$$g(\tau) = \ln(\tau^2),\ k(\tau,\eta) = \tau\eta,\ \tau,\eta \in [0,1],$$

$$(G\sigma)(\eta) = \frac{1}{9}\sigma(\eta),\ \eta \in [0,1], \sigma \in C[0,1].$$

The functions g and k are continuous and G is increasing linear transformation that satisfies in assumption (ii). To check assumption (iii), let's put

$$L(\tau,\eta) = \frac{1}{\Gamma(\frac{3}{2})}\int_{\eta}^{\tau}\mu\eta\sqrt{\tau-\mu}d\mu$$
$$= \frac{2\eta}{\sqrt{\pi}}\left[\frac{2\tau}{3}(\tau-\eta)^{\frac{3}{2}} - \frac{2}{5}(\tau-\eta)^{\frac{5}{2}}\right].$$

Since $\sup_{\tau\in[0,1]}\int_0^1 L^2(\tau,\eta)\,d\eta \le 1$, then applying Theorem 2, we deduce that Eq. (19) has a unique solution in χ.

REFERENCES

[1] Aghajani A, Jalilian Y, Trujillo JJ. (2012). On the existence of solutions of fractional integro-differential equations. Fractional Calculus and Applied Analysis 15: 44-69. http://dx.doi.org/10.2478/s13540-012-0005-4

[2] Alfifi HY, Ben Saad I, Turki S, El-Abidine ZZ. (2017). Existence and asymptotic behavior of positive solutions for a coupled system of semilinear fractional differential equations. Results in Mathematics 71: 705-730. http://dx.doi.org/10.1007/s00025-016-0528-9

[3] Anguraj A, Karthikeyan P, Trujillo JJ. (2011). Existence of solutions to fractional mixed integro-differential equations with nonlocal initial condition. Advances in Difference Equations 690653. https://doi.org/10.1155/2011/690653

[4] Baghani O, Gachpazan M, Baghani H. (2012). Existence, uniqueness and stability of solutions for a class of nonlinear integral equations under generalized Lipschitz condition. Indian Journal of Pure and Applied Mathematics 43: 309-321. http://dx.doi.org/10.1007/s13226-012-0019-y

[5] Balachandran K, Kiruthika S, Trujillo JJ. (2011). Existence results for fractional impulsive integro-differential equations in Banach spaces. Communications in Nonlinear Science and Numerical Simulation 16: 1970-1977. https://doi.org/10.1016/j.cnsns.2010.08.005

[6] Benyettou L. (2016). Performance evaluation of a multi-sensor system using fixed point DSP for water leak detection. Advances in Modelling and Analysis D 21: 78-87.

[7] Das S. (2008). Functional fractional calculus for system identification and controls. Springer-Verlag, Berlin, Heidelberg. https://doi.org/10.1007/978-3-540-72703-3

[8] Gautam GR, Dabas J. (2014). Existence result of fractional functional integro-differential equation with not instantaneous impulse. International Journal of Advances in Applied Mathematics and Mechanics 1: 11-21.

[9] Henderson J, Luca R. (2017). Existence of nonnegative solutions for a fractional integro-differential equation. Results in Mathematics 72: 747-763. http://dx.doi.org/10.1007/s00025-017-0655-y

[10] Kilbas AA, Srivastava HM, Trujillo JJ. (2006). Theory and Applications of Fractional Differential Equations. Elsevier, Amsterdam. https://doi.org/10.1016/S0304-0208(06)80001-0

[11] Klafter J, Lim SC, Metzler R. (2011). Fractional Dynamics in Physics: Recent Advances. World Scientific, Singapore. http://dx.doi.org/10.1007/s11012-012-9661-z

[12] Li G. (2017). A quintic spline collocation method for the fractional sub-diffusion equation with variable coefficients. Advances in Modelling and Analysis A 54: 40-49.

[13] Merdan M, Gökdoğan A, Yildirim A. (2013). On numerical solution to fractional non-linear oscillatory equations. Meccanica 48: 1201-1213.

[14] Nouri K, Baleanu D, Torkzadeh L. (2018). Study on application of hybrid functions to fractional differential equations. Iranian Journal of Science and Technology, Transactions A, Science 42: 1343–1350. https://doi.org/10.1007/s40995-017-0224-y

[15] Nouri K, Elahi-Mehr S, Torkzadeh L. (2016). Investigation of the behavior of the fractional Bagley-Torvik and Basset equations via numerical inverse Laplace transform. Romanian Reports in Physics 68: 503-514.

[16] Nouri K, Nazari M, Keramati B. (2017). Existence results for a coupled system of fractional integro-differential equations with time-dependent delay. Journal of Fixed Point Theory and Applications 19: 2927-2943. https://doi.org/10.1007/s11784-017-0463-8

[17] Wang F. (2012). Existence and uniqueness of solutions for a nonlinear fractional differential equation. Journal of Applied Mathematics and Computing 39: 53-67. https://doi.org/10.1007/s12190-011-0509-9