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Compromise solutions for a special type of stochastic multi-level linear multiple objective decision making problems

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ABSTRACT

We introduce the concept of the technique for order preference by similarity to ideal solution (TOPSIS) to develop a methodology to find compromise solutions for the multi-level linear multiple objective decision making (MLLMODM) problems of block angular structure with stochastic parameters in the right hand side of the independent constraints (SMLLMODM) of mixed (*Maximize/Minimize*)-type. We propose a modified formulas for the distance function from the positive ideal solution (PIS) and the distance function from the negative ideal solution (NIS). We present a new interactive hybrid algorithm based on the proposed TOPSIS approach, the chance constrained programming method and the decomposition method to generate a compromise solutions for these types of mathematical optimization problems. Also, we give an illustrative numerical example to clarify the main results developed in the paper. The solutions of the numerical example by the proposed interactive hybrid algorithm is compared with the solutions of the ideal point (IP) method. In general, the results show that, the proposed hybrid TOPSIS method is a good tool to generate compromise solutions for the SMLLMODM problems of mixed type.

1. INTRODUCTION

Compromise programming (CP) assumes that any decision maker (DM) seeks a solution as close as possible to the ideal point, [1-2]. TOPSIS is based on the principle that the solution should have the shortest distance from the (PIS) and the farthest distance from the (NIS), [3-8].

The non-centralized planning searches for a simultaneous compromise among the various objectives of the different levels. Multi-level programming, a tool for modeling non-centralized decisions, consists of the objective(s) of the manager at its first level and that is of the followers at the other levels. The decision-maker at each level seeks to optimize his individual objective functions, which depends in part on the variables controlled by the decision makers at the other levels and their final decisions are executed sequentially where the upper-level decision-maker makes his decision firstly, [9-10].

Caballero R. et al. [11], establish some relationships between different concepts of efficient solutions to problems of stochastic multiple objective programming.

BenAbdelaziz F., [12], presents a review for some methods and applications of the multiple objective stochastic programming models.

Masmoudi M. and BenAbdelaziz, F. [13], introduce a review for deterministic and stochastic multiple objective programming methods which can be used to solve portfolio selection problem.

A review on theory, applications and softwares of bi-level, multi-level multiple criteria decision making and TOPSIS approach is presented in [5].

An interactive decomposition algorithm for bi-level large scale linear multi-objective optimization problems with uncertain data using TOPSIS approach is given in [6].

In the following sections, the formulation of multi-level linear multiple objective decision making (MLLMODM) problems of block angular structure with stochastic parameters in the right hand side of the independent constraints (SMLLMODM) of mixed ($Maximize/Minimize$)-type is given in section (2). In section (3), we propose a modified formulas for the distance function from the positive ideal solution (PIS) and the distance function from the negative ideal solution (NIS) to modify the TOPSIS method to solve the SMLLMODM problems. We use the modified TOPSIS method, the chance constrained programming method, [14], and the decomposition method, [15], to introduce a new interactive hybrid algorithm to solve the SMLLMODM problems. In section (4), we provide a numerical example for the extended hybrid TOPSIS method. We compare the solutions of the proposed hybrid algorithm with the solution of ideal point (IP) method, [1-2]. In section (5), we present conclusions and future works.

2. FORMULATION OF THE PROBLEM

Consider the following Multi-Level Linear Multiple Objective Decision Making (MLLMODM) problem of block angular structure with Stochastic parameters in the right hand side of the independent constraints (SMLLMODM) of mixed (*Maximize/Minimize*)-type:

$[L_{DM_1}]$
$\underset{X_{I_1}}{Max/Min}(Q_{11}(X_{I_1},\dots,X_{I_h}),\dots,Q_{1k_1}(X_{I_1},\dots,X_{I_h}))$
$where\ X_{I_2} solves the\ 2^{nd} level$
$[L_{DM_2}]$
$\underset{X_{I_2}}{Max/Min}\left(Q_{21}(X_{I_1},\dots,X_{I_h}),\dots,Q_{2k_2}(X_{I_1},\dots,X_{I_h})\right)$
$where\ X_{I_3}\ solves\ the\ 3^{rd} level$
$[L_{DM_3}]$
$\underset{X_{I_3}}{Max/Min}\left(Q_{31}(X_{I_1},\dots,X_{I_h}),\dots,Q_{3k_3}(X_{I_1},\dots,X_{I_h})\right)$
$where\ X_{I_4}\ solves\ the\ 4^{th} level$

$\underset{X_{I_i}}{Max/Min}\left(Q_{i1}(X_{I_1},\dots,X_{I_h}),\dots,Q_{ik_i}(X_{I_1},\dots,X_{I_h})\right)$
$where\ X_{I_{i+1}}\ solves\ the\ (i+1)^{th} level$

$where X_{I_h} solves the\ h^{th} level$
$[L_{DM_h}]$
$\underset{X_{I_h}}{Max/Min}\left(Q_{h1}(X_{I_1},\dots,X_{I_h}),\dots,Q_{hk_h}(X_{I_1},\dots,X_{I_h})\right)$
subject to (1)

$$X \in \mathbb{M} = \{\sum_{j=1}^{q}\sum_{i=1}^{n} a_{ijh_0}x_{ijh_0} \le d_{h_0},$$
$$h_0 = 1,2,3,\dots,m_0,$$
$$P\left\{\sum_{i=1}^{n} b_{ijh_j}x_{ijh_j} \le v_{h_j}\right\} \ge \alpha_{h_j},$$
$$h_j = m_{j-1}+1, m_{j-1}+2,\dots,m_j,$$
$$x_{ij} \ge 0, i \in N, j = 1,2,\dots,q, q>1\}$$

where
a ,b and d are constants,
Q_{it_i} : objective functions for Maximization, $t_i \in K_i \subset K$, i=1,2,…,h,
Q_{iv_i} : objective functions for Minimization, , $v_i \in K_i \subset K$, i=1,2,…,h,
h : the number of levels,
k : the number of objective functions,
L_{DM_i} : i[th] level decision maker, i=1,2,…,h,
k_i : the number of objective functions of the L_{DM_i}, i=1,2,…,h,
n_{I_i} : the number of variables of the L_{DM_i},i=1,2,…,h.,
Q : the number of subproblems,
M : the number of constraints,
N : the numer of variables,
n_j : the number of variables of the j^{th}subproblem, $j=1,2,\dots,q,$
m_o : the number of the common constraints represented by

$$\sum_{j=1}^{q}\sum_{i=1}^{n} a_{ijh_0}x_{ijh_0} \le d_{h_0}$$

m_j : the number of independent constraints of the j^{th} subproblem represented by

$$\sum_{i=1}^{n} b_{ijh_j}x_{ijh_j} \le v_{h_j}, j = 1,2,\dots,q, q>1,$$

R : the set of all real numbers,
X : an n-dimensional column vector of variables,
X_{I_i} : an n_{I_i} - dimensional column vector of variables of the L_{DM_i}, i=1,2,…,h,
$K_i = \{1,2,\dots,k_i\}$, i=1,2,…,h,
$K = \{1,2,\dots,k\} = \bigcup_{i=1}^{h} K_i$,
$N = \{1,2,\dots,n\}$,
$H = \{1,2,\dots,h\}$,
$R^n = \{X=(x_1, x_2,\dots,x_n)^T : x_i \in R, i \in N\}$.

If the objective functions are linear, then the objective function can be written as follows:

$$Q_{i\theta_i} = \sum_{j=1}^{q} Q_{i\theta_i}^{j} = \sum_{j=1}^{q} C_{i\theta_i}^{j}X_j,$$
$$i = 1,2,\dots,h,\ \theta_i = 1,2,\dots,k_i, \tag{2}$$

where
$C_{i\theta_i}^{j}$: an n_j-dimensional row vector for the j^{th} subproblem in the i^{th} objective function,$i = 1,2,\dots,h,\ \theta_i = 1,2,\dots,k_i$,
X_j: an n_j -dimensional column vector of variables for the j^{th} subproblem, $j=1,2,\dots,q,$

In addition, P means probability, α_{h_j} are a specified probability levels, $h_j = m_{j-1}+1, m_{j-1}+2,\dots,m_j, j = 1,2,\dots,q,\ q>1$.
For the sake of simplicity, consider that the random parameters, v_{h_j} are distributed normally and independently of each other with known means E $\{v_{h_j}\}$ and variances Var$\{v_{h_j}\}$. Using the chance constrained programming method [14], the deterministic version of problem (1) can be written as follows:

$[L_{DM_1}]$
$\underset{X_{I_1}}{Max/Min}\left(Q_{11}(X_{I_1},\dots,X_{I_h}),\ \dots,Q_{1k_1}(X_{I_1},\dots,X_{I_h})\right)$
$where\ X_{I_2} solves the\ 2^{nd} level$
$[L_{DM_2}]$
$\underset{X_{I_2}}{Max/Min}\left(Q_{21}(X_{I_1},\dots,X_{I_h}),\dots,Q_{2k_2}(X_{I_1},\dots,X_{I_h})\right)$
$where\ X_{I_3}\ solves\ the\ 3^{rd} level$
$[L_{DM_3}]$
$\underset{X_{I_3}}{Max/Min}\left(Q_{31}(X_{I_1},\dots,X_{I_h}),\dots,Q_{3k_3}(X_{I_1},\dots,X_{I_h})\right)$
$where\ X_{I_4}\ solves\ the\ 4^{th} level$

$\underset{X_{I_i}}{Max/Min}\left(Q_{i1}(X_{I_1},\dots,X_{I_h}),\dots,Q_{ik_i}(X_{I_1},\dots,X_{I_h})\right)$
$where\ X_{I_{i+1}}\ solves\ the\ (i+1)^{th} level$

$where X_{I_h} solves the\ h^{th} level$
$[L_{DM_h}]$
$\underset{X_{I_h}}{Max/Min}\left(Q_{h1}(X_{I_1},\dots,X_{I_h}),\dots,Q_{hk_h}(X_{I_1},\dots,X_{I_h})\right)$
subject to (3)

$$X \in \mathbb{M}^{/} = \{\sum_{j=1}^{q}\sum_{i=1}^{n} a_{ijh_0}x_{ijh_0} \le d_{h_0},$$
$$h_0 = 1,2,3,\ldots,m_0,$$
$$\sum_{i=1}^{n} b_{ijh_j}x_{ijh_j} \le E\{v_{h_j}\} + k_{\alpha_j}\sqrt{Varv_{h_j}},$$
$$h_j = m_{j-1}+1, m_{j-1}+2,\ldots,m_j,$$
$$x_{ij} \ge 0, i \in N, j = 1,2,\ldots,q, q > 1\}.$$

where k_{α_j}, j=1,2,….,q, is the standard normal value such that $\Phi(k_{\alpha_j})$=1- α_j, j=1,…,q, and Φ represents the cumulative distribution function of the standard normal distribution.

3. OPSIS FOR (SMLLMODM) OF BLOCK ANGULAR STRUCTURE

Thus, we present the following hybrid algorithm of TOPSIS method to generate compromise solutions for MLLMODM problem of block angular structure with Stochastic parameters in the right hand side of the independent constraints SMLLMODM of mixed (*Maximize/Minimize*)-type**:**

The hybrid algorithm:

Phase (0):

Step 1.

Use the chance constrained programming method to transform problem (1) to the form of problem (3).

Step 2.

Let i=1 and go to phase (1)

Phase (1):

Step 3.

Use the decomposition method to construct the positive ideal solution (PIS) payoff table for the following problem:

$$[L_{DM_1}]$$
$$\underset{X_{I_1}}{Max/Min}\left(Q_{11}(X_{I_1},\ldots,X_{I_h}),\ldots,Q_{1k_1}(X_{I_1},\ldots,X_{I_h})\right)$$
$$\text{subject to} \tag{4}$$
$$X \in \mathbb{M}^{/}$$

and obtain $Q_1^{*L_{DM_1}} = \left(Q_{11}^{*L_{DM_1}}, Q_{12}^{*L_{DM_1}},\ldots,Q_{1k_1}^{*L_{DM_1}}\right)$ the individual positive ideal solutions, [6].

Step 4.

Use the decomposition method to construct the negative ideal solution (NIS) payoff table of problem (4) and obtain $Q_1^{-L_{DM_1}} = \left(Q_{11}^{-L_{DM_1}}, Q_{12}^{-L_{DM_1}},\ldots,Q_{1k_1}^{-L_{DM_1}}\right)$ the individual negative ideal solutions, [6].

Step 5.

Construct the distance functions $d_p^{PIS^{L_{DM_1}}}$ and $d_p^{NIS^{L_{DM_1}}}$ by using the above steps (3 & 4) and the following equations:

$$Q_1^{*L_{DM_1}} = \underset{X \in \mathbb{M}^{/}}{Max(orMin)} Q_{1t_1}^{L_{DM_1}}(X)\left(orQ_{1v_1}^{L_{DM_1}}(X)\right), \forall t_1(andv_1) \tag{5}$$

$$Q_1^{-L_{DM_1}} = \underset{X \in \mathbb{M}^{/}}{Min(orMax)} Q_{1v_1}^{L_{DM_1}}(X)\left(orQ_{1t_1}^{L_{DM_1}}(X)\right), \forall t_1(andv_1) \tag{6}$$

where $Q_1^{*L_{DM_1}} = \left(Q_{11}^{*L_{DM_1}}, Q_{12}^{*L_{DM_1}},\ldots,Q_{1k_1}^{*L_{DM_1}}\right)$ and $Q_1^{-L_{DM_1}} = \left(Q_{11}^{-L_{DM_1}}, Q_{12}^{-L_{DM_1}},\ldots,Q_{1k_1}^{-L_{DM_1}}\right)$ are the individual positive (negative) ideal solutions for the L_{DM_1}.

Thus, we obtain the following distance functions, [4, 6]:

$$d_p^{PIS^{L_{DM_1}}} = \left(\sum_{t_1\in K_1} w_{t_1}^p\left(\frac{Q_{1t_1}^{*L_{DM_1}} - Q_{1t_1}^{L_{DM_1}}(X)}{Q_{1t_1}^{*L_{DM_1}} - Q_{1t_1}^{-L_{DM_1}}}\right)^p + \sum_{v_1\in K_1} w_{v_1}^p\left(\frac{Q_{1v_1}^{L_{DM_1}}(X) - Q_{1v_1}^{*L_{DM_1}}}{Q_{1v_1}^{-L_{DM_1}} - Q_{1v_1}^{*L_{DM_1}}}\right)^p\right)^{1/p} \tag{7}$$

and

$$d_p^{NIS^{L_{DM_1}}} = \left(\sum_{t_1\in K_1} w_{t_1}^p\left(\frac{Q_{1t_1}^{L_{DM_1}}(X) - Q_{1t_1}^{-L_{DM_1}}}{Q_{1t_1}^{*L_{DM_1}} - Q_{1t_1}^{-L_{DM_1}}}\right)^p + \sum_{v_1\in K_1} w_{v_1}^p\left(\frac{Q_{1v_1}^{-L_{DM_1}} - Q_{1v_1}^{L_{DM_1}}(X)}{Q_{1v_1}^{-L_{DM_1}} - Q_{1v_1}^{*L_{DM_1}}}\right)^p\right)^{1/p} \tag{8}$$

where $w_i, i = 1,2,\ldots,k_1$, are the relative importance (weights) of objectives, and $p = 1,2,\ldots,\infty$.

Step 6.

Ask the L_{DM_1} to select p= p* ∈ {1, 2, …, ∞ },

Step 7.

Ask the L_{DM_1} to select $w_i = w_i^*, i = 1,2,\ldots,k_1$, where $\sum_{i=1}^{k_1} w_i = 1$,

Step 8.

Use steps (5, 6 and7) to compute $d_p^{PIS^{L_{DM_1}}}$ and $d_p^{NIS^{L_{DM_1}}}$.

Step 9.

Transform problem (4) to the following problem [4,6]:

$$Minimize\ d_p^{PIS^{L_{DM_1}}}(X)$$
$$Maximize\ d_p^{NIS^{L_{DM_1}}}(X)$$
$$\text{subject to} \tag{9}$$
$$X \in M^{/}$$

where $p = 1,2,\ldots,\infty$.

Step 10.

Construct the payoff table of problem (9):

At *p = 1*, use the decomposition method.

At *p ≥ 2*, use the generalized reduced gradient method, [15, 16], and obtain:

$$d_p^{-L_{DM_1}} = \left(\left(d_p^{PIS^{L_{DM_1}}}\right)^-, \left(d_p^{NIS^{L_{DM_1}}}\right)^-\right),$$
$$d_p^{*L_{DM_1}} = \left(\left(d_p^{PIS^{L_{DM_1}}}\right)^*, \left(d_p^{NIS^{L_{DM_1}}}\right)^*\right).$$

where

$$\left(d_p^{PIS^{L_{DM_1}}}\right)^* = \underset{X\in M^{/}}{Min/Max} d_p^{PIS^{L_{DM_1}}}(X)\ and\ the\ solution\ is X^{PIS^{L_{DM_1}}},$$
$$\left(d_p^{NIS^{L_{DM_1}}}\right)^* = \underset{X\in M^{/}}{Max/Min} d_p^{NIS^{L_{DM_1}}}(X)\ and\ the\ solution\ is X^{NIS^{L_{DM_1}}},$$
$$\left(d_p^{PIS^{L_{DM_1}}}\right)^- = d_p^{PIS^{L_{DM_1}}}\left(X^{NIS^{L_{DM_1}}}\right) and$$
$$\left(d_p^{NIS^{L_{DM_1}}}\right)^- = d_p^{NIS^{L_{DM_1}}}\left(X^{PIS^{L_{DM_1}}}\right).$$

Step 11.

Construct the following model for problem (9), [4,17,18]:

$$
\begin{aligned}
&Max\ \beta^{L_{DM_1}},\\
&\text{subject to} \qquad (10)\\
&\mu_1(X) \ge \beta^{L_{DM_1}},\\
&\mu_2(X) \ge \beta^{L_{DM_1}},\\
&\beta^{L_{DM_1}} \in [0,1],\\
&X \in M^{/},
\end{aligned}
$$

by using the following membership functions [18- 20]:

$$
\mu_1(X) = \begin{cases} 1, & if\, d_p^{PIS^{L_{DM_1}}}(X) < \left(d_p^{PIS^{L_{DM_1}}}\right)^*, \\ 1 - \dfrac{d_p^{PIS^{L_{DM_1}}}(X) - \left(d_p^{PIS^{L_{DM_1}}}\right)^*}{\left(d_p^{PIS^{L_{DM_1}}}\right)^- - \left(d_p^{PIS^{L_{DM_1}}}\right)^*}, & if \left(d_p^{PIS^{L_{DM_1}}}\right)^- \ge d_p^{PIS^{L_{DM_1}}}(X) \ge \left(d_p^{PIS^{L_{DM_1}}}\right)^*, \\ 0, & if\, d_p^{PIS^{L_{DM_1}}}(X) > \left(d_p^{PIS^{L_{DM_1}}}\right)^-, \end{cases} \tag{11}
$$

$$
\mu_2(X) = \begin{cases} 1, & if\, d_p^{NIS^{L_{DM_1}}}(X) > \left(d_p^{NIS^{L_{DM_1}}}\right)^*, \\ 1 - \dfrac{\left(d_p^{NIS^{L_{DM_1}}}\right)^* - d_p^{NIS^{L_{DM_1}}}(X)}{\left(d_p^{NIS^{L_{DM_1}}}\right)^* - \left(d_p^{NIS^{L_{DM_1}}}\right)^-}, & if \left(d_p^{NIS^{L_{DM_1}}}\right)^- \le d_p^{NIS^{L_{DM_1}}}(X) \le \left(d_p^{NIS^{L_{DM_1}}}\right)^*, \\ 0, & if\, d_p^{NIS^{L_{DM_1}}}(X) < \left(d_p^{NIS^{L_{DM_1}}}\right)^-, \end{cases} \tag{12}
$$

Step 12.

Solve problem (10) to obtain the satisfactory level $\beta^{*^{L_{DM_1}}}$ for the compromise solution $X^{*^{L_{DM_1}}}$, [1,2], (if exist). Otherwise, go to step (37).

Step 13.

Ask the L_{DM_1} to select the maximum negative and positive tolerance values τ_i^L and τ_i^R, $i = 1,2,\dots,n_{I_1}$ on the decision vector $X_{I_1}^{*^{L_{DM_1}}} = \left(x_{I_1 1}^{*^{L_{DM_1}}}, x_{I_1 2}^{*^{L_{DM_1}}}, \dots, x_{I_1 n_{I_1}}^{*^{L_{DM_1}}}\right)$, [6].

Step 14.

Let i=2 and go to phase (2).

Phase (2):

Step 15.

Use the decomposition algorithm to construct the PIS payoff table for the following problem:

$$
\begin{aligned}
&[L_{DM_2}]\\
&\underset{X_{I_2}}{Max/Min} \left(Q_{21}(X_{I_1},\dots,X_{I_h}),\dots,Q_{2k_2}(X_{I_1},\dots,X_{I_h})\right)\\
&\text{subject to} \qquad (13)\\
&X \in \mathbb{M}^{/}
\end{aligned}
$$

and obtain $Q_2^{*^{L_{DM_2}}} = \left(Q_{21}^{*^{L_{DM_2}}}, Q_{22}^{*^{L_{DM_2}}}, \dots, Q_{2k_2}^{*^{L_{DM_2}}}\right)$ the individual positive ideal solutions.

Step16.

Use the decomposition algorithm to construct the NIS payoff table of problem (13) and obtain $Q_2^{-^{L_{DM_2}}} = \left(Q_{21}^{-^{L_{DM_2}}}, Q_{22}^{-^{L_{DM_2}}}, \dots, Q_{2k_2}^{-^{L_{DM_2}}}\right)$ the individual negative ideal solutions.

Step 17.

Construct the distance functions $d_p^{PIS^{L_{DM_2}}}$ and $d_p^{NIS^{L_{DM_2}}}$ by using the above steps (15 &1 6) and the following equations:

$$
Q_2^{*^{L_{DM_2}}} = \underset{X \in \mathbb{M}^{/}}{Max(orMin)} Q_{2t_2}^{L_{DM_2}}(X) \left(or Q_{2v_2}^{L_{DM_2}}(X)\right), \forall t_2 (and v_2) \tag{14}
$$

$$
Q_2^{-^{L_{DM_2}}} = \underset{X \in \mathbb{M}^{/}}{Min(orMax)} Q_{2v_2}^{L_{DM_2}}(X) \left(or Q_{2t_2}^{L_{DM_1}}(X)\right), \forall t_2 (and\ v_2) \tag{15}
$$

where $Q_2^{*^{L_{DM_2}}} = \left(Q_{21}^{*^{L_{DM_2}}}, Q_{22}^{*^{L_{DM_2}}}, \dots, Q_{2k_2}^{*^{L_{DM_2}}}\right)$ and $Q_2^{-^{L_{DM_2}}} = \left(Q_{21}^{-^{L_{DM_2}}}, Q_{22}^{-^{L_{DM_2}}}, \dots, Q_{2k_2}^{-^{L_{DM_2}}}\right)$ are the individual positive (negative) ideal solutions for the L_{DM_1}.

Thus, we obtain the following distance functions:

$$
\begin{aligned}
d_p^{PIS^{L_{DM_2}}} = \Bigg(& \sum_{t_1 \in K_1} w_{t_1}^p \left(\frac{Q_{1t_1}^{*^{L_{DM_1}}} - Q_{1t_1}^{L_{DM_1}}(X)}{Q_{1t_1}^{*^{L_{DM_1}}} - Q_{1t_1}^{-^{L_{DM_1}}}}\right)^p + \\
& \sum_{v_1 \in K_1} w_{v_1}^p \left(\frac{Q_{1v_1}^{L_{DM_1}}(X) - Q_{1v_1}^{*^{L_{DM_1}}}}{Q_{1v_1}^{-^{L_{DM_1}}} - Q_{1v_1}^{*^{L_{DM_1}}}}\right)^p + \\
& \sum_{t_2 \in K_2} w_{t_2}^p \left(\frac{Q_{2t_2}^{*^{L_{DM_2}}} - Q_{2t_2}^{L_{DM_2}}(X)}{Q_{2t_2}^{*^{L_{DM_2}}} - Q_{2t_2}^{-^{L_{DM_2}}}}\right)^p + \\
& \sum_{v_2 \in K_2} w_{v_2}^p \left(\frac{Q_{2v_2}^{L_{DM_2}}(X) - Q_{2v_2}^{*^{L_{DM_2}}}}{Q_{2v_2}^{-^{L_{DM_2}}} - Q_{2v_2}^{*^{L_{DM_2}}}}\right)^p \Bigg)^{1/p}
\end{aligned} \tag{16}
$$

and

$$
\begin{aligned}
d_p^{NIS^{L_{DM_2}}} = \Bigg(& \sum_{t_1 \in K_1} w_{t_1}^p \left(\frac{Q_{1t_1}^{L_{DM_1}}(X) - Q_{1t_1}^{-^{L_{DM_1}}}}{Q_{1t_1}^{*^{L_{DM_1}}} - Q_{1t_1}^{-^{L_{DM_1}}}}\right)^p + \\
& \sum_{v_1 \in K_1} w_{v_1}^p \left(\frac{Q_{1v_1}^{-^{L_{DM_1}}} - Q_{1v_1}^{L_{DM_1}}(X)}{Q_{1v_1}^{-^{L_{DM_1}}} - Q_{1v_1}^{*^{L_{DM_1}}}}\right)^p + \\
& \sum_{t_2 \in K_2} w_{t_2}^p \left(\frac{Q_{2t_2}^{L_{DM_2}}(X) - Q_{2t_2}^{-^{L_{DM_2}}}}{Q_{2t_2}^{*^{L_{DM_2}}} - Q_{2t_2}^{-^{L_{DM_2}}}}\right)^p + \\
& \sum_{v_2 \in K_2} w_{v_2}^p \left(\frac{Q_{2v_2}^{-^{L_{DM_2}}} - Q_{2v_2}^{L_{DM_2}}(X)}{Q_{2v_2}^{-^{L_{DM_2}}} - Q_{2v_2}^{*^{L_{DM_2}}}}\right)^p \Bigg)^{1/p}
\end{aligned} \tag{17}
$$

where $w_i, i = 1,2,\dots,k_1 + k_2$, are the relative importance (weights) of objectives, and $p = 1,2,\dots,\infty$.

Step 18.

Ask the L_{DM_2} to select p= p* ∈ {1, 2, …, ∞ },

Step 19.

Ask the L_{DM_2} to select $w_i = w_i^*, i = 1,2,\dots,k_1 + k_2$, where $\sum_{i=1}^{k_1+k_2} w_i = 1$,

Step 20.

Use steps (17, 18 and19) to compute $d_p^{PIS^{L_{DM_2}}}$ and $d_p^{NIS^{L_{DM_2}}}$.

Step 21.

Transform problem (13) to the following problem:

$Minimize\, d_p^{PIS^{L_{DM_2}}}(X)$

$$Maximize\, d_p^{NIS^{L_{DM_2}}}(X)$$
subject to (18)
$X \in M^{/}$
where $p = 1,2, \ldots\ldots, \infty$.

Step 22.
Construct the payoff table of problem (18):
At $p = 1$, use the decomposition method.
At $p \geq 2$, use the generalized reduced gradient method and obtain:

$$d_p^{-L_{DM_2}} = \left(\left(d_p^{PIS^{L_{DM_2}}}\right)^-, \left(d_p^{NIS^{L_{DM_2}}}\right)^-\right), d_p^{*L_{DM_2}} = \left(\left(d_p^{PIS^{L_{DM_2}}}\right)^*, \left(d_p^{NIS^{L_{DM_2}}}\right)^*\right).$$

where

$$\left(d_p^{PIS^{L_{DM_2}}}\right)^* = \underset{X \in M^{/}}{Min/Max}\, d_P^{PIS^{L_{DM_2}}}(X) \text{ and the solution is } X^{PIS^{L_{DM_2}}},$$
$$\left(d_p^{NIS^{L_{DM_2}}}\right)^* = \underset{X \in M^{/}}{Max/Min}\, d_P^{NIS^{L_{DM_2}}}(X) \text{ and the solution is } X^{NIS^{L_{DM_2}}},$$
$$\left(d_p^{PIS^{L_{DM_2}}}\right)^- = d_p^{PIS^{L_{DM_2}}}\left(X^{NIS^{L_{DM_2}}}\right) \text{ and } \left(d_p^{NIS^{L_{DM_2}}}\right)^- = d_p^{NIS^{L_{DM_2}}}\left(X^{PIS^{L_{DM_2}}}\right).$$

Step 23.
Construct the following model for problem (18), [6]:

$$Maximize\, \beta^{L_{DM_2}},$$
subject to (19)
$$\mu_1(X) \geq \beta^{L_{DM_2}},$$
$$\mu_2(X) \geq \beta^{L_{DM_2}},$$
$$\beta^{L_{DM_2}} \in [0,1],$$
$$X \in M^{/},$$
$$\frac{x_{I_1 i} - \left(x_{I_1 i}^{*L_{DM_2}} - \tau_i^L\right)}{\tau_i^L} \geq \beta^{L_{DM_2}}, i = 1,2, \ldots, n_{I_1}$$
$$\frac{\left(x_{I_1 i}^{*L_{DM_2}} + \tau_i^R\right) - x_{I_1 i}}{\tau_i^R} \geq \beta^{L_{DM_2}}, i = 1,2, \ldots, n_{I_1}$$

by using the following membership functions:

$$\mu_1(X) = \begin{cases} 1, & if\, d_p^{PIS^{L_{DM_2}}}(X) < \left(d_p^{PIS^{L_{DM_2}}}\right)^*, \\ 1 - \dfrac{d_p^{PIS^{L_{DM_2}}}(X) - \left(d_p^{PIS^{L_{DM_2}}}\right)^*}{\left(d_p^{PIS^{L_{DM_2}}}\right)^- - \left(d_p^{PIS^{L_{DM_2}}}\right)^*}, \\ if \left(d_p^{PIS^{L_{DM_2}}}\right)^- \geq d_p^{PIS^{L_{DM_2}}}(X) \geq \left(d_p^{PIS^{L_{DM_2}}}\right)^*, \\ 0, & if\, d_p^{PIS^{L_{DM_2}}}(X) > \left(d_p^{PIS^{L_{DM_2}}}\right)^-, \end{cases} \tag{20}$$

$$\mu_2(X) = \begin{cases} 1, & if\, d_p^{NIS^{L_{DM_2}}}(X) > \left(d_p^{NIS^{L_{DM_2}}}\right)^*, \\ 1 - \dfrac{\left(d_p^{NIS^{L_{DM_2}}}\right)^* - d_p^{NIS^{L_{DM_2}}}(X)}{\left(d_p^{NIS^{L_{DM_2}}}\right)^* - \left(d_p^{NIS^{L_{DM_2}}}\right)^-}, \\ if \left(d_p^{NIS^{L_{DM_2}}}\right)^- \leq d_p^{NIS^{L_{DM_2}}}(X) \leq \left(d_p^{NIS^{L_{DM_2}}}\right)^*, \\ 0, & if\, d_p^{NIS^{L_{DM_2}}}(X) < \left(d_p^{NIS^{L_{DM_2}}}\right)^-, \end{cases} \tag{21}$$

Step 24.
Solve problem (19) to obtain the satisfactory level $\beta^{*L_{DM_2}}$ and the compromise solution $X^{*L_{DM_2}}$, (if exist). Otherwise, go to step (37)

Step 25.
Ask the L_{DM_2} to select the maximum negative and positive tolerance values τ_i^L and τ_i^R, $i = 1,2, \ldots, n_{I_2}$ on the decision vector $X_{I_2}^{*L_{DM_2}} = \left(x_{I_2 1}^{*L_{DM_2}}, x_{I_2 2}^{*L_{DM_2}}, \ldots\ldots, x_{I_2 n_{I_2}}^{*L_{DM_2}}\right)$, [6].

Step 26.
Let i=3 and go to phase (3).

Phase (h):

Step 27.
Use the decomposition method to construct the PIS payoff table for the following problem:

$[L_{DM_h}]$
$$\underset{X_{I_h}}{Max/Min} \left(Q_{h1}(X_{I_1}, \ldots, X_{I_h}), \ldots, Q_{hk_h}(X_{I_1}, \ldots, X_{I_h})\right)$$
subject to (22)
$X \in \mathbb{M}^{/}$

and obtain $Q_h^{*L_{DM_h}} = \left(Q_{h1}^{*L_{DM_h}}, Q_{h2}^{*L_{DM_h}}, \ldots\ldots, Q_{hk_h}^{*L_{DM_h}}\right)$ the individual positive ideal solutions.

Step 28.
Use the decomposition algorithm to construct the NIS payoff table of problem (22) and obtain $Q_h^{-L_{DM_h}} = \left(Q_{h1}^{-L_{DM_h}}, Q_{h2}^{-L_{DM_h}}, \ldots\ldots, Q_{hk_h}^{-L_{DM_h}}\right)$ the individual negative ideal solutions.

Step 29.
Construct the distance functions $d_p^{PIS^{L_{DM_h}}}$ and $d_p^{NIS^{L_{DM_h}}}$ by using the above steps (27 & 28) and the following equations:

$$Q_h^{*L_{DM_h}} = \underset{X \in \mathbb{M}^{/}}{Max(orMin)} Q_{ht_h}^{L_{DM_h}}(X)\left(or Q_{hv_h}^{L_{DM_h}}(X)\right), \forall t_h (and\, v_h) \tag{23}$$
$$Q_h^{-L_{DM_h}} = \underset{X \in \mathbb{M}^{/}}{Min(orMax)} Q_{hv_h}^{L_{DM_h}}(X)\left(or Q_{ht_h}^{L_{DM_h}}(X)\right), \forall t_h (and\, v_h) \tag{24}$$

where $Q_h^{*L_{DM_h}} = \left(Q_{h1}^{*L_{DM_h}}, Q_{h2}^{*L_{DM_h}}, \ldots\ldots, Q_{hk_h}^{*L_{DM_h}}\right)$ and $Q_h^{-L_{DM_h}} = \left(Q_{h1}^{-L_{DM_h}}, Q_{h2}^{-L_{DM_h}}, \ldots\ldots, Q_{hk_h}^{-L_{DM_h}}\right)$ are the individual positive (negative) ideal solutions for the L_{DM_h}.

Thus, we obtain the following distance functions:

$$d_p^{PIS^{L_{DM_h}}} = \left(\sum_{t_1 \in K_1} w_{t_1}^p \left(\frac{Q_{1t_1}^{*L_{DM_1}} - Q_{1t_1}^{L_{DM_1}}(X)}{Q_{1t_1}^{*L_{DM_1}} - Q_{1t_1}^{-L_{DM_1}}}\right)^p + \sum_{v_1 \in K_1} w_{v_1}^p \left(\frac{Q_{1v_1}^{L_{DM_1}}(X) - Q_{1v_1}^{*L_{DM_1}}}{Q_{1v_1}^{-L_{DM_1}} - Q_{1v_1}^{*L_{DM_1}}}\right)^p + \sum_{t_2 \in K_2} w_{t_2}^p \left(\frac{Q_{2t_2}^{*L_{DM_2}} - Q_{2t_2}^{L_{DM_2}}(X)}{Q_{2t_2}^{*L_{DM_2}} - Q_{2t_2}^{-L_{DM_2}}}\right)^p + \sum_{v_2 \in K_2} w_{v_2}^p \left(\frac{Q_{2v_2}^{L_{DM_2}}(X) - Q_{2v_2}^{*L_{DM_2}}}{Q_{2v_2}^{-L_{DM_2}} - Q_{2v_2}^{*L_{DM_2}}}\right)^p + \cdots +\right.$$

$$\sum_{t_h\in K_h} w_{t_h}^p \left(\frac{Q_{ht_h}^{*\,L_{DM_h}} - Q_{ht_h}^{L_{DM_h}}(X)}{Q_{ht_h}^{*\,L_{DM_h}} - Q_{ht_h}^{-\,L_{DM_h}}}\right)^p + \sum_{v_h\in K_h} w_{v_h}^p \left(\frac{Q_{hv_h}^{L_{DM_h}}(X) - Q_{hv_h}^{*\,L_{DM_h}}}{Q_{hv_h}^{-\,L_{DM_h}} - Q_{hv_h}^{*\,L_{DM_h}}}\right)^p\Bigg)^{1/p} \tag{25}$$

And

$$d_P^{NIS^{L_{DM_h}}} = \Bigg(\sum_{t_1\in K_1} w_{t_1}^p \left(\frac{Q_{1t_1}^{L_{DM_1}}(X) - Q_{1t_1}^{-\,L_{DM_1}}}{Q_{1t_1}^{*\,L_{DM_1}} - Q_{1t_1}^{-\,L_{DM_1}}}\right)^p + \sum_{v_1\in K_1} w_{v_1}^p \left(\frac{Q_{1v_1}^{-\,L_{DM_1}} - Q_{1v_1}^{L_{DM_1}}(X)}{Q_{1v_1}^{-\,L_{DM_1}} - Q_{1v_1}^{*\,L_{DM_1}}}\right)^p + \sum_{t_2\in K_2} w_{t_2}^p \left(\frac{Q_{2t_2}^{L_{DM_2}}(X) - Q_{2t_2}^{-\,L_{DM_2}}}{Q_{2t_2}^{*\,L_{DM_2}} - Q_{2t_2}^{-\,L_{DM_2}}}\right)^p + \sum_{v_2\in K_2} w_{v_2}^p \left(\frac{Q_{2v_2}^{-\,L_{DM_2}} - Q_{2v_2}^{L_{DM_2}}(X)}{Q_{2v_2}^{-\,L_{DM_2}} - Q_{2v_2}^{*\,L_{DM_2}}}\right)^p + \cdots + \sum_{t_h\in K_h} w_{t_h}^p \left(\frac{Q_{ht_h}^{L_{DM_h}}(X) - Q_{ht_h}^{-\,L_{DM_h}}}{Q_{ht_h}^{*\,L_{DM_h}} - Q_{ht_h}^{-\,L_{DM_h}}}\right)^p + \sum_{v_h\in K_h} w_{v_h}^p \left(\frac{Q_{hv_h}^{-\,L_{DM_h}} - Q_{hv_h}^{L_{DM_h}}(X)}{Q_{hv_h}^{-\,L_{DM_h}} - Q_{hv_h}^{*\,L_{DM_h}}}\right)^p\Bigg)^{1/p} \tag{26}$$

where $w_i, i = 1,2, \ldots, k_1 + k_2 + \cdots + k_h$, are the relative importance (weights) of objectives, and $p = 1,2, \ldots, \infty$.

Step 30.

Ask the L_{DM_h} to select $p = p^* \in \{1, 2, \ldots, \infty\}$,

Step 31.

Ask the L_{DM_h} to select $w_i = w_i^*, i = 1,2,\ldots,k_1 + k_2 + \cdots + k_h$, where $\sum_{i=1}^{k_1+k_2+\cdots+k_h} w_i = 1$,

Step 32.

Use steps (17, 18 and19) to compute $d_p^{PIS^{L_{DM_h}}}$ and $d_p^{NIS^{L_{DM_h}}}$.

Step 33.

Transform problem (22) to the following problem:

$$\begin{aligned} &\mathit{Minimize}\, d_p^{PIS^{L_{DM_h}}}(X) \\ &\mathit{Maximize}\, d_p^{NIS^{L_{DM_h}}}(X) \\ &\text{subject to} \\ &X \in M^{/} \end{aligned} \tag{27}$$

where $p = 1,2, \ldots, \infty$.

Step 34.

Construct the payoff table of problem (27):

At $p = 1$, use the decomposition method.

At $p \geq 2$, use the generalized reduced gradient method and obtain:

$$d_p^{-\,L_{DM_h}} = \left(\left(d_p^{PIS^{L_{DM_h}}}\right)^-, \left(d_p^{NIS^{L_{DM_h}}}\right)^-\right), d_p^{*\,L_{DM_h}} = \left(\left(d_p^{PIS^{L_{DM_h}}}\right)^*, \left(d_p^{NIS^{L_{DM_h}}}\right)^*\right).$$

where

$\left(d_p^{PIS^{L_{DM_h}}}\right)^* = \underset{X\in M^{/}}{Min/Max}\, d_P^{PIS^{L_{DM_h}}}(X)$ *and the solution is* $X^{PIS^{L_{DM_h}}}$, $\left(d_p^{NIS^{L_{DM_h}}}\right)^* = \underset{X\in M^{/}}{Max/Min}\, d_P^{NIS^{L_{DM_h}}}(X)$ *and the solution is* $X^{NIS^{L_{DM_h}}}$, $\left(d_p^{PIS^{L_{DM_h}}}\right)^- = d_p^{PIS^{L_{DM_h}}}\left(X^{NIS^{L_{DM_h}}}\right)$ *and* $\left(d_p^{NIS^{L_{DM_h}}}\right)^- = d_p^{NIS^{L_{DM_h}}}\left(X^{PIS^{L_{DM_h}}}\right)$.

Step 35.

Construct the following model for problem (27):

$$\begin{aligned} &\mathit{Maximize}\ \beta^{L_{DM_h}}, \\ &\text{subject to} \\ &\mu_1(X) \geq \beta^{L_{DM_h}}, \\ &\mu_2(X) \geq \beta^{L_{DM_h}}, \\ &\beta^{L_{DM_h}} \in [0,1], \\ &X \in M^{/}, \\ &\frac{x_{I_{\eta-1}i} - \left(X_{I_{\eta-1}i}^{*\,L_{DM_\eta}} - \tau_i^L\right)}{\tau_i^L} \geq \beta^{L_{DM_\eta}}, i = 1,2,\ldots,n_{I_{\eta-1}}, \\ &\quad \eta = 2,\ldots\ldots,h, \\ &\frac{\left(X_{I_{\eta-1}i}^{*\,L_{DM_\eta}} + \tau_i^R\right) - x_{I_{\eta-1}i}}{\tau_i^R} \geq \beta^{L_{DM_\eta}}, i = 1,2,\ldots,n_{I_{\eta-1}}, \\ &\quad \eta = 2,\ldots\ldots,h \end{aligned} \tag{28}$$

by using the following membership functions:

$$\mu_1(X) = \begin{cases} 1, & if\, d_p^{PIS^{L_{DM_h}}}(X) < \left(d_p^{PIS^{L_{DM_h}}}\right)^*, \\ 1 - \dfrac{d_p^{PIS^{L_{DM_h}}}(X) - \left(d_p^{PIS^{L_{DM_h}}}\right)^*}{\left(d_p^{PIS^{L_{DM_h}}}\right)^- - \left(d_p^{PIS^{L_{DM_h}}}\right)^*}, & if \left(d_p^{PIS^{L_{DM_h}}}\right)^- \geq d_p^{PIS^{L_{DM_h}}}(X) \geq \left(d_p^{PIS^{L_{DM_h}}}\right)^*, \\ 0, & if\, d_p^{PIS^{L_{DM_h}}}(X) > \left(d_p^{PIS^{L_{DM_h}}}\right)^-, \end{cases} \tag{29}$$

$$\mu_2(X) = \begin{cases} 1, & if\, d_p^{NIS^{L_{DM_h}}}(X) > \left(d_p^{NIS^{L_{DM_h}}}\right)^*, \\ 1 - \dfrac{\left(d_p^{NIS^{L_{DM_h}}}\right)^* - d_p^{NIS^{L_{DM_h}}}(X)}{\left(d_p^{NIS^{L_{DM_h}}}\right)^* - \left(d_p^{NIS^{L_{DM_h}}}\right)^-}, & if \left(d_p^{NIS^{L_{DM_h}}}\right)^- \leq d_p^{NIS^{L_{DM_h}}}(X) \leq \left(d_p^{NIS^{L_{DM_h}}}\right)^*, \\ 0, & if\, d_p^{NIS^{L_{DM_h}}}(X) < \left(d_p^{NIS^{L_{DM_h}}}\right)^-, \end{cases} \tag{30}$$

Step 36.

Solve problem (28) to obtain the satisfactory level $\beta^{*\,L_{DM_h}}$ and the compromise solution $X^{*\,L_{DM_h}}$, (if exist). Otherwise, go to step (37)

Step 37.

Stop.

4. ILLUSTRATIVE NUMERICAL EXAMPLE FOR THE HYBRID ALGORITHM

Consider the following three-Level Linear Multiple Objective Decision Making problem of block angular structure with stochastic parameters in the right hand side of the independent constraints of mixed (*Maximize/Minimize*)-type:

$[\mathrm{L}_{DM_1}]$
$\underset{x_1,x_2}{Maximize} f_{11}(X) = 6x_1 + 7x_2 + 3x_3 + 5x_4 + x_5 + x_6$
$\underset{x_1,x_2}{Minimize} f_{12}(X) = 3x_1 + 4x_2 + 2x_3 + 3x_4 + 2x_5 + x_6$
$where x_1 and x_2 solves the second level$
$[\mathrm{L}_{DM_2}]$
$\underset{x_3,x_4}{Maximize} f_{21}(X) = 13x_1 + 3x_2 + 5x_3 + 2x_4 + x_5 + 2x_6$
$\underset{x_3,x_4}{Minimize} f_{22}(X) = 10x_1 + 7x_2 + 4x_3 + 6x_4 + 2x_5 + 3x_6$
$where x_3 and x_4 solves the third level$
$[\mathrm{L}_{DM_3}]$
$\underset{x_5,x_6}{Maximize} f_{31}(X) = 12x_1 + 5x_2 + 6x_3 + 5x_4 + x_5 + x_6$
$\underset{x_5,x_6}{Minimize} f_{32}(X) = 9x_1 + 4x_2 + 5x_3 + 4x_4 + 3x_5 + 2x_6$
subject to

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 \le 50,\ P\{x_1 + x_2 \le v_0\} \ge 0.7257,$$
$$P\{2x_2 \le v_1\} \ge 0.4013, \quad P\{5x_3 + x_4 \le v_2\} \ge 0.5,$$
$$x_5 + x_6 \ge 5,\ x_5 + 5x_6 \le 50,\ x_1, x_2, x_3, x_4, x_5, x_6 \ge 0$$

Suppose that $v_i, i = 0,1,2$ are linearly independent normal distributed parameters with the following means and variances: $E(v_0) = 8,\ E(v_1) = 2\ ,\ (v_2) = 7\ ,\ Var(v_0) = 25, Var(v_1) = 4, Var(v_2) = 16$.

Solution:

By using problem (3), we can have:

$[\mathrm{L}_{DM_1}]$
$\underset{x_1,x_2}{Maximize} f_{11}(X) = 6x_1 + 7x_2 + 3x_3 + 5x_4 + x_5 + x_6$
$\underset{x_1,x_2}{Minimize} f_{12}(X) = 3x_1 + 4x_2 + 2x_3 + 3x_4 + 2x_5 + x_6$
$where\ x_1 and\ x_2\ solves\ the\ second\ level$
$[\mathrm{L}_{DM_2}]$
$\underset{x_3,x_4}{Maximize} f_{21}(X) = 13x_1 + 3x_2 + 5x_3 + 2x_4 + x_5 + 2x_6$
$\underset{x_3,x_4}{Minimize} f_{22}(X) = 10x_1 + 7x_2 + 4x_3 + 6x_4 + 2x_5 + 3x_6$
$where\ x_3\ and\ x_4\ solves\ the\ third\ level$
$[\mathrm{L}_{DM_3}]$
$\underset{x_5,x_6}{Maximize} f_{31}(X) = 12x_1 + 5x_2 + 6x_3 + 5x_4 + x_5 + x_6$
$\underset{x_5,x_6}{Minimize} f_{32}(X) = 9x_1 + 4x_2 + 5x_3 + 4x_4 + 3x_5 + 2x_6$
Subject to

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 \le 50,\ x_1 + x_2 \le 5,$$
$$2x_2 \le 2.5,\ 5x_3 + x_4 \le 7,\ x_5 + x_6 \ge 5,$$
$$x_5 + 5x_6 \le 50, \qquad x_1, x_2, x_3, x_4, x_5, x_6 \ge 0$$

Obtain *PIS* and *NIS* payoff tables for the $[\mathrm{L}_{DM_1}]$ of the Problem, (See APPENDIX-Table (1): PIS payoff table for L_{DM_1} problem of the example & Table (2): NIS payoff table for L_{DM_1} problem of the example). Thus,

PIS: $f^{*\mathrm{L}_{DM_1}}$= (104.25, 5) & *NIS*: $f^{-\mathrm{L}_{DM_1}}$= (5, 113.25)

- Next, construct equation and obtain the following equations:

$$d_P^{PIS^{\mathrm{L}_{DM_1}}} = \left[w_1^p \left(\frac{104.25 - f_{11}(X)}{104.25 - 5}\right)^p + w_2^p \left(\frac{f_{12}(x) - (5)}{113.25 - (5)}\right)^p \right]^{1/p}$$

$$d_P^{NIS^{\mathrm{L}_{DM_1}}} = \left[w_1^p \left(\frac{f_{11}(X) - 5}{104.25 - 5}\right)^p + w_2^p \left(\frac{113.25 - f_{12}(x)}{113.25 - (5)}\right)^p \right]^{1/p}$$

Thus, problem is obtained. In order to get numerical solutions, assume that $w_1^p = w_2^p = (0.5)^2$ and $p=2$. (See APPENDIX- Table (3): PIS payoff table for L_{DM_1} problem (p=2)). Thus, $d_2^{*\mathrm{L}_{DM_1}}$=(0.2391691, 0.5), $d_2^{-\mathrm{L}_{DM_1}}$=(0.5, 0.468276168).

Now, it is easy to compute problem (15):

$Maximize\ \beta^{\mathrm{L}_{DM_1}}$
Subject to

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 \le 50,\ \ x_1 + x_2 \le 11,$$
$$2x_2 \le 2.5,\ 5x_3 + x_4 \le 7,\ x_5 + x_6 \ge 5,$$
$$x_5 + 5x_6 \le 50,\ \ x_1, x_2, x_3, x_4, x_5, x_6 \ge 0$$
$$\left(\frac{d_2^{PIS^{\mathrm{L}_{DM_1}}}(X) - 0.2391691}{0.5 - 0.2391691}\right) \ge \beta^{\mathrm{L}_{DM_1}},$$
$$\left(\frac{0.5 - d_2^{NIS^{\mathrm{L}_{DM_1}}}(X)}{0.5 - 0.468276168}\right) \ge \beta^{\mathrm{L}_{DM_1}},$$
$$\beta^{\mathrm{L}_{DM_1}} \in [0,1]\ .$$

- The maximum "satisfactory level" ($\beta^{\mathrm{L}_{DM_1}}$ *=0.9865938)* is achieved for the solution $X_1^{*\mathrm{L}_{DM_1}}$*=zero,*$X_2^{*\mathrm{L}_{DM_1}}$*=zero,*$X_3^{*\mathrm{L}_{DM_1}}$*=zero,* $X_4^{*\mathrm{L}_{DM_1}}$*=zero,* $X_5^{*\mathrm{L}_{DM_1}}$*=5.890389,* $X_6^{*\mathrm{L}_{DM_1}}$*=zero.* Let the L_{DM_1} decide $X_1^{*\mathrm{L}_{DM_1}}$ *=zero* with positive tolerance $\tau^R = 0.00001$ and $\tau^l = 0.00001$ *and* $X_2^{*\mathrm{L}_{DM_1}}$ *=zero* with positive tolerance $\tau^R = 0.00001$and τ^l= 0.00001.
- j=2,

Obtain *PIS* and *NIS* payoff tables for the L_{DM_2}Problem, (Seee APPENDIX- Table (4): PIS payoff table for the L_{DM_2} problem & Table (5): NIS payoff table for the L_{DM_2} problem). Thus,

PIS: $f^{*\mathrm{L}_{DM_2}}$= (120, 10) & *NIS*: $f^{-\mathrm{L}_{DM_2}}$= (5, 171)

- Next, compute and obtain the following equations:

$$d_P^{PIS^{\mathrm{L}_{DM_2}}} = \left[w_1^p \left(\frac{104.25 - f_{11}(X)}{104.25 - 5}\right)^p + w_2^p \left(\frac{f_{12}(x) - (5)}{113.25 - (5)}\right)^p + w_3^p \left(\frac{120 - f_{21}(X)}{120 - 5}\right)^p + w_4^p \left(\frac{f_{22}(X) - (10)}{171 - (10)}\right)^p \right]^{1/p}$$

$$d_P^{NIS^{\mathrm{L}_{DM_2}}} = \left[w_1^p \left(\frac{f_{11}(X) - 5}{104.25 - 5}\right)^p + w_2^p \left(\frac{113.25 - f_{12}(x)}{113.25 - (5)}\right)^p + w_3^p \left(\frac{f_{21}(X) - 5}{120 - 5}\right)^p + w_4^p \left(\frac{171 - f_{22}(X)}{171 - (10)}\right)^p \right]^{1/p}$$

- Thus, problem is obtained. In order to get numerical solutions, assume that $w_1^p = w_2^p = w_3^p = w_4^p = (0.25)^2$ and p=2, (See APPENDIX- Table (6): PIS payoff table of L_{DM_2} problem (p=2)). Thus, $d_2^{*\mathrm{L}_{DM_2}}$ =(0.1998565, 0.3482764), $d_2^{-\mathrm{L}_{DM_2}}$=(0.346033, 0.3119787975).
- Now, it is easy to compute:

$Maximize\ \beta^{L_{DM_2}}$
Subject to

$$x_1+x_2+x_3+x_4+x_5+x_6\le 50,\ \ x_1+x_2\le 11,$$
$$2x_2\le 2.5,\ \ 5x_3+x_4\le 7,$$
$$x_5+x_6\ge 5,\ \ x_5+5x_6\le 50,\ \ x_1,x_2,x_3,x_4,x_5,x_6\ge 0$$
$$\left(\frac{d_2^{PIS^{L_{DM_2}}}(X)-0.1998565}{0.346033-0.1998565}\right)\ge\beta^{L_{DM_2}},$$
$$\left(\frac{0.3482764-d_2^{NIS^{L_{DM_2}}}(X)}{0.3482764-0.3119787975}\right)\ge\beta^{L_{DM_2}},$$
$$\left(\frac{(0+0.001)-x_1}{0.001}\right)\ge\beta^{L_{DM_2}}\ ,\ \left(\frac{x_1-(0-0.001)}{0.001}\right)\ge\beta^{L_{DM_2}}$$
$$\left(\frac{(0+0.001)-x_2}{0.001}\right)\ge\beta^{L_{DM_2}}\ ,\ \left(\frac{x_2-(0-0.001)}{0.001}\right)\ge\beta^{L_{DM_2}}\ ,$$
$$\delta^{L_{DM_2}}\in[0,1]\ .$$

- The maximum "satisfactory level" ($\beta^{L_{DM_2}}=1$) is achieved for the solution $X_1^{*L_{DM_2}}=0$, $X_2^{*L_{DM_2}}=0$, $X_3^{*L_{DM_2}}=0.8042670, X_4^{*L_{DM_2}}=2.978665$, $X_5^{*L_{DM_2}}=1.234568, X_6^{L_{DM_2}}=3.765432$, Let the L_{DM_2} decide $X_3^{*L_{DM_2}}=0.8042670, X_4^{*L_{DM_2}}=2.978665$ with positive tolerance $\tau^R=0.001$ and $\tau^l=0.001$.
- j=3,

Obtain *PIS* and *NIS* payoff tables for the L_{DM_2} Problem, (See APPENDIX- Table (7): PIS payoff table for the L_{DM_3} problem & Table (8): NIS payoff table for the L_{DM_3} problem). Thus,

PIS: $f^{*L_{DM_3}}=(133, 10)$ & *NIS*: $f^{-L_{DM_3}}=(5, 187)$

- Next, compute and obtain the following equations:

$$d_P^{PIS^{L_{DM_3}}}=\left[w_1^p\left(\frac{104.25-f_{11}(X)}{104.25-5}\right)^p+w_2^p\left(\frac{f_{12}(x)-(5)}{113.25-(5)}\right)^p+w_3^p\left(\frac{120-f_{21}(X)}{120-5}\right)^p+w_4^p\left(\frac{f_{22}(X)-(10)}{171-(10)}\right)^p+w_5^p\left(\frac{133-f_{31}(X)}{133-5}\right)^p+w_6^p\left(\frac{f_{32}(X)-(10)}{187-(10)}\right)^p\right]^{1/p}$$

$$d_P^{NIS^{L_{DM_3}}}=\left[w_1^p\left(\frac{f_{11}(X)-5}{104.25-5}\right)^p+w_2^p\left(\frac{113.25-f_{12}(x)}{113.25-(5)}\right)^p+w_3^p\left(\frac{f_{21}(X)-5}{120-5}\right)^p+w_4^p\left(\frac{171-f_{22}(X)}{171-(10)}\right)^p+w_5^p\left(\frac{f_{31}(X)-5}{133-5}\right)^p+w_6^p\left(\frac{223-f_{32}(X)}{187-(10)}\right)^p\right]^{1/p}$$

Thus, problem is obtained. In order to get numerical solutions, assume that $w_1^p=w_2^p=w_3^p=w_4^p==w_5^p=w_6^p=(1/6)^2$ and $p=2$, (See APPENDIX- Table (9): PIS payoff table of problem L_{DM_3} (p=2). Thus, $d_2^{*L_{DM_3}}=$ (0.1567556, 0.2858099), $d_2^{-L_{DM_3}}=$ (0.05997669727, 0.06691515737).

- Now, it is easy to compute:

$Maximize\ \beta^{L_{DM_3}}$
Subject to

$$x_1+x_2+x_3+x_4+x_5+x_6\le 50,\ \ x_1+x_2\le 11,$$
$$2x_2\le 2.5,\ \ 5x_3+x_4\le 7,$$
$$x_5+x_6\ge 5,\ \ x_5+5x_6\le 50,\ \ x_1,x_2,x_3,x_4,x_5,x_6\ge 0$$
$$\left(\frac{d_2^{PIS^{L_{DM_3}}}(X)-0.1567556}{0.05997669727-0.1567556}\right)\ge\beta^{L_{DM_3}},$$
$$\left(\frac{0.2858099-d_2^{NIS^{L_{DM_3}}}(X)}{0.2858099-0.06691515737}\right)\ge\beta^{L_{DM_3}},$$
$$\left(\frac{(0+0.001)-x_1}{0.001}\right)\ge\beta^{L_{DM_3}},\ \left(\frac{x_1-(0-0.001)}{0.001}\right)\ge\beta^{L_{DM_3}}$$
$$\left(\frac{(0+0.001)-x_2}{0.001}\right)\ge\beta^{L_{DM_3}}\ ,\ \left(\frac{x_2-(0-0.001)}{0.001}\right)\ge\beta^{L_{DM_3}}$$
$$\left(\frac{(0.803845+0.001)-x_3}{0.001}\right)\ge\beta^{L_{DM_3}}\ ,$$
$$\left(\frac{x_3-(0.803845-0.001)}{0.001}\right)\ge\beta^{L_{DM_3}}$$
$$\left(\frac{(2.9783+0.001)-x_4}{0.001}\right)\ge\beta^{L_{DM_3}}\ ,$$
$$\left(\frac{x_4-(2.9783-0.001)}{0.001}\right)\ge\beta^{L_{DM_3}},\ \beta^{L_{DM_3}}\in[0,1]$$

- The "satisfactory level" ($\beta^{L_{DM_3}}=0.5734662$) is achieved for the solution $X_1^{*L_{DM_3}}=0$, $X_2^{*L_{DM_3}}=0$, $X_3^{*L_{DM_3}}=0.8038405$, $X_4^{*L_{DM_3}}=2.978238$, $X_5^{*L_{DM_3}}=42.06675$,
- Table 1 presents a comparison among the proposed TOPSIS method, ideal point (IP) method and the ideal objective vector (IOV). Bold numbers represent better result. In general, the proposed TOPSIS algorithm is a good method to generate compromise solutions (at p=2).

Table 1. A comparison among the proposed TOPSIS method, IP method and the IOV

Objective	Proposed TOPSIS Method (p=2)	IP Method	Ideal Objective Vector	
			PIS	NIS
f_{11}	**59.3694615**	**5**	**104.25**	**5**
f_{12}	**94.6775895**	7.010214	**5**	**113.25**
f_{21}	**52.0424285**	**7.989786**	**120**	**5**
f_{22}	**105.21829**	12.989786	**10**	**171**
f_{31}	**61.780983**	**5**	**133**	**5**
f_{32}	**142.1324045**	12.010214	**10**	**187**

5. CONCLUSIONS AND FUTURE WORKS

In this paper:

We extend TOPSIS approach to find compromise solutions for the multi-level linear multiple objective decision making (MLLMODM) problems of block angular structure with stochastic parameters in the right hand side of the independent constraints (SMLLMODM) of mixed (*Maximize/Minimize*)-type.

We propose a modified formulas for the distance function from the positive ideal solution (PIS) and the distance function from the negative ideal solution (NIS).

We present a new interactive hybrid algorithm based on the proposed TOPSIS approach, the chance constrained programming method and the decomposition method to generate a compromise solutions for these types of mathematical optimization problems. Also, we give an illustrative numerical example to clarify the main results developed in the paper.

The solutions of the numerical example by the proposed interactive hybrid algorithm is compared with the solutions of the ideal point (IP) method. In general, the results show that, the proposed hybrid TOPSIS method is a good tool to generate compromise solutions for the SMLLMODM problems of mixed type.

In the future:

The scientists and the engineers can apply the presented hybrid algorithm to different practical SMLLMODM problems to obtain numerical solutions.
Based on the introduced hybrid algorithm, a MATLAB code can be built to solve SMLLMODM problems to obtain numerical compromise solutions.

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APPENDIX

Table 1. PIS payoff table for L_{DM_1} problem

	$f_{11}(X)$	$f_{12}(X)$	x_1	x_2	x_3	x_4	x_5	x_6
$\underset{x_1,x_2}{Max} f_{11}(X)$	104.25	113.25	3.75	1.25	0	7	38	0
$\underset{x_1,x_2}{Min} f_{12}(X)$	5	5	0	0	0	0	0	5

Table 2. NIS payoff table for L_{DM_1} problem

	$f_{11}(X)$	$f_{12}(X)$	x_1	x_2	x_3	x_4	x_5	x_6
$\underset{x_1,x_2}{Min} f_{11}(X)$	5	10	0	0	0	0	5	0
$\underset{x_1,x_2}{Max} f_{12}(X)$	104.25	113.25	3.75	1.25	0	7	38	0

Table 3. PIS payoff table for L_{DM_1} problem (p=2)

	$d_2^{PIS^{\propto-\mathrm{DM}_{L_1}}}$	$d_2^{NIS^{\propto-\mathrm{DM}_{L_1}}}$	$f_{11}(X)$	$f_{12}(X)$	x_1	x_2	x_3	x_4	x_5	x_6
$d_2^{PIS^{\propto-\mathrm{DM}_{L_1}}}$	0.24	0.47	72.17	43.17	3.75	1.25	0	7	0	5.92
$d_2^{NIS^{\propto-\mathrm{DM}_{L_1}}}$	0.5	0.5	5	5	0	0	0	0	0	5

Table 4. PIS payoff table for the $\mathrm{L}_{\mathrm{DM}_2}$problem

	$f_{21}(X)$	$f_{22}(X)$	x_1	x_2	x_3	x_4	x_5	x_6
$\underset{x_1,x_2}{Max} f_{21}(X)$	120	171	5	0	0	7	35	3
$\underset{x_1,x_2}{Min} f_{22}(X)$	5	10	0	0	0	0	5	0

Table 5. NIS payoff table for the $\mathrm{L}_{\mathrm{DM}_2}$ problem

	$f_{21}(X)$	$f_{22}(X)$	x_1	x_2	x_3	x_4	x_5	x_6
$\underset{x_1,x_2}{Min} f_{21}(X)$	5	10	0	0	0	0	5	0
$\underset{x_1,x_2}{Max} f_{22}(X)$	120	171	5	0	0	7	35	3

Table 6. PIS payoff table of L_{DM_2} problem (p=2)

	$d_2^{PIS^{\mathrm{L}_{DM_2}}}$	$d_2^{NIS^{\mathrm{L}_{DM_2}}}$	$f_{21}(X)$	$f_{22}(X)$	x_1	x_2	x_3	x_4	x_5	x_6
$Min.\, d_2^{PIS^{\mathrm{L}_{DM_2}}}$	0.2	0.312	86.322385	93.075	5	0	0.54	4.32	0	5
$Max.\, d_2^{NIS^{\mathrm{L}_{DM_2}}}$	0.35	0.35	10	15	0	0	0	0	0	5

Table 7. PIS payoff table for the L_{DM_3}problem

	$f_{31}(X)$	$f_{32}(X)$	x_1	x_2	x_3	x_4	x_5	x_6
$\underset{x_1,x_2}{Max} f_{31}(X)$	133	187	5	0	0	7	38	0
$\underset{x_1,x_2}{Min} f_{32}(X)$	5	10	0	0	0	0	0	5

Table 8. NIS payoff table for the L_{DM_3} problem

	$f_{31}(X)$	$f_{32}(X)$	x_1	x_2	x_3	x_4	x_5	x_6
$\underset{x_1,x_2}{Min} f_{31}(X)$	5	15	0	0	0	0	5	0
$\underset{x_1,x_2}{Max} f_{32}(X)$	133	187	11	0	2.4	7	38	0

Table 9. PIS payoff table of problem L_{DM_3} (p=2)

	$d_2^{PIS^{\mathrm{L}_{DM_3}}}$	$d_2^{NIS^{\mathrm{L}_{DM_3}}}$	$f_{31}(X)$	$f_{32}(X)$	x_1	x_2	x_3	x_4	x_5	x_6
$Min.\, d_3^{PIS^{\mathrm{L}_{DM_3}}}$	0.16	0.07	92.81	77.32	5	0	0.37852138	5.11	0	5
$Max.\, d_3^{NIS^{\mathrm{L}_{DM_3}}}$	0.0599	0.286	5	10	0	0	0	0	0	5