

## First-principles study of the binary intermetallics in the Cu-Lu and Cu-Pm systems

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[https://doi.org/10.18280.ama\\_a.550404](https://doi.org/10.18280.ama_a.550404)

### ABSTRACT

**Received:** 31 December 2017

**Accepted:** 25 June 2018

#### Keywords:

*Cu-Lu compounds, Cu-Pm compounds, First principles calculations, Rare earth alloys*

We present a first principle study of the structural, elastic and energetic properties of the  $\text{Cu}_a\text{X}_b$  ( $\text{X}=\text{Lu}, \text{Pm}$ ) compounds, within the first principles density functional theory (DFT). The equilibrium volume, lattice constant, enthalpy of formation and the elastic constant are calculated using the full-potential linearized augmented plane-wave [FP-LAPW] method in the generalized gradient approximation (GGA) scheme. The  $\text{CuLu}$ ,  $\text{Cu}_2\text{Lu}$ ,  $\text{Cu}_5\text{Lu}$ ,  $\text{CuPm}$ ,  $\text{Cu}_2\text{Pm}$ ,  $\text{Cu}_4\text{Pm}$ ,  $\text{Cu}_5\text{Pm}$  and  $\text{Cu}_6\text{Pm}$  were investigated in their similar Cu-Lanthanide structure prototype compounds observed experimentally. The  $\text{Cu}_7\text{Lu}_2$ ,  $\text{Cu}_9\text{Lu}_2$  and  $\text{Cu}_7\text{Pm}_2$  intermetallics reported without prototype structure, was also investigated by inspecting several hypothetical structures. The most stable structure for the  $\text{Cu}_7\text{X}_2$  compounds was found to be the orthorhombic structure in the  $\text{Ag}_7\text{Yb}_2$  prototype. For the  $\text{Cu}_9\text{Lu}_2$  compound the two structures studied have a positive enthalpy, implying that it is not a ground state for both tested.

## 1. INTRODUCTION

Copper is a very common substance that is naturally present in the environment and diffuses into the environment through natural phenomena. It is used enormously, for example in industry and in agriculture; on the other hand promethium Pm and lutetium Lu are counted among lanthanide compounds. The lanthanides alloys are widely used in the industries applications as catalysts, in streetlights, searchlights, and in the high-intensity lighting present in sports stadiums. They form alloys with many other metals, and these alloys exhibit a wide range of physical properties. We did not find an experimental study for the phase diagram of the Cu-Pm and Cu-Lu systems. The only theoretical studies previously proposed for Cu-Pm and Cu-Lu are from Subramanian and Laughlin [1, 2] which proposes a theoretical phase diagram based on the similarity of the binary phase diagrams of Cu-Lanthanide, as the Cu-Ce, Cu-Pr, Cu-Sm and Cu-Nd systems. Hence, they proposed the existence of the compounds  $\text{CuPm}$ ,  $\text{Cu}_2\text{Pm}$ ,  $\text{Cu}_4\text{Pm}$ ,  $\text{Cu}_5\text{Pm}$ ,  $\text{Cu}_6\text{Pm}$  and  $\text{Cu}_7\text{Pm}_2$ . The structure prototype of these compounds is inspired from the Cu-Ce binary system which has been studied by several authors [3-6]. All of these compounds crystallize in the orthorhombic structure except the  $\text{Cu}_5\text{Pm}$  which is hexagonal in  $\text{Cu}_5\text{Ce}$  prototype and the  $\text{Cu}_7\text{Pm}_2$  which they did not propose a prototype. The prototypes of the other compounds are also distinguished,  $\text{CuPm}$  and  $\text{Cu}_6\text{Pm}$  formed in FeB and  $\text{Cu}_6\text{Ce}$  prototype respectively;  $\text{Cu}_2\text{Pm}$  to  $\text{Cu}_2\text{Ce}$  as a prototype and the last  $\text{Cu}_4\text{Pm}$  crystallizes in a  $\text{Cu}_4\text{Ce}$  prototype. The Cu-Lu system is similar to Cu-heavy Lanthanide, and its compounds which have been proposed are  $\text{CuLu}$ ,  $\text{Cu}_2\text{Lu}$ ,  $\text{Cu}_5\text{Lu}$  in the prototypes structure form of the CsCl,  $\text{Cu}_2\text{Ce}$  and  $\text{AuBe}_5$  respectively, and  $\text{Cu}_7\text{Lu}_2$ ,  $\text{Cu}_9\text{Lu}_2$  without prototype. The crystal parameters for these intermediate phases were

estimated on the basis of the systematic of crystallographic data in the Cu- Lanthanide systems.

The aim of the present work, based on first-principles calculations, is to investigate the relative stabilities of the different compounds involved in the Cu-Lu and Cu-Pm systems, and predicting a stable structure for the  $\text{Cu}_7\text{X}_2$  ( $\text{X}=\text{Lu}, \text{Pm}$ ) compound. Having our calculations performed at 0 K, we will be mainly focusing on the determination of the ground state line of these systems.

## 2. COMPUTATIONAL DETAILS

We employed for the ground state energy the full potential linearized augmented plane wave [FP-LAPW] based on density functional theory (DFT) [7], as implemented in WIEN2K code [8, 9]. For structural and elastic properties, the exchange correlation potential was calculated using the generalized gradient approximation (GGA) in the form proposed by Perdew et al. [10]. In order to achieve energy eigenvalues convergence, the wave functions in the interstitial region were expanded in plane waves with a cut off of  $K_{\text{max}} = 9/R_{\text{mt}}$ , where  $R_{\text{mt}}$  denotes the smallest atomic sphere radius and  $K_{\text{max}}$  gives the magnitude of the largest k-vector in the plane wave expansion. The k-point meshes for Brillouin zone sampling were constructed using the MonkhorstPack scheme [11]; the (12x12x12) k-points mesh was used for all calculations. The structures of our studied compounds are optimized by calculating the total energy as a function of volume. The results are fitted according to the Murnaghan equation of state [12] to obtain the ground-state properties given by

$$E(V) = E_0 + \frac{B_0}{B'(B'-1)} \left[ V \left( \frac{V_0}{V} \right)^{B'} - V_0 \right] + \frac{B_0}{B'} (V - V_0) \quad (1)$$

where  $E(V)$  is the internal energy,  $V_0$  and  $E_0$  are the equilibrium volume and energy, at zero pressure,  $V$  is the deformed volume,  $B_0$  is the bulk modulus, and  $B'$  is the derivative of the bulk modulus.

The enthalpy of formation is obtained by calculating the difference between the energy of the compound and the energy of its constituents according to the following equation:

$$\Delta H_f(Cu_aX_b) = E_{tot}(Cu_aX_b) - [aE_{Cu} + bE_X] \quad (2)$$

where  $\Delta H_f(Cu_aX_b)$  is the enthalpy of formation of the compound  $Cu_aX_b$ .  $E_{tot}(Cu_aX_b)$ ,  $E_{Cu}$  and  $E_X$  are the ground state energies.

The elastic properties define how a material under stress deforms and then recover and return to its original shape after stress ceases. We used the calculation of the elastic stiffness constant  $C_{ij}$  to check the mechanical stability of the  $Cu_7X_2$  compounds. The strain–stress method [13] are used, in which the relaxed unit cell under hydrostatic stress tensor was linearly deformed, then the stress tensor was calculated self-consistently after relaxing forces.

### 3. RESULTS AND DISCUSSION

#### 3.1 Pure elements: Cu, Lu and Pm

We have evaluated the energies of the pure elements Cu, Lu and Pm in their observed structures at room temperature. The calculated lattice constants are summarized in table 1. We compared our results with the experimental data [14-16] and those obtained by the VASP code [17]. A good agreement has been obtained with the available results from literature.

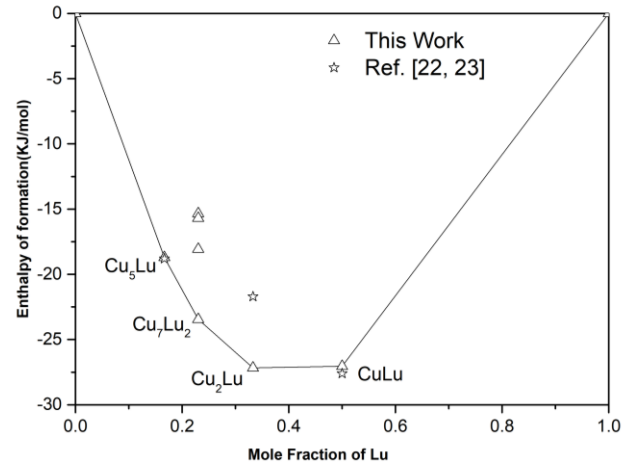
The promethium Pm has two structures,  $\alpha$  (hexagonal) and a  $\beta$  (cubic). Our calculation of the ground state energy indicate that the hexagonal form is the most stable one that's why we used it to calculate the enthalpy of formation

#### 3.2 Cu-Lu system

Only three compounds in the Cu-Lu systems are known: CuLu,  $Cu_2Lu$  and  $Cu_5Lu$ . Subramanian and Laughlin [18] have estimated phase diagram by systematic considerations of binary Cu-rare-earth systems and knowing the stoichiometry of some intermediate phases. Two intermetallics  $Cu_9Lu_2$  and  $Cu_7Lu_2$  were assumed to exist, for

analogous phases are present in the Cu-RE (RE = Gd, Dy and Er) systems.

Iandelli and Palenzona [19] reported the cubic  $AuBe_5$ -type structure for  $Cu_5Lu$ . Storm and Benson [20] reported the formation of  $Cu_2Lu$  with the orthorhombic  $CeCu_2$ -type structure, and Dwight [21] reported the cubic CsCl-type structure for CuLu compound.



**Figure 1.** Enthalpies of formation of Cu-Lu intermetallics

By minimizing the total energies, we obtained the equilibrium lattice constants of these compounds, and the results are summarized in table 2. The experimental results are reasonably reproduced with differences less than 1%. Calculated enthalpies of formation are presented in table 3 and figure 1 compared with the experimental data [22, 23], where a good agreement is obtained with the experimental uncertainties.

The existence of the intermetallic  $Cu_7Lu_2$  and  $Cu_9Lu_2$  has not confirmed in any work. There is no structural data of these compounds. To calculate the enthalpy of formation of this supposed compound, and since there are no structural data in literature, we tried out several structures. We have found six crystal structures for the  $A_7B_2$  compound and two crystal structures for the  $A_9B_2$ . The calculated enthalpies of formation of the  $Cu_9Lu_2$  compound in the two structures studied ( $Pd_9Si_2$  type, space group  $Pnma$  and  $Al_9Co_2$  type, space group  $P12_1/c1$ ) performed a positive enthalpy, which mean that it is not a ground state for both tested.

**Table 1.** Calculated lattice parameters and bulk modulus of the pure elements Cu, Lu and Pm

element	Pearson Symbol	Space group	Lattices Parameters (nm)			$B_0$ (GPa) (This Work)	
				This Work	VASP <sup>(a)</sup>		Exp.
Cu	cF4	$Fm-3m$	a	0.363	0.362	0.363 <sup>(b)</sup>	139.37
			b	0.349	0.351	0.350 <sup>(c)</sup>	
Lu	hP2	$P63/mmc$	b	0.349	0.351	0.350	47.11
			c	0.552	0.547	0.556	
			a	0.314	0.367	0.365 <sup>(d)</sup>	
$\alpha$ Pm	hP4	$P63/mmc$	b	0.314	0.367	0.365	36.33
			c	1.011	1.606	1.165	
			a	0.314	0.367	0.365	

(a) Ref. [17]

(b) Ref. [14]

(c) Ref. [15]

(d) Ref. [16]

**Table 2.** Calculated and experimental lattice parameters and bulk modulus of Cu-Lu intermetallics

Phases	Pearson Symbol	Space group	Lattice Parameters (nm)			B <sub>0</sub> (GPa) This work
				This work	Exp.	
Cu <sub>5</sub> Lu	cF24	$F\bar{4}3m$	a	0.7003	0.6970 <sup>(a)</sup>	115.14
			b	0.429	0.424 <sup>(b)</sup>	
Cu <sub>2</sub> Lu	oI12	$Imma$	b	0.666	0.663	93.20
			c	0.726	0.722	
CuLu	cP2	$Pm\bar{3}m$	a	0.340	0.339 <sup>(c)</sup>	78.09

(a) Ref. [19]

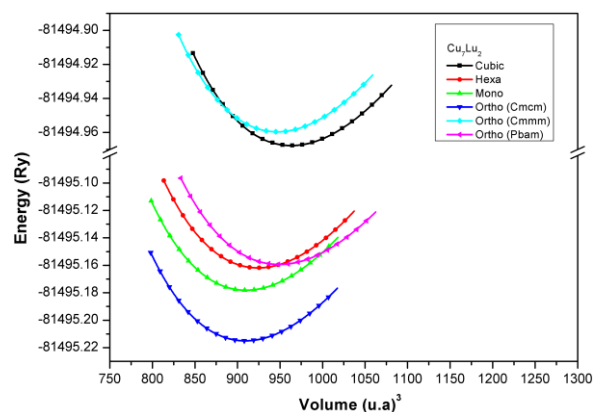
(b) Ref. [20]

(c) Ref. [21]

**Table 3.** Enthalpies of formation of Cu-Lu intermetallics (kJ/mol)

Compound	This Work	Exp. [22, 23]
Cu <sub>5</sub> Lu	-18.70	-18.8±1.9
Cu <sub>2</sub> Lu	-27.18	-21.7±5.3
CuLu	-27.04	-27.6±2.3

For the compound Cu<sub>7</sub>Lu<sub>2</sub>, we calculated the equations of state of the six proposed structures (Figure 2). As we can see the orthorhombic structure (space group Cmcm) is the stable phase. Lattice parameters and enthalpies of formation are listed in table 4. The lower enthalpy is obtained with orthorhombic structure. The results show that the enthalpy of formation of this structure breaks the convex hull (Figure 1). The different crystalline structures proposed for the Cu<sub>7</sub>Lu<sub>2</sub> are shown in figure 3.

**Figure 2.** Calculated total energies as a function of volume of the suspected structures for the Cu<sub>7</sub>Lu<sub>2</sub> compound**Table 4.** Enthalpies of formation and bulk modulus of Cu<sub>7</sub>Lu<sub>2</sub> in different suspected structures

Prototype	Pearson Symbol	Space group	Lattice Parameters (nm)	H <sup>for</sup> (kJ/mol)	B <sub>0</sub> (GPa)	
Al <sub>7</sub> Th <sub>2</sub>	oP18	$Pbam$	a	0.534	-15.33	101.89
			b	1.044		
			c	0.503		
Ni <sub>7</sub> Zr <sub>2</sub>	mC36	$C2/m$	a	0.492	-18.07	108.49
			b	0.862		
			c	1.276		
Ag <sub>7</sub> Yb <sub>2</sub>	oS36	$Cmcm$	a	0.853	-23.45	107.60
			b	0.497		
			c	1.267		
Ge <sub>2</sub> Li <sub>7</sub>	oC36	$Cmmm$	a	0.921	+13.80	90.97
			b	1.316		
			c	0.461		
Sb <sub>2</sub> Tl <sub>7</sub>	cI54	$Im\bar{3}m$	a	0.950	+12.63	89.65
Ce <sub>2</sub> Ni <sub>7</sub>	hP36	$P6_3/mmc$	a	0.503	-15.69	109.24
			c	2.497		

To verify the mechanical stability of this compound, we calculated its elastic stiffness constants and bulk modulus B<sub>H</sub> (Table 5), and we checked them with all stability conditions for orthorhombic structures given by Wu et al. [24]:

$$\left\{ \begin{array}{l} C_{11} > 0, \quad C_{22} > 0, \quad C_{33} > 0, \\ C_{44} > 0, \quad C_{55} > 0, \quad C_{66} > 0 \\ [C_{11} + C_{22} + C_{33} + 2(C_{12} + C_{13} + C_{23})] > 0 \\ (C_{11} + C_{22} - 2C_{12}) > 0 \\ (C_{11} + C_{33} - 2C_{13}) > 0 \\ (C_{22} + C_{33} - 2C_{23}) > 0 \end{array} \right. \quad (3)$$

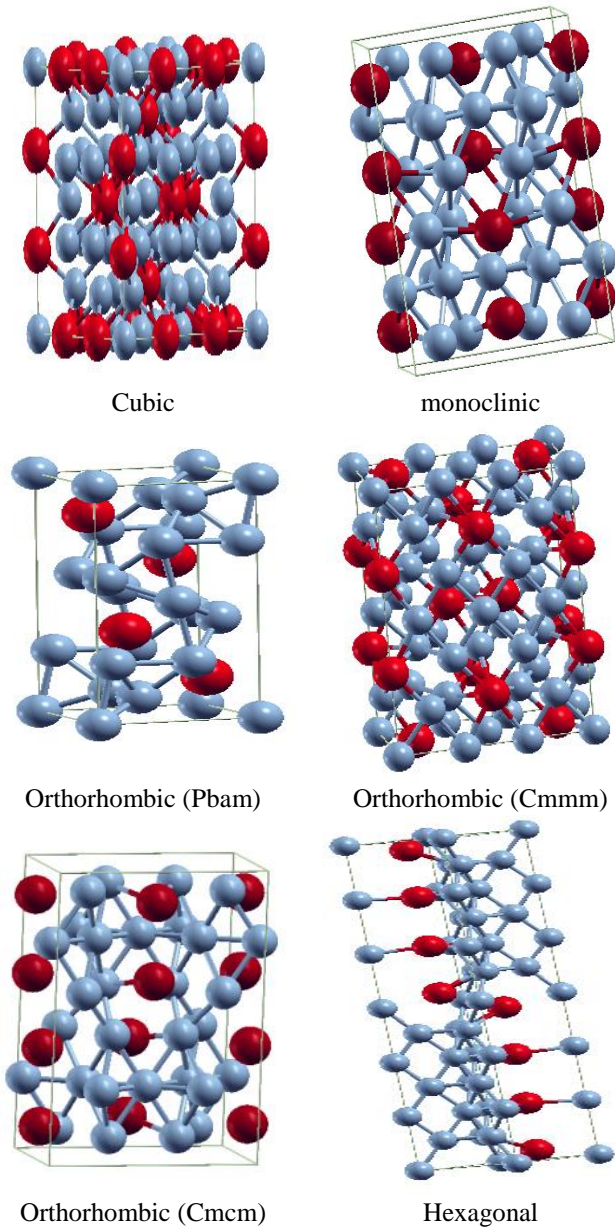
B<sub>H</sub> is the bulk modulus given according to the Hill approximation [13] as:

$$B_H = \frac{1}{2}(B_{\text{Reuss}} + B_{\text{Voigt}}) \quad (4)$$

where B<sub>Reuss</sub> and B<sub>Voigt</sub> are the bulk modulus given by Reuss [25] and Voigt [26] approximations respectively.

The obtained values satisfy all this stability conditions, which confirm its mechanical orthorhombic structure stability, and the bulk modulus B<sub>H</sub> is in good agreement with the one calculated by the Murnaghan equation.

To the best of our knowledge, there are no experimental and other theoretical data in literature for the elastic constants (C<sub>ij</sub>) and bulk modulus of this compound for comparison, so we consider the present results as prediction study which still awaits an experimental confirmation.



**Figure 3.** Schematic representation of the six proposed structures for the  $\text{Cu}_7\text{Lu}_2$  compound

**Table 5.** Elastic stiffness constants and bulk modulus of the  $\text{Cu}_7\text{Lu}_2$  and  $\text{Cu}_7\text{Pm}_2$  compound

	$\text{Cu}_7\text{Lu}_2$	$\text{Cu}_7\text{Pm}_2$
$C_{11}$	187.85	164.04
$C_{12}$	78.02	52.54
$C_{13}$	68.40	49.71
$C_{22}$	177.23	142.24
$C_{23}$	76.59	42.91
$C_{33}$	174.17	117.15
$C_{44}$	42.84	30.69
$C_{55}$	37.24	46.28
$C_{66}$	49.76	46.95
$B_H$	109.40	78.32
$B_0^{Murn}$	107.60	92.61

### 3.3 Cu-Pm system

This system has not been studied experimentally till now. By comparison with other Cu-rare-earth systems and interpolating the data which could be expected for the Cu-Pm system, Subramanian and Laughlin [1] have proposed a phase diagram with six compounds:  $\text{Cu}_6\text{Pm}$ ,  $\text{Cu}_5\text{Pm}$ ,  $\text{Cu}_4\text{Pm}$ ,  $\text{Cu}_2\text{Pm}$ ,  $\text{CuPm}$  and  $\text{Cu}_7\text{Pm}_2$ .

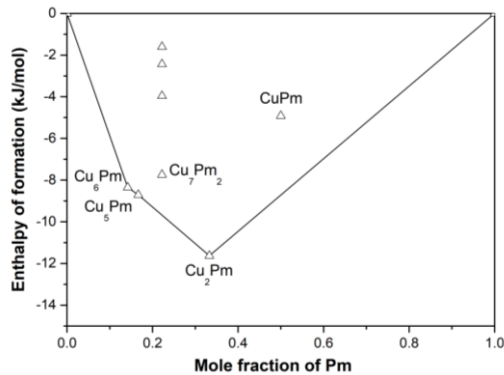
Lattice parameters for these intermetallics, except  $\text{Cu}_7\text{Pm}_2$ , were estimated by Subramanian and Laughlin [1] on the basis of the systematic of crystallographic data in the Cu lanthanide systems. We have not found any information for the  $\text{Cu}_7\text{Pm}_2$  compound.

Lattice parameters and enthalpies of formation of the five known compounds obtained from first principles calculations, are reported in table 6 and figure 4. As we can see the  $\text{Cu}_4\text{Pm}$  has a positive enthalpy of formation, so it's impossible to form this compound. While for the compound  $\text{CuPm}$  which is not in the ground state, it is at least no stable at 0 K.

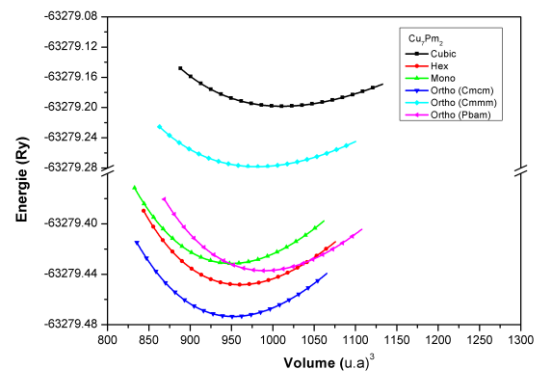
For the  $\text{Cu}_7\text{Pm}_2$ , since we have not found any information concerning its structure, six crystal structures for the  $\text{A}_7\text{B}_2$  compound like the  $\text{Cu}_7\text{Lu}_2$  intermetallics are calculated. The equations of state of the six structures are shown in figure 5, lattice parameters and enthalpies of formation are presented in table 7. The lower enthalpy is obtained with orthorhombic structure (space group  $\text{Cmcm}$ ). The results show that even with this structure the calculated enthalpy of formation does not break the convex hull (Figure 4).

**Table 6.** Lattice parameters, enthalpies of formation and bulk modulus of Cu-Pm intermetallics

Phases	Pearson Symbol	Space group	Lattice Parameters (nm)		$H^{\text{for}}$ (KJ/mol)	$B_0$ (GPa)	
			This work	Estim. [1]			
$\text{Cu}_6\text{Pm}$	oP28	$Pnma$	a	0.804	0.807	8.35	103.30
			b	0.506	0.505		
			c	1.007	1.008		
$\text{Cu}_5\text{Pm}$	hP6	$P6/mmm$	a	0.510	0.509	-8.70	100.89
			c	0.407	0.410		
$\text{Cu}_4\text{Pm}$	oP20	$Pnma$	a	0.485		+13.09	85.68
			b	0.470			
			c	1.386			
$\text{Cu}_2\text{Pm}$	oI12	$Imma$	a	0.432	0.437	-11.63	72.99
			b	0.689	0.696		
			c	0.729	0.704		
$\text{CuPm}$	oP8	$Pnma$	a	0.715	0.722	-4.92	50.21
			b	0.450	0.454		
			c	0.548	0.553		



**Figure 4.** Enthalpies of formation of Cu-Pm intermetallics



**Figure 5.** Calculated total energies as a function of volume of the suspected structures for the  $\text{Cu}_7\text{Pm}_2$

**Table 7.** Lattice parameters, enthalpies of formation and bulk modulus of  $\text{Cu}_7\text{Pm}_2$  in different suspected structures

Prototype	Pearson Symbol	Space group	Lattice Parameters (nm)	$H^{\text{for}}$ (KJ/mol)	$B_0$ (GPa)
$\text{Al}_7\text{Th}_2$	oP18	<i>Pbam</i>	a 0.5417 b 1.0587 c 0.5100	-2.44	85.77
$\text{Ni}_7\text{Zr}_2$	mC36	<i>C2/m</i>	a 0.4992 b 0.8750 c 1.2956	-1.60	93.15
$\text{Ag}_7\text{Yb}_2$	hP18	<i>Cmcm</i>	a 0.8663 b 0.5049 c 1.2863	-7.74	92.61
$\text{Ge}_2\text{Li}_7$	oC36	<i>Cmmm</i>	a 0.9326 b 1.3334 c 0.4673	+20.75	84.21
$\text{Sb}_2\text{Tl}_7$	cI54	<i>Im-3m</i>	a 0.9646	+32.41	74.17
$\text{Ce}_2\text{Ni}_7$	hP36	<i>P 63/mm c</i>	a 0.5094 c 2.5281	-3.9618	91.22

We have also calculated the elastic stiffness constants for this structure (Table 5). The obtained values satisfy all stability conditions (Eq. 3). So the stability of this compound at finite temperature may be explained by entropic effects that are related to vibrational state.

#### 4. CONCLUSIONS

Through first-principles local density functional calculations, we have investigated the ground state line of the Cu-Lu and Cu-Pm systems. We have calculated the enthalpies of formation of eight compounds which their structure are known, and three suspect intermetallics ( $\text{Cu}_7\text{Lu}_2$ ,  $\text{Cu}_9\text{Lu}_2$  and  $\text{Cu}_7\text{Pm}_2$ ). The established ground state shows that: (i) the compounds  $\text{CuLu}$ ,  $\text{Cu}_2\text{Lu}$ ,  $\text{Cu}_5\text{Lu}$  and  $\text{Cu}_7\text{Lu}_2$  are ground state; (ii) the  $\text{Cu}_7\text{Pm}_2$  is not stable at 0K but may be stable at finite temperature since its mechanically stable in the orthorhombic (*Cmcm*) structure; (iii) the compounds  $\text{Cu}_9\text{Lu}_2$ ,  $\text{Cu}_4\text{Pm}$  and  $\text{CuPm}$  are not stable at 0 K. The current results suggest that further experimental investigation is needed for the Cu-Lu and Cu-Pm systems

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## NOMENCLATURE

$V_0$	equilibrium volume
$E_0$	internal energy
$E_{\text{tot}}$	Total energy
$B_0$	bulk modulus
$B'$	derivative of the bulk modulus
$\Delta H^f$	enthalpy of formation of the compound
$C_{ij}$	Elastic stiffness constants