

Dusty Jeffrey fluid flow in a rotating system with volume fraction and Hall effect: An analytical approach

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ABSTRACT

Unsteady hydro-magnetic flow problem of dusty Jeffrey fluid through a rotating system has been studied in presence of Hall effect and volume fraction. The fluid motion experiences stability during the appliances of magnetic field along cross flow. The motion of dusty viscous fluid is governed by Saffman Model and Jeffrey fluid model. System is rotating with a constant angular velocity. The governing equations of motion are solved analytically using regular perturbation method. Velocity profile of the fluid motion has been discussed graphically for various values of flow parameters and shearing stresses are discussed numerically with a special emphasis is given on non-Newtonian parameters.

1. INTRODUCTION

The mechanism of non-Newtonian fluid flows has attracted many researchers because of its applications in chemical and allied processing industries. Examples of such complex non-Newtonian fluid flows are foams, slurries, emulsions, polymer melts and solutions, etc. [1-2]. Various non-Newtonian fluid models like the Rivlin- Ericksen second order model, Oldroyd model, Walters liquid, Johnson-Seagalman model etc. have been presented by many researchers in a diverse range of geometries using various analytical and computational schemes. These fluid flow mechanisms are used in various fields of petro-chemical, bio-medical and environmental engineering including polypropylene coalescence sintering, dynamically-loaded journal bearings, blood flow and geological flows etc. [3]. A particular class of non-Newtonian fluid model signifies the ratio of relaxation and retardation is Jeffrey fluid model.

Effects of thermal radiation on mixed convection flow of Jeffrey fluid towards a stretching sheet have been examined by Hayat et al. [4]. Also, they have investigated the impact of generation or absorption of heat in the Jeffrey fluid flow past a porous stretching sheet [5]. Hayat et al. [6] have discussed the three-dimensional flow of Jeffrey fluid over a linearly stretching surface with temperature dependent thermal conductivity. Qasim [7] have analyzed the effects of simultaneous thermal and mass diffusion of Jeffrey fluid flow over a stretching sheet under the influence of heat source or heat sink. Thermo-phoresis and joule heating effects on the Jeffrey fluid flow with radiation has been analyzed by Shehzad et al. [8].

The combination of viscous fluid and dust particles is a subject of interest because of its' occurrence in powder technology, transport of liquid slurries in chemical processing, nuclear processing and in different geophysical situations. The constancy of laminar flow of gas containing dust particles has been studied by Saffman [9] by neglecting volume fraction. Problem of gas flow containing dust particles has been investigated by Michael and Miller [10]. Inaccuracy of

governing gas flow by avoiding volume fraction has been discussed by Rudinger [11]. These leads to generalize the model of dust particle mixtures and Nayfeh [12] has formulated the problem by including volume fraction of dust. Gupta and Gupta [13] have studied the flow problem of dusty gas flow with time dependent pressure gradient. Singh [14] has analyzed time dependent dusty fluid flow through a rectangular channel with unsteady pressure gradient. The unsteady dusty viscous fluid flow under the influence of transverse magnetic field has been studied by Singh and Ram [15]. Effects of impulsive pressure gradient on a dusty fluid flow through a channel has been analysed by Prasad and Ramacharyulu [16]. Mechanics of time dependent non-Newtonian fluid flow with dust particles have been investigated by Gupta and Gupta [17]. Ajadi [18] has evaluated the isothermal flow of dusty fluid flow with oscillatory and non-oscillatory boundary motions. Time dependent Couette flow of viscous fluid with exponentially decreasing pressure gradient and thermal diffusion has been discussed by Attia et al. [19].

Rotating viscous fluid flow is governed by the action of Coriolis and viscous forces and its phenomenon is appeared in variety of industrial applications (e.g. the manufacturing of mono-disperse latex micro-spheres, hollow shells, the inference of molecular weights the thermo capillary fining of glass melts, electrophoresis of gas bubbles, the separation of valuable minerals (e.g. uranium), extraction of proteins and other macromolecules in biological and pharmaceutical operations, and industrial and municipal waste treatment) [20]. Dalal [21] has discussed the flow pattern of generalized Couette flow of dusty gas with impulsive pressure gradient. Singh and Singh [22] have formulated the flow problem of hydro-magnetic buoyancy driven dusty fluid with volume fraction through vertical parallel plates. Attia [23] and [24] has studied hydro-magnetic dusty fluid flow with thermal diffusion under various physical considerations. The problem of fluid flow in a rotating system has been studied by Ahmed et al. [25] considering the Hall current effect. A time dependent flow problem of dusty fluid through a rotating

horizontal channel has been studied by Singh et al. [26]. Dey [27-28] has studied the hydro-magnetic dusty stratified fluid flow through a porous medium with volume fraction and visco-elastic effects.

The constitutive equation for Jeffrey fluid is given by

$$\sigma_{ij} = -p\delta_{ij} + \tau_{ij} \quad (1)$$

$$\tau_{ij} = \frac{\mu}{1+\lambda_1} \left[R_{ij} + \lambda_2 \left\{ \frac{\partial}{\partial t} + v_k \frac{\partial}{\partial x_k} \right\} R_{ij} \right], R_{ij} = (V_{i,j} + V_{j,i}) \quad (2)$$

where μ be the dynamic viscosity, λ_1 be the ratio of relaxation time to the retardation time and λ_2 be the retardation time. For $\lambda_1 = 0$ & $\lambda_2 = 0$, the above model characterizes the mechanism of Newtonian fluid flow.

The objective of this work is to figure out the influences of Jeffrey fluid parameters on dusty viscous fluid flow in rotating system with volume fraction and Hall effects.

2. MATHEMATICAL FORMULATIONS

Let us consider hydro-magnetic unsteady dusty viscous fluid flow characterized by Jeffrey fluid model past a non-electrically conducting porous plate. The plate is subjected with a constant suction velocity $-w_0$ ($-ve$ sign characterizes the suction at the plate). In the present study, the following assumptions are made:

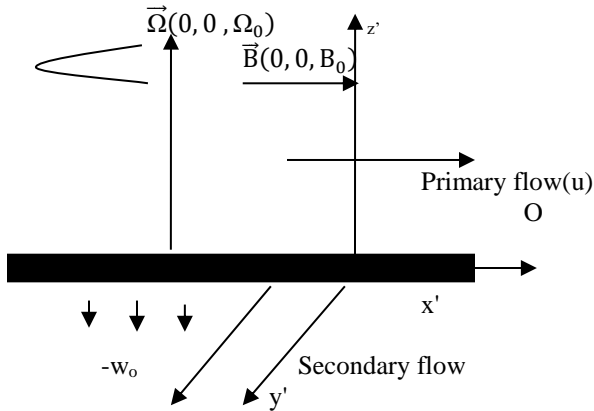


Figure 1. The physical configuration of the problem

- Shape of dust particles are assumed spherical and uniformly spread over the region.
- The temperature is consistent within the particle.
- Using Gauss's law of magnetism, the strength of magnetic of field is uniform and its application along cross flow direction generated Lorentz force. Assumption of low magnetic Reynolds number leads to ignorance of induced magnetic (Cramer and Pai [29]).
- Constant angular velocity Ω associated with the rotation of the system is so small that the centripetal acceleration $|\Omega \times (\Omega \times \mathbf{r})|$ is neglected.
- Due to infinite length of plate along x' and y' directions, all physical quantities except pressure are dependent only on z' and t' respectively. Geometry of the problem is given by Figure 1.

The governing equations of the fluid motion are:

Equation of Continuity:

$$v_{i,i} = 0 \Rightarrow w' = -W_0 \quad (3)$$

Equation of Continuity for dust particles:

$$v_{pi,i} = 0 \quad (4)$$

Momentum equation:

$$\rho \left[\frac{\partial v_i}{\partial t} + 2\varepsilon_{ijk}\Omega_j v_k + \varepsilon_{ijk}\varepsilon_{klm}\Omega_j\Omega_l x_m + v_k \frac{\partial v_i}{\partial x_k} \right] = -p_{,i} + \tau_{ij,j} + \varepsilon_{ijk}J_j B_k + \frac{KN}{1-\phi} (v_{pi} - v_i) \quad (5)$$

Momentum equation for dust particles:

$$m_p \left[\frac{\partial v_{pi}}{\partial t} + 2\varepsilon_{ijk}\Omega_j v_{pk} + \varepsilon_{ijk}\varepsilon_{klm}\Omega_j\Omega_l x_{pm} + v_{pk} \frac{\partial v_{pi}}{\partial x_k} \right] = \frac{\phi}{\rho} (-p_{,i} + \tau_{ij,j} + \varepsilon_{ijk}J_j B_k) + K(v_{pi} - v_i) \quad (6)$$

Generalized Ohm's law:

$$J_i + \frac{\omega_e \tau_e}{B_0} \varepsilon_{ijk} J_j B_k = \sigma \left(E_i + \varepsilon_{ijk} v_j B_k + \frac{1}{\epsilon_0} p_{e,i} \right) \quad (7)$$

Kirshoff's first law:

$$J_{i,i} = 0 \quad (8)$$

Gauss's law of Magnetism:

$$B_{i,i} = 0 \quad (9)$$

Using, the boundary layer approximation, we get the pressure gradients as

$$-\frac{1}{\rho} \frac{\partial p'}{\partial x'} = \frac{dU'}{dt'} + \frac{\sigma B_0^2 U'}{1+m^2} + \frac{KN}{\rho(1-\phi)} U' \quad (10)$$

$$-\frac{1}{\rho} \frac{\partial p'}{\partial y'} = 2U'\Omega_0 + \frac{\sigma B_0^2 m U'}{1+m^2} \quad (11)$$

where, $m = \omega_e \tau_e$ be the Hall parameter.

Introducing the following non-dimensional quantities

$$z = \frac{z' w_0}{v}, u = \frac{u'}{U_0}, v = \frac{v'}{U_0}, u_p = \frac{u'_p}{U_0}, v_p = \frac{v'_p}{U_0}, U = \frac{U'}{U_0}, t = \frac{t' w_0}{v}, \omega = \frac{\omega' v}{w_0^2}, \Omega = \frac{\Omega_0 v}{w_0^2}, J = \frac{\lambda_2 w_0^2}{\rho v}, R = \frac{KNv}{\rho w_0^2}, M = \frac{\sigma B_0^2 v}{\rho w_0^2}, G = \frac{Kv}{m_p w_0^2}, \bar{\epsilon} = \frac{1}{1-\phi}, \bar{\epsilon}_1 = \frac{\phi}{m_p} \quad (12)$$

The non-dimensional equations of the governing fluid motion are

$$\frac{\partial u}{\partial t} - 2\Omega v - \frac{\partial u}{\partial z} = \frac{1}{1+\lambda_1} \frac{\partial^2 u}{\partial z^2} + \frac{J}{1+\lambda_1} \left(\frac{\partial^3 u}{\partial z^2 \partial t} - \frac{\partial^3 u}{\partial z^3} \right) + \frac{M}{1+m^2} (mv - u + U) + R\bar{\epsilon}(u_p - u + U) \quad (13)$$

$$\frac{\partial v}{\partial t} + 2\Omega(u - U) + \frac{\partial v}{\partial z} = \frac{1}{1+\lambda_1} \left[\frac{\partial^2 v}{\partial z^2} + J \left(\frac{\partial^3 v}{\partial z^2 \partial t} - \frac{\partial^3 v}{\partial z^3} \right) \right] - \frac{M}{1+m^2} (mu + v + mU) + R\bar{\epsilon}(v_p - v) \quad (14)$$

$$\frac{\partial u_p}{\partial t} - 2\Omega v_p = \bar{\epsilon}_1 \left[\frac{1}{1+\lambda_1} \frac{\partial^2 u}{\partial z^2} + \frac{J}{1+\lambda_1} \left(\frac{\partial^3 u}{\partial z^2 \partial t} - \frac{\partial^3 u}{\partial z^3} \right) + \frac{M}{1+m^2} (mv - u + U) + RU + \frac{dU}{dt} \right] + \frac{1}{G} (u - u_p) \quad (15)$$

$$\frac{\partial v_p}{\partial t} + 2\Omega u_p = \bar{\epsilon}_1 \left[\frac{1}{1+\lambda_1} \frac{\partial^2 v}{\partial z^2} + \frac{J}{1+\lambda_1} \left(\frac{\partial^3 v}{\partial z^2 \partial t} - \frac{\partial^3 v}{\partial z^3} \right) + \frac{M}{1+m^2} (mu + v + mU) + 2U\Omega \right] + \frac{1}{G} (v - v_p) \quad (16)$$

The non-dimensional boundary conditions are

$$u = 0, v = 0, u_p = 0, v_p = 0 \text{ at } z = 0 \\ u \rightarrow 1 + \epsilon e^{i\omega t}, v \rightarrow 0, u_p \rightarrow 1 + \epsilon e^{i\omega t}, v_p \rightarrow 0 : z \rightarrow \infty \quad (17)$$

3. METHOD OF SOLUTION

The velocity components of fluid and dust particles of primary and secondary motions are combined using new functions. So, we consider

$$F = u + iv, F_p = u_p + iv_p \quad (18)$$

Then the equations (14)–(16) are reduced as follows:

$$\frac{\partial F}{\partial t} + i2\Omega F - 2U\Omega - \frac{\partial F}{\partial z} = \frac{1}{1+\lambda_1} \frac{\partial^2 F}{\partial z^2} + \frac{J}{1+\lambda_1} \left(\frac{\partial^3 F}{\partial z^2 \partial t} - \frac{\partial^3 F}{\partial z^3} \right) + \frac{M}{1+m^2} [-imF - F + U(1 - im)] + R\bar{\epsilon}(F_p - F + U) \quad (19)$$

$$\frac{\partial F_p}{\partial t} + i2\Omega F_p = \bar{\epsilon}_1 \left[\frac{1}{1+\lambda_1} \frac{\partial^2 F}{\partial z^2} + \frac{J}{1+\lambda_1} \left(\frac{\partial^3 F}{\partial z^2 \partial t} - \frac{\partial^3 F}{\partial z^3} \right) + \frac{M}{1+m^2} [-imF - F + U(1 - im)] + RU - i2U\Omega + \frac{dU}{dt} \right] + \frac{(F - F_p)}{G} \quad (20)$$

The corresponding boundary conditions are

$$F = 1 + \epsilon e^{i\omega t}, F_p = 1 + \epsilon e^{i\omega t} \text{ at } z \rightarrow \infty \\ F = 0, F_p = 0, \text{ as } z = 0 \quad (21)$$

To solve the equations, we use the perturbation scheme and are given as follows.

$$F = F_1(z) + \epsilon F_2(z) e^{i\omega t} + O(\epsilon^2) \\ F_p = F_3(z) + \epsilon F_4(z) e^{i\omega t} + O(\epsilon^2) \quad (22)$$

The perturbation parameter is taken as ϵ (amplitude of oscillations of velocity, small for stable motion). Using (22) in the equations (19) and (20) and equating the co-efficient of zeroth and first order terms, we get

$$JF_1''' - F_1'' - (1 + \lambda_1)F_1' + F_1(A_3 + iA_4) = A_5 + iA_6 \quad (23)$$

$$JF_1''' - F_1'' - (1 + \lambda_1)F_1' + F_1(A_3 + iA_4) = (A_5 + iA_6) \quad (24)$$

$$F_3(1 + i2\Omega G) = \bar{\epsilon}_1 \left[\frac{F_1''}{1+\lambda_1} - \frac{J}{1+\lambda_1} F_1''' + \frac{M}{1+m^2} (-imF_1 - F_1 - im) + R - 2i\Omega \right] \quad (25)$$

$$F_4(1 + i2\Omega G + i\omega G) = \bar{\epsilon}_1 \left[\frac{F_2'' + J(iF_2''\omega - F_2''')}{1+\lambda_1} + \frac{M}{1+m^2} (-imF_2 - F_2 + 1 - im) + R - 2i\Omega + i\omega \right] \quad (26)$$

4. RESULT AND DISCUSSIONS

Solving the equations, we get the velocity profile of fluid particles and are given as follows:

$$u = (1 - A_{22})e^{-\alpha_1 z} + A_{22} + \epsilon \cos \omega t [(1 - A_{34})e^{-\alpha_3 z} + A_{34}] - \epsilon \sin \omega t A_{35}(1 - e^{-\alpha_3 z}) \quad (27)$$

$$v = (1 - e^{-\alpha_1 z})A_{23} + \epsilon \cos \omega t [(1 - e^{-\alpha_3 z})A_{35}] + \epsilon \sin \omega t [A_{34}(1 - e^{-\alpha_3 z}) + e^{-\alpha_3 z}] \quad (28)$$

Knowing the velocity of primary and secondary flows, the dimensionless viscous drag or shearing stress is calculated as below:

$$\sigma_1 = \frac{\sigma'_{xz}}{\rho U_0 w_0} \Big|_{z=0} = \frac{1}{1+\lambda_1} \frac{\partial u}{\partial z} + \frac{J}{1+\lambda_1} \left(\frac{\partial^2 u}{\partial z \partial t} - \frac{\partial^2 u}{\partial z^2} \right) \Big|_{z=0} \quad (29)$$

$$\sigma_2 = \frac{\sigma'_{yz}}{\rho U_0 w_0} \Big|_{z=0} = \frac{1}{1+\lambda_1} \frac{\partial v}{\partial z} + \frac{J}{1+\lambda_1} \left(\frac{\partial^2 v}{\partial z \partial t} - \frac{\partial^2 v}{\partial z^2} \right) \Big|_{z=0} \quad (30)$$

A problem of unsteady rotating hydro-magnetic dusty Jeffrey fluid flow past a non-conducting porous plate has been investigated in presence of volume fraction. The non-zero values of λ_1 and j characterize the mechanism of governing Jeffrey fluid motion along with other flow parameters involved in the solution.

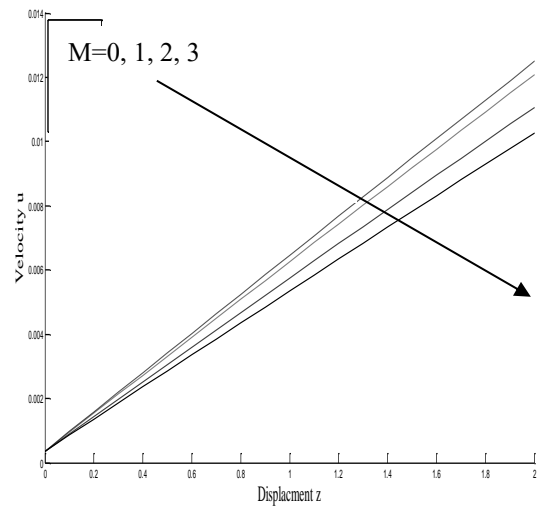


Figure 2. Velocity Profile u against the displacement variable z for various values of M and $m=0.5$, $\omega=1.5$, $\Omega=0.4$, $\epsilon=0.0005$, $\lambda_1=0.4$, $J=0.1$, $G=0.8$, $R=2$

Figures 2 to 5 represent the pattern of velocity profile of primary and secondary flows of dusty Jeffrey fluid for various values of the flow parameters. It is observed that the primary flow is accelerating with the increasing values of displacement variable (z) i.e. the maximum speed is noticed in the far away

of the porous plate. Effects of various combination of Jeffrey's parameters on both Primary and Secondary flows are shown in figure 3 and 5 respectively and it is seen that the Newtonian fluid ($\lambda_1 = 0$ and $j = 0$) shows a uniform nature in comparison to Jeffrey's fluid. The other non-zero combinations of λ_1 and j have a retarding effect on velocity profiles. If the retardation time of governing non-Newtonian fluid increases, then the speed of the motion will slow down but the maximum retardation effect in velocity profile is seen as we move away from the porous plate. The application of transverse magnetic field generates Lorentz force and it slows down the speed of fluid flow. As the strength of the magnetic field increases the impact of retardation will be enhanced along with the increasing values of displacement variable (figure 2). The effect of Hall parameter on the fluid flow is represented by figure 4 and it states that, Hall Effect accelerates the fluid motion.

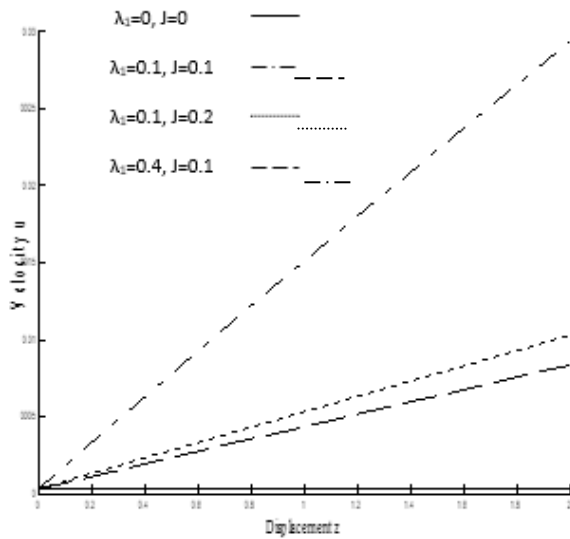


Figure 3. Velocity Profile u against the displacement variable z for various combination of Jeffrey fluid flow parameters (λ_1, J) and $m=0.5, M=2, \omega=1.5, \Omega=0.4, \epsilon=0.0005, G=0.8, R=2$

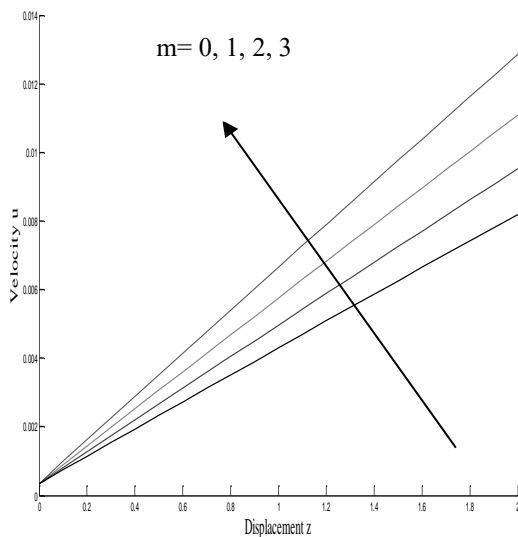


Figure 4. Velocity Profile u against the displacement variable z for various values of m and $M=2, \omega=1.5, \Omega=0.4, \epsilon=0.0005, \lambda_1=0.4, J=0.1, G=0.8, R=2$

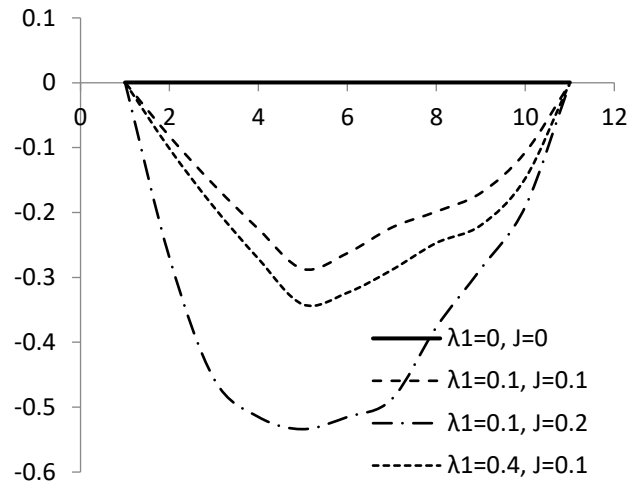


Figure 5. Secondary Velocity Profile against the displacement variable z for various combination of Jeffrey fluid flow parameters (λ_1, J) and $m=0.5, M=2, \omega=1.5, \Omega=0.4, \epsilon=0.0005, G=0.8, R=2$

The shearing stresses at the plate for both primary and secondary flows are calculated and represented it in tabular form. It is seen that the strength of shearing stress experienced by the primary flow is of lesser magnitude than the case of secondary flow. If the retardation time for Jeffrey fluid increases, the shearing stresses increase (Table 1). During the application of transverse magnetic field, if the strength of the magnetic rises, then the shearing stresses decrease (Table 2) but a reverse phenomenon is seen in case of Hall parameter (Table 3). The dimensionless relaxation time of dust particles (G) also has a negative impact on the strength of shearing stresses for both primary and secondary flows (Table 4).

Table 1. Shearing stresses σ_1 and σ_2 various combination of Jeffrey fluid flow parameters (λ_1, J) $m=0.5$ and $M=2, \omega=1.5, \Omega=0.4, \epsilon=0.0005, m=0.5, R=2, \lambda_1=0.4, J=0.1$

	σ_1	σ_2
$\lambda_1=0.1, J=0.1$	-0.0067	-0.0079
$\lambda_1=0.1, J=0.2$	-0.0252	-0.0296
$\lambda_1=0.4, J=0.1$	-0.0066	-0.0078

Table 2. Shearing stresses σ_1 and σ_2 various values of M and $G=0.8, \omega=1.5, \Omega=0.4, \epsilon=0.0005, m=0.5, R=2, \lambda_1=0.4, J=0.1$

	$M=0$	$M=1$	$M=2$	$M=3$
σ_1	0.0083	0.0068	0.0066	0.0066
σ_2	0.0100	0.0084	0.0078	0.0075

Table 3. Shearing stresses σ_1 and σ_2 various values of m and $M=2, \omega=1.5, \Omega=0.4, \epsilon=0.0005, G=0.8, R=2, \lambda_1=0.4, J=0.1$

	σ_1	σ_2
$m=0$	0.0045	0.0061
$m=0.5$	0.0066	0.0078
$m=0.6$	0.0070	0.0081
$m=0.7$	0.0074	0.0084
$m=0.8$	0.0078	0.0087
$m=0.9$	0.0081	0.0089
$m=1$	0.0085	0.0092

Table 4. Shearing stresses σ_1 and σ_2 various values of G and $M=2$, $\omega=1.5$, $\Omega=0.4$, $\varepsilon=0.0005$, $m=0.5$, $R=2$, $\lambda_1=0.4$, $J=0.1$

	σ_1	σ_2
G=0.3	0.0067	0.0079
G=0.4	0.0066	0.0079
G=0.5	0.0066	0.0079
G=0.6	0.0066	0.0078
G=0.7	0.0066	0.0078
G=0.8	0.0066	0.0078

5. CONCLUSIONS

A problem of unsteady rotating hydro-magnetic dusty Jeffrey fluid flow past a non-conducting porous plate has been investigated in presence of volume fraction. Some of the results are concluded as below:

- Maximum speed of Primary flow is seen in the neighborhood of the porous plate.
- The non-Newtonian parameters have retarding effect on the governing fluid motion.
- Fluid flows accelerate during the growth of Hall parameter.
- Strength of shearing stress experienced by the primary flow is of larger magnitude than the case of secondary flow.

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NOMENCLATURE

v_i & v_{pi} be the velocity of fluid particles and dust particles respectively, ϵ_{ijk} be the Levi-Civita symbol, Ω_i be the angular velocity, x_i be the displacement variable, p be the pressure, τ_{ij} be the viscous stress tensor, J_i be the current density, B_i be the magnetic induction vector, N the number of dust particle per unit volume, $K=6\pi\mu a$ (a = radius of dust particle) be the Stokes constant, ϕ the volume fraction, m_p be the average mass of dust particles, ω_e the electron frequency, τ_e electron collision time, n_e the number density of electron, p_e be the electron pressure, σ be the electrical conductivity of the fluid and E_i be the electric field, μ be the dynamic viscosity, λ_1 be the ratio of relaxation time to the retardation time and λ_2 be the retardation time, z be displacement variable, u, v velocities of fluid particles of primary and secondary motion, u_p, v_p velocities of dust particles of primary and secondary motions, t time, ω frequency of oscillations, Ω dimensionless angular velocity, M magnetic parameter, Ω_0 component of angular velocity along z -axis, m Hall current parameter, G dimensionless relaxation time of dust particles, J Jeffrey parameter, R mass concentration of dust particles.