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Prioritized induced heavy operators in open government metric in Mexico

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ABSTRACT

This operator combines an unbounded weighting vector, an induced vector and a prioritized vector in the same formulation that can be applied to the group decision-making process where the information provided by each decision maker does not have the same importance. Additionally, the characteristics and families of the PIHOWA operator are established, and the combination of the PIHOWA operator with the Open Government Metric (OGM) formula is developed. Finally, the article ends with an application of these formulations to rank the top 10 most transparent states in Mexico for 2017.

1. INTRODUCTION

In Mexico, the National Institute of Transparency (INAI for its Spanish acronyms) is a specialized public institution that analyzes transparency at the national level, including access to information and protection of personal data. Since 2007, every three years an evaluation of every state is performed to score their transparency [9]. To do this, an Open Government Metric (OGM) formula has been developed by the institution that takes into account 4 different elements: subindex of participation from the government's perspective, subindex of participation from the citizen's perspective, subindex of transparency from the government's perspective, and subindex of transparency from the citizen's perspective. The only issue with this formula is that it takes an average of the results of the four subindexes, and according to the state, or at the national level, there may be subindexes that are more important than another, or it is possible that the opinion of the government or the citizens can be higher or lower. To improve the formula, this work proposes the use of the ordered weighted average (OWA) operator [19] to create new formulas and approaches to evaluate transparency.

The OWA operator is an aggregation operator developed by Yager [19] that provides the ability to aggregate different types of information, such as the attitude and expectation of the decision maker. Among the extensions that have been developed and that will be discussed in this paper are the prioritized OWA (POWA) operator [21], which is used in a group decision-making process when not all the decision makers have the same importance in the final results, so to unify the criteria, a prioritized vector is applied. Additionally, the heavy OWA (HOWA) operator [20] is considered; this operator has characteristics that are not bounded to one such as those of the OWA operator; thus, it is possible to under or overestimate the results according to the expectations. Finally, the induced OWA (IOWA) operator [24] is applied. The main aspect of this operator is that the weighting vector is not assigned according to the maximum or minimum of the attributes but is assigned by an induced vector determined by the decision maker.

The main objective of the paper is to present a new extension of the OWA operator that will use in only one formulation the characteristics of the extensions described before. This new operator is called the prioritized induced heavy ordered weighted average (PIHOWA) operator. The main advantage of this operator is that it can consider in only one formulation a group decision-making problem with a weighting vector not bounded to one and assign the weights according to an induced vector. Additionally, a generalization of the PIHOWA operator is presented using quasi-arithmetic means. In this sense, many particular cases are presented that can be used according to the information and complexity of the problem.

The paper is organized as follows. In Section 2, we present some of the basic aggregation operators and the OGM formula. Section 3 presents the PIHOWA operator with its main characteristics, generalized cases and the application of the operator in the OGM formula. In Section 4, the use of the PIHOWA-OGM operator and its extensions are used to rank the top 10 most transparent states in Mexico for 2017. Finally, in Section 5, the conclusions of the paper are presented.

2. PRELIMINARIES

This section, review some of the basic concepts that are needed in the paper, such as OWA operator, POWA operator, IOWA operator, HOWA operator and some other extension, also, the OGM formula is presented.

2.1 OWA operator and some extensions

The OWA operator was introduced by Yager [19] and its main characteristic is that is possible to obtain the maximum and the minimum according to the reodring step of the operator. It can be defined as follows.

Definition 1. An OWA operator of dimension n is a mapping $OWA: R^n \rightarrow R$ with an associated weight vector W of dimension n such that $\sum_{j=1}^{n} w_j = 1$ and $w_j \in [0,1]$, according to the following formula:

$$OWA(a_1, \dots, a_n) = \sum_{j=1}^{n} w_j b_j, \quad (1)$$

being b_j the *j*th largest element of the collection a_i.

Among the extensions that will be taken into account in the paper is the Prioritized OWA (POWA) operator [22-23], that is a good aggregator operator in group decision making, this because usually in this kind of decisions not all the decision makers have the same importance or relevance in the decision, that is why sometimes the information provided for one person is valued higher than from another. The formulation is as follows

Definition 2. Assume that we have a collection of criteria portioned into q distinct groups, $H_1, H_2, \dots, H_q$ for which $H_i = \{C_{i1}, C_{i2}, \dots, C_{in}\}$ denotes the criteria of the *i*th category (*i=1,...,q*) and n_i is the number of criteria in the class. Furthermore, we have a prioritization between the groups as $H_1 > H_2 > \cdots > H_q$. That is, the criteria in the category H_i have a higher priority than those in H_k for all $i < k$ and $i, k \in \{1, \dots, q\}$. Denote the total set of criteria as $C = U_{i=1}^{q} H_i$ and the total number of criteria as $n = \sum_{i=1}^{q} n_i$. Additionally, suppose $X = \{x_1, \dots, x_m\}$ indicates the set of alternatives. For a given alternative x, let $C_{ij}(x)$ measure the satisfaction of the *j*th criteria in the *i*th group by alternative $x \in X$, for each $i = 1, \dots, q, j = 1, \dots, i_i$. The formula is as follows:

$$C_{(x)} = \sum_{i=1}^{q} \sum_{h=1}^{n_i} w_{ij} C_{ij}(x), \quad (2)$$

where w_{ij}, is the corresponding weight of the *j*th criteria in the *i*th category, $i = 1, \dots, q, j = 1, \dots, i_i$.

If the attention is put in the reordering process of the traditional OWA operator, it is possible not to do it according to the value of the arguments, but instead, according to the decision maker expectatives, this operator is known as the Induced OWA (IOWA) operator [24]. The definition is as follows

Definition 3. An IOWA operator of dimension n is an application $IOWA: R^n \times R^n \rightarrow R$ that has a weighting vector associated W of dimension n where the sum of the weights is 1 and $w_j \in [0,1]$, where an induced set of ordering variables are included (u_i) so the formula is

$$IOWA(\langle u_1, a_1 \rangle, \langle u_2, a_2 \rangle, \dots, \langle u_n, a_n \rangle) = \sum_{j=1}^{n} w_j b_j \quad (3)$$

where b_j is the a_i value of the OWA pair $< u_i, a_i >$ having the *j*th largest u_i. u_i is the order inducing variable and a_i is the argument variable.

Among the extensions of the OWA operator that put attention on the weighting vector there is the Heavy OWA (HOWA) operator [20]. In this extension, the weighting vector is not $\sum_{j=1}^{n} w_j = 1$, but in this case, is unbounded, so the weighting vector can be $1 \leq \sum_{j=1}^{n} w_j \leq n$. The formulation is as follows

Definition 3. A HOWA operator is a mapping $HOWA: R^n \rightarrow R$ which are associated to a weight vector w which $w_j \in [0,1]$ and $1 \leq \sum_{j=1}^{n} w_j \leq n$, so that

$$HOWA\ (a_1, a_2, \dots, a_n) = \sum_{j=1}^{n} w_j b_j, \quad (4)$$

being b_j the *j*th largest element of the collection a_i. It is also important to note, that in some cases it is possible that the weighing vector goes like $-\infty \leq \sum_{j=1}^{n} w_j \leq \infty$ being possible to under or overestimate the results according to the expectations of the decision maker.

It is important to note that Yager [20] has also developed characteristic of the HOWA operator that is called the beta value. This beta value can be defined as $\beta(W) = (|W| - 1)/(n-1)$. Note that if $\beta = 1$, we get the total operator and if $\beta = 0$, we get the usual OWA operator.

It is also possible add the attributes of the POWA operator and the IOWA operator in one formulation that not only change the reordering step to assign the weights to the arguments, but also if this is a group decision making process consider different levels of importance to each person. The definition is as follows [17].

Definition 4. A prioritized induced OWA (PIOWA) of dimension n is a mapping $PIOWA: R^n x\ R^n \rightarrow R$ that has an associated weight vector w of dimension n, where $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$, so that

$$PIOWA(\langle u_1, a_1 \rangle, \langle u_2, a_2 \rangle, \dots, \langle u_n, a_n \rangle) = \sum_{i=1}^{q} \sum_{h=1}^{n_i} b_j \hat{v}_{ij} C_{ij}(x), \quad (5)$$

where b_j is the *j*th element that has the largest value of u_i, u_i is the induced order of variables, $\hat{v}_{ij}$ is the corresponding weight of the *j*th criteria in the *i*th category, $i = 1, \dots, q, j = 1, \dots, i_i$ and $C_{ij}(x)$ measures the satisfaction of the *j*th criteria in the *i*th group by alternative $x \in X$, for each $i = 1, \dots, q, j = 1, \dots, i_i$.

Another extension that have been done is taking the reordering process of the IOWA operator and the unbounded weighting vector of the HOWA operator, this operator is called the induced heavy OWA (IHOWA) operator. The definition is as follows [14].

Definition 5. An IHOWA operator of dimension n is a mapping $IHOWA: R^n \times R^n \rightarrow R$ that has an associated weighting vector W of dimension n with $w_j \in [0,1]$ and $1 \leq \sum_{j=1}^{n} w_j \leq n$, such that

$$IHOWA(\langle u_1, a_1 \rangle, \langle u_2, a_2 \rangle, \dots, \langle u_n, a_n \rangle) = \sum_{j=1}^{n} w_j b_j, \quad (6)$$

where b_j is the a_i value of the IHOWA pair $< u_i, a_i >$ having the jth largest u_i. u_i is the order inducing variable and a_i is the argument variable. It is possible to expand the weighting vector from 1 to ∞ or even from $-\infty$ to ∞.

2.2 OGM formulation

The OGM measures how much a citizen can know what their goverments do and how much it can influence their decisions. The metric serves aas a line for the Open Government policies implemented by the INAI. The formulation is as follows [9].

Definition 6. The OGN is an average that can be defined as.

$$OGN = \frac{s_1 + s_2 + s_3 + s_4}{4}, \quad (7)$$

where s_1 is subindex of participation from the government's perspective, s_2 is subindex of participation from the citizen's perspective, s_3 is subindex of transparency from the government's perspective and s_4 is subindex of transparency from the citizen's perspective.

3. PRIORITIZED INDUCED HEAVY OWA OPERATOR IN OGM

3.1 PIHOWA operators and some particular cases

The prioritized induced heavy OWA (PIHOWA) operator is an aggregation operator that take into one formulation three characteristics that are: 1) the problem is group decision making and not all the people have the same importance in the final decision, 2) the reordering step is not based in the value of the argument and 3) the weighting vector is not bounded to a sum equal to one. The formula is as.

Definition 7. A PIHOWA operator of dimension n is a mapping $PIHOWA: R^n x\ R^n \to R$ that has an associated weight vector w of dimension n, where $w_j \in [0,1]$ and $1 \leq \sum_{j=1}^{n} w_j \leq n$, so that

$$PIHOWA(\langle u_1, a_1\rangle, \langle u_2, a_2\rangle, \dots, \langle u_n, a_n\rangle) = \sum_{i=1}^{q} \sum_{h=1}^{n_i} b_j \hat{v}_{ij} C_{ij}(x), \quad (8)$$

where b_j is the jth element that has the largest value of u_i, u_i is the induced order of variables, $\hat{v}_{ij}$ is the corresponding weight of the jth criteria in the ith category, $i = 1, \dots, q, j = 1, \dots, i_i$ and $C_{ij}(x)$ measures the satisfaction of the jth criteria in the ith group by alternative $x \in X$, for each $i = 1, \dots, q, j = 1, \dots, i_i$. It is important to note that sometimes the weighting vector can measure as $-\infty \leq \sum_{j=1}^{n} w_j \leq \infty$.

One particular case that can be obtained when the reordering step is not based on an induced value or if $u_i = 1/n$ for all arguments, then the PIHOWA operator becomes the prioritized heavy OWA (PHOWA) operator. The definition is as follows.

Definition 8. A PHOWA operator of dimension n is a mapping $PHOWA: R^n x\ R^n \to R$ that has an associated weight vector w of dimension n, where $w_j \in [0,1]$ and $1 \leq \sum_{j=1}^{n} w_j \leq n$, so that

$$PHOWA(a_1, \dots, a_n) = \sum_{i=1}^{q} \sum_{h=1}^{n_i} b_j \hat{v}_{ij} C_{ij}(x), \quad (9)$$

where b_j is the jth element that has the largest value of a_i, $\hat{v}_{ij}$ is the corresponding weight of the jth criteria in the ith category, $i = 1, \dots, q, j = 1, \dots, i_i$ and $C_{ij}(x)$ measures the satisfaction of the jth criteria in the ith group by alternative $x \in X$, for each $i = 1, \dots, q, j = 1, \dots, i_i$. It is important to note that sometimes the weighting vector can measure as $-\infty \leq \sum_{j=1}^{n} w_j \leq \infty$.

It is important to note that also if the weighting vector is bounded to one, the PIHOWA operator will become the PIOWA operator. Also, if the importance of all the decision makers is the same then the PIHOWA operator becomes the IHOWA operator.

Another way to generate more particular cases of the PIHOWA operator is when quasi-arithmetic mean is used. Using this technique, it is possible to generate cases that can be used when the problem that want to be solve is complex, and also, can be simplified if the topic to analyze doesn't have that complexity. That is why, the knowledge of different cases can help the decision maker to understand better situations that have more things to analyze [15-16, 11]. The quasi-PIHOWA operator, quasi-PHOWA operator, quasi-PIOWA operator and quasi-IHOWA operator are defined as follows.

Definition 9. A Quasi-PIHOWA operator of dimension n is a mapping $PIHOWA: R^n x R^n \to R$ that has an associated weight vector w of dimension n, where $w_j \in [0,1]$ and $1 \leq \sum_{j=1}^{n} w_j \leq n$, so that

$$Quasi - PIHOWA(\langle u_1, a_1\rangle, \dots, \langle u_n, a_n\rangle) = g^{-1} \sum_{i=1}^{q} \sum_{h=1}^{n_i} b_j \hat{v}_{ij} g(C_{ij}(x)), \quad (10)$$

where b_j is the jth element that has the largest value of u_t, u_t is the induced order of variables, $\hat{v}_{ij}$ is the corresponding weight of the jth criteria in the ith category, $i = 1, \dots, q, j = 1, \dots, i_i$, and $C_{ij}(x)$ measures the satisfaction of the jth criteria in the ith group by alternative $x \in X$, for each $i = 1, \dots, q, j = 1, \dots, i_i$ and $g(C_{ij}(x))$ is a continuous strictly monotonic function. It is important to note that sometimes the weighting vector can measure as $-\infty \leq \sum_{j=1}^{n} w_j \leq \infty$.

Definition 10. A Quasi-PHOWA operator of dimension n is a mapping $PHOWA: R^n x R^n \to R$ that has an associated weight vector w of dimension n, where $w_j \in [0,1]$ and $1 \leq \sum_{j=1}^{n} w_j \leq n$, so that

$$Quasi - PHOWA(\langle u_1, a_1\rangle, \dots, \langle u_n, a_n\rangle) = g^{-1} \sum_{i=1}^{q} \sum_{h=1}^{n_i} b_j \hat{v}_{ij} g(C_{ij}(x)), \quad (11)$$

where b_j is the jth element that has the largest value of a_i, $\hat{v}_{ij}$ is the corresponding weight of the jth criteria in the ith category, $i = 1, \dots, q, j = 1, \dots, i_i$, and $C_{ij}(x)$ measures the satisfaction of the jth criteria in the ith group by alternative $x \in X$, for each $i = 1, \dots, q, j = 1, \dots, i_i$ and $g(C_{ij}(x))$ is a continuous strictly monotonic function. It is important to note that sometimes the weighting vector can measure as $-\infty \leq \sum_{j=1}^{n} w_j \leq \infty$.

Definition 11. A Quasi-PIOWA of dimension n is a mapping $PIOWA: R^n x\ R^n \to R$ that has an associated weight vector w of dimension n, where $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$, so that

$$Quasi - PIOWA(\langle u_1, a_1\rangle, \dots, \langle u_n, a_n\rangle) = g^{-1} \sum_{i=1}^{q} \sum_{h=1}^{n_i} b_j \hat{v}_{ij} g\left(C_{ij}(x)\right), \quad (12)$$

where b_j is the jth element that has the largest value of u_i, u_i is the induced order of variables, $\hat{v}_{ij}$ is the corresponding weight of the jth criteria in the ith category, $i = 1, \dots, q, j = 1, \dots, i_i$, $C_{ij}(x)$ measures the satisfaction of the jth criteria in the ith group by alternative $x \in X$, for each $i = 1, \dots, q, j = 1, \dots, i_i$, and $g(C_{ij}(x))$ is a continuous strictly monotonic function.

Definition 12. A Quasi-IHOWA operator of dimension n is a mapping $IHOWA: R^n \times R^n \to R$ that has an associated weighting vector W of dimension n with $w_j \in [0,1]$ and $1 \leq \sum_{j=1}^{n} w_j \leq n$, such that

$$IHOWA(\langle u_1, a_1\rangle, \langle u_2, a_2\rangle, \dots, \langle u_n, a_n\rangle) = g^{-1} \sum_{j=1}^{n} w_j g(b_j), \quad (13)$$

where b_j is the a_i value of the IHOWA pair $< u_i, a_i >$ having the jth largest u_i. u_i is the order inducing variable and a_i is the argument variable and $g(b_j)$ is a continuous strictly monotonic function. It is possible to expand the weighting vector from 1 to ∞ or even from $-\infty$ to ∞.

In Table 1-4, some of the main particular cases based on the quasi-arithmetic mean for definition 9-12.

Table 1. Families of generalized PIHOWA operators

Particular case	Quasi-PIHOWA
$u_i = \frac{1}{n}, for\ all\ i$	Quasi-arithmetic prioritized heavy ordered average (Quasi-PHOWA)
$g(b) = b^{\lambda}$	Generalized PIHOWA
$g(b) = b$	PIHOWA
$g(b) = b^2$	Prioritized induced heavy ordered weighted quadratic average (PIHOWQA)
$g(b) \to b^{\lambda}, for\ \lambda \to 0$	Prioritized induced heavy ordered weighted geometric average (PIJOWGA)
$g(b) = b^{-1}$	Prioritized induced heavy ordered weighted harmonic average (PIHOWHA)
$g(b) = b^3$	Prioritized induced heavy ordered weighted cubic average (PIHOWCA)
$g(b) \to b^{\lambda}, for\ \lambda \to \infty$	Maximum operator
$g(b) \to b^{\lambda}, for\ \lambda \to -\infty$	Minimum operator

Table 2. Families of generalized PHOWA operators

Particular case	Quasi-PHOWA
$w_i = \frac{1}{n}, for\ all\ i$	Quasi-arithmetic prioritized heavy weighted average (Quasi-PHWA)
$g(b) = b^{\lambda}$	Generalized PHOWA
$g(b) = b$	PPOWA
$g(b) = b^2$	Prioritized heavy ordered weighted quadratic average (PHOWQA)
$g(b) \to b^{\lambda}, for\ \lambda \to 0$	Prioritized heavy ordered weighted geometric average (PHOWGA)
$g(b) = b^{-1}$	Prioritized heavy ordered weighted harmonic average (PHOWHA)
$g(b) = b^3$	Prioritized heavy ordered weighted cubic average (PHOWCA)
$g(b) \to b^{\lambda}, for\ \lambda \to \infty$	Maximum operator
$g(b) \to b^{\lambda}, for\ \lambda \to -\infty$	Minimum operator

3.2 Numerical example

In order to understand the proposed formulation, lets suppose that four different decision makers forecast the average sales for 2018 for a company that their work based in expectations of monthly sales for 2018. The data is presented in Table 5.

Table 5. Expectations of monthly sales in thousands of MXN of the company for 2018

Date	dm_1	dm_2	dm_3	dm_4
01-18	1,200	1,116	1,260	1,068
02-18	950	884	998	846
03-18	1,360	1,265	1,428	1,210
04-18	1,430	1,330	1,502	1,273
05-18	1,280	1,190	1,344	1,139
06-18	1,320	1,228	1,386	1,175
07-18	990	921	1,040	881
08-18	1,120	1,042	1,176	997
09-18	1,370	1,274	1,439	1,219
10-18	1,400	1,302	1,470	1,246
11-18	1,800	1,674	1,890	1,602
12-18	2,010	1,869	2,111	1,789

The importance of each decision maker to the final results is as follows

$dm_1 = 30\%, dm_1 = 20\%, dm_3 = 15\%, dm_4 = 35\%$

The induced vector is valued as $U = (1,3,6,4,2,5,9,12,11,7,8,10)$

The heavy weighting vector is $W = (0.05, 0.05,0.05,0.05,0.05,0.10,0.10,0.10,0.10,0.10,0.15,0.15)$

Using this information, the results of using the average, prioritized average, HOWA, PHOWA and PIHOWA operator is as follows.

	Average	PA	HOWA	PHOWA	PIHOWA
Sales	1,309	1,292	1,487	1,467	1,417

At can be seen considering different elements to the final average, it is possible to get a scenario that goes from 1,292 to 1,487 average sales for 2018. This helps the decision maker to get a better idea of what will come in the future, that from the perspective of the classical average can only know the 1,309 scenario.

3.3 OWA-OGM and its extensions

In the classical OGM formula all the subindex that compose the formula are considered that impact the same at the final grade for a state to measure their Open Government. In this sense, some States can work hard in a specific aspect only and the other can be low and get a good final score, but what happened if with the time passes and then not all the subindex are equally important or for some States one of them is more important to another. In order to get a better Open Government, score the use of OWA operator in order to aggregate information to the score and some of its extensions are presented. It is important to note that the actual formula only have four subindexes, but in the formula, they are leave it as *n*, because in the future could be possible to add more subindex, in this sense this proposed formulas can be used also in the future even if the number of subindex change. The formulations are as follows

Definition 13. An OWA-OGM operator of dimension n is a mapping $OWA - OGM: R^n \to R$ with an associated weight vector W of dimension n such that $\sum_{j=1}^{n} w_j = 1$ and $w_j \in [0,1]$, according to the following formula:

$$OWA - OGM(s_1, \dots, s_n) = \sum_{j=1}^{n} w_j b_j, \quad (14)$$

being b_j the jth largest element of the collection s_i.

Definition 14. A POWA-OGM operator of dimension n is a mapping $POWA - OGM: R^n x\, R^n \to R$ that has an associated weight vector w of dimension n, where $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$, so that

$$POWA - OGM(s_1, \dots, s_n) = \sum_{i=1}^{q} \sum_{h=1}^{n} b_j \hat{v}_{ij} C_{ij}(x), \quad (14)$$

where b_j is the jth element that has the largest value of s_i, $\hat{v}_{ij}$ is the corresponding weight of the jth criteria in the ith category, $i = 1, \ldots, q, j = 1, \ldots, i_i$ and $C_{ij}(x)$ measures the satisfaction of the jth criteria in the ith group by alternative $x \in X$, for each $i = 1, \ldots, q, j = 1, \ldots, i_i$.

Definition 15. An IOWA-OGM operator of dimension n is an application $IOWA - OGM: R^n \times R^n \to R$ that has a weighting vector associated W of dimension n where the sum of the weights is 1 and $w_j \in [0,1]$, where an induced set of ordering variables are included (u_i) so the formula is

$$IOWA - OGM(\langle u_1, s_1 \rangle, \ldots, \langle u_n, s_n \rangle) = \sum_{j=1}^{n} w_j b_j \quad (15)$$

where b_j is the s_i value of the OWA pair $< u_i, s_i >$ having the jth largest u_i. u_i is the order inducing variable and s is the subindex variable.

Definition 16. A HOWA-OGM operator is a mapping $HOWA: R^n \to R$ which are associated to a weight vector w which $w_j \in [0,1]$ and $1 \le \sum_{j=1}^{n} w_j \le n$, so that

$$HOWA - OGM\ (s_1, \ldots, s_n) = \sum_{j=1}^{n} w_j b_j, \quad (16)$$

being b_j the jth largest element of the collection s_i. It is also important to note, that in some cases it is possible that the weighing vector goes like $-\infty \le \sum_{j=1}^{n} w_j \le \infty$ being possible to under or overestimate the results according to the expectations of the decision maker

Definition 17. A PIOWA-OGM of dimension n is a mapping $PIOWA - OGM: R^n x\ R^n \to R$ that has an associated weight vector w of dimension n, where $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$, so that

$$PIOWA - OGM(\langle u_1, s_1 \rangle,, \ldots, \langle u_n, s_n \rangle) = \sum_{i=1}^{q} \sum_{h=1}^{n_i} b_j \hat{v}_{ij} C_{ij}(x), \quad (17)$$

where b_j is the jth element that has the largest value of s_i, u_i is the induced order of variables, $\hat{v}_{ij}$ is the corresponding weight of the jth criteria in the ith category, $i = 1, \ldots, q, j = 1, \ldots, i_i$ and $C_{ij}(x)$ measures the satisfaction of the jth criteria in the ith group by alternative $x \in X$, for each $i = 1, \ldots, q, j = 1, \ldots, i_i$.

Definition 18. A PHOWA-OGM operator of dimension n is a mapping $PHOWA - OGM: R^n x\ R^n \to R$ that has an associated weight vector w of dimension n, where $w_j \in [0,1]$ and $1 \le \sum_{j=1}^{n} w_j \le n$, so that

$$PHOWA - OGM(s_1, \ldots, s_n) = \sum_{i=1}^{q} \sum_{h=1}^{n_i} b_j \hat{v}_{ij} C_{ij}(x), \quad (18)$$

where b_j is the jth element that has the largest value of s_i, $\hat{v}_{ij}$ is the corresponding weight of the jth criteria in the ith category, $i = 1, \ldots, q, j = 1, \ldots, i_i$ and $C_{ij}(x)$ measures the satisfaction of the jth criteria in the ith group by alternative $x \in X$, for each $i = 1, \ldots, q, j = 1, \ldots, i_i$. It is important to note that sometimes the weighting vector can measure as $-\infty \le \sum_{j=1}^{n} w_j \le \infty$.

Definition 19. An IHOWA-OGM operator of dimension n is a mapping $IHOWA - OGM: R^n \times R^n \to R$ that has an associated weighting vector W of dimension n with $w_j \in [0,1]$ and $1 \le \sum_{j=1}^{n} w_j \le n$, such that

$$IHOWA - OGM(\langle u_1, s_1 \rangle, \ldots, \langle u_n, s_n \rangle) = \sum_{j=1}^{n} w_j b_j, \quad (19)$$

where b_j is the S_i value of the IHOWA-OGM pair $< u_i, S_i >$ having the jth largest u_i. u_i is the order inducing variable and S_i is the argument variable. It is possible to expand the weighting vector from 1 to ∞ or even from $-\infty$ to ∞.

Definition 20. A PIHOWA-OGM operator of dimension n is a mapping $PIHOWA - OGM: R^n x\ R^n \to R$ that has an associated weight vector w of dimension n, where $w_j \in [0,1]$ and $1 \le \sum_{j=1}^{n} w_j \le n$, so that

$$PIHOWA - OGM(\langle u_1, s_1 \rangle, \ldots, \langle u_n, s_n \rangle) = \sum_{i=1}^{q} \sum_{h=1}^{n_i} b_j \hat{v}_{ij} C_{ij}(x), \quad (20)$$

where b_j is the jth element that has the largest value of u_i, u_i is the induced order of variables, $\hat{v}_{ij}$ is the corresponding weight of the jth criteria in the ith category, $i = 1, \ldots, q, j = 1, \ldots, i_i$ and $C_{ij}(x)$ measures the satisfaction of the jth criteria in the ith group by alternative $x \in X$, for each $i = 1, \ldots, q, j = 1, \ldots, i_i$. It is important to note that sometimes the weighting vector can measure as $-\infty \le \sum_{j=1}^{n} w_j \le \infty$

In this section we briefly review the adequacy coefficient, the index of maximum and minimum level and the OWAWA operator.

5. CONCLUSIONS

The main purpose of this paper is to present a new extension of the OWA operator that we called the prioritized induced heavy ordered weighted average (PIHOWA) operator. The main characteristics of this new operator are that it can combine an unbounded weighting vector, an induced vector to assign to weights to the attributes and a prioritized vector to unify the opinions of the group decision-making process where not all the decision makers have the same importance in the final result.

These new formulations were combined with the traditional OGM formula presented by the INAI to score the transparency of each state in Mexico for 2017. These new formulations are explained and presented in the document and then are used to rank the top 10 states according to the information provided by the INAI and the expectation of each of the 4 experts with the use of the OWA operator and some of its extensions. Finally, the unification of the results obtained by each of 4 experts was performed using the PIHOWAD operator and some of its extensions.

With the analysis of the results, the changes realized by each of the top 10 ranking states are visible (with the exception of Ciudad de Mexico and Guanajuato, which were always numbers 1 and 2, respectively). In this sense, the use of different operators and experts' opinions helped to visualize that the states that were not as transparent as the traditional formulation showed and that the distance between the number 1 ranked state and a specific state can change drastically. This information will help to make better laws, processes, and politics that will help to achieve more transparent states.

For future research, more extensions of the OWA operator can be conceived with the use of distance operators [7], Bonferroni means [2-3], moving averages [10, 12] or logarithmic operators [1, 25].

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