum_{j=1}^{n} \sigma_{ij} x_i x_j \quad (1)$$

Subject to:

$$\sum_{i=1}^{n} r_i x_i \geq R \quad (2)$$

$$\sum_{i=1}^{n} x_i = 1 \quad (3)$$

$$\varepsilon_i z_i \leq x_i \leq \delta_i z_i, \quad \forall i \in \{1,2,\dots,n\} \quad (4)$$

$$0 \leq \varepsilon_i \leq \delta_i \leq 1, \quad \forall i \in \{1,2,\dots,n\}$$

(5)

$$z_i \leq M x_i, \quad \forall i \in \{1,2,\dots,n\} \quad (6)$$

$$p_i \leq z_i, \quad \forall i \in \{1,2,\dots,n\} \quad (7)$$

$$k_{min} \leq \sum_{i=1}^{n} z_i \leq k_{max} \quad (8)$$

$$z_i \in \{0,1\}, \quad \forall i \in \{1,2,\dots,n\} \quad (9)$$

The objective function (Equation (1)) quantifies the riskiness of the investment. Equation (2) ensures that the expected return greater or equal to the threshold return R. Equation (3) restrains investment to available resources. Further, an auxiliary variable is introduced to indicate whether the asset a_i is included in the solution ($z_i = 1$; $z_i = 0$ otherwise). For each asset a_i, Equation (4) sets a lower and an upper bound for x_i (ε_i and δ_i, respectively) in case a_i is selected (i.e., $z_i = 1$). These bounds range between zero and one inclusively (Equation 5). In Equation (6), M is a large positive value such that $Mx_i \geq 1$ for each asset a_i if $x_i > 0$. Equation (7) defines the pre-assignment constraint, where z_i depends on the parameter p_i. If the asset a_i is pre-selected ($p_i = 1$), it must be included in the solution (i.e., $z_i = 1$). Equation (8) describes the cardinality constraint. Finally, Equation (9) defines z_i as a binary variable.

Two modifications are applied to describe the realistic SPOP:

- Covariances (C_{ij}) in the objective function are assumed random following a certain probability distribution:

$$f(X) = \sum_{i=1}^{n} \sum_{j=1}^{n} C_{ij} x_i x_j \quad (10)$$

- Equation (2) is replaced by the following probabilistic constraint:

$$P\left(\sum_{i=1}^{n} R_i x_i \geq R\right) \geq P_0 \quad (11)$$

where R_i refers to the asset return modeled as a random variable. Equation (11) guarantees that the portfolio return will be no lower than the threshold R with a probability of, at least, P_0.

3. Related Work

The POP has been intensively studied for more than 6 decades, gaining popularity among both academics and practitioners. Traditionally, it has been addressed with exact methods (such as linear [6] and quadratic [7] programming methods) to determine optimal solutions. However, this approach usually requires making strong assumptions and simplifying formulations to render it solvable. Heuristics, used since the 1960s, tend to provide near-optimal solutions to complex COPs. Metaheuristics are an attractive alternative, since they are conceptually simple, relatively easy to implement and fast. A comprehensive and updated literature review on this problem may be found in [8].

By contrast the SPOP has been much less studied, but several approaches have been put forward. For instance, [9] applies uncertainty theory, relying on expert opinions to describe the returns. The author proposes a genetic algorithm for solving the mean-variance and mean-semivariance models. A different approach is suggested in [10], where a distinction is made between random and uncertain returns. The former are associated to securities for which ample historical information is available, while in the latter there is a lack of historical information. Several POPs are addressed in [11] assuming that returns behave as normal, rectangular and trapezoidal uncertain variables. As a result, they can be solved employing linear programming methods. In [12] the authors introduce an additional constraint based on triangular entropy to control the level of uncertainty. In the context of fuzzy theory, [13] model returns as random variables and formulate several POPs in hybrid uncertain decision systems. They present a solving methodology combining MCS and a particle swarm optimization metaheuristic. [14] also focus on fuzzy modeling and build a mean-semivariance-entropy approach. Their approach combines a genetic algorithm and a SA metaheuristic. The entropy measure for diversification is also employed in a mean-variance entropy approach by [15]. The author employs uncertainty theory and solves the model with an artificial bee colony metaheuristic.

4. Our Methodology

We propose a simheuristic methodology for addressing the realistic SPOP. Simheuristics constitute a natural yet powerful approach that contributes to reduce the gap in the literature regarding methodologies for addressing stochastic COPs. It proposes an extension of state-of the-art metaheuristics by introducing simulation techniques. While the metaheuristic searches promising solutions, simulation techniques are employed to assess their performance (checking also their feasibility if required). Our methodology combines a SA metaheuristic with MCS techniques. SA is a well-established metaheuristic employed in discrete and continuous optimization. It is inspired by the process of physical annealing with solids in which a crystalline solid is heated and then allowed to cool slowly until it achieves its most regular possible crystal lattice configuration, without crystal defects. A complete overview may be found in [16]. We employ the adaptive SA metaheuristic [17], which integrates a procedure to automatically adapt the temperature according to the experience. Our methodology is summarized in Fig. 1 and described next.

The algorithm has 4 parameters: t (minimum temperature), λ (rate of temperature rise), M (number of iterations) and β (parameter in solution building). First, the stochastic instance is transformed into a deterministic one by replacing random variables by their means. Second, an initial solution is built by means of the algorithm described in [18] (which uses β). It constructs a solution by combining the pre-assigned assets with high-return assets. The solution has to be feasible in a stochastic environment, i.e., the required return (R) has to be reached with a probability no smaller than P_0. MCS is employed to estimate this probability by means of the proportion of the cases in a sample of generated scenarios where the return obtained is at least as high as R and also to compute the expected risk as a sample mean of all risks. Each scenario is created by randomly drawing a value for each return in the original instance. Third, copies of the initial solution are stored as base solution and in a list of best stochastic solutions (i.e., those with the lowest expected risk). Afterwards, counters k and m are set to 0. While m is below M, the following steps are taken: *(i)* a new solution is selected from the neighborhood of the base solution; *(ii)* if the risk (using the modified instance) of the new solution is lower, the base solution and the list are updated (which requires estimating the risk in the stochastic environment), otherwise the base solution is also replaced with a probability that depends on the temperature (the higher the more likely); *(iii)* m is increased by one. Later, the stopping criteria (based on the time elapsed) is checked. If it is not satisfied, k is incremented by one (which leads to a temperature change) and m is reset to 0; otherwise a risk analysis is performed on the solutions in the list and the outputs are reported to a decision-maker. This analysis consists in obtaining more accurate estimates of the expected risk of the solutions by using a larger number of simulation scenarios and obtaining not only the expected value but also the variance, so that the decision-maker can choose the 'best' solution according to his/her risk aversion.

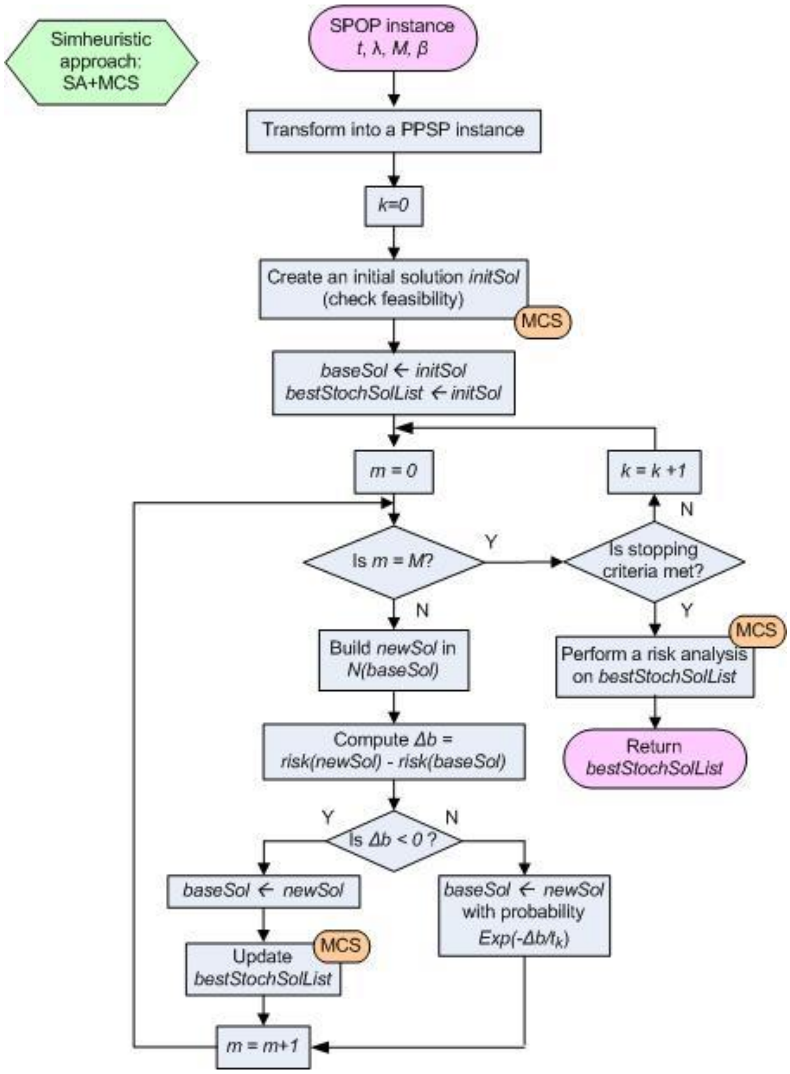

Fig.1. Flowchart of the proposed methodology.

5. Computational Experiment

Our methodology has been implemented as a Java application. A standard personal computer, Intel Core i5 CPU at 3.2 GHz and 4 GB RAM with Windows 7 has been employed. We have experimented with a stock market database from the repository ORlib (http://people.brunel.ac.uk/~mastjjb/jeb/orlib/portinfo.html), which was proposed in [19]. It represents the market index Hang Seng (Hong Kong) measured at weekly frequency spanning the period from March 1992 to September 1997. This instance gathers expected returns r_i , standard deviations σ_i and correlations ρ_{ij}. In order to assess our simheuristic methodology, the following variables are considered:

1. S_i (standard deviation) follows a $LN(\mu_S, \sigma_S)$, where LN represents a log-normal distribution, and μ_S and σ_S are the mean and the standard deviation of the variable natural logarithm, respectively. They are set to σ_i and $c \cdot \sigma_i$. c is assumed to take on three values; 0.01, 0.025, and 0.08.

2. P_{ij} (correlation) follows a $TN(\mu_P, \sigma_P, l, u)$, referring TN to truncated normal distribution, where the parameters are the mean, the standard deviation, and the lower and upper limit, respectively. μ_P is set to the original correlation ρ_{ij}. By definition, l and u are set to -1 and 1, respectively. The values of σ_P tested are: $\sqrt{0.00002}$, $\sqrt{0.0002}$, and $\sqrt{0.002}$.

3. R_i follows a $N(\mu_R, \sigma_R)$, where μ_R and σ_R are the mean and the standard deviation of the variable, and are set to r_i and S_i, respectively.

Our algorithm is executed 10 times using different seeds; only the best results are shown. Each execution is run for 20 seconds. Each combination of values has been tested using just one seed to tune parameters. Those values providing better results are employed. The ranges tested are: *β~U(a, b), (a,b)* $\in \{(0.1, 0.2), (0.2, 0.3), (0.3, 0.4)\}$; $t \in \{0.00000070, 0.00000075, 0.00000080\}$; $\lambda \in \{0.00000095, 0.00000100, 0.00000105\}$; and $M \in \{4,6,8\}$. The resulting values were: *β~U(0.3 0.4);* $t = 0.00000075$; $\lambda = 0.00000100$; and $M = 4$.

Table 1 and Fig. 2 summarize the results obtained. Table 1 compares the solutions provided by our algorithm with $P_0 = 0.40$ and $P_0 = 0.50$, considering 10 required returns (5 low and 5 high) and a scenario of a medium level of stochasticity (i.e., the second value tested of c and σ_P). Our results indicate that the higher the probability we require, the higher the risk for a given return. Fig. 2 shows the gaps of expected risks obtained between the best stochastic (i.e., considering random variables) and deterministic (i.e., with constant expected values) solutions found for $P_0 = 0.50$. As expected, the best stochastic solution Pareto dominates the best deterministic solution. Moreover, the higher the level of stochasticity, the larger is the difference between the best stochastic and deterministic solutions.

Table 1. Comparison of results for $P_0 = 0.40$ and $P_0 = 0.50$, considering a scenario of a medium level of stochasticity.

Required return	Exp. risk P_0: 0.40 (1)	Exp. risk P_0: 0.50 (2)	Gap (2)-(1) (%)
0.002861	0.0007872	0.0007873	0.015%
0.002942	0.0007873	0.0007873	0.000%
0.003023	0.0007882	0.0007890	0.094%
0.003104	0.0007899	0.0007934	0.438%
0.003185	0.0007913	0.0007921	0.101%
0.010542	0.0055269	0.0055803	0.965%
0.010622	0.0057234	0.0057805	0.996%
0.010703	0.0059276	0.0059884	1.024%
0.010784	0.0061395	0.0062040	1.050%
0.010865	0.0063331	0.0063772	0.696%

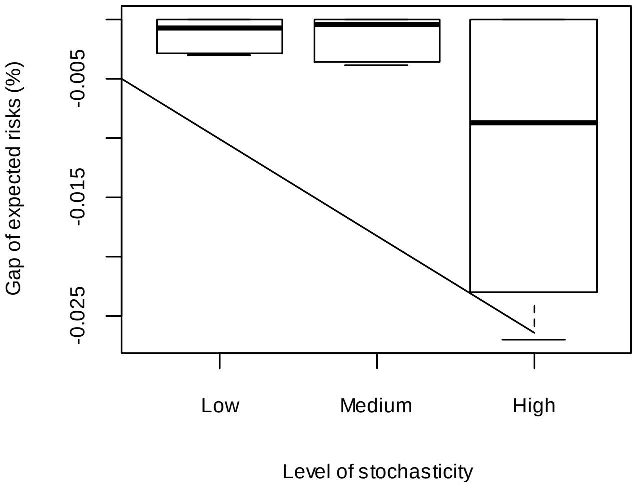

Fig.2. Multiple boxplot of gaps of expected risks between stochastic and deterministic solutions for each level of stochasticity.

6. Conclusions

This work solves the portfolio optimization problem (POP), which is a well-known combinatorial optimization problem with a wide range of applications. It consists of creating a portfolio of assets and setting their weights. We present a mathematical formulation for a rich POP with the pre-assignment constraints (based on investor's preference), quantity constraints (which keep each weight within user-specified floor and ceiling values) and cardinality constraints (providing a minimum and maximum value for the number of assets). Whereas researchers typically model covariances and expected returns with constant values, in our research they are assumed random. A NP-hard problem requires an approximate methodology for solving medium/high-sized instances in real time. Accordingly, we present a simheuristic algorithm. It combines an adaptive simulated annealing metaheuristic, which guides the search, with Monte Carlo simulation techniques, which check the feasibility and assess the goodness of promising solutions. A computational experiment is performed to illustrate its use and analyze how the solutions vary when changing several factors.

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Trigger Error in Parametric Earthquake Catastrophe Bonds: Minimization via Statistical Approaches

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Abstract

The losses inducted by earthquakes have led to the necessity of the insurance and reinsurance industry. This industry employs catastrophe (CAT) bonds to obtain coverage, which are financial instruments designed to transfer catastrophic risks to the capital markets. When an event occurs, a Post-Event Loss Calculation (PELC) process is initiated to determine the losses to the bond and the subsequent recoveries for the bond sponsor. Given certain event parameters such as magnitude of the earthquake and the location of its epicenter, the CAT bond may pay a fixed amount or not pay at all. This paper analyses three effective, simple to interpret and simple to implement statistical techniques for classification of events in order to identify which should trigger bond payments based on a large sample of simulated earthquakes. Numerical experiments are performed to illustrate their use, and to facilitate a comparison with a previously published evolutionary algorithm.

Key words

Catastrophe Bonds, Risk of Natural Hazards, Classification Techniques, Earthquakes, Insurance.

1. Introduction

Strategies to provide coverage for large losses ensuing after earthquakes through parametric CAT bonds have been implemented since the 1990s. These financial instruments allow insurers, reinsurers, governments, private entities and catastrophe pools to cede risks of losses to the capital markets via a transparent mechanism that determines payments based on certain quantifiable event features. These instruments bypass the claims adjusting process and therefore can provide a very fast recovery of funds to their sponsor after an event. The principal of the bond can also be collateralized thus reducing default risk, and most importantly, their price has been relatively competitive versus traditional premiums. In addition, the capital markets offer great capacity, although capacity has not been in short supply in the (re)insurance market lately.

Parametric CAT bonds employ triggers (or algorithms) to determine the payment that should take place when an earthquake occurs. These triggers rely on obtainable physical characteristics of the event [1, 2] and since no parties can manipulate this information, the risk transfer mechanism is transparent and reduces moral hazard (the risk that the parties involved can influence the payment outcome).

Earthquakes around the world cause enormous losses, which are rarely insured. These financial impacts end up distorting people's livelihoods and national economies severely. Parametric instruments have the potential to reduce this problem by making earthquake insurance more accessible. Therefore, this work focuses on improving the algorithms that can increase the quality of these transactions.

Our main objective is to explore three statistical techniques to automatically construct accurate triggers, i.e. triggers that induce payment when they should and that do not induce payment when they should not. Computational experiments are performed to analyze their performance, mainly in terms of accuracy and time required. In addition, our approach is compared with the metaheuristic-based one described in [5].

The remainder of this paper is organized as follows: Section 2 is devoted to describe the trigger mechanism in detail. Section 3 introduces three statistical techniques that can be applied. Afterwards, the experiments are performed in Section 4. Finally, Section 5 summarizes the main highlights of this paper.

2. The Trigger Mechanism

Consider a set of l earthquake events in a geographic region of interest A. An earthquake event i is characterized by a magnitude m_i, a hypocenter depth d_i, and epicenter coordinates (x_i, y_i) within A. A binary trigger will determine whether a payment is associated with event i. This response is represented by the variable B', whose values 1/0 indicate trigger/no-trigger (payment/no-payment). Two situations may arise: (a) at least one earthquake i triggers the bond ($B'_i = 1$) during its contract life, which means that the entire bond principal has to be disbursed and, as a consequence, the buyers of the bond lose their investment (the bond sponsors receive compensation), and (b) no earthquake triggers the bond during its life, in which case the principal is returned to the investors with interest.

Since the payment of a large sum of money is at stake, it is important that the trigger performs as desired, that is that the trigger responds positively to events that cause a large loss beyond a design threshold and that it does not respond for events that cause a loss below this threshold. The accuracy of the trigger determines its success in the market. Triggers that behave erratically erode the confidence of the markets in these tools and therefore jeopardize the risk transfer process. Hence the importance of designing accurate triggers that behave as they should.

To describe the accuracy of the trigger, first consider a reference variable *B* that represents the idealized behavior of the trigger, which depends on a measure based on the losses (typically monetary). For an earthquake event *i*, this variable can be described as follows:

$$B_i = \begin{cases} 0, & \text{if } L_i < L \\ 1, & \text{otherwise,} \end{cases} \tag{1}$$

where L_i is the actual loss caused and L is a loss threshold specified by the sponsor, usually expressed in terms of a specific return period. Events trigger this CAT bond only if the corresponding loss is above a given pre-specified threshold L.

The objective is to develop a mechanism that minimizes discrepancies between variables B and B' or the sum of errors , $(E = \sum_{i=1}^{l} I(B_i = B_i'))$ or in other words, the lack of correlation between the output of the trigger and the ideal trigger. A database including a set of events, their characteristics and the variable B for each event can be used to calculate trigger errors for this specific set of events. A measure of the loss has to be obtained or estimated to compute B.

Note that the physical characteristics suggested above do not constitute the only options to design a parametric trigger. However, the fact that they tend to be easily available from respected third parties makes them suitable for this purpose.

According to the description offered in this section, the development of a trigger mechanism can be labeled as a *binary classification problem*, allowing us to employ a wide range of techniques to address it. In the following sections, some of them are introduced and tested, and their use is illustrated.

3. Our Approaches

Classification techniques [7] constitute a set of procedures from statistics and machine learning (more specifically, supervised learning) to determine a category or class for a given observation. Having a dataset of *l* observations composed of independent or explanatory variables (X_1, X_2,…, X_n), and a dependent or response variable *Y*, these techniques attempt to explain the relationships between the variables and/or classify new observations based on the values of the explanatory variables.

Nowadays, there are plenty of classification techniques. Some of the most employed, e.g. Linear Discriminant Analysis or Logistic Regression, have been applied for more than five decades. These are mainly linear methods. Boosted by the computing advances in the 1980s and 1990s, non-linear methods such as Classification Trees, Neural Networks and Support Vector Machines emerged and/or started to attract attention more recently.

Tree well-known and powerful techniques to automatically design a trigger are presented here. The first and second one are Logistic Regression and Clusterwise Logistic Regression, which belong to the most typical linear methods, while the third one is Classification Trees, which belongs to the non-linear methods. The reader interested in more extensive comprehensive and practical descriptions is referred to [6] and [8].

- **Logistic Regression** techniques are designed to model the posterior probabilities of each class by means of linear functions. These probabilities must be non-negative and sum to one. Logistic Regression is especially useful when the aim is to be able to explain (i.e., not only classify) the outcome based on the explanatory variables. Non-linear functions can be considered including interactions and transformations of the original variables.
- **Clusterwise Logistic Regression** aims to combine two techniques in order to discover trends within data when more than one trend is likely to exist [3]. While Regression Analysis consists of fitting functions to analyze the relationship between variables, Clustering seeks subsets of similar observations (or variables) in a dataset. Clusterwise Logistic Regression is highly flexible because different functions can be estimated. This technique is considered a "white-box technique" in that its mathematical systems are not complex and its results are relatively easy-to-interpret.
- **Classification Trees**, contrary to global models (where a predictive formula is supposed to hold in the entire data space), tries to partition the data space into small enough parts where a simple model can be applied. The results can be represented as a tree composed of internal and terminal (or leaf) nodes, and branches. Its non-leaf part is a procedure to determine for each observation which model (i.e., terminal node) will be used to classify

it. At each internal node of the tree, the value of one explanatory variable is checked and, depending on the binary answer, the procedure continues to the left or to the right sub-branch. A classification is made when a leaf is reached.

The most relevant advantage of this classifier is the easiness to understand what trees represent. They seem more closely mirror human decision-making than other techniques. Furthermore, trees require little data preparation, are able to handle both numerical and categorical data, and perform well (i.e., use standard computing resources in reasonable time) with large datasets.

Although most researchers focus on accuracy, in real-life applications many other characteristics may play an important role when comparing the solutions given by these techniques. Examples are popularity, easiness-to-implement or to-explain to non-experts, and existence of graphical representations or summaries of the outputs, among many others. Even assuming we are only interested in the accuracy, the best technique will depend on the data at hand. Consequently, we can only present a general discussion about the performance of these techniques.

Logistic Regression, as a regression approach, is a well-established technique, which enables the understanding of the effects of the explanatory variables on the response. Clusterwise Logistic Regression aims to incorporate the strengths of Logistic Regression while offering more flexibility, which should lead to a better understanding of the relationships among variables and higher accuracy. Classification Trees constitute an efficient technique that only uses the most important variables, and results in a logic model. As other techniques studying non-linear relationships, they may easily overfit or underfit the model. Moreover, small changes in training data may lead to significant modifications. In addition, they may derive decisions that seem counterintuitive or are unexpected.

In conclusion, each technique has different characteristics that should be considered when addressing a classification problem.

4. Computational Experiments

This section illustrates the use of the techniques introduced, and compares the results with those obtained with the methodology proposed in [5]. The dataset analyzed is an earthquake catalog representing a sample of 10,000 years of seismicity, a total of 24,957 earthquakes, concerning Costa Rica. A more detailed description can be found in the aforementioned work.

As commented in Section 2, the construction of a trigger is driven by the minimization of the discrepancies between its outputs and those from a trigger with an idealized behavior. If the resulting trigger is expected to be useful for new or unseen observations, it is good to avoid employing the same observations for developing the trigger and assessing its performance. This could lead to a problem of overfitting (i.e., obtaining complex models that capture specificities of the data but do not generalize well for other observations). An effective and efficient technique to avoid this problem is to split the dataset into three different subsets: a training set used for constructing the triggers, a validation set employed to tune the parameters, and a test set required to assess their performance. We apply this approach considering the following weights, since they are the most typical used values: 50%, 25%, and 25%, respectively.

The statistical experiment has been performed with R (version 2.15.0) [9], a freely available and widely used language and environment for statistical computing and graphics. A confidence level of 0.95 is considered.

For each classifier, the corresponding confusion matrix (Table 1) is obtained, which summarizes the results. This matrix shows the number of times that a predicted value B' coincides or not with the real value B. Therefore, it is desirable that cells TP and TN contain high values (as high as the number of real triggers of each type), and cells FP and FN contain low values. The measured accuracy is employed to make comparisons. In addition, the sensitivity, the specificity, and the time required are reported. Both false positive and false negative are equally penalized, i.e. we only focus on minimizing the total number of errors.

Table 3. Structure of a confusion matrix

		Predicted Class	
		B' = 0	*B' = 1*
Actual Class	*B = 0*	True Positive (TP)	False Positive (FP)
	B = 1	False Negative (FN)	True Negative (TN)

4.1 Logistic Regression

Initially, a complete model including all variables and the interactions among pairs is built. Variables *xc* and *yc* correspond to the coordinates where the earthquake happens. An approach based on backward elimination has been implemented to select the final model. Accordingly, the probability of having to pay given the data observed for a new event is:

$$P(Y = 1 | M = m \cap D = d \cap XC = xc \cap YC = yc)$$
$$= \frac{e^{-2954.62 + 13.74m - 0.06d - 36.92xc + 271.89yc + 0.09m\cdot d + 0.50m\cdot x + 3.04m\cdot y - 0.07d\cdot y + 3.43x\cdot y}}{1 - 2954.62 + 13.74m - 0.06d - 36.92xc + 271.89yc + 0.09m\cdot d + 0.50m\cdot x + 3.04m\cdot y - 0.07d\cdot y + 3.43x\cdot y} \quad (2)$$

If this probability is above 0.5, the event is classified in the group 1; otherwise, it is assigned to group 0. Table 2 shows the confusion matrix, where we can see good values for True Positives and Negatives, as desirable, but we also have 26 times in which we obtain a false negative.

Table 2. Confusion matrix

	B' = 0	*B' = 1*
B = 0	6210	2
B = 1	26	1

4.2 Clusterwise Logistic Regression

Table 3 shows the confusion matrix for this technique, where we can see very good values for True Positives and Negatives, as desirable, but we also have 27 times in which we obtain a false negative.

Table 3. Confusion matrix

	B' = 0	*B' = 1*
B = 0	6212	0
B = 1	27	0

4.3 Classification Tree

In order to construct a Classification Tree, it is needed to specify the Complexity Parameter (a parameter to measure the tree cost-complexity). Values from 0.01 up to 0.20 have been tested. The best results correspond to the value 0.05.

Figure 1 shows the tree representation. The observations that satisfy the condition shown for each internal node go to the left, otherwise, they go to the right. The percentage shown at the bottom of each node indicates the proportion of observations that reach that node. The value above that percentage refers to the classification of the corresponding observations. Table 4 depicts the confusion matrix.

Table 4. Confusion matrix

	B' = 0	*B' = 1*
B = 0	6210	2
B = 1	13	14

4.4 Comparative Analysis

In order to validate the application of these techniques for the development of triggers for earthquake catastrophe bonds, we compare our results with those provided in [4]. In the paper, the author proposes the construction of binary "cat-in-a-box" trigger mechanisms, where the geographical space is discretized in square boxes or sub-regions of the same size. Each sub-region belongs to a specific zone denoted as k. This constitutes a relatively simple and popular approach, where the aim is to set the parameters of a trigger mechanism for each zone by mini-

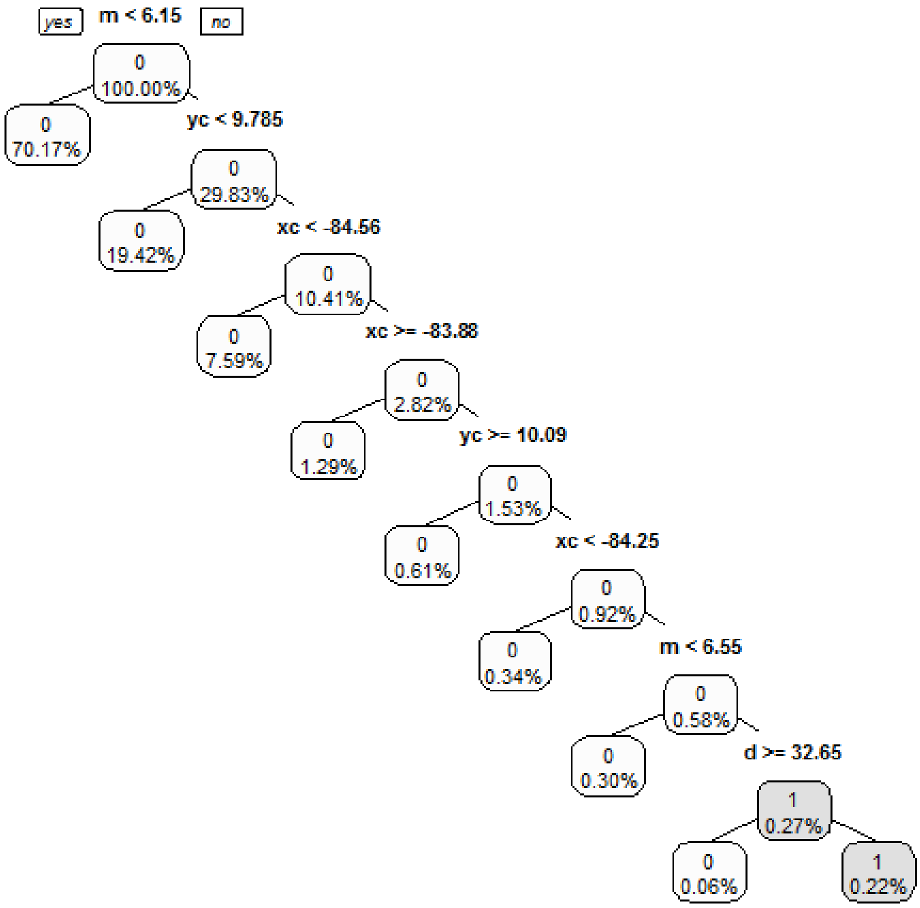

Figure 1 Classification Tree

mizing the trigger error. Concretely, the trigger mechanism has the following structure:

$$\forall (x_i, y_i) \in A_k, \qquad B_i' = \begin{cases} 0, & \text{If } m_i < M_k \quad \text{or} \quad d_i > D_k \\ 1, & \text{If } m_i \geq M_k \quad \text{or} \quad d_i \leq D_k \end{cases} \tag{3}$$

where M_k and D_k are the parameters to set and represent the magnitude and depth thresholds, respectively. An Evolutionary Algorithm (EA) is implemented to address this problem and is executed for different combinations of box side lengths and number of zones. Although the paper does not report computational times, it is clear that the authors dedicated several hours to develop this ad-hoc proposal and to perform the parameter fine-tuning.

In order to compare our results with those published in [5], we use the same design process, assuming the entirety of the catalog is available to fine tune the trigger. Table 5 summarizes the results found with our techniques, and the EA. According to our results, one of Classification Trees is able to obtain better performance than the EA in terms of accuracy and specificity. EA has a relatively good accuracy but it takes much longer in comparison. Notice that the sensitivity measures the proportion of positives that are correctly identified as such, the specificity measures the proportion of negatives that are correctly identified as such, and the accuracy measures the proportion of positives and negatives correctly identified as such.

Besides the computational time, statistical techniques present other relevant advantages regarding to:

- Scalability. While the structure of an EA has to be readjusted when more variables are taken into account, these techniques may be easily adapted to a larger parameter space. Being much faster, they may also work on bigger catalogs and still be able to provide results in a relatively short time.
- Implementation. There is a wide range of programs/ programming languages that enable a free and simple implementation of these techniques such as R, Octave [4] or SciLab [10]. To facilitate their use further, most do propose default parameters for their algorithms or functions to perform an automatic parameter fine-tuning.
- Understanding. EAs usually rely on concepts from the field of biological evolution such as reproduction, mutation, recombination and selection, which may seem difficult to understand for non-expert users. On the other hand, the proposed techniques are presented as optimization problems where the aim is to minimize an error-based function considering some assumptions without using terms from fields other than mathematics, statistics and computer science.

Table 5. Results of the experiment comparing our approach and that described in [5]

	Accuracy	Sensitivity	Specificity	Time (s)
Logistic Regression	0.9955	0.9958	0.3333	0.64
Clusterwise Logistic Regression	0.9956	0.9956	undefine	5.7
Classification Trees	**0.9975**	0.9979	**0.8750**	**0.34**
EA [4]	0.9966	**0.9986**	0.5556	*

* Data not available.

Conclusion

When natural catastrophes occur, large fractions of the population could be exposed to several financial impacts. The insurance and reinsurance industry, governments and catastrophe pools have started to employ financial instruments such as parametric CAT bonds to cede these catastrophic risks to the capital markets. For this process to be satisfactory to all parties, it is essential that the triggers or algorithms we employ to determine payments are accurate and minimize errors. We propose to address this trigger design process as a classification problem, employing well-known and powerful techniques from statistics.

Our results show that employing Logistic Regression, Clusterwise Logistic Regression, or Classification Trees, one can obtain results of equal or better accuracy than those published prior with the usage of evolutionary computation while also increasing the efficiency of the process. These observations point to the possibility that the state of the art in parametric risk transactions can be further enhanced by augmenting the parameter space used in the trigger design without further burdening the time required to design the trigger conditions.

Acknowledgments

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LIBOR Market Model Calibration with Separated Approach

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Abstract

From an economic perspective, interest rates constitute key tools for decision making in the financial sector as they have micro and macro impacts, making its risk management a crucial matter. The LIBOR Market Model (LMM) uses the yield curve of the British interbank rate LIBOR (forward) as its basic input. Unlike models that use instantaneous rates, those involved in the LMM are observable in the market. Furthermore, the model is consistent with and adjusts its parameters according to the option valuation on futures formula in the Black '76 fashion. This allows for efficient calibration and can be used to value various derivative financial instruments. While there are several approaches for calibration, this work uses the *separated* approach with *optimization*. It is implemented using a routine in MATLAB with data of european swaptions. This work concludes that the proposed algorithm is computationally efficient and the fit is satisfactory.

Keywords: interest rate model, optimization algorithm, financial derivative valuation, swaptions

1. Introduction

In order to illustrate the application of theoretical concepts developed in the LMM model[1], it is interesting to use swaptions on LIBOR [1]. This paper uses LIBOR market data to calibrate the LMM model by the optimized separation approach [2] and implement it in MATLAB. Model calibration consists in playing the market value of plain vanilla options [3].

Implementation poses a challenge which consists in transforming the unobservable values (such as forward rates and the rates volatility) into dynamics on observables [4]. Firstly, the

[1] For a more detailed development about stochastic calculus, see [1].

parameters are set by minimizing the quadratic residues of the difference between the theoretical model and the pre-valuation of the derivatives in question. This procedure requires from the analyst to have discretion regarding the data to be used and, therefore, market information needs to be available [5]. The remaining parts of the model's implementation process, are post-calibration which it reflects the use of the results, either for valuation of derivatives [6], or the associated cash flow, and in a context of less uncertainty.

The objective of this work[2] is to calibrate the LMM model using the separated approach and it is divided into two sections. The first one explains the LMM model and how to calibrate it. The latter applies this methodology to a database and calculates the parameters of the model in this context. After this, a conclusion is given, and future research is proposed.

2. The Model

Calibration of the LMM model implies finding parameters σ_i, $i = 1, \dots, N$. Theses parameters represent the volatility of certain derivative. In this research paper, swaptions on LIBOR rate agreements are used [6], [7]. For MATLAB, this procedure can take between less than a minute and fifteen minutes, depending on both the functional forms, and the number of iterations of the process. It is important to note that the process of calibrating is not something that should be done only once. An investor who uses these techniques to enhance their portfolio, should additionally think about how often it needs to re-calibrate the model [8], [9].

This section will be exposing some theoretical guidelines that justify the use of this approach for the calibration of the LMM model. This guidelines are taken from the book of Gatarek, Bachert and Maksymiuk [10] in which the model is studied in great detail. First, it is necessary to define the approach and the steps to outline the algorithm, which is then translated into computer language using MATLAB (2007a version).

As indicated by the authors mentioned, the algorithm belongs to the class of non-parametric calibration. The use of a matrix of volatilities allows for a straightforward implementation. The aim is to construct a covariance matrix forward LIBOR rate. To do this it will be necessary to compute different variants of the same approach depending on the set of parameters to be estimated: Λ_i. The relevant parameters are an array of lambdas associated with swaptions with various maturities [11]. An intermediate step is the construction of the covariance matrix (VCV) forward LIBOR rate by use of eigenvalues and eigenvectors. Here, sub-routines are necessary to correct data errors, so that we can assure the VCV is a positive definite matrix form. After this,

[2] We appreciate the comments and help from Pablo Matías Herrera, Joaquín Bosano and Tatiana Margulies.

one may proceed to minimize the mean square error between the theoretical valuation and market values.

The process creating a matrix of volatilities of swaptions [10] begins:

$$\Sigma^{SWPT} = \begin{bmatrix} \sigma_{1,2}^{swpt} & \sigma_{1,3}^{swpt} & \sigma_{1,4}^{swpt} & \cdots & \sigma_{1,m+1}^{swpt} \\ \sigma_{2,3}^{swpt} & \sigma_{2,4}^{swpt} & \sigma_{2,5}^{swpt} & \cdots & \sigma_{2,m+2}^{swpt} \\ \sigma_{3,4}^{swpt} & \sigma_{3,5}^{swpt} & \sigma_{3,6}^{swpt} & \cdots & \sigma_{3,m+3}^{swpt} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \sigma_{m,m+1}^{swpt} & \sigma_{m,m+2}^{swpt} & \sigma_{m,m+3}^{swpt} & \cdots & \sigma_{m,M}^{swpt} \end{bmatrix}_{m\times m} \quad \textbf{(1)}$$

Each matrix component, $\sigma_{i,j}^{swpt} = \sigma^{swpt}(t, T_i, T_j)$ is the volatility of a *swaption* with *maturity* T_i (assuming an underling swap T_i, T_j).

After that, it is possible to define the covariance matrix for the forward LIBOR as:

$$\Phi^i = \begin{bmatrix} \varphi_{1,1}^i & \varphi_{1,2}^i & \varphi_{1,3}^i & \cdots & \varphi_{1,m}^i \\ \varphi_{2,1}^i & \varphi_{2,2}^i & \varphi_{2,3}^i & \cdots & \varphi_{2,m}^i \\ \varphi_{3,1}^i & \varphi_{3,2}^i & \varphi_{3,3}^i & \cdots & \varphi_{3,m}^i \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \varphi_{m,1}^i & \varphi_{m,2}^i & \varphi_{m,3}^i & \cdots & \varphi_{m,m}^i \end{bmatrix}_{m\times m} \quad \textbf{(2)}$$

Source: Gatarek, Bachert y Maksymiuk [10].

Where

$$\varphi_{kl}^i = \int_0^{T_i} \sigma^{inst}(t, T_{l-1}, T_l)\, \sigma^{inst}(t, T_{k-1}, T_k)\, dt \quad ; \quad i < k \wedge i < l \quad \textbf{(3)}$$

And $\sigma^{inst}(t, T_{l-1}, T_l)$ is the instant volatility of the LIBOR rate $L_l(t, T_{l-1}, T_l)$.
It is assumed that parameters Λ_i exist, such that:

$$\varphi_{kl}^i = \Lambda_i \ \varphi_{kl} \quad \textbf{(4)}$$

Assuming that $\Lambda_i = \delta_{0,k} \ \forall\, k = 1, \dots, m$, it is possible to calculate parameters of the principal diagonal of Φ_{mxm}:

$$\varphi_{kk} = \frac{\delta_{0,k} \cdot \sigma^{swpt}(t, T_k, T_{k+1})^2}{\Lambda_k} \quad \textbf{(5)}$$

The next step is to define parameters $R^k_{i,j}(t)$

$$R^k_{i,j}(t) = \frac{B(0,T_{k-1}) - B(0,T_k)}{B(0,T_i) - B(0,T_j)} \tag{6}$$

Where $B(0,T_n)\ n = 1, \dots, M$ are the LIBOR discount factors, and vector B is defined as:

$$B = \begin{pmatrix} B(0,T_1) \\ \cdots \\ B(0,T_M) \end{pmatrix} \tag{7}$$

Please note that $R^k_{i,j}(t)$ depends on k, i, j and on the maturities selected in the calibration.

With these new definitions it is now possible to calculate the rest of matrix:

$$\varphi_{k,N-1} = \\ = \frac{\delta_k \cdot \sigma^2_{k,N} - \Lambda_k \left(\sum_{l=k+1}^{N}\sum_{i=k+1}^{N} R^i_{k,N}(0) \cdot \varphi_{i-1,l-1} \cdot R^l_{k,N}(0) - 2 \cdot R^{k+1}_{k,N}(0) \cdot \varphi_{k,N-1} \cdot R^N_{k,N}(0)\right)}{2 \cdot \Lambda_k \cdot R^{k+1}_{k,N}(0) \cdot R^N_{k,N}(0)}$$

Gatarek, Bachert y Maksymiuk [10] use the model proposed by Longstaff-Schwartz-Santa Clara [12] to give a numerical solution. Initially the authors proposed the creation of a new covariance matrix called Φ^M, similar to the original matrix but without the eigenvectors associated with negative eigenvalues.

This procedure consists in multiplying each eigenvector (e) by the square root of the associated eigenvalue $(\sqrt{\lambda_i})$. This is only done for positive λ_i. Then, a matrix is built with this changes. The matrix Φ^M is then, the result of the product between the modificated Φ^i by its transpose. This procedure fixes any problem related to negative values.

The development of this sub algorithm allows us to build the swaptions' theoretical values matrix by approaching their volatilities using, initial values of Φ^M.

Each component follows Gatarek, Bachert y Maksymiuk [10]:

$$\varphi^{M^i}_{kl} = \Lambda_i \varphi^M_{kl}$$

$$\delta_k \sigma^2_{k,N} \cong \Lambda_k \sum_{l=k+1}^{N} \sum_{i=k+1}^{N} R^i_{k,N}(0) \cdot \varphi^M_{i-1,l-1} \cdot R^l_{k,N}(0) \tag{8}$$

Since $\delta_k = \Lambda_k$ (Logstaff-Schwartz-Santa Clara model), then:

$$\sigma_{k,N}^2 \cong \sum_{l=k+1}^{N} \sum_{i=k+1}^{N} R_{k,N}^i(0) \cdot \varphi_{i-1,l-1}^M \cdot R_{k,N}^l(0) \tag{9}$$

Towards this assumptions, it is possible to build market volatilities and address the accuracy by the root mean square error between theoretical assumptions and market values:

$$RMSE = \sum_{i,j=1}^{m} \left(\sigma_{ij}^{Theoretical} - \sigma_{ij}^{Market}\right)^2 \tag{10}$$

With this expression described above, the separated approach can be optimized by nonlinear functions. RSME admits minimum only if VCV is defined positive. The theoretical model explains better the reality of the market if minimization is made. In the following section, the developed algorithm is shown with real data using MATLAB.

3. Calibration for European Swaptions

First of all, in order to implement LMM with real data it should be taken into consideration three steps: calibration, derivative payoff and cash flows related to the option. This work examines the procedure to obtain the parameters needed in the first step.

In the MATLAB routine it is possible to program a recursive process in six steps. The recursive character is configured by the fact that each step needs information of previous steps. The next figure shows all the process, data input and output required:

Step	Inputs	Outputs
1	Vector of Discount Factors **[B]**	The Matrix of Parameters **[R]**
2	i. **[R]** ii. Vector of Date **[T_num]** iii. Matrix of market swaption volatilities **[Sig]** iv. Vector of initial parameters **[Lambda]**	The matrix of covariances [**VCV]** as a function of parameters **[Lambda]**
3	**[VCV]**	i. The vector of EigenValues **[L]** as a function of parameters [Lambda] ii. The parix of eigenvestors **[E]** as a function of parameters [Lambda]
4	i. **[L]** ii. **[E]**	The modified covariance matrix **[VCV_M]**
5	i. **[R]** ii. **[VCV_M]**	Theoretical swaption volatilities **[Sig_theo]**
6	i. **[Sig_theo]** ii. **[Sig]**	RSME between theoretical and market swaption volatilities

Fig.1. Algorithm – separated approach. . (Source: compiled by authors based on Gatarek, Bachert y Maksymiuk [10])

As it was mentioned above, the recursive character is illustrated by the dotted arrow. The output data in each step is treated as input data of the next one.

This process is known as the separated approach and some initial data is necessary in order to execute the algorithm[3]: vector of dates, swaption market volatilities matrix and parameters vector (all belonging to step 2). Regarding to the first one, it is a column vector with schedule data according to the instruments tenors. If those *maturities* are expressed in years, vector has M dates of the same initial date but within a year difference. This paper is based on Gatarek, Bachert y Maksymiuk [10] data (european swaptions from january 2005).

As initial data it is necessary to put on a parameter vector: Λ_i. Taking into consideration this performing issues:

i. If an arbitrary vector is imposed, the MATLAB code will return an RSME that matches with this initial condition. Hardly this RSME will be the local minimum for all the problem.

ii. Rather than including an arbitrary vector *[Lambda]*, an optimization in which each component of the vector are control variables of the mentioned problem should be performed.

[3] A discount factor vector needs to be specified as well, this is easily calculated from interest rates and tenors.

This is why the separated approach with optimization is clearly superior than specifying an arbitrary vector. An initial condition is used as a vector “Lambda0” which components goes between 1 to 10 (in this case). In MATLAB it is allowed a function fminsearch to solve the mean square error as a nonlinear optimization:

```
options=optimset('MaxIter',1000);
Lambda0=[1 2 3 4 5 6 7 8 9 10];
[Lambda,f]=fminsearch(@FunciónObjetivo,Lambda0,options);
```

Then, routine will run an optimization performance with some objective values. The output it yields is the minimized RSME and an initial parameters vector which matches with this minimized error. Syntax, in this case, has RSME as objective function in its arguments. Initial parameters vector (Λ_i) is a constraint.

At the same time, since the process consists of a minimization with a searching objective, it is possible to set a specific number of iterations to generate a faster solution. This paper concludes that 10.000 iterations are enough to find a local minimum in our problem. To prove this affirmation, the program was run without any iterations imposed. Results were the same in both, the restricted and the unrestricted cases.

VCV=

k/l (years)	1	2	3	4	5	6	7	8	9	10
1	0,0312	0,0109	0,0044	0,0022	0,0012	0,0007	0,0005	0,0003	0,0002	0,0002
2	0,0109	0,0454	0,0256	0,0102	0,0051	0,0029	0,0018	0,0012	0,0008	0,0006
3	0,0044	0,0256	0,0528	0,0377	0,0149	0,0075	0,0042	0,0027	0,0018	0,0012
4	0,0022	0,0102	0,0377	0,0545	0,0489	0,0194	0,0097	0,0056	0,0035	0,0024
5	0,0012	0,0051	0,0149	0,0489	0,0679	0,0462	0,0183	0,0095	0,0053	0,0035
6	0,0007	0,0029	0,0075	0,0194	0,0462	0,0656	0,0587	0,0236	0,0119	0,0068
7	0,0005	0,0018	0,0042	0,0097	0,0183	0,0587	0,0638	0,0695	0,0274	0,014
8	0,0003	0,0012	0,0027	0,0056	0,0095	0,0236	0,0695	0,0835	0,0582	0,0234
9	0,0002	0,0008	0,0018	0,0035	0,0053	0,0119	0,0274	0,0582	0,0959	0,0535
10	0,0002	0,0006	0,0012	0,0024	0,0035	0,0068	0,014	0,0234	0,0535	0,1359

Fig 2. The matrix of covariances [VCV] as a function of parameters (Logstaff-Schwartz-Santa Clara model)

Since Fig.2, step 3 generates the eigenvalues vector and eigenvectors matrix as Λ_i functions:

Eigenvectors (L):

L^T = [-0.0147 0.0006 0.0179 0.0251 0.0339 0.0506 0.0727 0.1107 0.1549 0.2447]

E=

i	e_1i	e_2i	e_3i	e_4i	e_5i	e_6i	e_7i	e_8i	e_9i	e_10i
1	-0,0006	0,0213	-0,267	0,6756	-0,6152	-0,2504	-0,1615	-0,0583	0,0336	0,0107
2	0,0068	-0,1474	0,4869	-0,3956	-0,3148	-0,4639	-0,4567	-0,208	0,1296	0,0414
3	-0,0385	0,4361	-0,469	-0,0618	0,3768	-0,1297	-0,4712	-0,3531	0,2655	0,0923
4	0,1045	-0,6733	0,0558	0,2668	0,2121	0,2931	-0,1007	-0,3716	0,394	0,1646
5	-0,223	0,4878	0,3641	0,0431	-0,2504	0,3201	0,3005	-0,2474	0,4546	0,2337
6	0,4932	-0,0833	-0,371	-0,3318	-0,1948	-0,2442	0,3881	0,116	0,3601	0,3329
7	-0,6891	-0,1889	-0,064	0,0496	0,1717	-0,323	0,0691	0,3643	0,1652	0,4292
8	0,4525	0,2151	0,3562	0,3145	0,2348	0,0112	-0,2793	0,3958	-0,0495	0,4805
9	-0,1207	-0,0849	-0,2517	-0,3015	-0,3655	0,5259	-0,3248	0,0289	-0,3142	0,4569
10	0,018	0,016	0,0638	0,0882	0,1292	-0,2761	0,3226	-0,5707	-0,54	0,4162

Fig 3. Eigenvectors

Regarding the filter described above, it is possible to take away all eigenvectors associated with negative eigenvalues. In this case, only the first eigenvalue is negative. The VCV_M matrix or Φ^M is shown in Fig 4.:

VCV_M=

(years)	1	2	3	4	5	6	7	8	9	10
1	0,0312	0,0109	0,0044	0,0022	0,0012	0,0007	0,0005	0,0003	0,0002	0,0002
2	0,0109	0,0454	0,0256	0,0102	0,0051	0,0029	0,0017	0,0012	0,0008	0,0006
3	0,0044	0,0256	0,0528	0,0376	0,015	0,0072	0,0046	0,0024	0,0018	0,0012
4	0,0022	0,0102	0,0376	0,0547	0,0485	0,0202	0,0087	0,0063	0,0033	0,0024
5	0,0012	0,0051	0,015	0,0485	0,0686	0,0446	0,0206	0,008	0,0057	0,0034
6	0,0007	0,0029	0,0072	0,0202	0,0446	0,0692	0,0537	0,0269	0,011	0,007
7	0,0005	0,0017	0,0046	0,0087	0,0206	0,0537	0,0708	0,0649	0,0287	0,0138
8	0,0003	0,0012	0,0024	0,0063	0,008	0,0269	0,0649	0,0866	0,0574	0,0235
9	0,0002	0,0008	0,0018	0,0033	0,0057	0,011	0,0287	0,0574	0,0961	0,0535
10	0,0002	0,0006	0,0012	0,0024	0,0034	0,007	0,0138	0,0235	0,0535	0,1359

Fig 4. The modified covariance matrix [VCV_M] as a function of parameters

It is possible to generate theoretical volatilities matrix according to the last matrix and [R] matrixes calculated in the beginning of the algorithm. This is shown in formula (9).

Taking into account all the important information, the following outputs show theoretical swaption volatilities, the observables one (market information) and their differences:

Sig_theo=

Theoretical	1	2	3	4	5	6	7	8	9	10
1	22,70%	20,29%	19,96%	19,81%	19,94%	19,57%	19,24%	19,17%	18,89%	18,63%
2	22,40%	20,34%	19,26%	18,83%	18,06%	17,47%	17,19%	16,78%	16,42%	
3	20,90%	19,45%	18,70%	17,55%	16,72%	16,28%	15,76%	15,32%		
4	19,53%	19,61%	18,03%	16,93%	16,36%	15,69%	15,14%			
5	18,30%	16,64%	15,57%	14,96%	14,23%	13,64%				
6	17,93%	16,95%	16,34%	15,33%	14,54%					
7	17,61%	17,74%	16,45%	15,44%						
8	16,27%	15,07%	14,07%							
9	15,26%	14,28%								
10	14,50%									

Fig 5. Calculation of theoretical swaption volatilities [Sig_theo]

Sig=

Market	1	2	3	4	5	6	7	8	9	10
1	22,70%	23,00%	22,10%	20,90%	19,60%	18,60%	17,60%	16,90%	16,30%	15,90%
2	22,40%	21,50%	20,50%	19,40%	18,30%	17,40%	16,70%	16,20%	15,80%	
3	20,90%	20,10%	19,00%	18,00%	17,00%	16,30%	15,80%	15,50%		
4	19,50%	18,70%	17,70%	16,80%	16,00%	15,50%	15,10%			
5	18,20%	17,40%	16,50%	15,80%	15,10%	14,80%				
6	17,46%	16,74%	15,90%	15,24%	14,62%					
7	16,72%	16,08%	15,30%	14,68%						
8	15,98%	15,42%	14,70%							
9	15,24%	14,76%								
10	14,50%									

Fig 6. Matrix of market swaption volatilities [Sig]. (Source: compiled by authors based on Gatarek, Bachert y Maksymiuk [10]).

Hence, RSME between theoretical and market swaption volatilities [equation (10)] is:

$$RSME = \sum_{i,j=1}^{m} \left(\sigma_{ij}^{Theoretical} - \sigma_{ij}^{Market}\right)^2 = 0.005336$$

After running the program, initial parameters vector (control variables of the problem) are obtained:

Lambda= $(1.6495 \quad 2.2113 \quad 2.4858 \quad 2.7925 \quad 2.4413 \quad 2.7904 \quad 3.0697 \quad 2.4470 \quad 2.1824 \quad 1.5488)$

As was described, this outcome results in an optimization that matches RSME with the corresponded parameters. This information is an input to valuate the financial derivative and its future prices curve. If the proposed algorithm is computationally efficient and the fit is satisfactory (in the RSME sense), this mentioned future prices curve will be certainly close to the actual future prices. Algorithm complexity has sensitivity to data input and, specifically, to the derivatives considered. The higher the tenor considered, and the lesser partial autocorrelation has the time series, the lesser will be the prediction power of the parameters found.

It is possible to run the program with 100 and 1.000 iterations. The minimum reached is better when iterations increase. In the first case, RSME is 0.091659, while in second case 0.008107.

$$\underbrace{0.091659}_{RSME_{100}} > \underbrace{0.008107}_{RSME_{1.000}} > \underbrace{0.005336}_{RSME_{10.000}}$$

A RSME closer to zero means that LMM fits better to market data. A perfect fit is hard to achieve and has no sense to several financial analysts since reality has uncertainty and risks [13]. There is a challenge in reducing uncertainty of future events and using financial models for portfolio rebalance or creating value through investment strategies.

4. Conclusion

The LMM model aims to minimize root mean squared errors between the theoretical value and market data of European swaptions. This work analysed a calibration of this model using the methodology known as *separated approach with optimization* and implemented it in MATLAB.

This is especially important to understand the dynamics of the financial markets. In this complex environment, future is unpredictable, but investors could limit their risk exposure implementing routines like the one shown in this paper. The objective from the point of view of financial engineering is to assess the discrepancy between the market value and the theory of a financial asset, and minimize it using the "root mean squared error" methodology. Moreover, an investor who uses these techniques must, additionally, reflect on how often it should re-calibrate the model.

This work concluded that the proposed algorithm is computationally efficient and the fitting is appropriate. Particularly, the results of the non-linear minimization are sensitive to the number of iterations specified. Real-time valuation compromises its accuracy. Moreover, this procedure is not appropriate for valuing complex portfolios that include exotic derivatives due to the computational power needed for this type of valuations.

Future research aims to develop a framework for investors to evaluate different financial derivatives using LMM and also expand research on how to accelerate the algorithm presented.

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Metaheuristics for Real-Life Portfolio Optimization

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Abstract

Metaheuristic algorithms have recently received overwhelming attention in addressing combinatorial optimization problems in the area of computational finance. In particular, real-life formulations of financial problems, such as the popular portfolio optimization problem (POP), often require these optimization methods to address the complexity of constraints considered. By reviewing the scientific literature on the use of metaheuristics in solving rich variants of the POP and presenting an original numerical example, this paper demonstrates the capacity of these methods to provide high-quality solutions to complex POPs in short computing times, which might be a desirable property of solving methods that support real-time decision making.

Keywords

Metaheuristics, Portfolio optimization, Iterated Local Search

1. Introduction

Since the last century the direct relationship between financial decisions and increases in welfare through capital accumulation or economic development has been widely accepted [1]. In financial economics an optimization mentality is present in decision-makers. Traditionally, exact methods have been employed in determining optimal solutions to optimization problems. These methods, however, present some limitations when solving realistic and large-scale combinatorial optimization problems (COPs) of NP-hard nature, since under these circumstances they require either the use of simplifying (non-realistic) assumptions or extraordinarily long computing times. Thus, while the POP can be solved employing exact methods in small instances, it becomes NP-hard when realistic constraints are introduced [2]. As a result, metaheuristics have been developed as new solution approaches in optimization theory [3] and have been shown to provide solutions to problems for which traditional methods are not applicable [4] in addition to providing near-optimal solutions to complex-combinatorial-optimization problems. Metaheuristics have been developed to overcome the problem- or instance-specific nature of heuristics [5]. The term, first introduced by Glover [6], can be described as a set of guidelines or strategies to develop heuristic optimization algorithms. It is important to note that while metaheuristics (as frameworks) are domain-independent, their implementation is domain-specific. According to Feo and Resende [7], the effectiveness of these methods greatly depends on their ability to adapt to a specific instance to solve, to avoid getting stuck in local optima, and to exploit the structure of a problem. Hence, while metaheuristics do not guarantee finding a global optimal solution, with respect to exact methods that provide an optimal solution to approximated problem formulations, a near-optimal solution to the unrestricted problem combined with real-life constraints might be preferable [8].

Since Markowitz [9] developed the modern portfolio theory centered on the mean-variance approach the academic community has been highly engaged in advancing the tools for portfolio optimization. The theory is based on two constituting assumptions, namely: *(i)* the financial investors being concerned with the expected returns; and *(ii)* the risk of their respective investment. It is thus the goal to minimize the level of risk expressed through the portfolio variance for a given expected return level, resulting in the unconstrained efficient frontier. This established the portfolio optimization problem (POP), in which the risk is sought to be minimized based on a minimum expected return required by the investor. Fig. 1 presents the number of indexed publications over the years 2006 to 2015 that address the POP and solve it employing metaheuristics. The increasing popularity of the POP and the solving approach become clear.

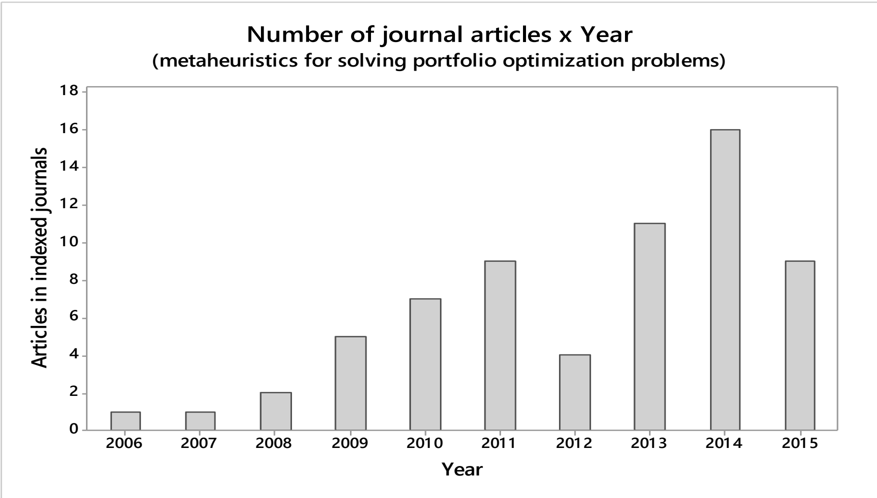

Fig. 1. Number of published articles per year on applications of metaheuristics in POP

Our paper focuses on a review of recent contributions of metaheuristics for solving realistic versions of the POP, including both single-objective and multi-objective optimization. We also present a computational example that illustrates the potential of metaheuristics in the field and the necessity for investors to look beyond stock markets.

2. The Single-Objective Portfolio Optimization Problem

While the original Markowitz problem can be solved using quadratic programming, metaheuristics have increasingly been employed to cope with the fact that the problem becomes NP-hard when real-life constraints are introduced [10]. These constraints include cardinality constraints (restricting the number of assets in the portfolio) and minimum proportional restrictions for inclusion of any asset. The classical version of the POP can be considered a single objective optimization problem with either one of the following model formulations: The investor minimizes the risk exposure subject to a minimum attainable expected return or the investor maximizes the expected return for a given level of risk. The classical POP can be formulated as follows [11]. The risk expressed as the portfolio variance is minimized:

$$\sum_{i=1}^{N} \sum_{j=1}^{N} w_i * w_j * \sigma_{ij} , \tag{11}$$

subject to a minimum return, the constraint that the weights have to add up to one and the constraint that all asset weights must lie between zero and one, inclusive.

$$\sum_{i=1}^{N} w_i * \mu_i \geq R^* , \tag{12}$$

$$\sum_{i=1}^{N} w_i = 1 , \tag{13}$$

$$0 \leq w_i \leq 1; \; i = 1, \dots, N , \tag{14}$$

where N is the total number of assets available, μ_i is the expected return of an asset i, R^* is the minimum required return, w are the respective weights of the assets making up the portfolio and σ_{ij} is the covariance between two assets i and j.

Chang et al. [11] solve the above classical problem definition using three different metaheuristic approaches in order to generate a cardinality-constrained efficient frontier: genetic algorithm (GA), tabu search (TS), and simulated annealing (SA). They suggest pooling the results from the different approaches because no single heuristic was uniformly dominating in all observed datasets. Following this suggestion and combining GA, TS, and SA, Woodside-Oriakhi et al. [12] further explore the pooling option. They find that, on average, SA contributes little to the performance of the process and that thus a pooled GA and TS heuristic is superior to single metaheuristic approaches at the expense of higher computational time. By combining exact and metaheuristic methods, they create matheuristics. However, Soleimani et al. [13] introduce sector capitalization and minimum transaction lots as further constraints and found that the GA they developed outperformed TS, and SA. Particle swarm optimization (PSO) was found to be competitive with all three, GA, TS, and SA for the cardinality-constrained portfolio selection problem and especially successful in low-risk portfolios [14]. Golmakani and Fazel [15] further introduce minimum transaction lots, bounds on holdings and sector capitalization in addition to cardinality constraints and apply a combination of binary PSO and improved PSO that they call CBIPSO and found that, especially for large-scale problems, CBIPSO outperforms GA in that it provides better solutions in less computing time.

Di Tollo and Roli [16] provide a survey concerned with the early applications of metaheuristics to the POP and some of the proposed constraints. They explicitly highlight the potential use of hybrid approaches. Such a hybrid method was proposed by Maringer and Kellerer [17] who employ hybrid local search algorithm, which combines principles of SA and evolutionary algorithms (EA), to optimize a cardinality-constrained portfolio. The option of hybrid approaches is further investigated by Di Gaspero et al. [18] who combine a local search metaheuristics with quadratic programming to optimize a portfolio while accounting for cardinality constraints, lower and upper boundaries for the quantity of an included asset and pre-assignment constraints. According to their results, the developed solver finds the optimal solution in several instances and is at least comparable to other state of the art methods.

3. The Multi-Objective Portfolio Optimization Problem

While single objective optimization methods consider either a minimal risk for a given return or a maximum risk for a given expected return or an objective function that weights the two goals and thus have to be run several times with the respective weights [19], multi-objective optimization methods find a set of Pareto solutions, while balancing two or more objective functions simultaneously. According to Streichert et al. [20] the problem can then be formulated as follows. For a multi-objective optimization it becomes necessary to minimize the portfolio risk expressed by the portfolio variance:

$$\sum_{i=1}^{N} \sum_{j=1}^{N} w_i * w_j * \sigma_{ij} \ , \tag{5}$$

while maximizing the return of the portfolio

$$\sum_{i=1}^{N} w_i * \mu_i \ , \tag{6}$$

subject to

$$\sum_{i=1}^{N} w_i = 1 \ , \tag{7}$$

$$0 \leq w_i \leq 1; \ i = 1, \ldots, N \ . \tag{8}$$

Zhu et al. [21] aim at a comparison of GA and particle swarm optimization (PSO) in solving the non-linear portfolio optimization problem with multi-objective functions. They argue that PSO overcomes the problem of increased convergence time in large instances expected for GA. They test their methodology on unconstrained, as well as constrained portfolios. While they do not include constraints other than a total weight equal to one, in addition to restricted portfolios, in which the short-selling of the portfolio's underlying assets is prohibited and thus all asset weights are positive, the authors also investigate unrestricted portfolios. However, they introduce the Sharpe ratio as a simultaneous measure of risk and return and thus turn the multi-objective optimization problem into a single objective optimization by optimizing an objective function that serves as a simultaneous measure of risk and return. The solution portfolios obtained with the PSO solver outperformed those constructed using GA for all test problems in terms of Sharpe ratio and the established efficient frontier was above that of GA portfolios in all but one instance. Enhanced PSO algorithms for solving the multi-objective POP have been proposed by Deng et al. [22] and He and Huang [23]. Cardinality and bounding constraints are incorporated by Deng et al. [22], who find that their algorithm mostly outperforms GA, SA, and TS algorithms as well as previous PSO approaches especially in the case of low-risk portfolios. It can be concluded that different findings unanimously favor PSO in situations when low-risk investment is demanded. Similarly, He and Huang [23] propose a modified PSO (MPSO) algorithm that outperforms regular PSO for their four different optimization sets. These sets consist of the traditional Markowitz mean-variance formulation and three alternative discontinuous objective functions that simultaneously account for minimizing risk while maximizing returns. More recently, they also developed a new PSO to further enhance discontinuous modeling of the POP and find that it generally outperforms PSO and also performs better than MPSO in larger search spaces [24]. Other population-based algorithms applied in optimizing portfolios include firefly algorithms (FA) [25] and artificial bee colony (ABC) algorithms. The authors developed these to address unconstrained portfolio optimization as well as portfolios with cardinality and bounding constraints. However, because the results were satisfactory at most even after modifications, the authors hybridized FA and ABC by incorporating the FA's search strategy into ABC to enhance exploitation and found that their data suggested superiority of the methodology compared to GA, SA, TS, and PSO [26] for unconstrained and cardinality-constrained portfolios.

Streichert et al. [20] account for further constraints, namely buy-in thresholds (acquisition prices) and roundlots (smallest volume of an asset that can be purchased). They treat the POP as a multi-objective optimization problem, in which they simultaneously minimize risk while maximizing expected returns through two multi-objective evolutionary algorithms (MOEA): GA and evolutionary algorithm (EA) enhanced through the integration of a local search that applies Lamarckism. They found that this enhancement greatly improved the reliability of the results, especially with respect to the additional constraints. However, there is a second point of criticism to the original Markowitz model, namely its assumption of normal financial returns, which, in reality are characterized by a leptokurtic and fat-tailed distribution [27], making it necessary to consider non-parametric risk measures. Such a measure is the value-at-risk as employed by Babaei et al. [10] who developed two multi-objective algorithms based on PSO to solve a cardinality- and quantity-constrained POP. Through splitting the whole swarm into sub-swarms that are then evolved distinctly their methodology outperformed similar benchmark metaheuristics. In order to optimize a non-parametric value-at-risk and to include further constraints, including a lower and upper bounds for the weights of included assets, a threshold for asset weight changes, lower and upper bounds for the weights of one asset class and a turnover rate that determines the maximum asset allocation changes possible at once, Krink & Paterlini [27] developed the differential evolution for multi-objective portfolio optimization (DEMPO) algorithm, partly based on differential evolution (DE). An extended version of a generalized DE metaheuristic is also employed in optimizing a highly constrained POP by Ayodele and Charles [28]. The included constraints consist of bounds on holdings, cardinality, minimum transaction lots, and expert opinion. An expert can form an opinion based on indicators beyond the scope of the analyzed data and influence whether or not an asset should be included. Their methodology shows improved performance when compared to GA, TS, SA, and PSO. Lwin et al. [29] considered cardinality, quantity, pre-assignment and round lot constraints and developed a multi-objective evolutionary algorithm that is improved through a learning-guided solution generation strategy, which promotes efficient convergence (learning-guided multi-objective evolutionary algorithm with external archive, MODEwAwL). It was shown that the developed algorithm outperforms four benchmark state of the art multi-objective evolutionary algorithms in that its efficient frontier is superior. An extensive review of the application of evolutionary algorithms is provided by Metaxiotis and Liagkouras [30].

4. A Numerical Example

Especially for portfolio investors, the extraordinary internationalization and integration of financial markets and institutions has not only caused the decision-making process to become more complex, but also diminished successful diversification of stock-only portfolios [31]. We will thus provide a second asset class and compare the solutions achieved through the application of an iterated local search algorithms combined with quadratic programming. In particular, we increase diversification by considering individual commodity futures contracts, which have been found to have low correlations with stocks [32] [33]. We employed an iterated local search framework, which chooses the assets to be included in the portfolio, and is then hybridized with a quadratic programmer to assign the optimal weights for a given subset of assets. As our approach is concerned with the comparison of an all-stocks and a stocks-and-futures portfolio, we obtained individual daily historical closing price data for the Dow Jones 30 constituents on the one hand and daily settlement prices for the 21 most actively traded futures prices in the United States and then compared the results for an all-stocks portfolio and one, in which the algorithm could include stocks and futures in the solutions. Table 1 presents the computational results for a subset of minimum required returns and compares the obtained risk and computational times for the different asset pools. It becomes obvious that computational times significantly increase from a pool of 30 assets to a pool of 51 assets, highlighting the complexity of the POP and the necessity of metaheuristics as solving approaches. Further, the associated risks of the best-found solutions as expressed by the weighted covariances of the constituent assets' returns are presented. As expected, they increase with increasing minimum return requested by the investor. However, more importantly, is the risk gap between the two portfolios for a given minimum return. A negative percentage risk gap indicates that the risk was successfully diversified and decreased when futures were included in the portfolio. This was the case for 89 out of the 94 return instances, while the remainder showed a gap equal to zero. Generally, the gap decreased with increasing minimum returns.

Table 4: Risk, risk gaps and computing times for a selected subset of instances

	Stocks Portfolio		**Stocks and Futures Portfolio**		**Gap**
Minimum Return [%]	**Risk (1)**	**Time [s]**	**Risk (2)**	**Time [s]**	**(2) - (1) [%]**
0.0000470255	0.0000892247	0.003	0.0000267271	2.915	-70.04521
0.0001645891	0.0000892247	0.013	0.0000267271	1.151	-70.04521
0.0002821527	0.0000892247	0.011	0.0000278297	15.610	-68.80938
0.0003997164	0.0000941372	0.128	0.0000347508	12.799	-63.08498
0.0005172800	0.0001057694	0.242	0.0000483057	6.547	-54.32922
0.0006348436	0.0001220270	0.377	0.0000655508	0.968	-46.28174
0.0007524073	0.0001451629	0.139	0.0000943327	2.886	-35.01597
0.0008699709	0.0001746331	0.068	0.0001393156	0.889	-20.22383
0.0009875345	0.0002132063	0.103	0.0001987092	3.642	-6.79955
0.0011050982	0.0003072844	0.002	0.0003072844	0.078	0.00000
Overall Average		0.184		3.685	-45.08945

Figure 2 presents a box plot of the overall percentage risk gaps. The diamond represents the mean of -45.09%. The immense diversification potential is provided not only by the high maximum value of -70.05%, but also the high values associated with the different quantiles.

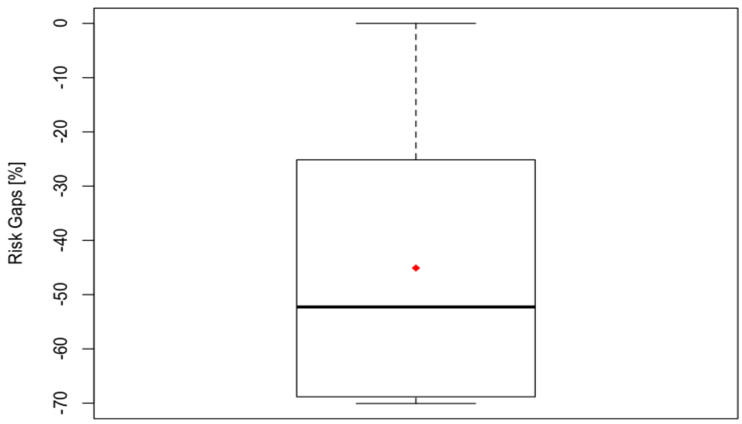

Fig. 2. Risk gaps between the best stocks portfolios and the best futures portfolios

We can thus conclude that, on the one hand, our algorithm is able to find portfolios that fulfill the investor's constraints in reasonable computing times of a few seconds and, on the other, the diversification benefits of individual futures contracts are immense. This has been shown by the risk level gaps between an all-stocks portfolio and one that included futures that were significant and never negative.

5. Conclusions

This paper analyzes the role of metaheuristic-based approaches in solving realistic variants of the well-known portfolio optimization problem, either with single or with multiple objectives. As discussed in the paper, metaheuristic algorithms are gaining popularity to solve these rich variants of the problem, since they might be used even in those scenarios in which exact methods cannot provide optimal solutions in reasonable computing times. An example of application illustrates the usefulness of metaheuristics in finding high-quality solutions in short computing times and their capacity to address further research questions with regards to diversification.

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Determinants of Corruption

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Abstract

In this paper we study determinants of the level of certain types of corruption in a society. To that end, we apply the simplest technique used in Evolutionary Game Theory, namely, the replicator dynamics with two types of agents, corrupted, and not corrupted. Through a learning interpretation of that technique, we obtain the main determinants of corruption are the initial proportion of corrupted people, and the relative pecuniary gain of being corrupted, relative to the pecuniary gain of being not corrupted. Under some weak conditions, the model applies to all types of corruption for which the larger the proportion of corrupted people is, the larger the expected payoff of being corrupted will be, and the lower the expected payoff of being not corrupted will be.

Key words

Systemic Corruption, Informal Economies, Dynamics of Corruption

1. Introduction

The main objective of this paper is to find determinants of corruption in general, although we concentrate on what is called systemic corruption and micro-corruption (see O'Hara (2014)), that is, widespread corruption, and hence we essentially concentrate on those types of corruption in which the more corrupted people there is in the society, the more beneficial to be corrupted is, and less beneficial to be not corrupted is.

The importance of studying corruption in economics comes mainly from the fact that corruption and social capital are related issues, both of which are very important in order to promote growth and development, the former in negative terms, the latter in positive terms. In the paper by Mauro (1995) it is shown that corruption is associated with low income, although it has not been established the direction of causation yet. Also, corruption has been related to political institutions and political outcomes (see Persson et. al (2003)), which in turn are crucial to growth, as argued by North (1991). For more on the negative correlation between corruption and growth, see Besley (2006) ---Figure 1.6, among others---. Similarly, there is evidence that there exists a positive correlation between social capital and growth, as it is shown by Temple and Johnson (1998). They construct an index of social capital which turned out to be a good predictor of economic growth. One of the main ingredients of that index is again the level of corruption in the society. See also Fukuyama (1995) for more arguments in favour of the positive correlation between social capital, honest behaviour (not corrupted behaviour) and growth.

In this paper, however, we strongly depart from standard studies on corruption, as most of them concentrate on public corruption by officials, governments, power elites, tax havens, abuse of power, etc. That is, for us corruption is not merely vested interests (of the most powerful people of the society) against the common good (See O'hara (2014)). We will say that an action is corrupted, if that action violates or breaks the rules prevailing in the society. An agent is corrupted, then, if it commits a corrupted action. Therefore, in particular, all the informal economy is a form of corruption. This last form of corruption is not always denounced in the literature. Moreover, that form of tax evasion may account for huge quantities of value. For instance, if the informal sector is of 25% of GNP, and taxes are on average 30% or 40% for small firms, the tax evasion may account for at least between 7.5% and 10% of the GNP of a country. Estimates tell that, for instance, the informal economy in Spain is more than 22%, and in Italy or Greece are more than 25%. For countries in Africa, for instance Zimbabwe, the informal economy is more than 59% of GNP. See, for these numbers and many others, Schneider (2002).

Other forms of not honest behaviour can be taken into account in our model as corruption, as for instance, at the university, students copying from one another, or even stealing exams from professors. These last types of dishonest behaviours may also have, of course, a higher pecuniary payoff than the honest behaviour, in expected future returns. A priory, we conjecture that for both tax evasion, and dishonest behaviour at the university, it is true that the larger the number of people willing to cheat is, the larger its expected payoff is, as the probabilities of being discovered in those cases would be lower when there is a lot of corrupted people in the society. So our model applies to those types of corruption.

Thus, our model also aims at describing when an agent is willing to cheat the firm in which it is working. For instance, when an unwatched worker is not working properly, precisely because not all workers can be watched. For instance, in a supermarket, when a worker is willing to steal food, alcohol, or even money if possible. On the other hand, workers might also be cheated by bosses, for instance, when they are not paying extra hours, though this behaviour is partly conceded or consented by workers, as otherwise they may lose the job. These types of corrupted behaviour seem to be exacerbated in crises times, as in current times in Spain for instance. Unfortunately, we have not found verifiable data (a common problem when trying to measure corruption, as workers would not denounced it, of course, for fear of losing the job), however news journals in Spain have been reporting that even more than 50% of extra hours are not paid in current times. It is clear in these last examples, that the larger the number of corrupted workers is, the more difficult to watch them is.

We obtain then that the main determinants of corruption in the long run are the proportion of corrupted people there is in the society at the initial moment of analysis, and the relative average gain in pecuniary terms of being corrupted, relative to the average gain of all the population, corrupted and not corrupted. Having only two types, the payoff of a type, say not corrupted, is larger than the average, if and only if its payoff is larger than the payoff of the other type, say corrupted people. Therefore, the second determinant is the relative gain of being not corrupted, relative of the gain of being corrupted. Our interpretation is the following. A given agent, at a given moment of its life, is faced with the election of following the rules, or not. Then, we postulate that the given agent will tend to cheat if it is profitable in pecuniary terms, and it will be profitable depending on how much corrupted people there is in the society at the given moment. If there are too few corrupted people, it is expected that to cheat is less profitable than not to cheat, and conversely, if there is a lot of corrupted people, it is expected to cheat is more profitable than not to cheat. Roughly speaking, in our model, corruption is lust for value.

The rest of the paper is as follows. In Section II we roughly revise the related literature. Section III lays down the model and Section IV presents the results. In Section V we discuss the results, and finally the conclusions are presented.

2. Related Literature

The closest studies to ours are clearly those which searched for the determinants of corruption. Perhaps the most recent and related to our study is given in Salih (2013). As it can be inferred from that last paper, most of the studies on the issue have concentrated on empirical relationships between corruption and the proposed determinant variables. Among these variables, as reported in Salih (2013), we have: 1) Income (negatively correlated); 2) Government size (negatively correlated); 3) Foreign Direct Investment (potential negative correlation); 4) Economic freedom (not clear sign of the correlation); 5) Quality of Judiciary system (negatively correlated); 6) Import share (negatively correlated); 7) Trade openness (negatively correlated); 8) Foreign aid (not clear sign of the correlation); 9) Inflation (positively correlated). In sharp contrast to these literature, we propose two theoretical determinants, both of which appear as we apply the replicator dynamics to the issue.

In relation to dynamics of corruption, there are works studying that issue, however they are concentrated on governmental corruption, as far as we know. See, for instance, Schumacher (2013), and Hauk and Saez-Marti (2002).

Some comments are in order. Observe that for most of the determinants above named, it is heavily suggested that in rich countries, corruption is lower than in poor countries. This indicates some contradictions in the issue, which is to say, most studies in corruption concentrate on the type of corruption powerful people exercises, but most powerful people, owners or managers of big firms for instance, are in rich countries presumably. To our understanding, the data suggests the following interpretation: In rich countries corruption stays mainly at high levels of the society, but lower income people of those rich countries, tend to be less corrupted, than those lower income people of poor countries. In poor countries, corruption would be present everywhere, low income people, and high income people.

Now we come to the model.

3. The Model

There are two types in the society $j \in \{C,NC\}$, where C denotes the corrupted type, and NC denotes the not corrupted type. Denote by $(\rho(t),1-\rho(t))$ the profile of types, where $\rho(t)$ is the proportion of people of type C at t, and an analogous definition to $1-\rho(t)$ applies. Time is continuous, and the horizon is infinite. We may think that at a given moment of time, there are a proportion of adult corrupted people, and some new people is born, in a way that the existing ones and the new born are learning the type they will adopt. Alternatively, we may assume a constant infinitely lived agents. The former interpretation just proposed is, to our understanding, more realistic.

The type C is characterized by a function of the proportion of types C of the form $U_C:[0,1]\rightarrow\mathbb{R}_{++}$, a function which represents the expected pecuniary payoff that a corrupted agent expects to gain, if at t there are $\rho(t)$ proportion of corrupted people. Similarly, the type NC is characterized by a function of the proportion of types C of the form $U_{NC}:[0,1]\rightarrow\mathbb{R}_{++}$, which represent the expected pecuniary payoff that a not corrupted agent expects to gain, if at t there are $\rho(t)$ proportion of corrupted people. Observe that both pecuniary payoffs are strictly positive, which is assumed for simplicity (to avoid details when a denominator is zero).

Observe that we use expected values, to emphasize the idea that it contains both, benefits and counter benefits of being corrupted, including the risk of going to jail, but measured in pecuniary terms. Moreover, as most probably not all forms of corruption have the same payoff, we prefer to fix the type of corruption, as it was insinuated in the Introduction.

Finally, we impose a replicator dynamics over the types of the model. For a general presentation of the replicator dynamics, see, for instance, Vega-Redondo (1996). To that end, let us define $U(\rho(t))=\rho(t)U_C(\rho(t))+(1-\rho(t))U_{NC}(\rho(t))$, which is the average pecuniary payoff in the population. Therefore, we postulate that $\rho(t)$ is determined by the following equation:

$$d\rho(t)/dt=\rho(t)(U_C(\rho(t))-U(\rho(t))/U(\rho(t),$$

where $d\rho(t)/dt$ denotes the derivative of $\rho(t)$ with respect to t, evaluated at t.

That is, the proportion of corrupted people increases at t, if and only if, there are corrupted people in the society at t, and the pecuniary gain of corrupted people outnumber the average pecuniary gain. As we have said, we do not impose a biological interpretation on the equation, that is, we do not postulate people are automata who acquire a genotype which determines its future behaviour. Instead, we propose a learning interpretation: People tend to imitate those who are the most successful in the society, so if to be corrupted generates higher payoff than average, people will tend to follow that behaviour and, vice versa, if people observe that to be not corrupted generates higher gains than average, people will tend to imitate that behaviour.

It is a simple exercise to see that the previous differential equation is equivalent to:

$$d\rho(t)/dt =\rho(t)(1-\rho(t))(U_C(\rho(t))- U_{NC}(\rho(t))/U(\rho(t).$$

Notice that if $\rho(t)=1$ at some t, then we have $d\rho(t)/dt =0$, that is, the proportion of corrupted people does not change at t, as it must be: A given type only can decrease if there exists another competing type.

Therefore, provided there are some corrupted people in the society, the proportion of corrupted people increases, if and only if, the expected pecuniary gain of being corrupted is higher than the pecuniary gain of being not corrupted.

Now we come to the assumptions.

A1 $U_C:[0,1]\rightarrow\mathbb{R}_{++}$ is increasing and $U_{NC}:[0,1]\rightarrow\mathbb{R}_{++}$ is decreasing, and both functions are continuous.

A2 $U_{NC}(0)> U_C(0)$, and $U_{NC}(1)< U_C(1)$.

Both assumptions A1 and A2 would be satisfied if tax evasion is the type of corruption under study, as we commented before. We assume continuity just for simplicity of the exposition, as in this case we can apply standard theorems to prove existence of solutions to ordinary differential equations.

Some standard definitions follows. A steady state is a profile (ρ',1- ρ') with ρ'=ρ(t') such that dρ(t')/dt=0 for some t' not lower than zero (assuming the economy started at t=0). This profile is said to be globally stable on [0,1], if for any $\rho(t_0)\in(0,1)$ ($\rho(t_0)$ the initial condition for the system), then ρ(t) tends to ρ', as t tends to infinite. That is, the proportion of corrupted people converges to the state ρ'. Similarly, this profile is said to be asymptotically stable, if there exists an ε>0 (it may be arbitrarily small) such that for any $\rho(t_0)\in(\rho'-\varepsilon, \rho'+\varepsilon)$, then ρ(t) tends to ρ', as t tends to infinite. Observe that both $\rho_1^*=0$, and $\rho_2^*=1$ are steady states of the system.

On the other hand, notice that given A1-A2, there exists a unique $0<\rho^*<1$ such that $U_C(\rho^*)- U_{NC}(\rho^*)=0$. Thus, that ρ^* is the unique interior steady state of the system, if A1 and A2 are assumed.

Now we come to the main theorem of this paper.

4. The Result

Theorem Assume A1 and A2, and the replicator dynamics on ρ(t). Therefore, given the initial condition $\rho(t_0)$, there exists a unique function ρ(t) satisfying the replicator dynamics and that initial condition. Moreover, we have that: a) If $\rho(t_0)<\rho^*$, then the unique solution of the equation, ρ(t), tends to ρ_1^*; thus, in particular, $\rho_1^*=0$ is asymptotically stable; b) If $\rho(t_0)>\rho^*$, then the unique solution of the equation, ρ(t), tends to $\rho_2^*=1$, as t tends to infinite; thus, in particular, $\rho_2^*=1$ is asymptotically stable.

Proof: The proof is as follows. First, the existence of a unique solution satisfying the initial condition and the replicator dynamics follows at once from A1. This is a standard result in the issue of dynamical systems. See, for instance, Arnold (1978), or any standard text book in the topic of ordinary differential equations. Now we come to the more specific results, for which it is sensible to prove it, once we know there exists a solution of the differential equation. Consider first Item (a). Notice that due to A1-A2, we have that $U_C(\rho)$- $U_{NC}(\rho)<0$ for all $\rho<\rho^*$, and therefore $d\rho(t)/dt <0$ for all $t>t_0$, which can be proven by contradiction. Thus, $\rho(t)$ is decreasing for all $t>t_0$, and hence tends to $\rho_1^*=0$ as t tends to infinite, as if $\rho(t)$ tended to a non-zero level, $\rho(t)$ would continue decreasing, as that level must be lower than ρ^*. The proof of (b) is analogous, and hence omitted.

Some technical comments are in order. Notice that, assumed A1, A2 is a crucial assumption to obtain a unique interior steady state. Indeed, given A1, If we dropped A2, either ρ_1^* or ρ_2^* may become globally stable. If we required $U_{NC}(1)> U_C(1)$, for instance, ρ_2^* would be globally stable. And if we required $U_{NC}(0)< U_C(0)$, ρ_1^* would be globally stable. On the other hand, if we dropped the monotonicity requirements in A1, other interior steady states may appear, and therefore there may exist a society in which coexist corrupted and not corrupted people in the long run. Perhaps for some types of corruption that result might be the appropriate one.

5. Discussion

As said it, our objective is to find determinants of corruption. In the Theorem above, Item (a) states: If the proportion of not corrupted people is large enough at the initial moment of analysis, say t_0 (normally, $t_0=0$), that is, if $\rho(t_0)<\rho^*$, and the pecuniary gain of being not corrupted is larger than the pecuniary gain of being corrupted ((A2) and $\rho(t_0)<\rho^*$), then in the long run there will not be corrupted people in the society. On the other hand, Item (b) states: If the proportion of corrupted people is large enough at the initial moment of analysis, that is, if $\rho(t_0)>\rho^*$, and the pecuniary gain of being corrupted is larger than the pecuniary gain of being not corrupted ((A2) and $\rho(t_0)>\rho^*$), then in the long run there will be only corrupted people in the society.

Finally, as commented in the previous section, if the monotonicity requirement in A1 was dropped, other interior steady states may appear, being some of them asymptotically stable. Therefore, other types of corruption may be studied under our set-up, by correcting the assumptions appropriately.

From our result it is possible to infer recommendations to governments, if the objective is to reduce corruption in the society. Indeed, given the proportion of corrupted people at a given moment of time, governments may strengthen punishments to minor faults, so that to lower the expected pecuniary gain of being corrupted. In the end, any strengthen of punishments may reduce corruption. However, if it is done to minor faults, this may imply punishments to juvenile people, and thus harsh punishments in the future for more serious faults become more credible. Definitely, one lesson of our paper is the following: If to be corrupted is profitable (due to weak punishments, for instance), we can hardly expect to have a not corrupted society.

Moreover, our approach heavily suggests that the problem of corruption in general, it is a problem of the whole society. The previous paragraph suggested a possible mechanism to fight against corruption implemented by governments which essentially places the issue on education and learning, learning from incentives. That learning would be much more effective if parents, from the very beginning of children's life, are teaching to be honest, by example, by effectively lowering the expected payoff of being not honest, as the replicator dynamics suggests, under our interpretation.

This way, if the society is successful in lowering the expected payoff of corrupted people, firms and workers would be beneficed if there is less micro-corruption in the society. Perhaps those corrupted agents that are not punished would have a very high return, however those that are punished, are severely punished.

Of course, our study suggests the question: Is it possible to reduce the type of corruption we study here without punishments? Perhaps the answer is yes, but our model does not provide the answer. The model is not saying that governments and parents must severely punish, but it is suggesting the reason why that behaviour may work. The crucial message is not to punish, but to lower the expected payoff of corrupted people, given the proportion of corrupted people there is in the society at the given moment of analysis.

Conclusion

In this paper we use the standard replicator dynamics in continuous time to find determinants of certain types of corruption. We find two determinants, one of them being the relative gain of being corrupted in relation to the gain of being not corrupted, and the other is the initial proportion of corrupted people.

This conclusion applies for certain types of corruption which satisfies our two assumptions. If those assumptions does not hold, then it may appear situations in which, depending on the initial proportion of corrupted people, the society may converge to an interior steady state in which there is both, corrupted and not corrupted people.

The type of corruption for which our model can be applied are those that normally causes much unrest in the society, both for firms and workers in particular. Indeed, a firm which needs not to spend resources to watch workers, will have higher benefits. Presumably, those benefits will go to workers as well, at least in part. Although one may strongly doubt about that, the data we have already commented here suggests that is the case. Indeed, that is what the negative correlation between corruption and income suggests, as in general rich countries (with a high average income per-capita), is where corruption is less present.

Therefore, the results depend on the type of corruption is considered. This begs the question: Is it possible to set general model in order to study all types of corruption? We leave for future research to answer that question. However, our intuition is that it may be possible, by constructing an appropriate unified corruption index.

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Towards Agent-Based Simulation Support for Training Teachers

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Abstract

The quality of teacher training is crucial as it influences the academic performance of students. The last decades have witnessed a steep increase in the use of information applications for improving the training of teachers. In this line of work, the current approach proposes to use agent-based simulation for supporting the training of teachers. The current work focuses on the capability of teachers in designing appropriate teaching strategies with the corresponding schedules of learning activities. In particular, the current approach uses FTS-SOCI (an agent-based Framework for simulating Teaching Strategies with evolutions of Sociograms) for simulating the social repercussions of certain teaching strategies. Teachers can pursue obtaining groups with high cohesion, which is normally related with a high academic performance. This article illustrates the current approach with the training of a computer science teacher for the programming subject.

Key words

Agent-based simulation, agent-oriented software engineering, multi-agent system, social simulation, teacher training

1. Introduction

Several studies show that the quality of teachers' training influences their productivity in promoting student achievement [1, 2]. The last decades have witnessed an increasing demand of reducing the costs of training teachers through information and communication technologies (ICT) [3]. Most of the current works use general-purpose tools in the training of teachers. Examples of these tools vary from the ones for sharing documents [4] to social media applications [5].

One of the keys for achieving a proper training of teachers is to acquire practice with real students or at least with simulated situations that are similar to the real ones. Before becoming autonomous teachers, candidates can practice by participating in learning activities of some ongoing courses. However, it is less common that teachers' candidates can design a teaching strategy with a schedule of learning activities and put it into real practice. In this particular task of designing teaching strategies, we propose agent-based simulation as a possible tool support for letting teacher candidates get this practice.

Teaching strategies can include both individual and collaborative learning activities. Among others, the latter ones can be discussions in class, teamwork in pairs or larger groups, or role-play activities. These collaborative activities can help students in knowing each other and establishing new social relations. It is well known that the cohesion of a group in terms of social relations can positively influence in the performance of their teamwork [6]. Thus, teachers could promote social relations to have group of students with a high cohesion, which will probably imply a high academic performance.

In this context, the current approach proposes that teachers are trained by designing different teaching strategies and simulating their repercussion in the social relations of the group. For this purpose, Agent-Based Simulators (ABSs) have shown to be especially useful for simulating social repercussions [7]. In particular, the current work proposes to use FTS-SOCI (an agent-based Framework for simulating Teaching Strategies with evolutions of Sociograms) [8] and the associated technique for designing teaching strategies [9]. In this way, teachers can learn to design teaching strategies that will probably promote the class cohesion obtaining a high academic performance in average.

2. Background

2.1 Training teachers with new technologies

The teaching training has been supported with different technologies. Some technologies have been specifically designed for training teachers, while other studies just use general-purpose tools for it.

In the group of works about the former kind of technologies, Ke and Xu [10] present a mixed-reality integrated learning environment where university teaching assistants can practice. Their system uses a Kinect interface, where teachers and students were represented with virtual avatars. They concluded that their system improved the sense of presence and supported the performance of a wide range of virtual teaching tasks and actions. Moreover, Jiménez and O'Shanahan [11] use a web-based tutorial program for reading instruction, called ``Letra'', for training teachers. In the web-based tutorial, teachers can learn how children can reach a good level of phonological awareness. Teachers can access to the digital tutorials from a row of books in a virtual library environment. Brubacher et al. [12] provided online training for teachers in which they learned how to effectively question children in suspicious circumstances to detect child abuse. They practiced simulated interviews with a virtual avatar.

In the case of applying general-purpose tools for teachers' training, Hassan et al. [4] present a training program of teachers that follows a specific way of using certain open source technologies. Among other tools, they used the Google Form, Google Calendar and Google Drive. They show the benefits of using these tools in the training of teacher, who then were able to use these tools with their students. In this line of research, Igual et al. [13] discuss some free software simulation tools that teachers use for preparing the learning activities in engineering higher education. Furthermore, Salminen et al. [5] described the training of nurse teacher candidates with social media applications.

In this context, the current approach belongs to the group of works in which an application is specifically designed for training teachers. More concretely, the current approach is focused on providing a simulation environment in which the teachers can practice different schedules of learning activities. The used application is an ABS, and the bases of this kind of systems are presented in the next section

2.2 Agent-based simulators: foundations and practical applications

A proper introduction of ABSs needs a brief discussion of their precursors, the multi-agent systems (MASs). The paradigm of MASs probably emerged from the need of distributing the techniques of artificial intelligence [14]. These systems were composed of autonomous entities, which were called agents. From the beginning of this paradigm, the definitions of agents were extensively discussed with different definitions. Franklin and Graesser [15] collected the most relevant definitions. For instance, the MuBot definition focused on the autonomy and the ability of reasoning. By contrast, the AIMA definition indicated the agent was an entity that was able to perceive the environment and to act on it. Other definitions, such as the KidSim definition introduced the concept of goals of agents (also referred as agendas). This last kind of agents was aligned with the well known Belief-Desire-Intention (BDI) architectures [16].

The most relevant features of agents were not only their individual features, but also their social activities when they interact among each other [17]. For instance, Fitoussi and Moshe [18] discussed the way of choosing social laws for reaching coordination.

In this context, from a software engineering point of view, the last decades have witnessed the proposal of a large variety of agent-oriented software engineering processes and methodologies. In particular, Cernuzzi et al. [19] discuss different process models that can be followed for developing MASs. Due to the variety of agent-development processes, a tool was developed for specifically defining agent-oriented development processes using a subset of the Software & Systems Process Engineering Metamodel (SPEM) [20]. Regarding the agent-oriented methodologies, the Gaia methodology [21] introduced a well-sound theoretical basis that many of the later works about MASs considered. From a practical point of view, the Ingenias and Prometheus methodologies provided alternative ways of developing agents following a model-driven approach, with respectively the Ingenias Development Kit [22] and the Prometheus Design Tool [23]. Practitioners can define a MAS model with some graphical interface, and a considerable part of the programming code is automatically generated from it, by means of templates and/or model transformations [24]. In the former methodology, the Ingenias Agent Framework was extended to support simulations [25], so the Ingenias language included many aspects that were necessary for fully defining ABSs.

In this context, some MASs were developed for simulating groups of entities with social interactions, and the number of this kind of systems increased until a subfield of MASs appeared with the name of agent-based social simulation [26]. In this subfield, ABSs started to have some specific needs. For instance, they needed to run large number of agents or analyze large amounts of data, like for example when simulating the political elections with the citizens of a city [27], the discussions of experts with the knowledge based on large sets of Wikipedia documents [28], or all the tourists that come to a city in a year [29]. However, these systems do not normally need some features of some non-simulation MASs such as the possibility of physical distribution of agents (i.e. each agent in a different device or server). In this context, we proposed a Process for developing Efficient Agent-Based Simulators (PEABS) [30]. This process is based on the common needs of ABSs. It uses the expressivity of the Ingenias language with its modeling tool, and at the same time it provides a framework for developing simulations with large amounts of agents in a reasonable response time. There are other processes for developing efficient MASs like the one of Di Stefano and Santoro [31], but this process does not use a graphical modeling language specific for ABSs, which is useful in the early design of these systems. Therefore, FTS-SOCI was developed with PEABS, and the next section presents its practical application.

3. Training teachers with FTS-SOCI

In the current approach for training teachers, firstly they must be aware that the cohesion of a group is normally related with its performance. Thus, teachers should know that if most students get on well with each other, they will probably be able to learn and work with enthusiasm and without worries about negative social relations.

Secondly, teachers need to get used to the user interface of FTS-SOCI [8]. In the main interface, teachers or candidate teachers select the number of students of each type. The available types of students are quiet students (also known as passive), participant students (also known as active), tangent students (i.e. they derive discussions to tangent topics that are irrelevant for the subject), joker students (they make jokes cheering the class) and occasional participant students (with an intermediate activity level). In the training phase, normally teachers do not have a real group of students yet, so they should simulate common distribution of student types. For example, in a group of 20 students, the current approach suggests a common distribution of five quiet students, four participant students, one tangent student, one joker student, two obstructive students and seven occasional participants. However, there can be other common distributions regarding their institutions, the subjects, the academic level, and so on. Thus, teachers can use other distributions if for example they know the common proportions of students in their universities and subjects.

In addition, the number of iterations must be set to the number of class hours. If they are training for a specific subject, they should enter the number of class hours of the subject. Otherwise, teachers are recommended to select 60, which is a common number of hours for a subject.

Regarding the simulation speed, in a first phase of training they are recommended to select between 0.5 to 2 iterations per second. In this manner, teachers can slowly observe the evolution of the sociometric status of the class represented with a sociogram. They can understand the particular influences of certain learning activities in different parts of the subject schedule. In the initial phase of training, teachers are recommended to run only one or very few simulations at each execution.

In the selection of the teaching strategy in the interface, teachers should first try to simulate one of the default teaching strategies. Then, teachers are recommended to design their own teaching strategies. They should choose the number of learning activities and their types (i.e. teamwork in pairs or larger groups, discussions and role play activities). In the case of teamwork, they should also select the number of members for each group. They need to think about the content of these activities so that the teaching strategy is feasible and aligned with the goals of the particular subject. After designing a teaching strategy, the teacher defines this strategy by means of the framework of FTS-SOCI by simply extending a Java object-oriented class and implementing one method where they can call the inherited methods in the appropriate iterations. The interested readers can read further about the definition of strategies with FTS-SOCI in our previous work [8].

When teachers simulate different possible teaching strategies, they should observe the evolution to understand common repercussions of certain kinds of activities. They should also assess the final cohesion (i.e. the IAg sociometric). As some recommended standards, teachers can consider an appropriate cohesion when $IAg \geq 0.040$ in groups around 20 students or $IAg \geq 0.060$ in groups around 10 students.

Since FTS-SOCI is nondeterministic, teachers can run a battery simulations with the same input to reduce the bias of the results occurred by chance, in a second phase of practice. In fact, the obtained average cohesion is more reliable than the particular cohesion of each singular simulation. In this phase, teachers are recommended to set the number of simulations to a number from 10 to 100, and increase the simulation speed to the maximum available option of the tool (currently the maximum available speed is 200 iterations per second).

4. Case study: training of a computer science teacher

The current approach was applied to train a teacher of computer science. The teacher designed several teaching strategies for a Programming subject of the first grade of Computer Science. He analyzed the advantages and drawbacks of each strategy by considering both (a) his own opinion about the right way of teaching the subject with reasonable schedules of activities, and (b) the evolutions and outputs of FTS-SOCI.

Finally, he chose one teaching strategy and he concluded it was probably the most appropriate strategy from the ones he had considered. The schedule of this strategy contained teamwork in pairs from almost the beginning. He observed that the sooner the collaborative activities start, the more cohesion is achieved. The individual activities were kept to minimum, and these were orally presented to the classmates in classes so there were some interactions between students. There was also a final discussion between all the students some days before finishing the subject.

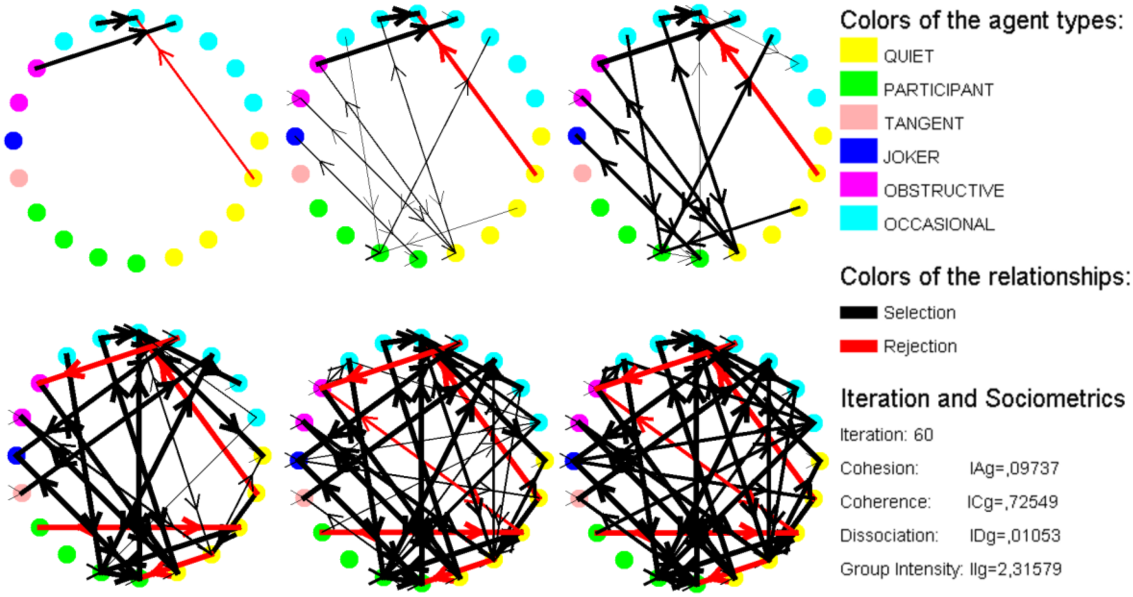

Fig. 1 Evolution of the simulated sociometric status with a teaching strategy for the Programming subject and the recommended distribution of student types

Fig. 1 shows the evolution of the sociometric status with this teaching strategy in FTS-SOCI. One can observe that the simulated cohesion was higher than the recommended standard, so the results advocated that the teaching strategy was appropriate. This was further assessed by running 100 simulations. The average cohesion was 0.0736 (SD=0.0205) in these simulations. Thus, the previous results were confirmed since the simulated average cohesion was also higher than the proposed standard.

4. Conclusions and future work

The current approach supports teachers in training for the particular task of designing teaching strategies in terms of scheduling learning activities. Since candidate teachers normally cannot implement their teaching strategies, simulations can probably be one of the most appropriate solutions.

The current work is planned to be extended in several ways. First, the current approach will be further experienced in more than one teacher, in order to obtain more reliable results about the utility of the current approach. Second, we will explore other interfaces for defining teaching strategies, since the current way of defining these might not be intuitive or easy enough for non-technical teachers.

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Increasing Transparency and Integrity in the Financial and Non-Financial Communication to Construct a Sustainable Corporate Identity

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Abstract

The aim of this project is to analyze the quality of information about sustainable behavior at international level in the framework of financial and non-financial communication that e-commerce companies nowadays are implementing. Specifically, it will focus on the case of the two international companies, Amazon and Alibaba, the largest and most valuable e-commerce companies in the world, and both trading in the NASDAQ and the New York Stock Exchange (NYSE). The objective is to establish an international reference framework in order to compare the transparency and reliability of their communication.

The analysis is based on qualitative methodology such as Content Analysis (CA), defined by Krippendorff (2013). The units selected for analysis of corporate identity values will be the corporations 'website texts on mission, vision and corporate values, including their communications in the social networks; then a comparison will be established with the reports published by the companies.

Key words

Sustainability, business management, corporate communication, stakeholders' engagement, CSR implementation

1. Introduction

As a result of the current economic, social and cultural crisis, companies must make a greater effort to recover and reinforce the confidence that society places in them, in the sense of ratifying their "license to operate". That is why acquiring the commitment or engagement with their different stakeholders is a key to ensure the long-term survival of the company, implying a proper management of the short term, with an honest two-way dialogue with their interest groups (customers, suppliers, employees, associations and institutions, investors...). If organizations support this real commitment with all their stakeholders, it will contribute to a better identification and mitigation of risks, but also to identify new business opportunities and to the improvement of their economic performance, all from a sustainable, transparent and two-way point of view.

In this regard, further research is needed because, as Ihlen et al (2011) note, the commitment of organizations to improve social welfare through proper management of their relationships with various stakeholders is increasingly relevant for a larger number of companies. So, organizations must interact and generate a two-way communication to make a sustainable and responsible business management.

After studying the state-of-the-art, we have found out that a limited number of case studies have explored how to understand, establish, develop and preserve these relationships with actors.

In this sense, the main objective will be establishing which is the level and quality of information in the framework of financial and non-financial companies' communication process.

The purpose will be to acquaint how these companies are implementing their communication plan through their integrated reporting and the rest of their communication actions as management tools.

2. Literature review

Focusing in the theoretical framework, first of all, we need to know what means sustainability in business. For some authors, it is an attitude of the company towards society and its social, labor and ecological environment. It includes a set of practices that go beyond the applicable legal minimum and must acquire a dimension within the overall strategy, including it in the set of operational processes and business management.

As Porter and Krammer (2006) point, this situation could be an opportunity to generate competitive advantages, extending beyond risk management or corporate reputation. To achieve this, the company must be understood as a "multipolar entity", as a kind of open circle including local governments, consumers, trade unions, media, NGOs, administrations, agencies…

It is commonly accepted that communication helps the corporation to understand which expectations exist and which demands stakeholders are making. At the same time, it can ideally improve corporate decision-making, stakeholder engagement, and corporate governance. In addition, communication with stakeholders is also a matter of earning legitimacy, provided the information that the stakeholders receive, must be relevant, reliable, honest, reciprocal and open. It must fulfil the following principles: integrity, transparency, sincerity, mutual respect, cooperation and mutual benefit (Tata and Prasad, 2014).

But, what does the principle of Transparency involve? It means that corporations should have clear and visible missions, policies, procedures and guidelines; including financial, social and environmental performance. This aspect will affect their identity, a concept extensively researched in numerous theoretical studies. We emphasize the definitions made by the authors van Riel-Balmer and Hatch -Schultz. For van Riel (1995) identity is "the manifestation of a bundle of characteristics, which form a kind of shell around the organization, displaying its personality". Later van Riel and Balmer (1997) conceived the concept also "indicates the ways a company presents itself through behavior as well as through symbolism, to internal and external audiences. It is rooted in the behavior ofindividual firm members, expressing the firm's ''sameness over time'', ''distinctness'' and ''centrality''.

Furthermore, Hatch and Schultz (1997) viewed that "the symbolic construction of corporate identity is communicated to organizational members by top management, but is interpreted and enacted by organizational members based on the cultural patterns of the organization, work experiences and social influence from external relations on the environment. Thus, organizational identity emerges from the ongoing interactions between organizational members (including middle-level managers) as well as from top management influence".

Finally, we define Sustainable Corporate Identity as the articulation of "what the firm does for stakeholders" and the voluntary integration into "what the firm does" (Otubanjo, 2013). It should be integrated through corporate symbols, organizational structure, corporate culture, corporate strategy, corporate behavior and the formal corporate communications. So, we can establish that identity is "what the company indubitably are". In this sense, organizations must ensure that the projected image and identity are appropriate (Jahdi and Acikdilli, 2009).

3. Materials and methods

Our aim is to analyse the quality of information about sustainable behaviour at international level in the framework of financial and non-financial communication that e-commerce companies nowadays are implementing. We have chosen this sector due to their remarkable turnover and geographic expansion in recent years, as the leading global management consulting A.T. Kearney shows in its last report "The 2015 Global Retail E-commerce Index": "Across the world, the past year brought a continuation of the impressive growth of retail e-commerce around the world. Sales increased more than 20 percent worldwide in 2014 to almost $840 billion, as online retailers continued expanding to new geographies and physical retailers entered new markets through e-commerce".

Specifically, it will focus on the case study of the two international companies (Yin, 2011), Amazon and Alibaba, the largest and most valuable e-commerce companies in the world, and both trading in the New York Stock Exchange (NYSE) and the NASDAQ Global Select Market. The sample is considered relevant for their volume of business and the stakeholders they reach. The objective is to establish an international reference framework in order to compare the transparency and reliability of their external communication and information pieces included in their corporate websites.

The analysis is also based on qualitative methodology such as Content Analysis (CA), defined by Krippendorff (2013) as "a research technique for making replicable and valid inferences from texts (or other meaningful matter) to the contexts of their use". The units selected for analysis of corporate identity values will be their last annual reports and their corporate website in English, including the companies'website texts on mission, vision and corporate values; then a comparison will be established with the reports published by the companies. Corporate reporting can disseminate the information in different ways, including both voluntary and mandatory information in their annual reports and accounts (Bebbington et al, 2008; Campopiano and De Massis, 2015).

We expect to acquaint their values, requirements and expectations of their stakeholders and to figure out how these companies are implementing their communication plan through their integrated reporting and the rest of their communication actions.

4. Results and discussion

In this section, we show the findings of our analysis of sustainability and CSR reporting in the two top e-commerce companies.

First of all, we have analysed the information needs and expectations of the different stakeholders of Amazon and Alibaba, according to the report “Sustainability Topics for Sectors: What do stakeholders want to know" published by Global Reporting Initiative (GRI) in 2013. This project records the main topics of interest identified by the stakeholders in relation to different business activities. More than 190 organizations, including business associations, workers' representatives, civil society, information users and experts, have participated in this study and have identified 1612 subjects.

Following GRI classification, we have identified Amazon and Alibaba with the retailing sector:

> “*Companies providing retail services primarily on the internet. Owners and operators of department stores. Owners and operators of stores offering diversified general merchandise. Retailers specialized mainly in apparel and accessories. Owners and operators of consumer electronics, computers, video and related products retail stores. Owners and operators of specialty retail stores not classified elsewhere. Includes jewelry stores, toy stores, office supply stores, health & vision care stores, and book & entertainment stores*”.

GRI divides the topics in three different sustainability categories, environmental (E), social (S) and others (O). It proposes 18 specific topics: Food waste management (E); Waste management (E); Packaging (E); Packaging and waste management (E); Transport of products (E); Labour conditions (S); Local community engagement (S); Land use (S); Persons’ with special needs access to products, services and facilities (S); Product information (S); Marketing and consumer communication (S); Product safety (S); Screening of product safety (S); Corporate governance (O); Supplier screening (O); Relations with suppliers (O); Sourcing strategy and policies (O); and Store lease (O). On the other hand, GRI distinguishes between six important groups: Business; Business, Civil Society. Organization; Business, Financial Markets & Information Users; Financial Markets & Information Users; Mediating Institution; and Civil Society Organization.

Then, Table 1 shows the topics that interest the different members of the Amazon and Alibaba communities:

Sustainability Category	***Proposed Topic***	***Topics Specification (if available)***	***Constituency***
ENVIRONMENTAL	*Materials sourcing*	Product certification (e.g: timber, palm oil, cotton)	Business
		Wood-based products from responsibly managed forests	Business, Civil Society. Organization
	Energy consumption	Owned and leased retail facilities	Business, Financial Markets & Information Users
			Business
	Water consumption	Owned and leased retail facilities	Financial Markets & Information Users
			Business
	Waste water management	Firefighting water run-off in facilities storing products with a potential pollution potency	Mediating Institution
	Emissions to air - GHG emissions	Refrigeration and cooling technology	Business, Financial Markets & Information Users
		Transport	Business
		Transport and retail facilities	Business
	Food waste management		Business
	Waste management		Financial Markets & Information Users
		Owned and leased retail facilities	Business
	Packaging	Use and disposal	Financial Markets & Information Users
			Financial Markets & Information Users
	Packaging and waste management	Packaging	Civil Society Organization
	Transport of products	Fuel consumption	Business
SOCIAL	*Labor conditions*	Restructuring, low wages and part-time labor	Business
	Local community engagement	Owned and leased facilities and operations	Business
	Land use	Owned and leased retail facilities	Business

	Persons' with special needs acces to productos, services and facilities		Civil Society Organization
	Product information	Life Cycle Assessment (LCA) of products	Business
	Marketing and consumer communication	Sustainability education and promotion	Business
	Product safety	Risk assessment of products and response policy	Business
	Screening of product safety	Supply chain	Business
OTHER	*Corporate governance*	Gender participation on governance bodies	Financial Markets & Information Users
	Supplier screening	Social standards in the supply chain - Ethical Trading Initiative (ETI) Base Code	Business
		Ethical and environmental practices	Financial Markets & Information Users
		Product quality and safety	Business
	Relations with suppliers	Social ethics - (e.g., payment terms, contract fairness, price, bargaining power)	Business
	Sourcing strategy and policies	Supply chain and outsourced manufacturing facilities	Business
	Store lease	Adherence to sustainability criteria	Business

Table 1. Interest in topics (developed by authors)

Once we have studied their main communication actions, we can conclude that Amazon and Alibaba work their sustainable and corporate social responsibility plan, in order to improve social welfare through proper management of their social, environmental and economic impacts as well as its relations with stakeholders.

During the period analyzed, information published in 2015 and 2014 annual reports, we can notice an interest to communicate beyond the minimum applicable legal. On one hand, in the specific case of AMAZON, society demanded more information about sustainability issues and the company has a new section on its website since last May 2016. Although the community claimed for a sustainability report during a long time, the specific website dedicated to sustainability has recently been developed. Apparently, this form of sustainability reporting is promoted by the Chief Executive and could be considered as morally responsible, good business, and attractive to socially responsible investors. It works in order to engage all their stakeholders.

On the other hand, ALIBABA has a specific website focused in sustainability but their contents are only available in Chinese language, so the access to this information is more limited. Moreover, it exists evidence of a low level in communication management on Sustainability and CSR issues addressed to all stakeholders. Both companies seem to obviate the opportunities that corporate website and social networks create. Although Amazon and Alibaba are working properly with the compulsory information, they need to establish a responsible model for corporate communications to ensure transparency and integrity.

The communication of financial and non-financial information has acquired an operational dimension of management within their global strategies. In any case, it is true that should be further developed because communication is a key factor creating value, making decisions and sticking to medium and long term. This situation means an opportunity to our research to develop a proposal with specific indicators for measuring Sustainable Corporate Identity (including image and reputation).

The main results are in line with previous studies, identifying target groups, interests, contents and use of Global Reporting Initiative models. However, as unexpected result, this preliminary study finds that social media are not so significant in order to disseminate sustainable contents.

A limitation of this first stage study is that empirical findings are conditioned by the sample and the availability of information. Larger samples, extending this study to European Union companies and a longer time scale, are clearly needed to test the robustness of the results.

Conclusion

In summary, we would like to emphasize that we are in the first stage of the research, in a starting process of comprehensive review of the state of art. This study advances in the understanding of financial and non-financial communication by exploring the variables that may interest the stakeholders in the global financial market.

The next phase of this work would be establish an international reference framework in order to compare the transparency and reliability of their communication and information pieces. Finally, another further research objective would be creating a system model to enable the analysis of the evolution of the integrated reporting and other elements and pieces of information disclosed in the corporation. Hence, we could extract their communication management model.

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A bibliometric Study of Production and Operations Management

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Abstract

This paper presents an overview of production and operations management between 1990 and 2014. The study uses a bibliometric approach considering the most productive and influential countries in this field. It also considers the bibliographic coupling of the most significant journals for production and operations management. The analysis considers a global perspective, a temporal evolution and the results found in some selected journals. The findings of the paper indicate that the USA is the most productive and influential country in this area. China is in the second position and growing a lot and the UK obtains the third place in the ranking. The expectations for the future are that soon China will become more influential and close to the results of the USA. Focusing on the selected journals, the results indicate that the journals tend to have a local nature where the local countries tend to publish more than the average.

Key words

Production and operations management, bibliometrics, Web of Science, countries.

1. Introduction

Production and operations management is a research field that has become independent after growing for several decades inside operations research and management science [2]. Today, there are many journals strictly dedicated to the field and many other general journals regularly publish some paper in the field. In order to obtain a general picture of this research field a general literature reviews are needed. A methodology for doing so is by developing bibliometric studies which are defined as the quantitative study of the bibliographic material [1]. They have been implemented in many fields [7].

In production and operations management there are many bibliometric works. For example, Sower et al. [11] study the classical issues concerning this field. Pilkington and Meredith [10] analyze the evolution of the discipline between 1980 and 2006 by using a citation analysis approach. Hsieh and Chang [5] present a general overview of production and operations management by considering the publications in twenty leading journals. Some other studies focus on journal rankings including Holsapple and Lee-Post [4] that focus on the knowledge dissemination channels in the area and Stonebraker et al. [12] in the importance that the impact factor has in assessing journals in this field.

The aim of this paper is to analyze the leading countries in production and operations management between 1990 and 2014. For doing so, we use the Web of Science (WoS) database which is usually regarded as the most significant one for scientific research. The analysis also considers the evolution of the citations throughout time and the bibliographic coupling of the most relevant and influential journals in the field. The results indicate that the USA is the most productive and influential country. However, Singapore and Taiwan presents better results per person. In general, English speaking countries, Western Europe and East Asia lead the ranking. However, some developing country also appears in the list.

The rest of the paper is organized as follows. Section 2 briefly describes the bibliometric methods used in the paper. Section 3 presents the results including the annual citation evolution, the bibliographic coupling, and the country analysis. Section 4 summarizes the main findings and conclusions of the paper.

2. Bibliometric Methods

In order to develop a bibliometric analysis, we need to search for the bibliographic material needed. For doing so, we use the WoS database which is the most popular one for dealing with academic research. WoS includes more than 50 million articles and 15 thousand journals. This material is considered to be the leading one worldwide. The search process requires the use of keywords. In this study, we use a double search which combines keywords with journals. The keywords are: production management; operations management; total quality management; supply chain management; logistics; TQM; JIT; and MRP. The journals selected fully are those that appear in Table 2 of Muller and Merigó [9] with 100% in the %POM column. The time period considered goes from 1990 until 2014. With the results obtained with this search, we develop an additional filter that only considers the research areas of Business Economics, Engineering and Operations Research and Management Science. Inside these areas, we only consider the following Web of Science categories: Operations Research & Management Science; Engineering – Industrial; Engineering – Manufacturing, Management; Business; Business Finance; Economics; Agricultural Economics; Industrial Relations and Planning & Development. Note that the search process was developed between April and May 2015.

This article considers many bibliometric indicators in order to consider a wide range of perspectives that can adapt to the specific interests of the readers. For the country analysis we focus on productivity considering the number of papers published [7]. However, many other indicators are considered including the number of citations, the citations per paper, the papers and citations per person and the *h*-index [3]. The *h*-index is a modern measure that combines articles with citations. If a set of papers have an *h*-index on 10, it means that inside the set, 10 papers have received 10 citations or more but there are not 11 articles with 11 or more citations. Note that this study follows the methodology of WoS so it has similar limitations [8] such as the problem of giving one unit to all the articles independently of the importance of the journal considered. However, they are not very significant and form a general point of view, this approach allows the reader to get a general overview of the research developed in this field.

3. Results

This Section presents the results of the paper. First, the study considers the evolution of the annual citations in the field and the bibliographic coupling of the most relevant journals in this discipline. Next, the work presents the leading countries over the last twenty-five years. Finally, some selected journals are studied individually.

3.1 Citation structure in production and operations management

Over the last years, production and operations management has become a huge field publishing a lot of papers every year. The impact of these papers can be measured by looking to the number of citations received. For doing so, Figure 1 presents the number of citations received by articles published between 1990 and 2014.

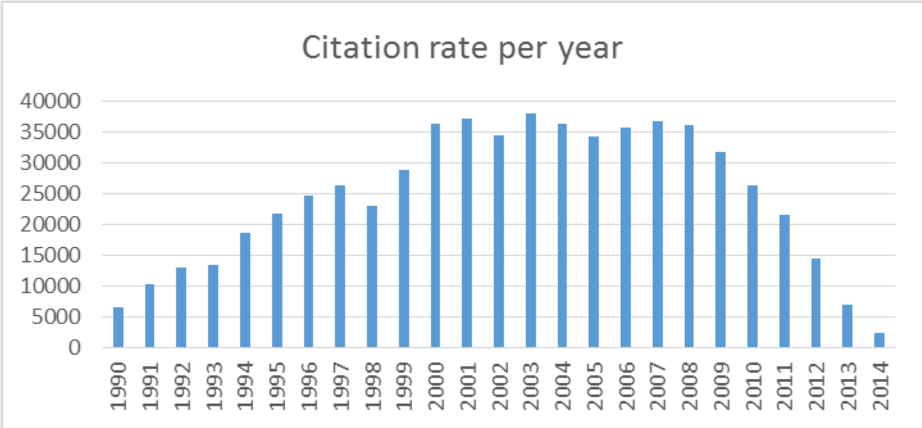

Fig. 2. Annual number of citations in production and operations management

As we can see, the articles from the first decade of the millennium have received the highest number of citations. The main reason for this is that in the nineties there were not so many papers published in the field so there is less potential for receiving many citations.

Another interesting issue to consider is the journals with the highest impact in the field. A method for dealing with this issue is bibliographic coupling. It appears when two different documents reference a common third study in their bibliographies [6]. Figure 2 shows the bibliographic coupling generated between journals by the five thousand most cited papers in production and operations management.

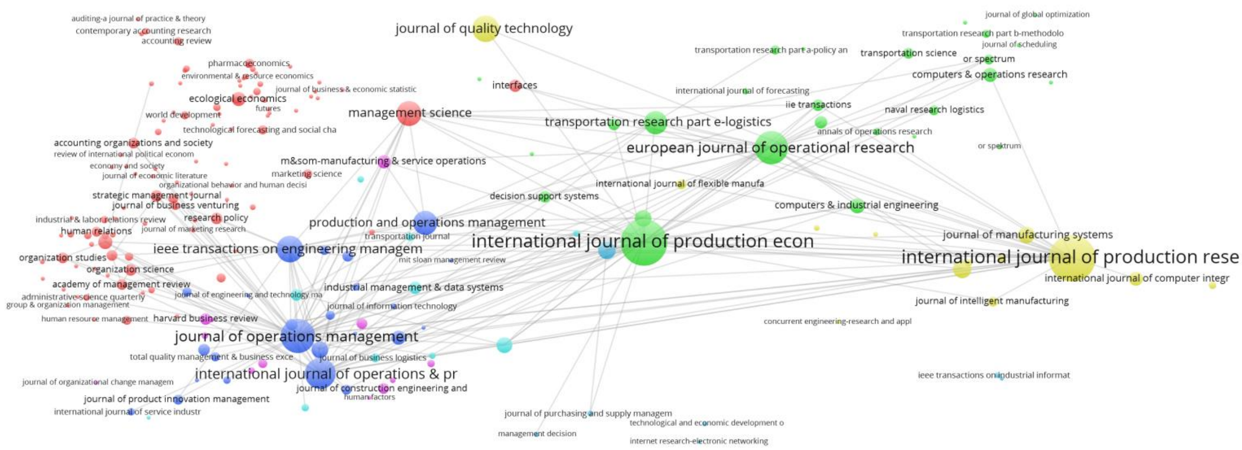

Fig. 2. Bibliographic coupling of journals in production and operations management

The International Journal of Production Economics and the International Journal of Production Research are the most significant journals in the field with the widest presence and network. The Journal of Operations Management, the European Journal of Operational Research and the International Journal of Operations & Production Management also present an important network. Note that figure 2 only shows the 200 most important connections between journals.

3.2 Leading countries in production and operations management

Many countries have made significant contributions to the field of production and operations management through the publication of contributions by their researchers working in their institutions. Table 1 presents the leading countries between 1990 and 2014. Note that they are ranked by the number of publications.

The USA is the most productive and influential country in this area. China is already the second most productive country although the UK still has more influence. The expectation for China is that it will continue growing in the future. Taiwan also obtains very remarkable results according to his size and appears in the fourth position. After Singapore that appears in the sixteenth position, it is the most productive and influential country per person in this field.

3.3 Leading countries in selected journals in production and operations management

In this section, let us look into the leading countries in each of the leading journals of the field. Following [9], we have selected sixteen journals with a strong focus on the field and seven journals that regularly publish papers in this area. Table 2 and 3 present the results for the sixteen journals strongly focused on production and operations management.

The USA is the leader in most of the journals. The only exceptions are IJOPM and SCM that are led by the UK and TQMBE that is led by Taiwan. Some journals like IJPR and IJPE are diverse and publish papers from most of the countries around the World. However, there are some selective journals were non-English speaking countries publish a very low number of articles including JOM, JQT, MSOM, JSCM, IJPDL and JBL.

Conclusion

This study has presented a bibliometric overview of production and operations management by developing a country analysis between 1990 and 2014. The data has been collected from the WoS database which is usually regarded as the most influential one for academic research. The results indicate that the USA is the most productive and influential country. The UK is also very productive and influential according to his size although currently China is becoming more relevant in the field. Taiwan has also shown very remarkable results being in the fourth position of the global ranking and with the second highest per person results after Singapore. In general, English speaking countries and Western Europe obtain very good results. East Asia is growing a lot and it is remarkable that India already obtains the thirteenth position according to productivity. Some other developing countries appear in the top 30 including Turkey, Iran, Brazil and Malaysia.

Table 5. Most productive countries in production and operations management

R	Country	TP	TC	H	TC/TP	≥100	≥50	TP/Pop
1	USA	18512	314409	178	16,98	484	1368	5,86
2	China	6592	71073	81	10,78	50	206	0,49
3	UK	5385	75225	93	13,97	76	293	8,40
4	Taiwan	2806	30883	60	11,01	19	91	12,18
5	Canada	2638	37358	76	14,16	48	151	7,50
6	Germany	1876	21337	60	11,37	18	77	2,33
7	Netherlands	1574	22137	60	14,06	22	83	9,37
8	France	1519	17530	54	11,54	21	61	2,30
9	Italy	1391	14990	50	10,78	5	49	2,31
10	Spain	1378	12335	45	8,95	8	33	2,96
11	Australia	1351	14152	50	10,48	9	46	5,84
12	S. Korea	1344	14815	50	11,02	7	50	2,68
13	India	1270	13464	48	10,60	10	45	0,10
14	Sweden	1006	10751	42	10,69	10	26	10,48
15	Turkey	994	12093	48	12,17	6	42	1,33
16	Singapore	875	11936	46	13,64	9	37	16,21
17	Japan	858	9072	39	10,57	6	24	0,67
18	Iran	635	5260	31	8,28	4	15	0,82
19	Finland	600	5743	35	9,57	5	17	11,03
20	Brazil	595	3891	27	6,54	3	7	0,30
21	Greece	538	5821	37	10,82	3	21	4,88
22	Denmark	524	5157	34	9,84	1	13	9,33
23	Belgium	510	7990	44	15,67	11	32	4,56
24	Israel	466	4530	29	9,72	3	10	5,78
25	Switzerland	436	4754	32	10,90	3	16	5,39
26	Portugal	375	3849	28	10,26	4	9	3,59
27	Norway	351	3270	28	9,32	3	8	6,91
28	New Zealand	327	3435	27	10,50	2	9	7,36
29	Austria	272	2431	25	8,94	0	7	3,21
30	Malaysia	262	2041	21	7,79	1	10	0,88

Abbreviations: TP, TC = Total papers and citations; H = H-index; TC/TP = cites per paper; ≥200, ≥100, ≥50 = Number of papers with more than 200, 100 and 50 citations; TP/Pop, TC/Pop = Papers and citations per person.

Table 2. Leading countries in eight selected journals

R	Country	IJPR	IJPE	JOM	IJOPM	JCP	JQT	TEM	POM	PPC
1	USA	2184	1133	548	407	387	623	890	701	285
2	China	788	534	35	8	350	35	91	88	80
3	UK	565	497	48	509	256	19	69	34	215
4	Taiwan	565	347	2	23	58	27	44	5	116
5	Canada	391	312	50	57	153	64	78	76	66
6	Germany	199	259	8	25	179	11	31	22	62
7	Netherlands	151	273	22	81	210	24	21	32	62
8	France	266	217	6	17	120	2	15	41	94
9	Italy	208	230	13	79	177	13	11	1	108
10	Spain	147	115	22	71	260	8	11	13	42
11	Australia	118	91	14	78	218	6	23	7	41
12	S. Korea	323	141	9	10	48	26	35	14	53
13	India	385	148	5	20	93	10	13	11	101
14	Sweden	65	188	1	56	229	2	11	6	28
15	Turkey	243	143	1	7	50	3	9	17	40
16	Singapore	217	84	8	14	15	29	52	27	16
17	Japan	140	180	0	10	108	0	20	2	30
18	Iran	155	57	1	3	45	1	4	0	13
19	Finland	24	131	6	14	85	1	9	1	33
20	Brazil	71	78	1	25	144	9	4	4	18
21	Greece	51	81	1	6	56	4	7	7	17
22	Denmark	28	66	2	31	92	4	12	1	27
23	Belgium	49	78	6	15	64	15	5	7	13
24	Israel	111	71	4	4	6	6	18	9	14
25	Switzerland	39	45	12	10	71	4	13	7	23
26	Portugal	57	25	6	13	82	0	4	5	17
27	Norway	12	16	1	9	32	2	6	1	32
28	New Zealand	45	19	4	20	29	8	4	2	5
29	Austria	26	57	0	2	61	1	5	2	7
30	Malaysia	28	23	0	5	99	2	1	0	6

Abbreviations: IJPR = Int. J. Prod. Research; IJPE = Int. J. Prod. Economics; JOM = J. Operations Manage.; IJOPM = Int. J. Prod. & Operations Manage.; JCP = J. Cleaner Prod.; JQT = J. Quality Technology; TEM = IEEE Trans. Engin. Manage.; POM = Prod. Oper. Manage.; PPC = Prod., Planning and Control.

Table 3. Leading countries in the second group of eight selected journals

R	Country	TQMBE	IJCIM	JMS	SCM	MSOM	JSCM	IJPDL	JBL
1	USA	141	287	502	130	314	138	115	173
2	China	46	230	61	32	37	9	7	4
3	UK	122	209	21	166	13	12	48	7
4	Taiwan	226	119	51	27	0	0	6	0
5	Canada	26	66	64	20	35	14	4	2
6	Germany	20	49	8	18	4	19	40	13
7	Netherlands	21	25	6	25	15	6	12	0
8	France	5	113	10	4	15	1	6	0
9	Italy	20	50	11	18	0	1	7	0
10	Spain	88	41	6	31	5	4	2	0
11	Australia	27	15	4	42	3	3	14	3
12	S. Korea	21	79	55	12	1	1	3	0
13	India	48	34	31	12	6	2	2	0
14	Sweden	72	17	15	24	0	1	34	5
15	Turkey	35	26	18	9	6	1	3	1
16	Singapore	5	30	15	4	17	0	3	2
17	Japan	7	18	11	1	2	0	1	0
18	Iran	11	36	44	3	0	0	2	0
19	Finland	4	8	2	20	0	1	13	2
20	Brazil	4	16	6	6	0	1	4	2
21	Greece	29	18	0	7	2	0	1	0
22	Denmark	17	7	6	8	0	3	16	9
23	Belgium	5	10	7	6	1	1	2	1
24	Israel	6	10	10	0	4	1	1	0
25	Switzerland	9	12	3	9	1	5	8	8
26	Portugal	31	29	2	0	1	0	1	0
27	Norway	0	2	1	14	0	0	9	0
28	New Zealand	21	16	4	8	0	0	1	0
29	Austria	5	1	1	1	1	0	2	1
30	Malaysia	32	4	11	4	0	0	2	0

Abbreviations: TQMBE = Total Quality Manage. & Bus. Excellence; IJCIM = Int. J. Computer Integrated Manufact.; JMS = J. Manufact. Syst.; SCM = Supply Chain Manage.; MSOM = Manufact. & Service Oper. Manage.; JSCM = J. Supply Chain Manage.; IJPDL = Int. J. Phys. Distrib. Logistics Manage.; JBL = J. Bus. Logis.

This paper represents a first step in the analysis of productive and influential countries in production and operations management. However, many other issues should be taken into account in order to provide a complete overview. In future research, we expect to generalize this approach providing deeper results and considering supranational regions. Some other variables will be considered including the evolution of the leading countries throughout time.

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Citation Analysis of Fuzzy Research Journals

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Abstract

Since the publication of the article Fuzzy Sets in Information and Control, by Lofti A. Zadeh in 1965, thousands of applications and extensions have appeared following the multi-valued logic proposed. The consolidation of fuzzy methodologies and techniques has been captured in several high quality journals. A visualization of the influence generated not only by publications, but also for universities, countries and research areas can be obtained following citations trends of journals. The purpose of the present paper is to analyze, with the utilization of bibliometric tools, the general influence that eight journals with a strong focus on fuzzy science have had in the past 50 years.

Key words

Bibliometrics, Fuzzy Research, Web of Science

1. Introduction

The Theory of Fuzzy Sets [1] has been recognized as an intuitive approach capable of generating new insights to ambiguous problems in a wide variety of fields [2-4]. The capacity of bounding sets that are not clear, well defined or subjective is the main characteristic that has given the fuzzy sets theory criticism, but also notoriety.

In the past 50 years, researchers from all around the world have been publishing many studies with applications in diverse fields of knowledge, establishing fuzzy sets as a well-known science, recognized and leaded by several professional associations, such as North American Fuzzy Information Processing Society (NAFIPS), International Fuzzy Systems Association (IFSA), or the Institute of Electrical and Electronics Engineers Computational Intelligence Society (IEEE CIS).

The evolving consolidation of the fuzzy sciences have been shaped in several international scientific journals e.g. Fuzzy Sets and Systems (1978), the Journal of Japan Society for Fuzzy Theory and Intelligent Informatics (1989); the IEEE Transactions on Fuzzy Systems (1993); the International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems (1993); the Journal of Intelligent & Fuzzy Systems (1993); the Fuzzy Economic Review (1995); the International Journal of Fuzzy Systems (1999); Fuzzy Optimization and Decision Making (2002); and the Iranian Journal of Fuzzy Systems (2003).

Bibliometric analysis is a research field that analyzes publications, citations and their sources [5]. The use of information technologies has allowed a flexible and dynamic scope of a scientific field. The structured gathered material shows a broader picture of the contributions that have shaped the area.

The objective of the present research is to generate a citation analysis utilizing bibliometric tools of eight journals with a strong focus on fuzzy research. Thus offering an initial picture on the question "*who is citing the journal?"* And how does that impact the conformation of research groups, leading universities around the world, top countries that explore fuzzy techniques and prolific research areas on each of the journals selected.

2. Methodology

The information gathered to build the research has as foundation the core collection of the Thomson & Reuters Web of Science database (WoS). The selection of the database responds to the need for articles published in high quality journals. Some other databases such as SCOPUS and Google Scholar are available, however this paper has focused on the WoS for being considered the more complete, neutral and rigorous database.

To develop the search we have used the name of each of the studied journals: 1) "Fuzzy Sets and Systems"; 2) "Expert Systems with Applications"; 3) "Information Sciences"; 4) "Knowledge-Based Systems"; 5) "International Journal of Approximate Reasoning"; 6) "Applied Soft Computing"; 7) "International Journal of Intelligent Systems"; and 8) "International Journal of Uncertainty Fuzziness and Knowledge-Based Systems”. The search was conducted for the option “All years”, which include studies made from 1864 until 2016. In order to include only research studies, a search filter was used, considering only articles, reviews, letters or notes. The search was carried out in January 2016. This study classifies the material using the approach found in Merigó et al. [5]. The information is structured in three main categories: Universities, Countries and Research Areas. The top 20 results are shown for each category. The studies for all the selected journals including the filters sum a total of 242,606 articles see Table 1 for a descriptive analysis of the results obtained.

Table 1. Total Articles Citing the Studied Journals

Journal	Total Studies
Fuzzy Sets and Systems	57,671
Expert Systems with Applications	59,764
Information Sciences	58,249
Knowledge-Based Systems	14,029
International Journal of Approximate Reasoning	13,258
Applied Soft Computing	19,939
International Journal of Intelligent Systems	11,469
International Journal of Uncertainty Fuzziness and Knowledge-Based Systems”	8,227

Source: Self-elaborated.

3. Results

The citation analysis is a useful tool to view the sources of influence of each journal [6]. In the present research we will focus on the 20 top influencers on the categories of University, Country and Research Areas.

In general, a total of 41,074 studies from 160 universities are displayed. The Islamic Azad University leads with 3,651 studies, next we find the University of Granada (3,494), the Chinese Academy of Sciences (2,482), the Hong Kong Polytechnic University (1,594), the Harbin Institute of Technology (1,590), the Northeastern University (1,576), the Shanghai Jiao Tong University (1,506), the Indian Institute of Technology (1,488), the University of Tehran (1,457) and the Polish Academy of Sciences (1,446).

A total of 244,350 studies from 26 different countries were retrieved, from them the Peoples Republic of China leads with a total of 68,600 studies, secondly the United States of America (23,880), following Taiwan (20,124), Spain (17,377), Iran (13, 548), India (12,364), England (9,644), Turkey (8,631), Canada (8,414) and France (7,886).

There are a total of 32 Research Areas comprising 379,946 studies. The most influential area is Computer Science (155,803), Engineering (84,900), Mathematics (31,110), Operations Research Management Science (25,563), Automation Control Systems (20,207), Business Economics (10,136), Telecommunications (5,782), Physics (4,358) and Environmental Sciences Ecology (4,355). The specific results for each journal are displayed in the next Tables 2 – 9.

Table 2. Results for Fuzzy Sets and Systems

R	University	TS	Country	TS	Research Area	TS
1	Univ Granada	793	PR China	14957	Computer Science	38060
2	Islamic Azad Univ	790	Taiwan	5332	Engineering	18634
3	Polish Acad Sci	574	USA	5278	Mathematics	12711
4	Harbin Inst Tech	489	Spain	3966	Automation Control Systems	6844
5	Northeastern Univ	458	Iran	3025	Operations Research Management	4993
6	Natl Chiao Tung Univ	458	India	2704	Business Economics	2226
7	Univ Alberta	453	France	2226	Robotics	1171
8	Natl Cheng Kung Univ	439	Canada	2105	Environmental Sciences Ecology	1135
9	Indian Inst Tech	424	Japan	1913	Science Technology Other Topics	1048
10	Natl Taiwan Univ Sci Tech	408	England	1912	Physics	948
11	Univ Ghent	394	Turkey	1864	Instruments Instrumentation	917
12	Hong Kong Polytech Univ	383	South Korea	1860	Mechanics	822
13	Univ Tehran	382	Italy	1823	Imaging Science Photographic	768
14	Univ Oviedo	381	Poland	1689	Telecommunications	753
15	City Univ Hong Kong	371	Germany	1288	Materials Science	663
16	Chinese Acad Sci	365	Australia	1168	Water Resources	603
17	Dalian Univ Tech	354	Czech R.	1032	Energy Fuels	475
18	Sichuan Univ	322	Belgium	943	Transportation	439
19	Southeast Univ	317	Brazil	848	Geology	412
20	Shanghai Jiao Tong Univ	307	Greece	730	Remote Sensing	322

Table 3. Results for Expert Systems with Applications

R	University	TS	Country	TS	Research Area	TS
1	Islamic Azad Univ	1195	PR China	16834	Computer Science	31595
2	Univ Tehran	742	Taiwan	6418	Engineering	27150
3	Chinese Acad Sci	696	USA	5053	Operations Research Management	10368
4	Hong Kong Polytech Univ	625	Iran	4294	Business Economics	4039

5	Natl Taiwan Univ Sci Tech	576	Spain	3513	Automation Control Systems	3865
6	Natl Cheng Kung Univ	570	India	3313	Mathematics	3233
7	Huazhong Univ Sci Tech	528	Turkey	2986	Materials Science	1717
8	Natl Chiao Tung Univ	520	South Korea	2480	Environmental Sciences Ecology	1679
9	Harbin Inst Tech	504	England	2452	Telecommunications	1676
10	Shanghai Jiao Tong Univ	489	Malaysia	1961	Energy Fuels	1495
11	Zhejiang Univ	459	Canada	1691	Science Technology Other Topics	1196
12	Univ Malaya	446	Australia	1664	Physics	1154
13	City Univ Hong Kong	439	France	1227	Mechanics	1142
14	Indian Inst Tech	401	Italy	1216	Instruments Instrumentation	1140
15	Univ Granada	400	Brazil	1140	Information Science Library Science	1102
16	Tsinghua Univ	384	Germany	887	Mathematical Computational Biology	896
17	Northeastern Univ	379	Greece	861	Chemistry	857
18	Amirkabir Univ Tech	376	Japan	839	Robotics	819
19	Nanyang Tech Univ	374	Poland	824	Transportation	773
20	Dalian Univ Tech	359	Singapore	714	Medical Informatics	769

Table 4. Results for Information Sciences

R	University	TS	Country	TS	Research Area	TS
1	Chinese Acad Sci	772	PR China	17638	Computer Science	39576
2	Islamic Azad Univ	714	USA	6919	Engineering	17946
3	Univ Granada	711	Taiwan	4325	Mathematics	8381
4	Northeastern Univ	497	Spain	3563	Automation Control Systems	5191
5	Harbin Inst Tech	494	India	2987	Operations Research Management	4289
6	Huazhong Univ Sci Tech	477	Iran	2738	Telecommunications	1889
7	Polish Acad Sci	468	Canada	2219	Business Economics	1587
8	City Univ Hong Kong	449	England	2095	Science Technology Other Topics	1260
9	Dalian Univ Tech	412	Japan	1939	Physics	1243
10	Indian Inst Tech	393	South Korea	1902	Robotics	994
11	Shanghai Jiao Tong Univ	385	France	1881	Instruments Instrumentation	856
12	Tsinghua Univ	382	Turkey	1806	Mechanics	803
13	Zhejiang Univ	372	Italy	1661	Imaging Science Photographic	728
14	Natl Chiao Tung Univ	371	Australia	1602	Mathematical Computational Biology	722
15	Univ Alberta	368	Poland	1560	Materials Science	682
16	Nanyang Tech Univ	363	Germany	1177	Environmental Sciences Ecology	603
17	Xidian Univ	348	Malaysia	938	Optics	594
18	Hong Kong Polytech Univ	336	Brazil	915	Energy Fuels	467

19	Univ Elect Sci Tech China	333	Singapore	677	Chemistry	420
20	Southeast Univ	331	Belgium	652	Information Science Library Science	408

Table 5. Results for Knowledge-Based Systems

R	University	TS	Country	TS	Research Area	TS
1	Chinese Acad Sci	203	PR China	4662	Computer Science	9926
2	Univ Granada	160	USA	1402	Engineering	4317
3	Islamic Azad Univ	135	Spain	999	Operations Research Management	1480
4	Cent S Univ	124	Taiwan	934	Mathematics	771
5	Shanghai Jiao Tong Univ	120	England	865	Automation Control Systems	721
6	Huazhong Univ Sci Tech	116	Australia	521	Business Economics	713
7	Sichuan Univ	110	Iran	505	Telecommunications	330
8	Southeast Univ	106	India	481	Information Science Library Science	279
9	Hong Kong Polytech Univ	103	Canada	439	Science Technology Other Topics	273
10	Harbin Inst Tech	103	South Korea	421	Robotics	249
11	Zhejiang Univ	100	France	386	Materials Science	226
12	Xi An Jiao Tong Univ	92	Japan	348	Environmental Sciences Ecology	194
13	Tsinghua Univ	92	Malaysia	344	Education Educational Research	179
14	Univ Tehran	89	Turkey	340	Medical Informatics	172
15	Natl Chiao Tung Univ	89	Italy	334	Physics	170
16	Univ Malaya	87	Germany	291	Mathematical Computational Biology	164
17	Nanjing Univ Aeronaut Astronaut	85	Poland	225	Psychology	141
18	Nanyang Tech Univ	80	Brazil	224	Imaging Science Photographic	134
19	City Univ Hong Kong	80	Singapore	207	Mechanics	129
20	Tongji Univ	78	Greece	207	Social Sciences Other Topics	126

Table 6. Results for International Journal of Approximate Reasoning

R	University	TS	Country	TS	Research Area	TS
1	Univ Granada	518	PR China	3178	Computer Science	10264
2	Univ Ghent	169	Spain	1585	Engineering	3433
3	Univ Oviedo	165	USA	1424	Mathematics	1879

4	Polish Acad Sci	158	France	911	Operations Research Management	956
5	Univ Jaen	151	England	747	Automation Control Systems	897
6	Univ Regina	133	Italy	638	Business Economics	326
7	Southeast Univ	132	Canada	622	Science Technology Other Topics	309
8	Chinese Acad Sci	115	Taiwan	550	Robotics	269
9	Univ Alberta	111	Poland	497	Environmental Sciences Ecology	188
10	Islamic Azad Univ	102	Iran	454	Imaging Science Photographic	183
11	Univ Tech Compiegne	97	India	364	Telecommunications	156
12	Univ Toulouse 3	92	Japan	357	Physics	144
13	Univ Ostrava	89	Germany	354	Mathematical Computational Biology	134
14	Univ Kansas	86	Belgium	291	Instruments Instrumentation	103
15	Sichuan Univ	76	Czech Republic	275	Medical Informatics	98
16	Xi An Jiao Tong Univ	73	Australia	275	Mechanics	90
17	Univ Manchester	73	Brazil	236	Water Resources	82
18	Southwest Jiaotong Univ	72	Turkey	225	Remote Sensing	80
19	Tsinghua Univ	71	South Korea	162	Materials Science	80
20	Univ Politecn Madrid	70	Netherlands	159	Energy Fuels	72

Table 7. Results for Applied Soft Computing

R	University	TS	Country	TS	Research Area	TS
1	Islamic Azad Univ	548	PR China	6080	Computer Science	11002
2	Indian Inst Tech	270	India	2006	Engineering	8310
3	Chinese Acad Sci	261	Iran	1974	Operations Research Management	1824
4	Univ Tehran	244	Taiwan	1483	Mathematics	1537
5	Amirkabir Univ Tech	204	Spain	1192	Automation Control Systems	1490
6	Huazhong Univ Sci Tech	200	USA	1179	Energy Fuels	636
7	Dalian Univ Tech	199	Turkey	973	Materials Science	615
8	Univ Granada	197	Malaysia	772	Mechanics	600
9	Iran Univ Sci Tech	194	England	730	Science Technology Other Topics	540
10	Univ Teknol Malaysia	174	Canada	557	Telecommunications	514
11	Nanyang Tech Univ	165	Australia	542	Physics	488
12	Natl Inst Tech	159	South Korea	422	Business Economics	479
13	Univ Malaya	153	Brazil	401	Mathematical Computational Biology	410
14	Natl Taiwan Univ Sci Tech	152	France	389	Instruments Instrumentation	404
15	Northeastern Univ	151	Italy	371	Robotics	365
16	Hong Kong Polytech Univ	147	Mexico	312	Environmental Sciences Ecology	358
17	Cent S Univ	141	Poland	305	Chemistry	310

18	Natl Cheng Kung Univ	133	Japan	296	Thermodynamics	287
19	Xi An Jiao Tong Univ	127	Greece	262	Water Resources	271
20	Shanghai Jiao Tong Univ	126	Singapore	228	Optics	189

Table 8. Results for International Journal of Intelligent Systems

R	University	TS	Country	TS	Research Area	TS
1	Univ Granada	470	PR China	2969	Computer Science	9066
2	Iona Coll	166	USA	1537	Engineering	3053
3	Univ Jaen	160	Spain	1482	Mathematics	1266
4	Polish Acad Sci	154	Italy	567	Operations Research Management Science	942
5	Southeast Univ	146	England	563	Automation Control Systems	780
6	Univ Barcelona	103	Taiwan	559	Business Economics	406
7	Univ Ghent	101	France	527	Robotics	210
8	City Univ Hong Kong	98	Canada	486	Science Technology Other Topics	191
9	Northeastern Univ	91	Poland	392	Telecommunications	156
10	Sichuan Univ	89	Australia	311	Imaging Science Photographic Technology	135
11	Cent S Univ	86	Japan	303	Environmental Sciences Ecology	119
12	Pla Univ Sci Tech	81	Iran	275	Physics	116
13	Tsinghua Univ	80	India	270	Information Science Library Science	116
14	Islamic Azad Univ	80	Germany	267	Medical Informatics	91
15	Univ Manchester	79	Turkey	250	Psychology	89
16	Shanghai Jiao Tong Univ	79	Belgium	248	Mechanics	85
17	Univ Alberta	74	South Korea	188	Mathematical Computational Biology	84
18	Csic	74	Netherlands	150	Instruments Instrumentation	84
19	Chinese Univ Hong Kong	70	Czech R.	145	Neurosciences Neurology	78
20	Chinese Acad Sci	70	Brazil	132	Social Sciences Other Topics	72

Table 9. Results for International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems

R	University	TS	Country	TS	Research Area	TS

1	Univ Granada	245	PR China	2282	Computer Science	6314
2	Univ Ghent	127	USA	1088	Engineering	2057
3	Slovak Univ Tech Bratislava	100	Spain	1077	Mathematics	1332
4	Polish Acad Sci	92	Taiwan	523	Operations Research Management Science	711
5	Univ Jaen	91	Italy	363	Automation Control Systems	419
6	Hebei Univ	91	France	339	Business Economics	360
7	Univ Oviedo	90	Canada	295	Telecommunications	308
8	Islamic Azad Univ	87	Iran	283	Science Technology Other Topics	133
9	Natl Cheng Kung Univ	86	England	280	Mechanics	133
10	Tsinghua Univ	85	Australia	242	Physics	95
11	Southeast Univ	84	India	239	Information Science Library Science	92
12	Univ Barcelona	78	Poland	228	Robotics	86
13	Sichuan Univ	78	Belgium	213	Environmental Sciences Ecology	79
14	Chongqing Univ Arts Sci	70	Japan	211	Medical Informatics	76
15	Univ Publ Navarra	66	Germany	195	Acoustics	57
16	Univ Rovira Virgili	63	Turkey	187	Imaging Science Photographic Technology	53
17	Tianjin Univ	63	Czech R	186	Mathematical Computational Biology	50
18	Univ Roma La Sapienza	62	Slovakia	150	Transportation	47
19	Univ Ostrava	59	South Korea	107	Instruments Instrumentation	46
20	Iona Coll	56	Austria	104	Mathematical Methods In Social Sciences	45

A deep analysis of the results shows that from the category of universities for the 8 Journals, 3 organizations appeared in the first place: University of Granada (4), the Chinese Academy of Sciences (2) and the Islamic Azad University (2).

In the category of countries The Peoples Republic of China appeared in the first position of all journals. In the second place we find the United States of America (4), Taiwan (2), India (1) and Spain (1). In the third we find the United States of America (3), Spain (3), Taiwan (1) and Iran (1).

In the category of research areas Computer Science appeared in the first position of all journals. In the second place Engineering appeared in all eight journals. In the Third place Mathematics (5) and Operations Research Management Science (3).

Conclusion

A citation analysis of eight journals indexed in the WoS strongly related to fuzzy research was conducted utilizing bibliometric analysis tools. The results are structured in 3 categories: universities, countries and research areas. The top 20 categories with more studies are displayed in each of the 8 tables presented. A total of 242,606 studies were retrieved, each of them corresponds to a contribution citing the journals selected. Results convey that in the category of Universities 3 organizations appear in the first places of the journals: University of Granada, the Chinese Academy of Sciences and the Islamic Azad University. In the category of countries the Peoples Republic of China resulted first in all the selected journals. Finally in the category of Research Areas, Computer Science resulted as first in each of the journals. The present work is a first approach to an in depth categorization of the influencers of each of the selected journals. Future research is needed; firstly expand the categories in order to include authors and years, and secondly a cross-citation analysis in order to find trends between journals. Although there are several limitations to this study, this bibliometric analysis identifies the main trends in citations for the selected journals.

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Predicting Defection in Non-Life Motor and Home Insurance

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Abstract

We focus on real cases of the motor and home insurance sector and we evaluate the predictive capacity of three different churn-models by comparing different performance measures. Our comparison includes an analysis of a logistic regression, a conditional tree and a support vector machine for these two insurance products. Results show that predictive performance is higher for home insurance than for motor insurance and that the optimal prediction method may differ in each case depending on the performance criteria. In our illustration, the support vector machine methods outperform the others when prediction the cancellation of home insurance contracts.

Key words

Customer retention, predictive analytics, cancellation, renewal, lapse, loyalty, classification methods, ROC curves, performance measures.

1. Introduction

Insurance companies have to keep their customers satisfied to increase the chances that contracts are renewed at the expiring date. Therefore, customer loyalty is one of the most important priorities for insurers, which guarantees the stability of their business market share. Different researchers argue that retaining a customer is less costly than gaining a new one (Zeithaml, 1996; Fornell and Wernerfelt, 1987; 1988), and this seems to be the case because in order to attract new clients, insurers have to invest in advertising. Moreover, there is no information in the files of the company about new customers, while this knowledge is available for existing clients. Personal information of insured clients is valuable and is being used intensively with data analytics to predict the future expected behaviour of the undertaken risk (in Thuring et al. (2013) and Verhoef (2003) are shown some examples about this topic). Past information allows calculating a fair price of the insurance coverage. In addition, long-term customers tend to be more engaged with the company (Thuring et al., 2012) and take less of the company's time than attracting new customers (Keaveney, 1995; Brockett et al., 2008). Moreover, customer satisfaction has a direct link with future revenue (Fornell, 1992) and can be affected among other things by not investing in facilitating complaints of dis-satisfied customers (Fornell, 1988), increased premiums, poor claims management and delays in payment of claims (some models for claims prediction are shown in Boucher et al., 2007; 2009 and Frees et al., 2016).

In this context, there is an increasing interest for insurers to find predictive models that anticipate customers' decisions regarding the renewal of contracts. Companies are interested in finding the reasons why customers cancel their policies and in identifying factors that explain why these customers switch to another company (Guillen, 2012). So, the prediction of defection is central in nowadays management of an insurance firm and it is also central to the global strategy of the firm. Predictive analytical methods are based on past information about clients who decided to defect versus those who stayed in the company. These models are needed for implementing short-term actions, which are oriented to improve customer satisfaction and reverse the intention to leave the company (Guillen, 2008; Guelman and Guillen, 2014; Guelman, Guillen and Pérez-Marín, 2014; 2015a; 2015b).

We aim to contribute to the existing literature on this topic and to fill an existing gap, because in general the available contributions on customer churn in insurance do not compare the criteria used to select between modelling alternatives. Moreover, to the best of our knowledge no previous work compares the model performance between two different lines of insurance business. In this paper, we propose performance measures and threshold values (cut-offs) that can guide practitioners to decide which is the best model in their particular context. We also present an illustration from a Spanish motor and home insurance data set (see Bolancé et al., 2016 for a previous illustration focused in motor insurance).

In our case of study, we focus on non-life insurance products, which are underwritten to protect objects or properties and to cover liabilities. Specifically we chose to concentrate on two of the most important personal line products: automobile, the largest non-life insurance market, and homeowner insurance, each of which represents a line of business with some intrinsic characteristics: For instance, the percent of policies that are cancelled each year is quite different for these two types of insurance coverage.

In general, for motor insurance the rate of cancellation is higher than the rate observed for home insurance. The reason why motor policies tend to be cancelled at a higher frequency is that there exists a larger market for motor insurance and a fiercer competition between firms than in home insurance. So, consumers can find opportunities for motor insurance that are more convenient to their own preferences (in motor insurance the premium is fundamental, see, for example, Yeo et al., 2001; Pinquet et al. 2001; Bolancé, et al., 2003; 2008 and Guillen et al., 2013). On the other hand, in home insurance, the competition between insurers is lower than in motor insurance and so, consumers seem to lose their interest for changing from one company to another.

We want to analyse the methods that can serve to calculate the renewal probability of each customer in the insurance company for each sample. We have considered the following classical predictive models (or classifiers): logistic regression, decision tree and support vector machine. These methods allow classifying each new client by means of the probability of belonging to the group of customers that ‘stay’ versus those who ‘churn’. In fact, we consider two differentiated groups in the classifying algorithm: retainers and churners.

We conduct our analysis by comparing the receiver operating characteristics (ROC) curves obtained in every method (Guillen, 2014). ROC curves are one of the most common tools for evaluating and comparing classifications models (Fawcett, 2006), but can be misleading when their trajectories overlap. Then interpretation of performance leads to an ambiguous result (Gigliarano et al., 2014). In this paper we propose criteria that characterise the predicting methods and therefore we are able of to identify particular characteristics that measure the classification performance for the different models as applied to different insurance products.

In this study, we use three different criteria: two of them are based on metrics generated directly from a confusion matrix and the third one is based on the area under the ROC curve (AUC). We will present alternatives regarding the choice of the best model according to the company's specific goals, based on these criteria. In this sense, we look for balanced results in the two groups and globally.

We use two large databases, provided by an insurance company, that contain relevant information about policies at the individual level. All analyses are performed using the R language.

The structure of this paper is as follows. In Section 2, we present the predictive models to estimate the probabilities of defection and the performance measures for the comparative analysis. In Section 3, we introduce the data used in our study. In Section 4, we show the main results. Finally, in Section 5, we summarize the main conclusions of this application.

2. Models issues

We are interested in modelling the probability of renewal for each policy based on the customer and policy observed features. Let Y_{ij} be the outcome random variable and assume that y_{ij} is its categorical observed response given by individual *i* and policy *j*, which gives us information about whether or not the policyholders renew their insurance policy with the company. That is, if the policy is renewed, the variable takes the value $y_{ij} = 1$ with probability p_{ij} and, otherwise, it takes the value $y_{ij} = 0$ with probability $(1 - p_{ij})$, where $p_{ij} \in (0,1)$. Furthermore, let $X_i' = (x_{0i}, x_{1i}, \ldots, x_{ki})$ be a vector of k covariates, associated with the ith policy. This vector contains information about the characteristics of interest.

The predictive models that are being implemented here are focused on a binary response (for further applications in actuarial science see Frees, 2009 and Frees et al., 2013). These methods estimate class probability p_{ij} for each policy which subsequently allows customers to be classified in two classes Y={Renew, Not renew}. Predicted classes are defined by comparing p_{ij} with different classification thresholds, $t \in [0, 1]$. For a fixed cut-off level t, the performance of each classifier is described in a confusion matrix where there are four possible outcomes, as shown in Table 1, which can be evaluated using the following statistical measures: sensitivity, specificity, false positive rate, false negative rate and accuracy (for more details see Fawcett, 2006).

Table 1: Confusion matrix for a given cut-off level t.

		Predicted	
		Not renew	Renew
Real	Not renew	True negative(TN)	False positive(FP)
	Renew	False negative(FN)	True positive(TP)

From Table 1 the following measures can be defined in order to assess the predictive performance of the classification models that are being implemented below:

- Sensitivity is a measure of the proportion of customers that are classified in the renewal group among those who renew their policies. So, this corresponds to TP/TP+FN.
- Specificity is a measure of the proportion of customers that are classified in the non-renewal group among those who cancel their policies. So, TN/TN+FP.

- Accuracy measures the proportion of clients that are correctly classified. Namely, (TP+TN)/(TP+FP+TN+FN)

2.1 Predictive models

In this section, we briefly describe logistic regression, decision trees and support vector machine methods, three popular approaches used for classification in this study. These classification techniques allow generating a score from 0 to 100 for each policy, i.e. a numerical measure for classifying each customer according to their propensity to defect or abandon, which means to leave the company instead of renewing the existing policy contract (see Kumar and Garg, 2013 and Vafeiadis et al., 2015 for a review on data mining techniques for churn prediction).

Logistic regression is a classic predictive model for dichotomous outcomes. Specifically, it is a particular case of a generalized linear model, which is composed by a random variable, a systematic linear predictor and a link function (for further details, see McCullagh and Nelder, 1989). This model is recognized for being simple to understand, easy to explain and their fitted values can be interpreted easily (see Frees et al., 2014). A main disadvantage of this method is the possibility of overfitting, that is, predictions are too accurate and therefore they cannot be extrapolated out of the sample.

Decision trees are characterized for being a powerful classification and visualization tools. In particular, we design a conditional inference tree using the “ctree” function of the party package of statistical software R. In each step, the algorithm evaluates the overall hypothesis of independence between the dependent variable and each covariate, or in other words, the association between Y_{ij} and each input X's is tested and a new split is generated (Hothorn et al., 2006). The main advantages of decision trees are that: they are easy to interpret, they can be used for clustering, they can handle nonlinear relations between features and classes, and they can also properly handle numerical and categorical inputs (see Friedl, 1997). Important differences between training and test samples give raise to poor robust results (see also Kim et al. 2005).

Support vector machine (SVM) models represent an alternative to the classical predictive methods. They were first introduced by Boser et al. (1992) and Cortes (1995) and they are useful for classification, regression analysis and outlier detection (Meyer, 2001). In this method, the data are mapped into a higher dimensional space where individuals’ classes are separated by an optimal separating hyperplane (Suykens, 1999, Meyer 2001; 2012 and Hornik et al., 2006). In both cases, a radial kernel handles nonlinear relations between class and attributes, and represents a reasonable first choice according to Hsu et al. (2003). Among others, some advantages of SVM in the context of classification are: the choice of the boundary separating groups becomes flexible when introducing kernels, good out-of-sample generalizations can be obtained and the method is practical in terms of producing scores. However, this approach is sensible to the adjustment of kernel parameters and may also be difficult to interpret.

2.2 Performance measures

From the evaluation measures presented in Section 2, we propose three different criteria in order to assess the performance of the classifiers.

The following indicators summarize in a single scalar value the performance of each model:
Criteria 1(C1): maximum (sensitivity + specificity)
Criteria 2(C2): maximum (accuracy)
Criteria 3(C3): maximum (AUC)

3. Data

The data sets are provided by a major insurance company based in Spain and they include clients who have one or more motor or home insurance policies. The databases do not share common customers, i.e, data sets have been designed so that customers with car insurance policies do not have home insurance policies and vice versa. Moreover, the records contain variables related to the policy and the customer as exposed in Table 2 and specific variables linked to the risk object as shown in Tables 3 and 4. For our purposes, we consider three key driver variables during the whole analysis: date of entry into force, cancellation date and the status of the policy.

Table 2: Some common variables used in the case study on modeling churn in motor and home insurance.

Related to	Variables
Policyholder	Sex, log(Age), Number of policies in force, Total number of policies in force in other lines of business insurance, Premiums paid
Policy	Policy status (C= cancelled, I= in force), New premium, Old premium, Change in premium (new vs previous), Discounts applied or Bonus-Malus level, Payment scheme, Mediator

Source: Own data set, 2015.

Table 3: Some variables used in the case study on modeling churn in motor insurance.

Related to	Variables

Object of risk	Vehicle's type, Main driver's age, Second driver (Yes , No), Power, Number of seats

Source: Own data set, 2015.

Table 4: Some variables used in the case study on modeling churn in home insurance.

Related to	Variables
Object of risk	Type of home, continent, content, home size

Source: Own data set, 2015.

The study is carried out in the usual two phases: pre-processing and modelling process. Data pre-processing is an extensive but necessary process previous to variable selection and model implementation that includes: identification of outliers, appropriate transformation and creation of new variables from combining existing ones. In general, some good rules of thumb are: to make a manual selection of features to be kept, to ignore all those variables that provide a priori information about the possible decision of the insured, to avoid variables that are interrelated and to discard unnecessary predictors.

In this illustration, two samples are considered, one for each line of business. The motor data set has about 80% proportion of policies in force versus 20% cancelled, while the home data set has 90% of current policies versus 10% of cancelled policies. In both cases, we decide to use complete-case analysis (for further details, see Gelman, 2006), so cases with missing information are deleted. This gives rise to a total of 11, 000 motor and 4,000 home policies, approximately.

We compare the predictive accuracy of our models by dividing the former samples randomly into two groups (namely, the training set and the test set) in a ratio of 70% - 30% respectively, that is, we perform the analysis on the training set and validate it on the test set. With the modelling results, we generate a confusion matrix for each case taking into account all possible cut-offs t and we calculate the different criteria proposed in the sub-section 2.2.

4. Results

In Figure 1 we show the results based on the area under the ROC curves for the test sample of each business line. In the case of automobile insurance, we obtain similar results for all classifiers, although the support vector machine model slightly outperforms the other methods. On the other hand, we note that for home insurance the discriminating ability is clearly higher for the support vector machine than for the other methods. Likewise, both methods have an approximate probability of misclassification equal to 26%.

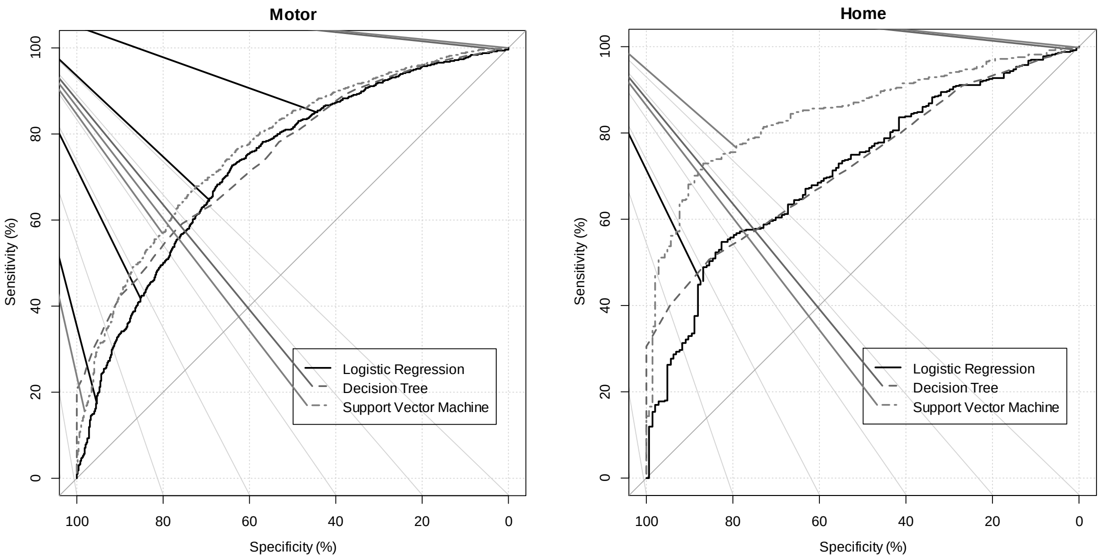

Figure 3: Intersecting ROC curves

Tables 3 and 4 show the results for the motor and the home insurance data sets, respectively. For each method we obtain the threshold t which maximizes C1 and C2. In addition, to compare models, we calculate C3 that does not depend on t. For motor insurance we find similar results in the performance measures using the different criterion, although we find slight differences regarding the support vector machine model. Finally, the largest area under the curve corresponds to the support vector machine approach, followed by conditional tree and the logistic regression method. For home insurance, where the percentage of policyholders who renew their policy is over 90%, support vector machine considerably improves the results obtained with the logistic regression and the conditional tree. In Table 6, it is interesting to note that the results of C1 and C3 for support vector machine improves considerably those obtained with logistic regression and conditional tree; however, the results of C2 are still the same, i.e. the overall percent of correct classification is similar for all the methods.

Once the predicting models are applied and based on the predicted churn probability for all the cases, we must decide which groups of customers have to be the ones to be addressed when implementing specific actions in order to increase the odds that they remain with the company. So, the optimal selection of a threshold is the next key step to be considered. The last two rows of Tables 5 and 6 give us possible alternatives about the cut-off point choice.

Table 5: Performance criteria for each model in the test data set for motor insurance

		Model		
		Logistic regression	**Conditional Tree**	**Support vector machine**
Criterion	C1	1.36	1.35	1.40
	C2(%)	79	79	80
	C3(%)	73	75	77
Optimal threshold	C1	0.77	0.78	0.96
	C2	0.56	[0.39,0.54]	0.81

Source: Own calculations

Table 6: Performance criteria for each model in the test data set for home insurance.

		Model		
		Logistic regression	**Conditional Tree**	**Support vector machine**
Criterion	C1	1.35	1.36	1.53
	C2(%)	92	92	92
	C3(%)	71	73	85
Optimal threshold	C1	0.92	[0.89,0.93]	0.97
	C2	[0.01,0.37]	[0.01,0.76]	[0.01,0.88]

Source: Own calculations

It should be noted that, in Table 6 we see wide threshold intervals and the same values for criterion C2 are obtained. These results are related with disproportionate levels of sensitivity and specificity. As we said in Section 3, we generate a confusion matrix for each possible cut-off, so we can find the evidence of such disparity.

Conclusions

In the previous section we used three different churn-modelling methods to predict the probability of renewal of motor and home insurance policies. From the estimated probabilities we generate a score, which is useful to predict customer groups in order to identify the ones that seem to be likely to leave the company. Selecting the best model is not a trivial choice, at least for car insurance, while for home insurance it seems to be easier. However, independently of the business line, the support vector machine is as good as the other methods or outperforms them in these samples and with all the criteria.

The preference for one model or another should be linked to the selection of the criterion that gives a classification as equilibrated as possible of the clients that stay and those who leave the company. But the criterion is not necessarily unique.

Performance criteria have one or more associated thresholds that can serve as a guide for customer classification. In this sense, threshold selection must be performed carefully and it must be compared with the sensitivity and specificity obtained in each case. Otherwise this would lead to poor classifications. The fact that the insured with home insurance have over 90% renewal rates, affects the results obtained with C1 and C2. In this sense, in the case of C1 the only method that finds an equal percent of sensitivity and specificity is the SVM. Moreover, the maximum percentage of correct classification (C2) obtained with each of the three models is achieved when the sensitivity is exactly equal to 100% and specificity is equal to 0%, so that if we chose the thresholds associated with the maximization of C2, then all customers would be classified in the renewal group and so, we would expect that they all continue the contractual relationship with the company for the following year. We consider that the prediction that none would cancel the policy is nonsense.

Further research should include novel metrics for performance comparison in order to evaluate the results of the predictive models. We suggest trying different time-frame renewal intervals, with the purpose of adapting the previous analysis to the characteristics of each business line. Seasonality is a fundamental aspect to consider when defining intervals of renewal for insurance policies, because customers tend to behave differently in specific months (i.e. before holidays and so on). Moreover, a regular monitoring of outcomes and cut-offs would be necessary.

In future work, we plan to repeat this analysis with clients who have contracted both auto and home policies simultaneously and we will model the insurance company expected losses as well.

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On the Effects of Considering Realistic Constraints for Freight Delivery Routes in Mountainous Regions

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Abstract

A careful planning of the delivery routes for freight transportation in mountainous regions must consider several particularities: remote areas cannot be accessed by large trucks and the costs associated to traveling may differ significantly depending on the sense taken. In this work, we present a solution method based on successive approximations for solving the site-dependent capacitated vehicle routing problem with heterogeneous fleet. We test the proposed heuristic on a set of benchmark instances. We offer several comparisons between different solutions and metrics, including an estimate of the CO_2 emissions.

Key words

Heterogeneous Site-dependent VRP; Successive Approximations Method; Clarke-and-Wright Savings algorithm.

1. Introduction

The delivery of goods plays an important role within the logistics and transportation sector. With the increase of demand, promoted by the growth of e-commerce or the development of new logistic strategies such as just-in-time, the number of batches to deliver also grows. Therefore, an accurate planning of the delivery routes is extremely important.

In this paper we focus on the planning of routes for the delivery of goods in mountainous regions. Mountainous regions may have special characteristics related to the topography. Some of the customers may be accessible only by regional roads, or after crossing mountain passes which in winter times may require that vehicles are equipped adequately. City centers, which have narrow streets and limited parking areas, may have even harder driving conditions (such as streets with slopes). All these characteristics limit the type of vehicles that can access certain areas. In particular, large trucks may be unable to access downtown areas or may experience difficulties in driving up and down the mountains. In such situation smaller vehicles seem more appropriate to be used to serve such customers.

The classical Capacitated Vehicle Routing Problem (CVRP) with heterogeneous fleet is extended in order to incorporate the condition that some customers cannot be served by a particular type of vehicle. The particular characteristic of this application is that most of the customers are accessible for any type of vehicle except a reduced subset. This defines the Site-Dependent Vehicle Routing Problem, SDVRP.

The vast majority of the literature dedicated to solve VRPs assumes a symmetric cost matrix, thus neglecting the cost differences associated to drive in one direction or the other. It is clear that in mountainous regions, there can be large differences in cost if the route is uphill or downhill. The variant considered in this work looks for routing all the nodes using an heterogeneous fleet within an oriented network (asymmetric costs) and ensuring that customers and delivery vehicles are compatible (site-dependent), HSDA-VRP (Heterogeneous Site-Dependent with Asymmetric costs VRP).

The contributions and aims of this work are: (1) to present a heuristic procedure to solve the presented problem, and (2) to compare the different solutions obtained with symmetric and asymmetric costs. We also provide an estimate of the CO_2 emissions for the resulting routes.

The rest of the paper is organized as follows. Section 2 reviews the vehicle routing problem and its extensions, and the solution approaches presented in the literature. It follows Section 3 with the description of the problem. In Section 4 we present our choice for solving this problem. Section 5 presents the benchmark instances used to test the model and compares different solution configurations. To conclude, the main findings are enumerated together with future extensions to this work

2. Literature Review

In this section, we first provide a general picture of the VRP and its variants and later we discuss some of the solution approaches.

2.1 The Vehicle Routing Problem and its variants

The Vehicle Routing Problem is a classical Combinatorial Optimization Problem (COP) present in many applications. The most popular VRP variant is the Capacitated Vehicle Routing Problem. Given a fleet of identical vehicles located at a depot, the goal is to provide routes to supply a set of customers with known demand so the total delivery cost is minimized. Each customer can be visited only once and each vehicle covers only one route. The total demand served by a particular route cannot exceed the vehicle capacity.

The VRP has been extended in many directions in order to incorporate more realistic characteristics. Regarding the properties of the fleet, this can be homogeneous (all vehicles have the same characteristics) or heterogeneous (include a combination of vehicles with different capacities), vehicles can have a limited driving range [2] or drivers have scheduled breaks [3]. The network structure may be defined based on symmetric costs (usually Euclidean distances are used) or asymmetric when other factors such as fuel consumption or time used define the cost structure [4]. In the multi-depot VRP, customers are served from one of the several depots. Regarding the quality of service, there can be conditions on the time when the customers can be visited, *i.e*. time-windows are defined for each customer (VRPTW), or the customers are not only receivers of demand but there is pick-up and delivery (VRPPD). In the site-dependent VRP (SDVRP), a subset of customers is incompatible with some vehicles of the fleet. For an extensive review of these and other VRP variants and applications, the reader is referred to [5]–[10].

2.2 Solution Approaches

Different approaches have been proposed to solve vehicle routing problems, both exact methods and heuristics. Regarding exact methods, Kallehauge [11] reviews several formulations of the VRP and exact solution methods. Baldacci *et al* [12] provide different formulations of two classes of VRP variants. The set partitioning formulation shows to be the most appropriated when exact methods are used which combined with column-generation methods solve to optimality instances of up to one hundred customers.

Since the SDVRP belongs to the NP-hard category of problems [13], heuristic and metaheuristic algorithms such as Tabu Search, Variable Neighborhood Search, Scatter Search or Genetic Algorithms are usually used in practice. The description of site-dependent VRPs was first proposed in [14] and presented several simple heuristics to solve them. Chao *et al* [15] developed a Tabu Search heuristic enhanced with deterministic annealing to widen the search space. Cordeau *et al* [13] derived the SDVRP as a special case of the periodic VRP and solved it with a Tabu Search heuristic for the periodic VRP. Pisinger *et al* [16] presented a unified heuristic based on the Adaptive Large Neighborhood Search framework capable to solve five variants of the VRP, being the site-dependent VRP one of them. To the best of our knowledge, there are few contributions for solving the VRP combining asymmetric costs with heterogeneous fleet and site-dependent characteristics. Recently, Yusuf [17] has presented a three-phase heuristic for solving the routing of ships considering two objectives, accessibility and profitability, as a multi-depot heterogeneous site-dependent VRP with asymmetric costs. The method proposed falls into the 'cluster first and route second' category.

3. Description of the Problem

The HSDA-VRP is a natural extension of the classical capacitated VRP. Formally, the HSDA-VRP is defined over a complete graph $G = (N, A)$ where $N = \{0,1,\ldots,n\}$ is the set of nodes where node 0 is assumed to be the depot and the rest represent the customers that need to be visited. Each customer has a (deterministic) demand d_i. The set $A = \{(i,j): i,j \in N, i \neq j\}$ contains the arcs that represent the road network. The set *A* considers that any pair of nodes is connected, and if this differs from the reality a large cost will be associated to it. The cost of travelling on the arc (i,j) is represented by C_{ij}. In the heterogeneous version, the cost of going from i to j is different from the cost of going from j to i. This cost can be related to distance, travelling time, fuel costs or other measures depending on the interest of the application.

Customer demands are carried by one of the vehicles available in the fleet. The set F includes all available vehicles. Each vehicle k will have particular characteristics. Most notable, the total cost of travelling from *i* to *j* will not only depend on the cost of the arc but also depends on the type of vehicle used. The cost of the arc is multiplied by a factor that depends on the vehicle type, v_k, (larger vehicles have higher variable cost than smaller ones), being then the total cost of the arc a three-index parameter, $C_{ij}^k = v_k \times C_{ij}$. Moreover, each route includes a fixed cost for using a vehicle, f_k. Therefore, the cost of a route corresponds to the sum of the fixed and variable costs of the arcs belonging to the route. Parameter Q_k denotes the maximum load that vehicle k can carry. Vehicles can serve only compatible customers, C_k, where $C_k \subseteq N \setminus \{0\}$ is the set of nodes that vehicle k can reach.

The AHSD-VRP goal is to find the routes that will serve all customers demand and minimize the total cost of travel. This problem can easily accommodate the main characteristics present when planning routes for freight transportation by road in mountainous regions: some type of vehicles may not be able to serve a subset of particular clients (thus requiring a fleet with multiple types of vehicles) and travelling costs depend on the direction of the route and type of vehicle. For a complete mathematical formulation of this problem, the reader should refer to [1].

4. Successive Approximations Method Algorithm

We are interested in taking advantage of the existing algorithms for solving simpler versions of the VRP. The solution approach that we propose is based on the Successive Approximations Method (SAM), based on the ideas presented in [1]. The SAM algorithm is a multi-round process in which a solution is constructed in several steps using a randomized version of the classical Clarke & Wright Savings heuristics. In this version, the sorted savings lists is calculated using the weighted average of arcs (i, j) and (j, i) in order to consider asymmetric costs as in [4].

The number of rounds is limited by the number of types of vehicles. At each round a vehicle type is selected and routes are built to serve the subset of compatible nodes. When more routes than available vehicles are scheduled, a subset of routes is randomly discarded so no more vehicles than the available ones are used. Such routes are saved as a partial solution, and the nodes belonging to the discarded routes are extracted so they can be routed in the next round. In successive rounds, another type of vehicle is selected and a new VRP is solved assuming an unlimited number of vehicles. Side conditions (such as banning vehicles from visiting incompatible customers or the vehicle capacity) are ensured within the SAM algorithm. Improvements on the routing costs due to the orientation of the routes are done locally after the assignment of the nodes that a vehicle with visit is done.

Figure 1 shows the pseudocode of the SAM algorithm used to solve the heterogeneous, asymmetric and site-dependent VRP. In addition, we estimate CO_2 emissions for the best solution found by our procedure. In order to have accurate estimations, we apply the model proposed in [19]. This model proposes that emissions are function of vehicle load and distances. It is to note, that at this stage, any other valid method to calculate CO_2 emissions could be applied.

```
Procedure SAM (Nodelist, Veh_Types, maxTime)
    initialize (PartialSol, CurrentSol, BestSol)
    While (maxTime not reached)
        initialize(UnroutedNodes)
        While (UnroutedNodes.size>0)
            Select aVehType from Veh_Types
            CompatibleNodeList <- Selectcompatiblenodes(UnroutedNodes, aVehType)
            PartialSol <-applyRandCWS(CompatibleNodeList, aVehType)
            If (#Routes in PartialSol >= #vehicles of aVehType)
                CurrentSol <- CurrentSol.add(removeExcedingRoutes(PartialSol))
                Veh_Types <- Veh_Types.remove(aVehType)
            else
                CurrentSol <-CurrentSol.add(PartialSol)
            end If
            Update (UnroutedNodes)
        end While
        BestSol<-BestSol.update(CurrentSol)
        CurrentSol <- curentSol.remove(RandomRoutes)
        Update (UnroutedNodes)
        Update (Veh_Types)
    end While
    return BestSol
end Procedure
```

Figure 4 Pseudocode of the SAM for the asymmetric, heterogeneous and site-dependent vehicle routing problem.

5. Experiments

The aforementioned procedure was coded as a Java application. In order to test the potential of our method we carried out three different experiments: (i) homogeneous VRP with asymmetric instances, (ii) heterogeneous VRP with asymmetric distances and, (iii) heterogeneous VRP with asymmetric distances and site-dependency (*i.e.* some customers cannot be served by the vehicles with the largest capacity). Each experiment was carried over all selected instances using 5 different random seeds and was executed using a 2.4 GHz core i5 personal computer with 8 GB RAM. The execution time was established in 120 seconds per run.

5.1 Test Cases

In order to perform the tests described above, we have randomly selected 4 instances with Euclidean distances from classical Homogeneous VRP instances available at [20]. These instances have been modified to generate AHSD-VRP. These modified instances are available upon request to the corresponding author of this paper.

5.2 Results and Analysis

Table 1 summarizes, for each instance, the results obtained during the experiments. For each instance we report the best solution obtained in terms of total costs (vehicle fixed cost + vehicle variable cost) and its corresponding distance-based cost. Note that our objective function is the minimization of the total costs, but we used the distance-based cost only for comparison purposes with respect to the best known solutions for the symmetric case.

The left side of the table allows the comparison of the asymmetric and symmetric versions of the homogeneous VRP. In this case, since we have only one type of vehicles, the optimization of the total costs has the same solution than the optimization of routing distances. Our mean gap in terms of distance cost is 0.20% which shows the competitiveness of our approach.

The right side of the table shows the results obtained for the heterogeneous VRP with and without site-dependency, which corresponds to experiments (ii) and (iii) respectively. In the case of non-site dependency, we can see that, in average, the heterogeneous version outperforms the homogeneous version in terms of total costs (average gap of -1.47%) while the distance cost is 8.89% higher, in average. Since there are less available vehicles with larger capacities, solutions are forced to use smaller vehicles (with lower fixed and variable costs) and to perform more trips. In the case of site-dependency (see Table 2), which restricts even more the usage of larger trucks, the total costs increases (average gap of 2.89%) with respect to the homogeneous case, while the distance costs increase, on average 20.71%. Figure 2 shows the differences among the best solutions obtained for a given instance when using symmetric and asymmetric costs. Variations in the sense of traveling of some routes combined with different route configurations can be appreciated due to the inclusion of this more realistic assumption.

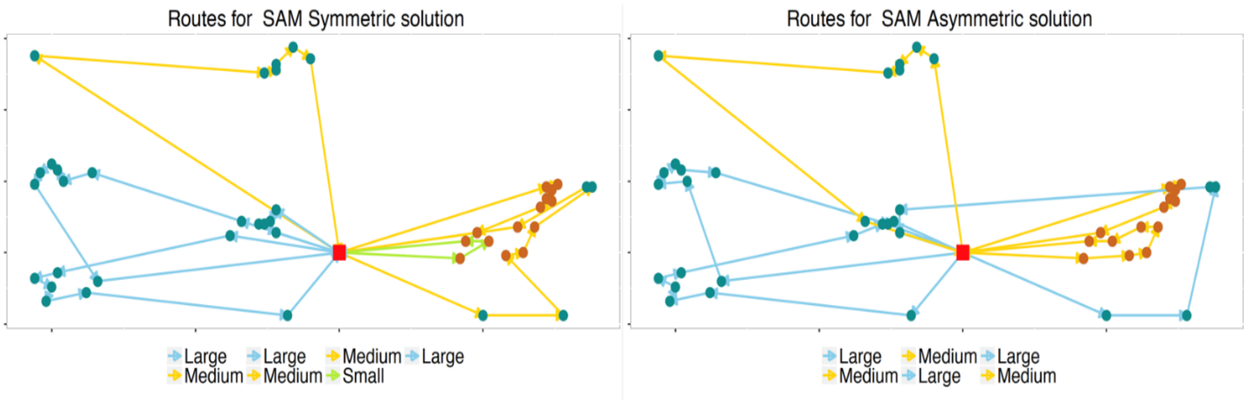

Figure 5 Graphical comparison of OBS for instance B-n45-k5 among symmetric (left) and asymmetric (right) costs.

6. Conclusions and Future Enhancements

We have introduced an efficient, fast and easy to implement multi-round algorithm for planning goods delivery in mountainous regions with heterogeneous fleet. This situation is represented in this article by the so-called Heterogeneous Site-Dependent with Asymmetric Costs Vehicle Routing Problem (HSDA-VRP). The proposed algorithm is based on a randomized version of the CWS heuristic which assigns a higher probability of being chosen to the most promising movements. Preliminary tests carried out show that our approach seems promising in order to solve more realistic versions of the VRP.

Table 6 Summary of results without site dependency

Instances	Distance based costs (symmetric)	Homogeneous Case					Heterogeneous Case																	
							Instance Information			Non-site Dependent							Site Dependent							
	BKS	Avail Trucks	OBS Total Cost (2)	OBS Distance-based Cost (3)	Used Trucks	GAP% (3)-(1)	Available Trucks / Capacity			OBS Total Cost (4)	OBS Distance-based Cost (5)	GAP% (4)-(2)	GAP% (5)-(3)	Used trucks			OBS Total Cost (6)	OBS Distance-based Cost (7)	CO_2 emis.	GAP% (6)–(2)	GAP% (7)-(3)	Used trucks		
							T1	T2	T3					T1	T2	T3						T1	T2	T3
P-n40-k5	461.73	8	2115.45	471.82	5	2.18	4 / 140	2 / 105	3 / 70	2045.87	475.62	-3.29	0.81	4	1	0	1988.35	559.52	487.37	-6.01	18.58	3	2	1
B-n41-k6	834.92	13	3089.90	829.97	6	-0.59	5 / 100	6 / 75	6 / 50	3095.45	975.26	0.17	17.51	3	4	0	3520.48	1075.19	942.84	13.93	29.54	4	4	0
B-n45-k5	754.22	8	2729.91	743.30	5	-1.44	4 / 100	3 / 75	3 / 50	2579.33	791.69	-5.51	6.51	3	3	0	2719.50	888.42	781.52	-0.38	19.52	2	4	1
A-n80-k10	1766.50	16	6334.66	1778.22	10	0.66	8 / 100	6 / 75	6 / 50	6508.37	1969.47	2.74	10.76	8	3	0	6588.70	2048.27	1818.41	4.01	15.19	6	5	0
		AVERAGE GAP				0.20						-1.47	8.89							2.89	20.71			

Further research efforts could be oriented to include real-life data or conditions (*i.e.* customer locations, real distance-based costs, other vehicle types and associated costs, uncertain demand, uncertain travel times, etc.)

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Heating Uniformity Evaluation in Domestic Cooking using Inverse Modelling

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Abstract

The heating uniformity of a domestic cooker is decisive for a proper food elaboration. In this work the use of inverse thermal modelling for the calculation of the power distribution transferred from a domestic cooker to the pan is presented. The most common cooking stoves (induction, electric resistance and gas), have been used. The proposed method using Cartesian coordinates, a rectangular thin metallic piece and an infrared camera, improves the accuracy and reduces the computational cost with respect to previous works. Moreover, it can help to improve the design process of cookers, providing more reliable results than current methods based in experimental tests with real food.

Key words

Thermal modelling, inverse modelling, power distribution, domestic cookers

1. Introduction

Manufacturers of domestic cooking appliances are continuously trying to improve the performance of their products in order to offer better services to the users at lower cost. Apart from the energy consumption, efficiency and cost, one of the main characteristics which determine the good performance of the cookers is the uniformity of the temperature distribution generated in the bottom of the cooking vessel. A uniform heating ensures a good cooking result, whereas an uneven heating can originate a partial burning of the food in contact with the hottest zones of the pan. For this reason, the evaluation of the heating uniformity of each cooker during the design process becomes a necessity.

As known, in the current domestic cooktops different technologies are used as heating elements, essentially gas burners, electric resistors, or inductors. With each technology the heat is transferred to the cooking vessel by a different phenomenon, which determines the temperature uniformity and the cooking result. Current producers carry out the evaluation of the heating performance of such different systems by means of experimental tests, cooking real food [1]. These tests are costly and not easily reproducible, since the results are sensitive to the ingredients and the food composition. Thus, to determine the heating uniformity by means of other methods is of high interest. A direct method is the theoretical calculation of the power distribution generated by each cooker considering its particular configuration (geometry, power supply, etc.). For example, in the case of gas burners, by means of complex combustion models including chemical reactions and computational fluid dynamics (CFD) a heat transfer distribution in the pan base can be estimated [2]. This distribution is highly dependent on the velocity of the gas flowing through the burner outlet, which increases the complexity of the problem. In the case of electric stoves with resistors, the heat is transferred to the pan through the contact with the ceramic glass, which is heated by radiation from the hot resistance. The solution of the radiative heat transfer is typically complex to obtain due to the strong dependence with the geometry of the system and the nonlinearity of the problem [3]. Finally, the induction heating used in modern electric cooktops has been studied in several works [4]–[6]. Most of them have carried out numerical calculations of the coupled electromagnetic-thermal problem using finite element methods (FEM), which require high computational cost.

In order to establish a method which allows the comparison of the heating uniformity of the cookers, regardless the type of heating element, an inverse model can be used [7]. The inverse modelling has been widely used in heat transfer applications [8]–[10]. The main advantage which offers is the possibility of calculating non-measurable parameters such as the heat flux, heat transfer coefficients or temperatures in inaccessible locations, from the measurement of the temperature in determined positions. Examples applied to gas burners have been presented in [11], [12], in which the steady-state power distribution of the flame on the heated material is obtained with an inverse method based in analytical expressions of the temperature evolution. The applied inverse method is based in iterative calculations of the heat flux using a least squares fitting method.

In the present work, a numerical method based in finite differences is used to solve the inverse problem and compute the power distribution. This method allows the calculation of the power distribution during the transient and the steady-state modes, with different common technologies, such as inductors, gas burners and electric stoves. A heated piece with rectangular geometry is considered, and a Cartesian coordinate system is used to develop the finite differences method. Thus, the accuracy of the calculated power distribution is increased and the numerical problem is easier to implement and solve. Experiments using a gas cooker, an electric stove with a resistor and an induction hob are carried out, and the power distribution which heats the cooking vessel is calculated with the inverse model. The obtained results can be used to establish comparisons between the heating performance and to evaluate the suitability of each cooker to guarantee good cooking results.

2. Inverse Thermal Model

In [7] we presented the inverse modelling of pan heating in domestic cooktops, considering the most common pans with round shape. The model was numerically solved using a finite differences method in polar coordinates. The temperature distribution on the top surface of the bottom of the pan was measured using an infrared camera, and the calculated power distribution of different cookers was presented. One of the main problems found in the calculation of the inverse problem is the strong influence of the measurement error over the results. Though it can be mitigated using image filtering, the coordinate transformation of the original rectangular thermal images into polar coordinates also introduces a noise component which produces higher deviations in the results.

The most common cooking vessels in the world's market have a round shape. Therefore, the geometry of most cookers is usually circular in order to better adapting to the pans. However, in this work we consider that the shape of the heated piece is rectangular, and the size is larger than the cooker. Hence, the geometry of the piece has negligible influence on the calculated power distribution and it is independent of the shape of the cooker. The heated piece is modelled as a thin sheet of homogeneous ferromagnetic material with known thermal properties and geometry. The length in x direction is L_x, in y direction is L_y and the thickness is e.

The temperature evolution on each point of the heated piece, placed over a heat source, obeys the well-known heat equation:

$$P + \lambda \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) = \rho \, C_p \, \frac{\partial T}{\partial t} \tag{15}$$

where P is the equivalent volumetric power density generated by the heating source and transferred to the piece in W/m^3. In the term of heat conduction, the Laplacian of the temperature

(T) is expressed in Cartesian coordinates (x, y, z), λ is the heat conductivity of the material. The term on the right side is the variation of the energy stored in the system with time, where the material has density ρ and specific heat capacity C_p. The inverse modelling in this thermal problem consists in obtaining the value of the power density distribution in each point of the domain and each instant, from the temperature distribution on the surface, measured during a heating process (for example with an infrared camera).

Due to the small thickness of the piece, the temperature inside the material in the z direction is considered constant. On the top surface of the plate we consider that the heat flux in z direction is equal to the losses to the environment due to convection and radiation

$$-\lambda \frac{\partial T}{\partial z} = h\,(T - T_a)\Big|_{z=e},$$
$$h = h_{conv} + h_{rad} = h_{conv} + \sigma\varepsilon\,(T^2 + T_a^2)(T + T_a) \tag{16}$$

where T_a is the ambient temperature, h_{conv} is the convection coefficient on the plate, $\sigma = 5.67 \cdot 10^{-8}\ \mathrm{W/m^2K^4}$ is the Stefan Boltzmann constant and $\varepsilon = 1$ is the surface emissivity, considering that the radiant surface behaves as a black body and that the surrounding surfaces are at ambient temperature. The heat flux through the bottom surface is included in the equivalent power generation term P in the case of a gas burner and the radiative electric stove

$$-\lambda \frac{\partial T}{\partial z}\Big|_{z=0} = P \cdot e. \tag{17}$$

In the case of an induction hob the bottom surface is considered adiabatic, since the gap between the coil and the heated piece will be filled with an insulating material. The power generation term embodies the heat generated from the dissipation of the induced eddy currents and the losses due to magnetic hysteresis.

The boundary conditions in the borders of the heated piece are Neumann conditions, which specify the heat flux considering a heat losses coefficient h_b including convection and radiation

$$\lambda \frac{\partial T}{\partial x}\Big|_{x=0} = h_b(T - T_a)\,|_{x=0}, \qquad \lambda \frac{\partial T}{\partial y}\Big|_{y=0} = h_b(T - T_a)\,|_{y=0},$$
$$-\lambda \frac{\partial T}{\partial x}\Big|_{x=L_x} = h_b(T - T_a)\,|_{x=L_x}, \qquad -\lambda \frac{\partial T}{\partial y}\Big|_{y=L_y} = h_b(T - T_a)\,|_{y=L_y}. \tag{18}$$

In order to calculate the power distribution, the problem is discretized applying the finite differences method, using a rectangular grid. The sizes of each element i,j are $\Delta x = \mathrm{L}_x/n$, $\Delta y = \mathrm{L}_y/m$, where n, m are the number of elements in the grid in the x, y directions, respectively. A central differences scheme is used for the discretization of the spatial derivatives and a regressive approach is used to discretize the time, obtaining an implicit differences method which ensures stability and convergence. The size of the time step selected in the discretization of the temporal dimension is Δt. The power distribution in each element $P_{i,j}$ for each instant is related with the temperature measured on each element $T_{i,j}$ with the expression

$$P_{i,j} = a_{i,j}\,T_{i,j} + b_i^+\,T_{i+1,j} + b_i^-\,T_{i-1,j} + c_j^+\,T_{i,j+1} + c_j^-\,T_{i,j-1} - \frac{\rho C_p}{\Delta t}\,T_{i,j}^{\tau} - a_{i,j}^*\,T_a \qquad (19)$$

where $T_{i,j}^{\tau}$ is the temperature of the element i,j in the previous instant. The coefficients $a_{i,j}, a_{i,j}^*, b_i^+, b_i^-, c_j^+, c_j^-,$ take different values in the boundaries and inside the grid, see Table 7. The effect of the radiation increases nonlinearly with the temperature of the material, as seen in (16). Thus, the value of the losses factor is recalculated on each iteration, using the temperature values from the previous instant.

Table 7. Coefficients in the equation of the inverse model.

Index	$a_{i,j}$	$a_{i,j}^*$	b_i^+	b_i^-	c_j^+	c_j^-
$1 < i < n$, $1 < j < m$	$\frac{2\lambda}{\Delta x^2} + \frac{2\lambda}{\Delta y^2} + \frac{h}{e} + \frac{\rho\, c_p}{\Delta t}$	$\frac{h}{e}$	$-\frac{\lambda}{\Delta x^2}$	$-\frac{\lambda}{\Delta x^2}$	$-\frac{\lambda}{\Delta y^2}$	$-\frac{\lambda}{\Delta y^2}$
$i = 1$, $1 < j < m$	$\frac{2\lambda}{\Delta x^2} + \frac{2\lambda}{\Delta y^2} + \frac{h}{e} + \frac{\rho\, c_p}{\Delta t} + \frac{2\, h_b}{\Delta x}$	$\frac{h}{e} + \frac{2\, h_b}{\Delta x}$	$-\frac{\lambda}{\Delta x^2}$	0	$-\frac{\lambda}{\Delta y^2}$	$-\frac{\lambda}{\Delta y^2}$
$i = n$, $1 < j < m$	$\frac{2\lambda}{\Delta x^2} + \frac{2\lambda}{\Delta y^2} + \frac{h}{e} + \frac{\rho\, c_p}{\Delta t} + \frac{2\, h_b}{\Delta x}$	$\frac{h}{e} + \frac{2\, h_b}{\Delta x}$	0	$-\frac{\lambda}{\Delta x^2}$	$-\frac{\lambda}{\Delta y^2}$	$-\frac{\lambda}{\Delta y^2}$
$i = 1$, $j = 1$	$\frac{2\lambda}{\Delta x^2} + \frac{2\lambda}{\Delta y^2} + \frac{h}{e} + \frac{\rho\, c_p}{\Delta t} + \frac{2\, h_b}{\Delta x} + \frac{2\, h_b}{\Delta y}$	$\frac{h}{e} + \frac{2\, h_b}{\Delta x} + \frac{2\, h_b}{\Delta y}$	$-\frac{\lambda}{\Delta x^2}$	0	$-\frac{\lambda}{\Delta y^2}$	0
$i = n$, $j = 1$	$\frac{2\lambda}{\Delta x^2} + \frac{2\lambda}{\Delta y^2} + \frac{h}{e} + \frac{\rho\, c_p}{\Delta t} + \frac{2\, h_b}{\Delta x} + \frac{2\, h_b}{\Delta y}$	$\frac{h}{e} + \frac{2\, h_b}{\Delta x} + \frac{2\, h_b}{\Delta y}$	0	$-\frac{\lambda}{\Delta x^2}$	$-\frac{\lambda}{\Delta y^2}$	0

$1<i<n$, $j=1$	$\frac{2\lambda}{\Delta x^2}+\frac{2\lambda}{\Delta y^2}+\frac{h}{e}+\frac{\rho\, c_p}{\Delta t}+\frac{2\, h_b}{\Delta y}$	$\frac{h}{e}+\frac{2\, h_b}{\Delta y}$	$-\frac{\lambda}{\Delta x^2}$	$-\frac{\lambda}{\Delta x^2}$	$-\frac{\lambda}{\Delta y^2}$	0
$1<i<n$, $j=m$	$\frac{2\lambda}{\Delta x^2}+\frac{2\lambda}{\Delta y^2}+\frac{h}{e}+\frac{\rho\, c_p}{\Delta t}+\frac{2\, h_b}{\Delta y}$	$\frac{h}{e}+\frac{2\, h_b}{\Delta y}$	$-\frac{\lambda}{\Delta x^2}$	$-\frac{\lambda}{\Delta x^2}$	0	$-\frac{\lambda}{\Delta y^2}$
$i=1$, $j=m$	$\frac{2\lambda}{\Delta x^2}+\frac{2\lambda}{\Delta y^2}+\frac{h}{e}+\frac{\rho\, c_p}{\Delta t}+\frac{2\, h_b}{\Delta x}+\frac{2\, h_b}{\Delta y}$	$\frac{h}{e}+\frac{2\, h_b}{\Delta x}+\frac{2\, h_b}{\Delta y}$	$-\frac{\lambda}{\Delta x^2}$	0	0	$-\frac{\lambda}{\Delta y^2}$
$i=n$, $j=m$	$\frac{2\lambda}{\Delta x^2}+\frac{2\lambda}{\Delta y^2}+\frac{h}{e}+\frac{\rho\, c_p}{\Delta t}+\frac{2\, h_b}{\Delta x}+\frac{2\, h_b}{\Delta y}$	$\frac{h}{e}+\frac{2\, h_b}{\Delta x}+\frac{2\, h_b}{\Delta y}$	0	$-\frac{\lambda}{\Delta x^2}$	0	$-\frac{\lambda}{\Delta y^2}$

The values of the losses coefficient h_{conv} and h_b will be estimated using measured data from experiments. Knowing the value of these coefficients and from the measured temperature, the power density supplied to each element can be obtained.

3 Analysis and Model Identification

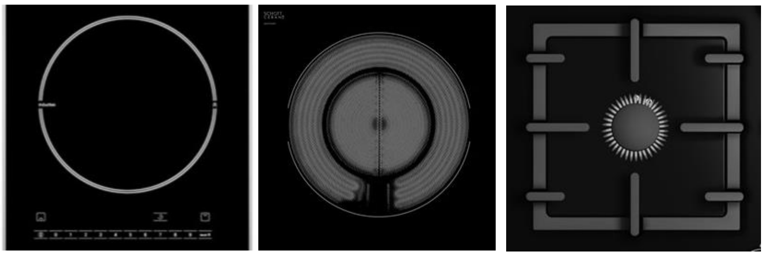
The model presented in the previous section requires knowing the evolution of the temperature on the top surface of the heated piece. In order to use the model to study real cookers, experiments with real stoves have been carried out, heating ferromagnetic steel sheets and measuring the temperatures with an infrared camera. The data is used to estimate the losses coefficients on top and borders of the piece.

3.1 Experimental Set-up

Three conventional cooktops are used to perform the experiments: an induction cooker with diameter of 210 mm, an electric stove with a double resistance with diameters 123 and 213 mm, and a gas cooker with 120 mm of diameter, Fig. 2. A sheet of laminated ferromagnetic steel of 415x380 mm and 0.5mm of thickness is heated with each cooktop. The thermal properties of the steel are $\lambda = 55\ \mathrm{W\ m^{-1}K^{-1}}$, $\rho = 7950\ \mathrm{kg\ m^{-3}}$ and $C_p = 500\ \mathrm{J\ kg^{-1}K^{-1}}$. The top surface of the piece is coated with a thermo-resistant paint with emissivity $\varepsilon = 1$. The measurement of the sheet temperature has been carried out with an infrared camera (FLIR A650), which records the temperature evolution at each point with resolution of 640x480 pixels, at 3 fps and accuracy of

± 2 ºC. The camera is located at a controlled distance and oriented perpendicularly to the metallic piece in order to maximize the image resolution.

The thermographies recorded with the infrared camera in the experiments with the induction hob, the electric stove and the gas cooker, contain on each pixel the mean temperature measured in the corresponding pixel area. This information must be processed before computing the inverse model with two operations: a homography and a noise-reduction filtering. Both operations have been performed using the Image Processing toolbox of Matlab. The homography is needed to compensate the image distortion introduced by the camera lens, and it is calculated using a standard algorithm. The noise-reduction filtering is necessary to improve the calculation of the temperature derivatives appearing in the inverse model, which have an amplification effect on the noise of the measured signal. In this work we propose to use a smooth filter proposed by Savitzky & Golay, [13], which is one of the most commonly used filters in image smoothing due to the improved results with respect to other filters, such as mean or median filters. The filtering is carried out in the two spatial dimensions and the filter is tuned in order to reduce the noise as much as possible.



INDUCTION **RADIATION** **GAS**

Fig. 2. Cookers employed in the experiments: induction hob (PIE375N14E), electric stove (3EE721LS), gas stove (PPQ716B91E).

3.2 Parameter Identification

In order to simplify the identification of the losses coefficients only the induction hob is used, because the input power can be measured and controlled with higher accuracy. A power analyser (Yokogawa PZ4000) is connected in the output of the power stage which feeds the inductor and the real heating power dissipated in the sheet is estimated as the 97% of the power measured, which is the typical efficiency of the inductor [14]. The ceramic glass which typically covers the inductor in an induction cooker is replaced with an insulating blanket of the same thickness, which is 4 mm. Thus, the bottom surface of the sheet is isolated and only the heat losses on the top surface and the borders take place.

In order to calculate the heat losses coefficient in the borders h_b, the previous expressions (18) are used, calculating the derivatives numerically from the measured temperature in the normal direction to each border during the heating process. The average value of the coefficient obtained from different experiments is $h_b = 236\ \mathrm{W/m^2K}$. On the other hand, the average heat loss coefficient on the top surface h_{conv} is obtained from a power balance considering the whole metallic sheet during a heating process with low power supply from the inductor, which allows for reaching a steady state at temperatures below 200 ºC. At the steady state the power supplied, which is measured with the power analyser, equals the heat losses. This can be formulated as

$$P_T - \sum_{\substack{1\le i\le n\\1\le j\le m}} \left(h_{conv}(T_{i,j} - T_a) + \sigma\varepsilon(T_{i,j}^4 - T_a^4)\right)\Delta x\Delta y - \sum_{\substack{1\le i\le n\\j=1,m}} \left(h_b(T_{i,j} - T_a)\right)e\Delta x - \sum_{\substack{i=1,n\\1\le j\le m}} \left(h_b(T_{i,j} - T_a)\right)e\Delta y = 0 \quad (20)$$

where P_T is the total measured power in W. Solving the equation, the average value of the convection coefficient obtained is $h_{conv} = 9.55\ \mathrm{W/m^2K}$. An example of the fitting obtained is shown in Fig. 3. It can be seen how the total power losses approach to the power supplied as the steady state is reached. Moreover, the magnitude of the losses due to radiation and convection on the top surface is similar, and the losses in the borders can be neglected, as expected due to the slight thickness of the sheet.

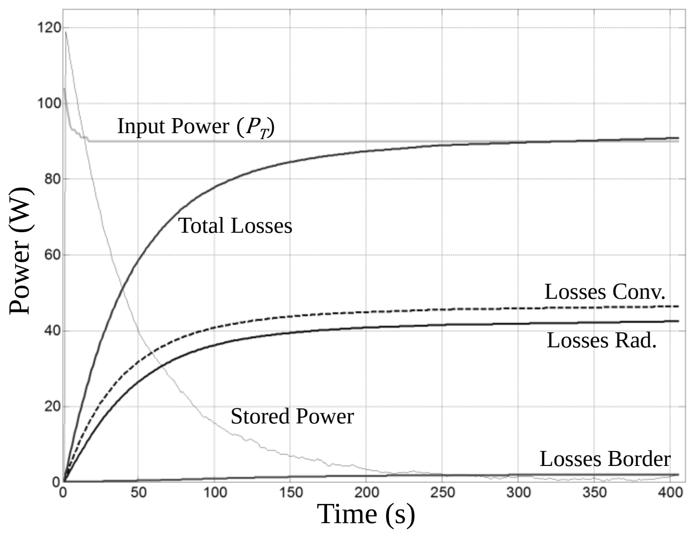


Fig. 3. Example of the power balance during the heating process up to a stationary regime using the calculated values of the heat losses coefficients.

4 Power Distribution of Domestic Cookers

The power density distributions in different heating situations with the three technologies considered in this work are calculated with the proposed inverse model. The results obtained are shown in Fig. 4, where the power distributions in the metallic piece using the induction cooker, the electric stove with a double resistance and the gas cooker are presented. The obtained results are similar to the ones presented in [7]. However, the processing of the thermographies and the calculation is simpler and more accurate.

The calculated power distributions can help to analyse the influence of the configuration of each cooker on the heating uniformity. For example, in the case of the induction hob, the power distribution is null in the centre of the coil and has two maximums in the radial direction, which are produced by the particular distribution of wire turns of the coil. It is also observed the effect of the 8 ferrite bars beneath the coil, which amplify the magnetic field over its position, increasing the induced currents. In the double cooking zone of the electric stove, the power distribution shows a peak in the central area of the plate, originated by the higher power density radiated by the internal resistance. It can be also observed the effect of the gap between resistances, which produces a low-power area with circular shape. The power distribution transferred in the gas cooker shows a maximum where the temperature of the flame is higher, and a large central zone in which the burnt gas has a lower incidence. The metallic structure which supports the sheet above the burner isolates the bottom surface from the flame and generates gaps in the power distribution.

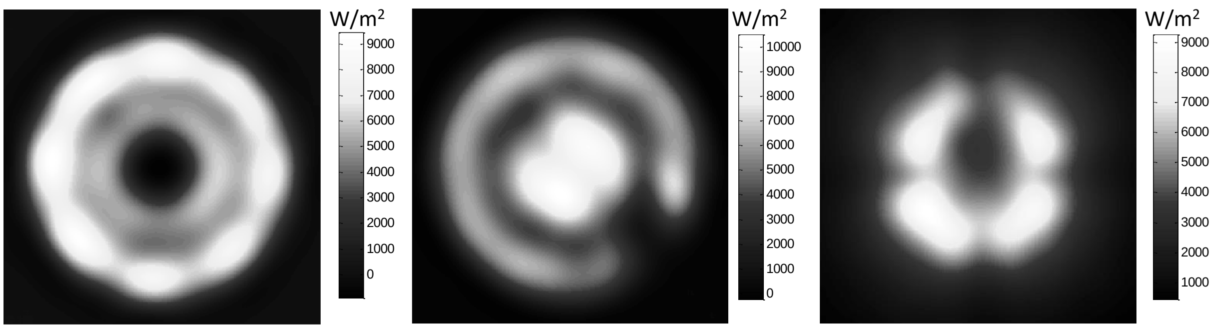


Fig. 4. Power distributions of an induction cooker (*left*), an electric stove (*middle*) and a gas cooker (*right*), calculated with the proposed inverse model.

Conclusions

In this work the use of the inverse thermal modelling for the calculation of the power distribution transferred from a cooker to the pan has been presented. The most common cooking stoves, with different heating technologies (induction, electric resistance and gas), have been analysed. The proposed method, using a rectangular thin metallic piece and an infrared camera can be used to improve the accuracy and to simplify the calculation, with respect to previous works in which a polar geometry was employed. The proposed method can help to improve the design process of cookers, providing more reliable results than methods based in experimental tests with real food.

Acknowledgments

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