

Modal Parameter Identification for Transmission Tower Based on Point Spectrum Correlation Function of Time-Frequency Distribution

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Abstract

Modal parameter identification is the foundation of structure model correction, damage diagnosis and safety evaluation of the transmission tower. To obtain modal parameter identification method with clear physical meaning and practicability for transmission towers, the point spectrum correlation function of time-frequency distribution was used to deduce analytic expression of damped free vibration response of the structure. The deduction in this paper reveals the response function relationship between point spectrum correlation function values and structural modal parameters, and then the square root value of point spectrum correlation function is proposed to identify modal shape of the transmission tower. This method does not need time domain or frequency domain information from external excitation. Numerical example of transmission tower structure results show that the modal frequencies can be accurately achieved by spectrum and time-frequency analysis of vibration response signal. The maximum identification error of the first order modal shape of the structure is 0.54%, and the maximum identification error of second order modal shape is 1.52% using square root value of time-frequency spectrum correlation function. The structure modal frequency identification is not affected under the noise level of 10%, the maximum identification error of first order modal shape is 2.53%, and the maximum identification error of second order modal shape is 4.55%.

Key words: Modal Parameter, Transmission Tower, Point Spectrum Correlation Function, Time-Frequency Distribution

1. Introduction

Structural modal identification is generally divided into analytical modal identification and experimental modal identification. Generally, analytical modal identification is under a giving structure geometry, material properties and boundary conditions, which is under a giving mass matrix, stiffness matrix and damping matrix of the structure. And then the modal parameters, such as each order natural frequency, damping coefficient and modal vibration mode are determined with these enough information of the structure. Experimental modal identification is a kind of signal processing method and modal parameter identification method based on dynamic input and output data from measurement some points on structure. The ultimate purpose is to identify the modal parameters of the system, and to provide a basis for structure dynamic analysis, dynamic optimization design and damage detection forecast.

At present, there are many methods for the structural modal parameter identification. Those methods can be roughly divided into the time domain identification method, frequency domain identification method and the time -frequency domain identification method [1-2]. Time domain identification method [3-4] is the parameterized method, which firstly get the middle temporal sequence by signal processing and then the classical modal identification algorithm under time domain is utilized to calculate. Frequency domain identification method [5-6] belongs to the non parametric method. Frequency domain identification method usually conduct classical modal analysis under frequency domain with known input signal. To do that, the frequency response function will be obtained by using input and output signals, and then the law of peak value of frequency response function near the system natural frequency is used to identify the model parameter. Time-frequency domain identification method [7-8] utilizes the two dimensional function of the time and frequency, which can also reflect the information of vibration signal in time and frequency domain, especially suitable for solving the nonlinear problem, but it is still in the initial stage of the theoretical research.

Transmission tower is the main structure for transmission and transformation engineering of the power, and its security is the foundation of reliability of regional large-scale power system.

But in recent years, the destruction of transmission tower construction accidents often happens at both home and abroad, and it seriously affected the social normal order of production and living. The modal parameter identification method is the theory basis for damage diagnosis and safety early warning of the transmission tower structure. Existing research [9-12] work point out that dynamic response of transmission tower is far more complicated than the general engineering structure due to the complicated environment and load conditions, plenty of node number, stiffness mutation of the tower and tower line coupling. All those factors led to the current more effective structural modal parameter identification method difficult to be directly applied.

To obtain the modal parameter identification method for transmission towers with clear physical meaning and good practicability, the structure modal parameter analysis method and time-frequency analysis method for signal are combined. The analytic expression of damped free vibration response of the structure was deduced by the time-frequency point spectrum correlation function, and the response function relation between the structural modal parameter and the point spectrum correlation function value was also revealed. The square root value of point spectrum correlation function was proposed to identify modal shape of the transmission tower, the validation and analysis of numerical example were conducted.

2. Point spectrum correlation function

For non-stationary signal $s(t)$, the bilinear time-frequency distribution is defined as:

$$R_z(t, \tau) = \int_{-\infty}^{\infty} \varphi(u-t, \tau) z(u+\tau/2) z^*(u-\tau/2) du \quad (1)$$

The upper formula is also called the local correlation function. where, $\varphi(t, \tau)$ is the window function, $z(t)$ is the analytical signal by Hilbert transform of $s(t)$, $z^*(t)$ is the complex conjugate of $z(t)$, parameter t is the time and τ is the time delay.

Let $\varphi(u-t, \tau) = \delta(u-t)$ and get instantaneous value in the time domain, there is no restriction to the time delay τ , and the instantaneous correlation function will be obtained as follows:

$$k_z(t, \tau) = \int_{-\infty}^{\infty} \delta(u-t) z(u+\tau/2) z^*(u-\tau/2) du = z(t+\tau/2) z^*(t-\tau/2) \quad (2)$$

To do a Fourier transforming of time delay τ for formula (2), and Wigner-Ville distribution (WVD) will be obtained as follows:

$$W_z(t, f) = \int_{-\infty}^{\infty} z(t+\tau/2) z^*(t-\tau/2) \cdot e^{-j \cdot 2\pi \tau \cdot f} d\tau \quad (3)$$

To do a Fourier transforming of time t for formula (3), another two dimensional time-frequency distribution function named the fuzzy function will be obtained as follows:

$$A_z(\tau, \nu) = \int_{-\infty}^{\infty} z(t + \tau/2)z^*(t - \tau/2) \cdot e^{j2\pi t\nu} dt \quad (4)$$

where, the parameter ν is the frequency deviation. A kind of two dimensional distribution will be obtained by choosing any two parameters from τ , t , f and ν . It can be seen from the expression of Wigner-Ville distribution and the fuzzy function that Wigner-Ville distribution indicates the energy field plane with parameter t and f , while the fuzzy function indicates the correlation field plane with parameter τ and ν , and the instantaneous correlation function is with the parameters t and τ . So the distribution with parameters f and ν is the so called point spectrum correlation function:

$$K_z(f, \nu) = z(f - \nu/2)z^*(f + \nu/2) \quad (5)$$

Wigner-Ville distribution, instantaneous correlation function, fuzzy function and point spectrum correlation function form the four basis time-frequency of Cohen distribution. The distribution properties are reflected on the constraint of kernel function. The choice of different kernel functions for integral transform forms different time-frequency distribution.

Wigner-Ville distribution and fuzzy function form the Fourier transforming pairs. According to the definition, the Wigner-Ville distribution is obtained by Fourier transforming on frequency deviation for point spectrum correlation function, that is to say, the Fourier inverse transformation of Wigner-Ville distribution is point spectrum correlation function.

$$K_z(f, \nu) = \int_{-\infty}^{\infty} W_z(t, f) \cdot e^{j2\pi t\nu} dt = \iint z(t + \tau/2)z^*(t - \tau/2) \cdot e^{-j2\pi\tau f} e^{j2\pi t\nu} d\tau \cdot dt \quad (6)$$

3. Modal parameter identification

For structure system with n degrees of freedom, the motion differential equation can be generally expressed as:

$$M\ddot{x} + C\dot{x} + Kx = f(t) \quad (7)$$

where, M , C and K represent mass matrix, stiffness matrix and damping matrix respectively, and $f(t)$ is the load vector. Under initial displacement condition by suddenly unloading, the damped free vibration displacement response is available by applying the method of modal decomposition method:

$$x_k = \sum_{i=1}^n \phi_{ik} \phi_i^T Mx_0 \cdot e^{-\xi_i \omega_i t} \cdot \cos(\omega_{di} t + \varphi_i) \quad (8)$$

where, ϕ_{ki} is the component k of the i^{th} modal shape, ϕ_i is i^{th} modal shape, ω_i is i^{th} order modal frequency, ω_{di} is i^{th} damped frequency, φ_i is the i^{th} initial phase of displacement response, x_0 is the initial displacement vector.

As for the real frequency modulation signal $A(t) = a(t) \cdot \cos \omega(t)$, the corresponding analytical signal is:

$$A(t) = a(t) \cdot \cos \omega(t) + jH[a(t) \cdot \cos \omega(t)] = a(t)e^{j\omega(t)} \quad (9)$$

where, $H[a(t)]$ is the Hilbert transforming of $a(t)$, and it is defined as:

$$H[a(t)] = \frac{1}{\pi} \int_{-\infty}^{\infty} a(u) \frac{du}{t-u} \quad (10)$$

According to formula (3), (8) and (9), the Wigner-Ville distribution of the damped free vibration displacement response signal on an arbitrary structure measurement point k :

$$W_{z_k}(t, f) = \int_{-\infty}^{\infty} \left(\sum_{i=1}^n \phi_{ki} \phi_i^T Mx_0 \cdot e^{-\xi_i \omega_i t} \cdot e^{j(\omega_{di} t + \varphi_i)} \cdot e^{-\xi_i \omega_i \frac{\tau}{2} + j\omega_{di} \frac{\tau}{2}} \right) \cdot \left(\sum_{l=1}^n \phi_{kl} \phi_l^T Mx_0 \cdot e^{-\xi_l \omega_l t} \cdot e^{-j(\omega_{dl} t + \varphi_l)} \cdot e^{\xi_l \omega_l \frac{\tau}{2} + j\omega_{dl} \frac{\tau}{2}} \right) \cdot e^{-j2\pi f \tau} d\tau \quad (11)$$

where, i and l are used to identify the signal $z_k(t)$ and its conjugate and the value is chosen from 1 to n . From formula (6), the point spectrum correlation function of structural damped free vibration response signal can be written as:

$$K_{Z_k}(f, \nu) = \sum_{i=1}^n \phi_{ik} \phi_i^T Mx_0 \cdot \sum_{l=1}^n \phi_{lk} \phi_l^T Mx_0 \cdot e^{-(\xi_i \omega_i + \xi_l \omega_l) \nu} \cdot \delta\left(\frac{\omega_{di} - \omega_{dl}}{2\pi}\right) \delta\left(\frac{\omega_{di} + \omega_{dl}}{4\pi} - f\right) \quad (12)$$

where, $\delta(\cdot)$ is Dirac function. According to formula (12), the point spectrum correlation function distribution of the damped free vibration displacement response signal on point k of the structure is a series point on the plane of frequency f and frequency deviation ν .

When $\omega_{di} \neq \omega_{dl}$, namely $f = (\omega_{di} + \omega_{dl})/4\pi$, the value of point spectrum correlation function is the sum of the product of different order modal shape parameter. The physical meaning is the cross term of structural vibration signal components in the domain. It is a kind of false signals coming from the interactions between different signal component of the multi-component signal, which will disturb the structural modal parameter identification and can be eliminated through the determination of each order frequency values and control of frequency deviation.

When $\omega_{di} = \omega_{dl} = \omega_u$, $i = l = u$, namely $f = \omega_u / 2\pi$, the value of point spectrum correlation function is the product of the same order modal shape parameter. The physical meaning is the

signal part of structure vibration signal components at the u order modal damped frequency ω_u . The value of point spectrum correlation function on point k of the structure is the square of u order modal shape parameter:

$$K_{z_k}(f, \nu) = (\phi_{ku} \phi_u^T M x_0 \cdot e^{-\xi_u \omega_u \nu})^2 \quad (13)$$

So, u order modal shape parameter can be obtained through square root vector of the point spectrum correlation function on all measurement points:

$$\tilde{\phi} = \beta \cdot \{\phi_{1u} \quad \phi_{2u} \quad \dots \quad \phi_{nu}\}^T = \left\{ |K_{z_1}(f, \nu)|^{1/2} \quad |K_{z_2}(f, \nu)|^{1/2} \quad \dots \quad |K_{z_n}(f, \nu)|^{1/2} \right\}^T \quad (14)$$

where, $\beta = \phi_i^T M x_0 \cdot e^{-\xi_u \omega_u \nu}$. The normalized modal shape can be obtained through the square root vector divided by the maximum square root value of the component.

The derivation above is based on the displacement response signal of structure vibration, while the acceleration response signal is the most common parameters in structure dynamic test. According to the formula (8), the acceleration response can be obtained by determining the second derivative of time t :

$$a_k(t) = \ddot{x}_k(t) = \sum_{i=1}^n \phi_{ki} \phi_i^T M x_0 \cdot e^{-\xi_i \omega_i t} \cdot \cos(\omega_{di} t + \theta_i) \quad (15)$$

where, θ_i is the i th initial phase of acceleration response. So, the expression of acceleration response has the same cosine function expression with that of the displacement response. And the derivation above can also be applied to acceleration response, which brings great convenience for structural modal parameter identification of the actual testing work.

To utilize the time-frequency spectrum correlation function to identify structure modal parameter by damped free vibration response signal, the steps are as follows: Firstly, pick up the displacement or acceleration signal of damped free vibration at each measuring point by suddenly unloading, determine its conjugate complex analytical signal according to the Hilbert transformation. Then, analyze the spectrum and time-frequency Wigner-Ville distribution of vibration response signal and get the modal frequency information of the structure. Thirdly, obtain the square root value of point spectrum correlation function on the structural modal frequency points by vibration response signal. Finally, on reference to the corresponding points of the maximize point spectrum correlation function square root value, construct the normalized the square root value vector of point spectrum correlation function to obtain the normalized structure modal shape.

4. Numerical example

To verify the effectiveness of the modal parameter identification method, taking a large span transmission tower structure as an example, the finite element software was used to simulate the transmission tower structure. The transmission tower is free-standing glass steel pipe combination tower, plane shape of the tower is square. It is connected by flange between the main rod, and it is connected by cross board between the main rod and web member with CHS joints. Because of the biaxial symmetry, mass center and stiffness center overlap, and torsion effect can be neglected. Nonlinear finite element calculation model is established as the beam-link mixed model in this paper. Four main chord choose beam element, the web member and cross bar choose link element, the finite element model for transmission tower is shown in Fig.1.

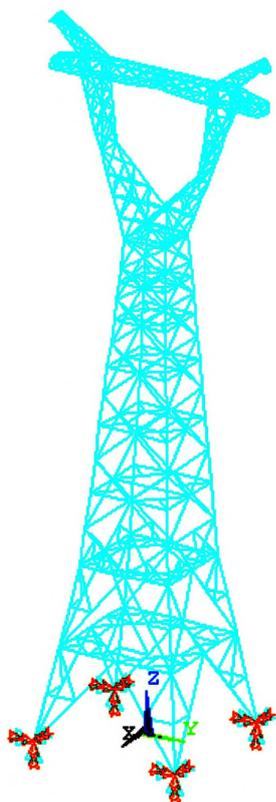


Fig.1 The model of transmission tower

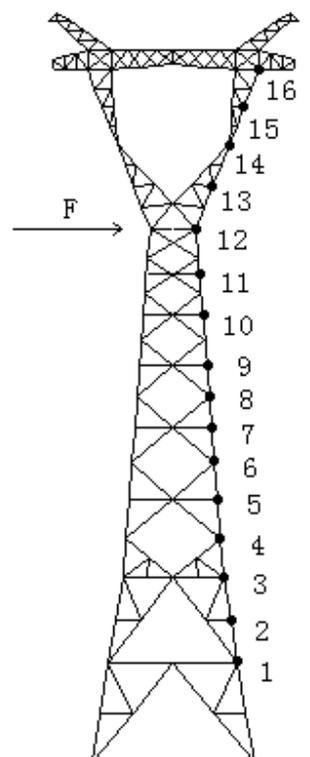


Fig.2 Arrangement of measure and incentive point

The numerical example is mainly to identify the bending modal along y direction. To call for the high order modal as much as possible and avoid reverse, suddenly unloading excitation amplitude is 80 KN and is evenly applied to four nodes of tower head and tower junction chord. The arrangement of measuring points and suddenly unloading excitation location are shown in Fig.2. Displacement response of the transmission tower structure under suddenly unloading

excitation is obtained through the finite element transient analysis. Fig. 3 and Fig. 4 give the typical structure vibration response and frequency spectrum diagram at measuring point 11.

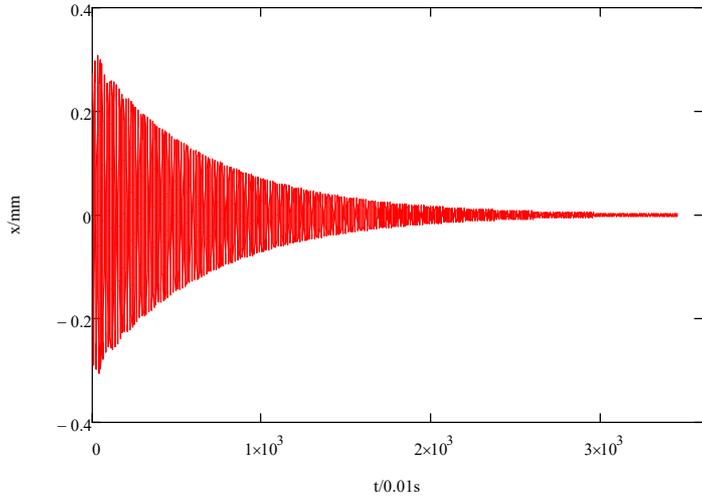


Fig.3 Structure vibration response

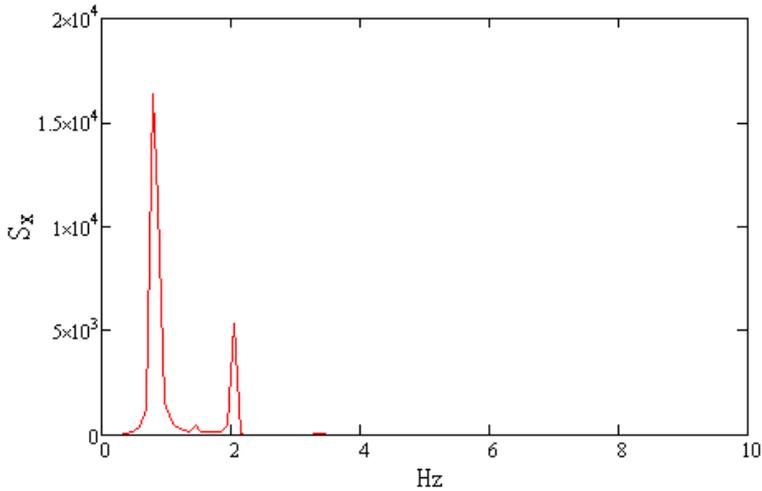


Fig.4 Spectrum of structure vibration response

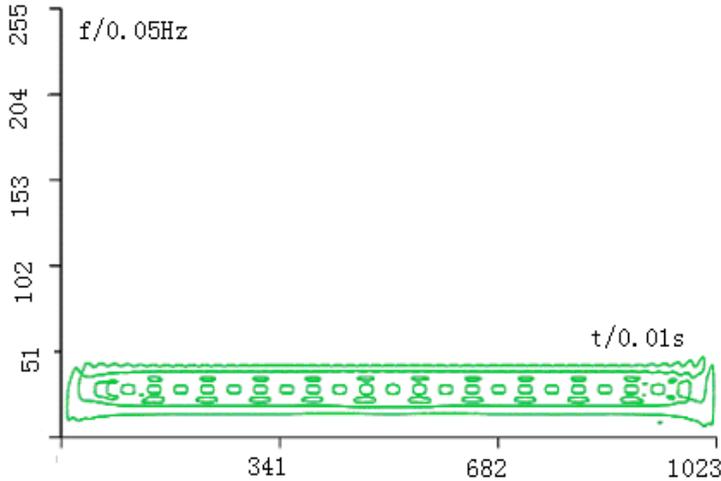


Fig.5 Time-frequency Spectrum of WVD

Analytical signal of vibration and Wigner-Ville distribution are calculated by the compiled program. Fig. 5 shows the Wigner-Ville distribution spectrum of the structural vibration response, in which the horizontal axis is axis of time with time resolution of 0.01s and the vertical direction is the frequency axis with the frequency resolution 0.05Hz. Furthermore, the horizontal continuous back parts are the signal-terms, and the discontinuous point parts are the cross-terms. From the response spectrum and the spectrum diagram of time-frequency distribution, it is can be found that structure vibration response had only be called for the first two order modal component, because of the great power transmission tower structure size. The first order modal frequency of the structure along y direction is 0.8262Hz, compared with the simulation calculation result 0.82415Hz, the identification error is only 0.25%. The second order modal frequency is 2.0264 Hz, compared with the simulation calculation result 2.0359Hz, the identification error is only 0.47%. The modal frequency identification error mainly comes from the data truncation of structure vibration response signal.

As a result, with reference to the biggest square root value, calculate the point spectral correlation function value of the first two order modal frequency at each measuring point, and get the modal vibration shape after normalized processing and the final results are shown in the Tab.1 and Tab.2. The maximum error of first order modal shape identification is 0.54%, the maximum error of second order modal shape identification is 1.52%. Because of the transmission tower structure size is huge, the second order modal vibration is inadequate, and the second order modal shape identification error is significantly higher than that of first order. The error of modal shape identification mainly comes from the vibration signal data truncation error, the time-frequency energy interference of cross terms and oscillation of time-frequency distribution. Compared with the wavelet identification method, the maximum error is 5% and the identification precision of the proposed method is greatly increased.

Tab.1 The first order modal shape identification result

Measure point	First order modal shape			Error (%)
	Simulation value	Point Spectrum value	Identification value	
16	1.00000	42787.06055	1.00000	0.00000
15	0.86234	31819.71054	0.86237	0.00330
14	0.79079	26761.63691	0.79086	0.00870
13	0.71684	21990.94038	0.71691	0.01025

12	0.63383	17192.50681	0.63389	0.00867
11	0.54761	12832.18483	0.54764	0.00545
10	0.46279	9164.09893	0.46280	0.00005
9	0.37257	5938.17786	0.37254	-0.00790
8	0.28419	3454.33464	0.28414	-0.01952
7	0.20261	1755.11599	0.20253	-0.03637
6	0.12774	697.09327	0.12764	-0.07769
5	0.06172	162.41376	0.06161	-0.18461
4	0.04421	83.29280	0.04412	-0.20827
3	0.03014	39.28244	0.03030	0.54373
2	0.01789	13.82933	0.01798	0.49591
1	0.01555	10.40917	0.01560	0.31233

Tab.2 The second order modal shape identification result

Measure point	Second order modal shape			Error (%)
	Simulation value	Point Spectrum value	Identification value	
16	0.99616	8373.78603	0.99859	0.24332
15	1.00000	8397.48509	1.00000	0.00000
14	0.99233	8251.36093	0.99126	-0.10749
13	0.97434	7938.88349	0.97231	-0.20831
12	0.95039	7536.40882	0.94734	-0.32067
11	0.90814	6867.44016	0.90432	-0.42089
10	0.86050	6152.71452	0.85597	-0.52695
9	0.78736	5141.29150	0.78246	-0.62273
8	0.70893	4159.51641	0.70380	-0.72442
7	0.61215	3095.28034	0.60712	-0.82162
6	0.51835	2214.42627	0.51352	-0.93222
5	0.41195	1395.34934	0.40763	-1.04941
4	0.31175	795.83131	0.30785	-1.25061
3	0.23566	453.24538	0.23232	-1.41568
2	0.14577	173.70381	0.14382	-1.33707
1	0.05868	28.03978	0.05778	-1.51743

To test the ability to resist noise on structural modal parameter identification by square root value of the spectrum correlation function, the noise with level 10% is added into the vibration signal and the structure vibration signal with noise is obtained as follows[13]:

$$x^*(t) = x(t) \cdot (1 + r|x(t)_{\max}| \rho) \quad (16)$$

where, r is the white noise with mean value of 0 and variance of 1, and ρ is the noise level taken as 10%.

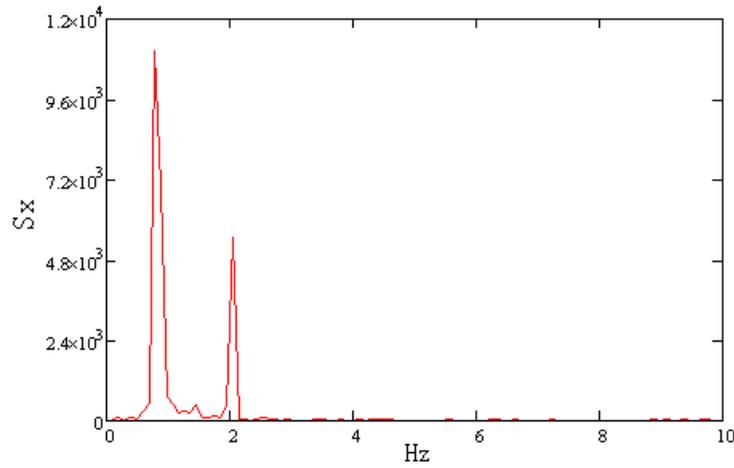


Fig.6 Spectrum of structure vibration response under the influence of noise

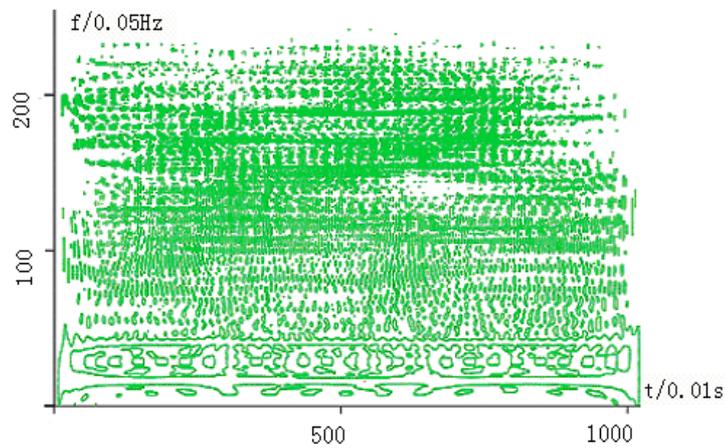


Fig.7 Time-frequency spectrum of WVD under the influence of noise

Fig.6 and Fig.7 give the vibration response spectrum and Wigner-Ville distribution spectrum of transmission tower structure respectively with influence of noise. From the spectrum diagram, it can be seen that the first and second order modal frequency of the structure along y direction can be clearly identified, the modal frequency identification values are the same as that without noise. From Wigner-Ville distribution spectrum diagram, due to the effects of environmental noise, the ingredients of the time-frequency distribution of structure vibration response signal have become more complex, in addition to the signal-term distribution and cross-term distribution, which are the continuous back and discontinuous point parts, there is a wide range of environmental noise signal composition distribution in time and frequency domain.

The identification result of transmission tower structure modal shape based on square root value of point spectrum correlation function under the influence of noise is presented in Tab.3 and Tab.4. It can be seen that the structural modal shape can be accurately identified, however the

identification error increased slightly compared with that without noise. In the case of environmental noise level is 10%, the maximum error of first order modal shape identification is 2.53%, the maximum error of second order modal shape identification is 4.55%. Compared with the maximum error 10% of wavelet identification method under the influence of the noise, the anti-noise ability of the proposed identification method is more superior.

Tab.3 The first modal shape identification result under noise

Measure point	First order modal shape			Error (%)
	Simulation value	Point Spectrum value	Identification value	
16	1.00000	43938.01978	1.00000	0.00000
15	0.86234	33289.87699	0.87043	0.93861
14	0.79079	28189.00144	0.80098	1.28810
13	0.71684	22055.05927	0.70849	-1.16480
12	0.63383	17899.73199	0.63827	0.70018
11	0.54761	12859.08824	0.54098	-1.20989
10	0.46279	9416.81817	0.46295	0.03406
9	0.37257	5798.84570	0.36329	-2.49146
8	0.28419	3511.57170	0.28270	-0.52322
7	0.20261	1713.51980	0.19748	-2.53169
6	0.12774	698.27134	0.12606	-1.31186
5	0.06172	172.79934	0.06271	1.60729
4	0.04421	83.21904	0.04352	-1.56023
3	0.03014	39.96050	0.03016	0.05804
2	0.01789	14.46542	0.01814	1.42266
1	0.01555	10.40328	0.01539	-1.04575

Tab.4 The second modal shape identification result under noise

Measure point	Second order modal shape			Error (%)
	Simulation value	Point Spectrum value	Identification value	
16	0.99616	8784.20186	1.00000	0.38548
15	1.00000	8599.02448	0.98940	-1.05965
14	0.99233	8448.02494	0.98068	-1.17420
13	0.97434	8395.99733	0.97765	0.34008
12	0.95039	7630.60107	0.93203	-1.93222
11	0.90814	7093.86113	0.89865	-1.04508

10	0.86050	6145.23844	0.83641	-2.79976
9	0.78736	4961.63091	0.75156	-4.54741
8	0.70893	4156.34775	0.68787	-2.97103
7	0.61215	3080.58186	0.59220	-3.25971
6	0.51835	2212.47477	0.50187	-3.18004
5	0.41195	1413.70510	0.40117	-2.61686
4	0.31175	806.94215	0.30309	-2.77822
3	0.23566	458.56479	0.22848	-3.04654
2	0.14577	175.38506	0.14130	-3.06581
1	0.05868	28.19410	0.05665	-3.45321

5. Conclusions

The point spectrum correlation function of time-frequency distribution is used to deduce analytic expression of free vibration response of the structure, the deduction reveals the response function relationship between point spectrum correlation function values and structural modal parameters, and then the square root value of point spectrum correlation function is proposed to identify modal shape of the transmission tower. This method does not need time domain or frequency domain information from external excitation.

Results of numerical example of transmission tower show that the structure modal frequency can be accurately achieved by spectrum and time-frequency analysis of vibration response signal. The maximum identification error of the first order modal shape using the square root value of time-frequency spectrum correlation function is 0.54%, and the maximum identification error of the second order modal shape is 1.52%. Under the influence of the noise level of 10%, the structure modal frequency identification is not affected, the maximum identification error of the first order modal shape is 2.53%, and the maximum identification error of the second order modal shape is 4.55%.

Acknowledgments

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