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Double finite sine transform method for deflection analysis of isotropic sandwich plates under uniform load

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ABSTRACT

In this study, the double finite Fourier sine transform method was used to solve the governing partial differential equation of equilibrium of isotropic sandwich plates with all edges simply supported for the case of uniformly distributed transverse load over the entire plate domain. The governing equation solved for the sandwich plate domain was obtained by Liaw Boen Dar by ignoring the linear terms in the Reissner's plate equation to have a fourth order partial differential equation. Application of the double finite Fourier sine transformation, and use of the simply supported Dirichlet boundary conditions simplified the problem to an algebraic problem in terms of the double finite Fourier sine of the unknown deflection in the transform space. Inversion of the double finite Fourier sine transform yielded the unknown deflection in the physical domain space. Specific problem of uniformly distributed transverse load over the entire plate yielded rapidly convergent solutions for the deflection. The deflections are found to be expressible in terms of the sum of flexural and shear deflections for the case of general distributed load, and for the specific case of uniformly distributed load. Maximum deflections, found to occur at the plate center were found to be decomposed into flexural and shear components.

1. INTRODUCTION

Sandwich plates have become increasingly attractive in structural applications due to their ability to provide high bending stiffness and their low weight. Sandwich plates are composite plates structures composed of (i) two thin stiff strong skins, called covers, faces or facings, (ii) a thick core made of light weight material to separate the skins and carry the load and (iii) an adhesive attachment capable of transmitting shear and axial loads to and from the core [1-5]. Typical three dimensional (isometric) and two dimensional cross-sectional) views of a typical sandwich plate are presented in Figures 1 and 2. Sandwich plates are thus composite plates made of three layers. The top and bottom layers, called the faces are usually thin and are made from high strength materials. The third, middle layer, called the core is made from a comparatively low weight and low strength material [3-4, 6].

In general, the skins are laminated with thickness h_1 for the lower skin and thickness h_2 for the upper skin, while the core has a thickness denoted by h. The coordinate system is defined as shown in Figure 3 such that the xy coordinate plane is the middle plane [4].

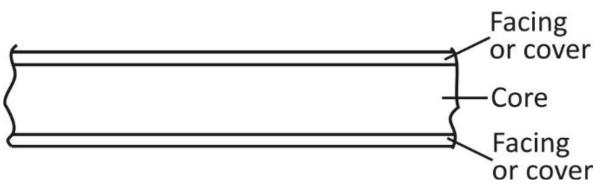

Figure 1. Typical cross-section of a sandwich plate

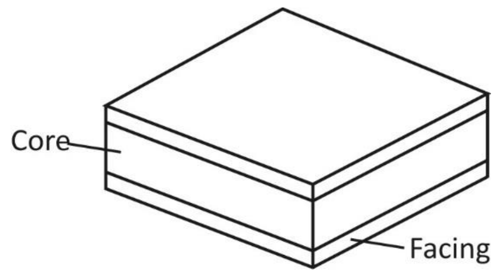

Figure 2. 3 Dimensional view of isotropic sandwich plate

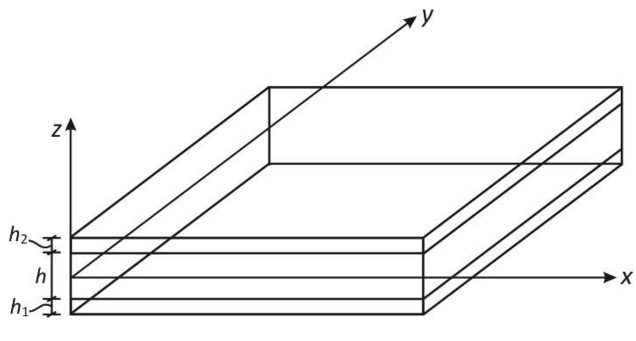

Figure 3. Notations for a sandwich plate

Typically, the core is made up of materials with reduced stiffness, resulting in the introduction of shear deformation effects which need to be accounted for in the governing equations [8]. Chirac and Vrabie [8] have used the first order shear deformation theory (FSDT) also called the Mindlin – Reissner theory to formulate the governing equations of sandwich plates. The obvious disadvantage of the use of the

FSDT is that the kinematics assume a global transverse shear strain considered constant over the plate thickness which violate the known parabolic distribution of transverse stresses in the theory of elasticity.

In order to make transverse shear stress resultants agree with the parabolic distribution resultants of the theory of elasticity, a shear correction factor is introduced into the expression for the transverse shear stress resultant, and the accuracy of the shear stress correction factor becomes vital for the validation of the results using the FSDT.

Magnueka – Blandzi [9] and Magnueka – Blandzi and Wittenberg [6] formulated from first principles, the mathematical equations governing the equilibrium of a sandwich circular plate made of two faces (skins, covers) and a core with variable mechanical properties. They derived the governing equations using the principle of extremization of the total potential energy functional. Wang [10] also formulated from fundamental principles, the governing equations of equilibrium of sandwich plates using the Reissner – Mindlin shear deformation plate theory, and presented exact relationships between the deflections of isotropic sandwich plates and their corresponding classical thin plates. Kormenikova and Manuzic [11] used the shear deformation plate laminate theory for sandwich plates by ignoring the membrane and flexural deformations in the core material and the shear deformation in the facings. Liaw Boen Dar [12] derived the governing partial differential equation of equilibrium of rectangular sandwich plates with orthotropic cores and also derived the governing differential equation of equilibrium of rectangular sandwich plates with isotropic cores by neglecting the non-linear terms in the stress based Reissner's plate shear deformation theory, to obtain a fourth order linear governing partial differential equation.

In this work, Liaw Boen Dar's governing equation for isotropic sandwich plates is adopted as the mathematical model for the sandwich plate; and the equation is solved using the double finite Fourier sine transform method for the case of simply supported edges and uniformly distributed transverse load over the entire plate domain.

2. THEORETICAL FRAMEWORK

The governing partial differential equation of equilibrium for rectangular sandwich plates, obtained by ignoring the non-linear terms in the Reissner's plate equation is given by Liaw Boen Dar, [12] as follows:

$$D\nabla^2\nabla^2 w(x,y) = q(x,y) - \frac{D}{D_s}\nabla^2 q(x,y) \tag{1}$$

where $D_s = hG_s$ (2)

$$D = \frac{Eh^3}{12(1-\mu^2)} \tag{3}$$

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \tag{4}$$

$$\nabla^4 = \nabla^2\nabla^2 = \frac{\partial^4}{\partial x^4} + 2\frac{\partial^4}{\partial x^2 \partial y^2} + \frac{\partial^4}{\partial y^4} \tag{5}$$

$w(x, y)$ is the transverse deflection of arbitrary points (x, y) on the plate domain, $q(x, y)$ is the distribution of transversely applied load over the plate region; x and y are the in-plane Cartesian coordinates describing points on the plate domain, μ is the Poisson's ratio, E is the Young's modulus of elasticity; h is the plate thickness, G_c is the modulus of shear rigidity of the core material.

$$G_c = \frac{E}{2(1+\mu)} \tag{6}$$

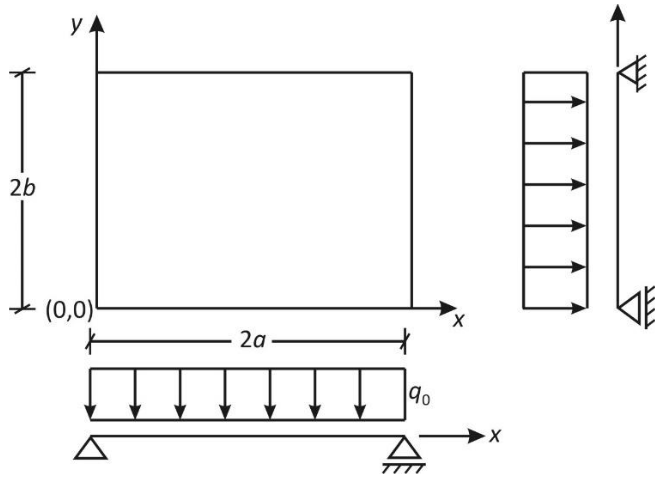

Figure 4. Simply supported isotropic sandwich plate under uniformly distributed load

D_s is the shear modulus of the core, D is the flexural modulus, ∇^2 is the Laplacian, and ∇^4 is the biharmonic differential operator.

Equation (1) can be expressed as:

$$\nabla^4 w(x,y) = \frac{q(x,y)}{D} - \frac{1}{D_s}\nabla^2 q(x,y) \tag{7}$$

or, in expanded form as:

$$\left(\frac{\partial^4 w(x,y)}{\partial x} + \frac{\partial^4 w(x,y)}{\partial x^2 \partial y^2} + \frac{\partial^4 w(x,y)}{\partial y^4}\right) = \frac{q(x,y)}{D} - \frac{1}{D_s}\left(\frac{\partial^2 q(x,y)}{\partial x^2} + \frac{\partial^2 q(x,y)}{\partial y^2}\right) \tag{8}$$

For simply supported edges $x = 0$, $x = 2a$, $y = 0$, $y = 2b$, as shown in Figure 3, the boundary conditions are:

$$w(x=0,y) = 0 \tag{9}$$

$$w(x=2a,y) = 0 \tag{10}$$

$$w(x,y=0) = 0 \tag{11}$$

$$w(x,y=2b) = 0 \tag{12}$$

$$w_{xx}(x=0,y) = \frac{\partial^2 w}{\partial x^2}(x=0,y) = 0 \tag{13}$$

$$w_{xx}(x=2a,y)=\frac{\partial^2 w}{\partial x^2}(x=2a,y)=0 \quad (14)$$

$$w_{yy}(x,y=0)=\frac{\partial^2 w}{\partial y^2}(x,y=0)=0 \quad (15)$$

$$w_{yy}(x,y=2b)=\frac{\partial^2 w}{\partial y^2}(x,y=2b)=0 \quad (16)$$

3. METHODOLOGY

A finite sine transformation kernel $k(r, s)$ suitable for the simply supported Dirichlet boundary conditions-Equations (9 – 16) at the edges $x = 0, 2a, y = 0, 2b$ is:

$$k(r,s)=\sin\frac{r\pi x}{2a}\sin\frac{s\pi y}{2b} \quad (17)$$

where $r = 1, 3, 5, 7, \ldots$; $s= 1, 3, 5, 7, \ldots$

Hence, the Fourier sine transformation of the governing domain equation is given by

$$\int_0^{2a}\int_0^{2b}\left(\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial x^2\partial y^2}+\frac{\partial^4 w}{\partial y^4}\right)\sin\frac{r\pi x}{2a}\sin\frac{s\pi y}{2b}dxdy=$$
$$\int_0^{2a}\int_0^{2b}\frac{q(x,y)}{D}\sin\frac{r\pi x}{2a}\sin\frac{s\pi y}{2b}dxdy$$
$$-\int_0^{2a}\int_0^{2b}\frac{1}{D_s}\left(\frac{\partial^2 q}{\partial x^2}+\frac{\partial^2 q}{\partial y^2}\right)\sin\frac{r\pi x}{2a}\sin\frac{s\pi y}{2b}dxdy \quad (18)$$

By the linearity property of the Fourier sine transformation, and using the Dirichlet conditions at the edges, the transformation simplifies to:

$$\left(\left(\frac{r\pi}{2a}\right)^4+2\left(\frac{r\pi}{2a}\right)^2\left(\frac{s\pi}{2b}\right)^2+\left(\frac{s\pi}{2b}\right)^4\right)\times$$
$$\int_0^{2a}\int_0^{2b}w(x,y)\sin\frac{r\pi x}{2a}\sin\frac{s\pi y}{2b}dxdy$$
$$=\frac{1}{D}\int_0^{2a}\int_0^{2b}q(x,y)\sin\frac{r\pi x}{2a}\sin\frac{r\pi y}{bh}dxdy-$$
$$\frac{1}{D_s}\int_0^{2a}\int_0^{2b}\left\{\left(\frac{r\pi}{2a}\right)^2+\left(\frac{s\pi}{2b}\right)^2\right\}q(x,y)\times$$
$$\sin\frac{r\pi x}{2a}\sin\frac{s\pi y}{2b}dxdy \quad (19)$$

Let

$$\int_0^{2a}\int_0^{2b}w(x,y)\sin\frac{r\pi x}{2a}\sin\frac{s\pi y}{2b}dxdy=w(r,s) \quad (20)$$

$$\int_0^{2a}\int_0^{2b}q(x,y)\sin\frac{r\pi x}{2a}\sin\frac{s\pi y}{2b}dxdy=q(r,s) \quad (21)$$

where $w(r, s)$ is the double Fourier sine transform of $w(x, y)$ and $q(r, s)$ is the double finite sine transform of $q(x, y)$.

The transformation then becomes:

$$\left(\left(\frac{r\pi}{2a}\right)^2+\left(\frac{s\pi}{2b}\right)^2\right)^2 w(r,s)=\frac{1}{D}q(r,s)+$$
$$\frac{1}{D_s}\left(\left(\frac{r\pi}{2a}\right)^2+\left(\frac{s\pi}{2b}\right)^2\right)q(r,s) \quad (22)$$

Solving, for $w(r, s)$

$$w(r,s)=\frac{\frac{q(r,s)}{D}}{\left(\left(\frac{r\pi}{2a}\right)^2+\left(\frac{s\pi}{2b}\right)^2\right)^2}+\frac{\frac{q(r,s)}{D_s}\left(\left(\frac{r\pi}{2a}\right)^2+\left(\frac{s\pi}{2b}\right)^2\right)}{\left(\left(\frac{r\pi}{2a}\right)^2+\left(\frac{s\pi}{2b}\right)^2\right)^2} \quad (23)$$

$$w(r,s)=\frac{q(r,s)}{D\left(\left(\frac{r\pi}{2a}\right)^2+\left(\frac{s\pi}{2b}\right)^2\right)^2}+\frac{q(r,s)}{D_s\left(\left(\frac{r\pi}{2a}\right)^2+\left(\frac{s\pi}{2b}\right)^2\right)} \quad (24)$$

By inversion,

$$w(x,y)=\frac{4}{(2a)(2b)}\sum_r^\infty\sum_s^\infty w(r,s)\sin\frac{r\pi x}{2a}\sin\frac{s\pi y}{2b}=$$
$$\frac{1}{ab}\sum_r^\infty\sum_s^\infty w(r,s)\sin\frac{r\pi x}{2a}\sin\frac{s\pi y}{2b} \quad (25)$$

$$=\frac{1}{ab}\sum_r^\infty\sum_s^\infty\frac{q(r,s)\sin\frac{r\pi x}{2a}\sin\frac{s\pi x}{2b}}{D\left(\left(\frac{r\pi}{2a}\right)^2+\left(\frac{s\pi}{2b}\right)^2\right)^2}+$$
$$\frac{1}{ab}\sum_r^\infty\sum_s^\infty\frac{q(r,s)\sin\frac{r\pi x}{2a}\sin\frac{s\pi x}{2b}}{D_s\left(\left(\frac{r\pi}{2a}\right)^2+\left(\frac{s\pi}{2b}\right)^2\right)^2} \quad (26)$$

where $q(r, s)$ is found from Equation (21).

4. RESULTS

For uniformly distributed loads,

$$q(x,y)=q_0 \quad (27)$$

$$q(r,s)=\int_0^{2a}\int_0^{2b}q_0\sin\frac{r\pi x}{2a}\sin\frac{s\pi y}{2b}dxdy \quad (28)$$

$$q(r,s)=q_0\int_0^{2a}\sin\frac{r\pi x}{2a}dx\int_0^{2b}\sin\frac{s\pi y}{2b}dy$$
$$=q_0\left\{\frac{-2a}{r\pi}\cos\frac{r\pi x}{2a}\right]_0^{2a}\left\}\left\{\frac{-2b}{s\pi}\cos\frac{s\pi y}{2b}\right]_0^{2b}\right\}$$
$$q(r,s)=q_0\left\{\frac{4ab}{rs\pi^2}(\cos r\pi-\cos 0)(\cos s\pi-\cos 0)\right\} \quad (29)$$

$$q(r,s) = \frac{4abq_0}{rs\pi^2}(\cos r\pi - 1)(\cos s\pi - 1) =$$

$$q(r,s) = \frac{4abq_0}{rs\pi^2}((-1)^r - 1)((-1)^s - 1) \quad (30)$$

$$q(r,s) = \frac{4abq_0}{rs\pi^2}(-2)(-2);\ r = 1, 3, 5,$$

$$q(r,s) = \frac{16abq_0}{rs\pi^2} \quad (31)$$

$$w(x,y) = \frac{1}{ab}\sum_r^\infty\sum_s^\infty \frac{\frac{16abq_0}{rs\pi^2}\sin\frac{r\pi x}{2a}\sin\frac{s\pi y}{2b}}{D\left(\left(\frac{r\pi}{2a}\right)^2 + \left(\frac{s\pi}{2b}\right)^2\right)^2} + \frac{1}{ab}\sum_r^\infty\sum_s^\infty \frac{\frac{16abq_0}{rs\pi^2}\sin\frac{r\pi x}{2a}\sin\frac{s\pi y}{2b}}{D_s\left(\left(\frac{r\pi}{2a}\right)^2 + \left(\frac{s\pi}{2b}\right)^2\right)} \quad (32)$$

$$w(x,y) = \frac{16q_0}{\pi^2 D}\sum_r^\infty\sum_s^\infty \frac{\sin\frac{r\pi x}{2a}\sin\frac{s\pi y}{2b}}{rs\left(\left(\frac{r\pi}{2a}\right)^2 + \left(\frac{s\pi}{2b}\right)^2\right)^2} + \frac{16q_0}{\pi^2 D_s}\sum_r^\infty\sum_s^\infty \frac{\sin\frac{r\pi x}{2a}\sin\frac{s\pi y}{2b}}{rs\left(\left(\frac{r\pi}{2a}\right)^2 + \left(\frac{s\pi}{2b}\right)^2\right)} \quad (33)$$

$$w(x,y) = \frac{16q_0}{\pi^6 D}\sum_r^\infty\sum_s^\infty \frac{\sin\frac{r\pi x}{2a}\sin\frac{s\pi y}{2b}}{rs\left(\left(\frac{r}{2a}\right)^2 + \left(\frac{s}{2b}\right)^2\right)^2} + \frac{16q_0}{\pi^4 D_s}\sum_r^\infty\sum_s^\infty \frac{\sin\frac{r\pi x}{2a}\sin\frac{s\pi y}{2b}}{rs\left(\left(\frac{r}{2a}\right)^2 + \left(\frac{s}{2b}\right)^2\right)} \quad (34)$$

Let $\frac{b}{a} = \alpha$ (35)

$$w(x,y) = \frac{16q_0}{\pi^6 D}\sum_r^\infty\sum_s^\infty \frac{\sin\frac{r\pi x}{2a}\sin\frac{s\pi y}{2b}}{rs\left(\frac{(r^2\alpha^2 + s^2)^2}{16a^4\alpha^4}\right)} + \frac{16q_0}{\pi^4 D_s}\sum_r^\infty\sum_s^\infty \frac{\sin\frac{r\pi x}{2a}\sin\frac{s\pi y}{2b}}{rs\left(\frac{r^2\alpha^2 + s^2}{4a^2\alpha^2}\right)} \quad (36)$$

$$w(x,y) = \frac{16q_0(16a^4\alpha^4)}{\pi^6 D}\sum_r^\infty\sum_s^\infty \frac{\sin\frac{r\pi x}{2a}\sin\frac{s\pi y}{2b}}{rs(r^2\alpha^2 + s^2)^2} + \frac{16q_0(4a^2\alpha^2)}{\pi^4 D_s}\sum_r^\infty\sum_s^\infty \frac{\sin\frac{r\pi x}{2a}\sin\frac{s\pi y}{2b}}{rs(r^2\alpha^2 + s^2)} \quad (37)$$

$$w(x,y) = \frac{256q_0a^4\alpha^4}{\pi^6 D}\sum_r^\infty\sum_s^\infty \frac{\sin\frac{r\pi x}{2a}\sin\frac{s\pi y}{2b}}{rs(r^2\alpha^2 + s^2)^2} + \frac{64q_0a^2\alpha^2}{\pi^4 D_s}\sum_r^\infty\sum_s^\infty \frac{\sin\frac{r\pi x}{2a}\sin\frac{s\pi y}{2b}}{rs(r^2\alpha^2 + s^2)} \quad (38)$$

$$w(x,y) = w(x,y)^f + w(x,y)^s \quad (39)$$

where,

$$w(x,y)^f = \frac{256q_0a^4\alpha^4}{\pi^6 D}\sum_r^\infty\sum_s^\infty \left\{\frac{\sin\frac{r\pi x}{2a}\sin\frac{s\pi y}{2b}}{rs(r^2\alpha^2 + s^2)^2}\right\} \quad (40)$$

and $w(x,y)^s = \frac{64q_0a^2\alpha^2}{\pi^4 D_s}\sum_r^\infty\sum_s^\infty \frac{\sin\frac{r\pi x}{2a}\sin\frac{s\pi y}{2b}}{rs(r^2\alpha^2 + s^2)}$ (41)

$w(x, y)^f$ is the flexural component of the transverse deflection, and $w(x, y)^s$ is the shear component of the transverse deflection.

Maximum deflection

Maximum deflection occurs at the center of the plate where $x = a$, $y = b$ and $\sin\frac{r\pi x}{2a} = \sin\frac{r\pi}{2} = (-1)^{\frac{r-1}{2}}$, $\sin\frac{s\pi y}{2b} = \sin\frac{s\pi}{2} = (-1)^{\frac{s-1}{2}}$,

$$w_{\max} = w(a,b) = \frac{256q_0a^4\alpha^4}{\pi^6 D}\sum_r^\infty\sum_s^\infty \frac{(-1)^{\frac{r+s-2}{2}}}{rs(r^2\alpha^2+s^2)^2} + \frac{64q_0a^2\alpha^2}{\pi^4 D_s}\sum_r^\infty\sum_s^\infty \frac{(-1)^{\frac{r+s-2}{2}}}{rs(r^2\alpha^2+s^2)} \quad (42)$$

$$w_{\max} = w_{\max}^f + w_{\max}^s \quad (43)$$

where,

$$w_{\max}^f = \frac{256q_0a^4\alpha^4}{\pi^6 D}\sum_r^\infty\sum_s^\infty \frac{(-1)^{\frac{r+s-2}{2}}}{rs(r^2\alpha^2 + s^2)^2} \quad (44)$$

$$w_{\max}^s = \frac{64q_0a^2\alpha^2}{\pi^4 D_s}\sum_r^\infty\sum_s^\infty \frac{(-1)^{\frac{r+s-2}{2}}}{rs(r^2\alpha^2 + s^2)} \quad (45)$$

The values of the maximum flexural and shear components of the deflection at convergence of the double series are calculated for various plate aspect ratios α, and for various values of D/D_s and presented in Table 1. The convergence properties of the double sine series functions for the flexural and shear components of the deflection, and hence the resultant deflection function is demonstrated with the convergence study done for square isotropic plates and presented in Table 2.

Table 1. Maximum deflection coefficients for the centre of uniformly loaded sandwich plates ($2a \times 2b$) with simply supported edges ($x = 0$, $x = 2a$, $y = 0$, $y = 2b$)

b/a	$w_{\max}^{(f)}$ $\left(\times \frac{q_0 a^4}{D}\right)$	$w_{\max}^{(s)}$ $\left(\times \frac{q_0 a^2}{D_s}\right)$	$\frac{D_s a^2}{D}=1$ $w_{\max}^{s}$ $\left(\times \frac{q_0 a^4}{D}\right)$	$\frac{D_s a^2}{D}=10$ $w_{\max}^{s}$ $\left(\times \frac{q_0 a^4}{D}\right)$	$\frac{D_s a^2}{D}=20$ $w_{\max}^{s}$ $\left(\times \frac{q_0 a^4}{D}\right)$	$\frac{D_s a^2}{D}=50$ $w_{\max}^{(s)}$ $\left(\times \frac{q_0 a^4}{D}\right)$
1.0	6.496×10^{-2}	28.480×10^{-2}	28.48×10^{-2}	2.848×10^{-2}	1.424×10^{-2}	0.5696×10^{-2}
1.2	9.024×10^{-2}	34.720×10^{-2}	34.720×10^{-2}	3.4720×10^{-2}	1.7360×10^{-2}	0.69440×10^{-2}
1.4	11.280×10^{-2}	38.680×10^{-2}	38.68×10^{-2}	3.868×10^{-2}	1.934×10^{-2}	0.7736×10^{-2}
1.6	13.280×10^{-2}	41.680×10^{-2}	41.68×10^{-2}	4.168×10^{-2}	2.084×10^{-2}	0.8336×10^{-2}
1.8	14.896×10^{-2}	43.920×10^{-2}	43.92×10^{-2}	4.392×10^{-2}	2.196×10^{-2}	0.8784×10^{-2}
2	16.208×10^{-2}	45.560×10^{-2}	45.56×10^{-2}	4.556×10^{-2}	2.278×10^{-2}	0.9112×10^{-2}
3	19.568×10^{-2}	49.080×10^{-2}	49.08×10^{-2}	4.9080×10^{-2}	2.454×10^{-2}	0.9816×10^{-2}
4	20.512×10^{-2}	49.800×10^{-2}	49.80×10^{-2}	4.980×10^{-2}	2.49×10^{-2}	0.996×10^{-2}
5	20.752×10^{-2}	49.960×10^{-2}	49.96×10^{-2}	4.996×10^{-2}	2.498×10^{-2}	0.9992×10^{-2}
∞	20.832×10^{-2}	50×10^{-2}	50×10^{-2}	5×10^{-2}	2.5×10^{-2}	1×10^{-2}

Table 2. Convergence characteristics of w^f and w^s for $b/a = 1$

r	s	$w_{\max}^{f}\left(\times 10^{-2}\frac{q_0 a^4}{D}\right)$	$w_{\max}^{s}\left(\times 10^{-2}\frac{q_0 a^4}{D_s}\right)$
1	1	6.65703	32.851143
3	1	6.56824	30.661067
1	3	6.47948	28.470305
3	3	6.488612	28.87587467
3	5	6.487076352	28.747666
5	3	6.485540705	28.618838
5	5	6.485966755	28.6714
5	7	6.48582778	28.64603
7	5	6.485688866	28.62066
7	7	6.48574545	28.63434

5. DISCUSSION

The governing partial differential equation of equilibrium of isotropic sandwich plates given by Equation (1) has been solved using the double finite Fourier sine transform method. Simply supported isotropic sandwich plates ($2a \times 2b$) with origin at a corner of the plate and edges at $x = 0$, $x = 2a$, $y = 0$, $y = 2b$ was studied. The general arbitrary load distribution $q(x, y)$ was considered, as well as the specific case of uniformly distributed load over the entire plate domain $(0 \le x \le 2a,\ 0 \le y \le 2b)$. Application of the double finite Fourier sine transformation, and use of the simply supported (Dirichlet) boundary conditions simplified the boundary value problem to the algebraic problem given in terms of the unknown deflection in the Fourier transform space as Equation (22). Solution of the algebraic problem yielded the unknown deflection in the Fourier sine transform space as Equation (24). Inversion of Equation (24) yielded the general solution for the unknown deflection in the physical domain space coordinate variables as Equation (26) for general arbitrary load distribution.

The solution for the particular problem of uniformly distributed load was obtained as Equation (24) or alternatively as Equation (38). Equation (38) is seen to be expressible as the sum of transverse deflection and shear deflection components as shown in Equations (39), (40) and (41). The maximum deflection was found to occur at the plate center for the case of uniformly distributed load and was found as Equation (42). The equation for maximum deflection was similarly found to be expressible as the sum of transverse and shear components as given by Equation (43), (44) and (45). The series obtained for maximum deflection, maximum flexural deflection components and maximum shear deflection components were found to be rapidly converging double series with sufficient convergence obtained with five terms of the series.

Table 1 which presents maximum deflection coefficients for the center of simply supported isotropic sandwich plates for various values of b/a and D_s/D shows that the contribution of shear deflection to the overall deflection reduces as the ratio D_s/D increases.

Table 2 which presents the convergence studies of the deflection expression illustrates that the flexural component of deflection converges faster than the shear component. Convergence of the flexural component of the deflection is achieved for $r = s = 5$, while that for shear component is achieved for $r = s = 9$. The solutions obtained in this study were in exact agreement with solutions obtained for simply supported isotropic sandwich plates by Plantema (1966) who used a Navier method.

6. CONCLUSION

The following conclusions are made from the study:

(i) The double finite Fourier sine transform method has been successfully used to solve the governing partial differential equation of isotropic sandwich plate with simply supported edges $(x = 0,\ x = 2a,\ y = 0,\ y = 2b)$ and subject to arbitrary load distribution.

(ii) The double finite Fourier sine transform method has been successfully used to solve the governing boundary value problem of simply supported isotropic sandwich plate for the specific case of uniformly distributed load over the entire plate domain.

(iii) The double finite Fourier sine transform method yielded solutions for the unknown deflection as a rapidly convergent double trigonometric sine series of infinite terms.

(iv) The deflection was found to be decomposable or expressible as flexural and shear deflection components

(v) The contribution of the shear deflection to the total deflection reduces as the ratio of D_s/D increases.

(vi) This paper will hopefully enhance our understanding of the deflection behaviour of simply supported isotropic sandwich plates under uniformly distributed transverse loads.

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NOMENCLATURE

$w(x, y)$	transverse deflection of arbitrary points (x, y) on the plate domain
$q(x, y)$	distribution of transversely applied load over the plate region
x, y	in-plane Cartesian coordinates describing points on the plate domain.
μ	Poisson's ratio of plate material
E	Young's modulus of elasticity
h	plate thickness
G_c	modulus of shear rigidity of the core material
D_s	shear modulus of the core
D	flexural modulus
$k(r, s)$	finite sine transformation kernel
$2a$, $2b$	dimensions of the sandwich plate in the x and y coordinate axes respect (i.e. breadth and length)
r, s	integers
∞	infinity
$w(r, s)$	deflection in the finite sine transform space
$q(r, s)$	distributed transverse load in the finite sine transform space
q_0	intensity of uniformly distributed load
α	plate aspect ratio
FSDT	first order shear deformation theory

Subscripts

max	maximum
f	flexural
s	shear

Mathematical symbols

∇^2	Laplacian
$\nabla^4 = \nabla^2\nabla^2$	biharmonic operator
$\int$	integration sign or integral
$\iint$	double integration sign or double integral
Σ	summation
$\Sigma\Sigma$	double summation
$\frac{\partial}{\partial x}$	partial derivative with respect to x
$\frac{\partial}{\partial y}$	partial derivative with respect to y
$\frac{\partial^2}{\partial x \partial y}$	mixed partial derivative