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The Probability Lattice in the Random Decision Formal Context

C. Feng [1], X. Li [1], J. Liang [2]

[1.] Department of Basic Sciences and Applied Technique, Guangdong University of Science and Technology, 523083 Guangdong P. R. China (57079540@qq.com)

[2.] School of Applied Mathematics, Guangdong University of Technology, 510006 Guangzhou P. R. China (LiangJP@126.com)

Abstract

Concept lattice theory, based on binary logic, establishes the binary relation between object and attribute in the form background, forming concept and constructing concept lattice. The data of concept lattice analysis were generally described in the formal context. As a powerful tool for conceptual data analysis and knowledge processing, the formal concept analysis was widely used in data mining, knowledge discovery of artificial intelligence, and many other areas. In different formal contexts, the concept lattice structure, rule extraction, object reduction and attribute reduction are the hot issues of research. In this paper, we proposed the concept of the probability lattice in the random decision formal context, the corresponding theorems and algorithm. Also, we proved its effectiveness and application with the examples, especially in solving the risk decision problem.

Key Words

Concept lattice, random decision formal context, probability lattice in the random decision formal context, risk decision problem.

1. Introduction

Theory of concept lattice was German professor Wille who first proposed in 1982 as a mathematical theory. It based on the binary logic, established the binary relation between object and attributes in the formal context, and constructed the concept lattice. As a powerful tool for conceptual data analysis and knowledge processing, the formal concept analysis was widely used in data mining, knowledge discovery of artificial intelligence, and many other areas [5-7].

With the development of technology, in most cases, the information that human dealing with was often uncertain, which were classified as fuzzy information and random information. For fuzzy information, the fuzzy concept [2] of fuzzy relation between object and property is presented, basing on fuzzy logic. And it had been widely used in the data analysis, knowledge processing and information retrieval. Since 2005, when the professor W. Zhang proposed the decision formal context, all kinds of extended decision formal context had been put forward, such as: fuzzy decision formal context, real value decision formal context and incomplete decision formal context [9-12]. In different contexts, the concept lattice structure, rule extraction, object reduction and attribute reduction are the hot issues of research. In 2013, professor B. Liu and Y. Li put forward the random concept [3]. So far, there are very few studies that use the concept lattice to describe and process random information, but it is a development trend.

In this paper, we construct the probability lattice in the random decision formal context. As a mathematical theory, it can easily and efficiently reflect and process random information.

Definition 1 [4]

If a binary relation $\leq$ is called a partial order relationship, when it meets:

(1) Reflexive: $\forall a \in A, \ a \leq a$;

(2) Anti-symmetric: $\forall a, b \in A, \ a \leq b, b \leq a \Rightarrow a = b$;

(3) Transitive: $\forall a, b, c \in A, \ a \leq b, b \leq c \Rightarrow a \leq c$.

Then, $(A, \leq)$ is called poset.

Theorem 2 [4]

Let L is a non-empty set. $\vee, \wedge$ are the binary operations of L. If it's satisfy:

Idempotent law: $a \vee a = a, a \wedge a = a$;

Commutative law: $a \vee b = b \vee a, a \wedge b = b \wedge a$;

Associative law: $a \vee (b \vee c) = (a \vee b) \vee c, a \wedge (b \wedge c) = (a \wedge b) \wedge c$;

Absorption law: $a \wedge (a \vee b) = a, a \vee (a \wedge b) = a$.

And there is: $\forall a, b \in L, a \le b \Leftrightarrow a \vee b = b$,

Then， $(L, \vee, \wedge)$ is a lattice.

Definition 3[4]

Let $(L, \vee, \wedge)$ is a lattice, if $\forall x, y, z \in L$, there is $x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)$ (or $x \vee (y \wedge z) = (x \vee y) \wedge (x \vee z)$). Then $(L, \vee, \wedge)$ is a distributive lattice.

Definition 4[2]

Let (U, A, I) is a formal context, of which $U = \{x_1, x_2, \cdots, x_n\}$ is an object set and each $x_i (i \le n)$ is called an object; For the condition attribute set $A = \{a_1, a_2, \cdots, a_m\}$, each $a_j (j \le m)$ is called a condition attribute; Let I is the binary relationship from U to $A, I \subseteq U \otimes A$. If $(x, a) \in I$, it means x has a condition attribute a; If $(x, a) \notin I$, it means x has no condition attribute a. We define that 1 represents $(x, a) \in I$, and 0 represents $(x, a) \notin I$.

Definition 5[2]

Let $(U, A, I,)$ and $(U, C, J,)$ are two formal contexts, with the same area of discussion, then (U, A, I, C, J) is called the decision formal context.

Definition 6[3]

Let (U, A, I, B, J) is a random decision formal context, of which $U = \{x_1, x_2, \cdots, x_n\}$ is an object set and each $x_i (i \le n)$ is called an object; For the condition attribute set $A = \{a_1, a_2, \cdots, a_m\}$, each $a_j (j \le m)$ is called an condition attribute; For the decision attribute set $B = \{b_1, b_2, \cdots, b_k\}$， each $b_q (q \le k)$ is called a decision attribute; Let I is the binary relationship from U to $A, I \subseteq U \otimes A$. If $(x, a) \in I$, it means x has a condition attribute a; If $(x, a) \notin I$, it means x has no condition attribute a. Let J is the binary relationship from U to $B, J \subseteq U \otimes B$. If $(x, b) \in J$, it means x may make decision b; If $(x, b) \notin J$, it means

x not make decision b.

We define 1 represents $(x,a) \in I$, and 0 represents $(x,a) \notin I$. When $(x,b) \in J$, let $p \in (0,1]$ be the probability of x to make decision b; and 0 represents $(x,b) \notin J$. Thereof, the random decision formal context transforms into a table in $[0,1]$. If all probability p is 0 or 1, the random decision formal context is degraded to a classical formal context.

Example 1 [3]

Let $U = \{1,2,3,4\}$, $A = \{a_1, a_2, a_3\}$ and $B = \{b_1, b_2, b_3\}$. We get the table 1. (Noticing the decision attributes is not independent of each other, so the sum of the probabilities of each row is not necessarily equal to 1).

Table 1, the random decision formal context of example 1

	Condition attributes A			Decision attributes B		
U	a_1	a_2	a_3	b_1	b_2	b_3
1	1	1	0	0.7	0.2	0.8
2	0	1	1	0.1	0.3	0.9
3	1	0	1	0.2	0.9	0.7
4	0	1	0	0.1	0.3	0.6

2. Main Results

Definition

Let (U,A,I,B,J) is a random decision formal context. $X^A = (A,\cap,\cup)$ is a condition attribute space. $X^B = (B,\cap,\cup)$ is a decision attribute space. We call the set P_{X^A,X^B} is the probability set in the random decision formal context:

$$P_{X^A,X^B} = \{p_{s,t}(x) \mid \forall x \in U, \forall s \in X^A, \forall t \in X^B, (x,s) \in I, (x,t) \in J\}.$$

In which, $p_{s,t}(x)$ is the probability of x, that has the condition attribute s, to make decision t.

Obviously, there is $P_{X^A,X^B} \subseteq (0,1]$.

Definition 8

Let P_{X^A,X^B} is the probability set in the random decision formal context. Let the "$\le$" is the less than or equal to in real number field R. Then, $(P_{X^A,X^B}, \le)$ is a poset.

Theorem 9

Let P_{X^A,X^B} is a probability set in the random decision formal context. Let the "$\le$" is the less than or equal to in real number field R. Let for $\forall p_1, p_2 \in P_{X^A,X^B}$, there are $p_1 \vee p_2 = \max\{p_1, p_2\}$, $p_1 \wedge p_2 = \min\{p_1, p_2\}$. Then, $(P_{X^A,X^B}, \vee, \wedge)$ is a lattice, and it is a distributive lattice.

Proof:

(1) In $(P_{X^A,X^B}, \le)$, for $\forall a, b, c \in P_{X^A,X^B}$, there is

Idempotent law: $a \vee a = \max\{a, a\} = a$, $a \wedge a = \min\{a, a\} = a$;

Commutative law: $a \vee b = \max\{a, b\} = b \vee a$, $a \wedge b = \min\{a, b\} = b \wedge a$;

Associative law:

$$a \vee (b \vee c) = \max\{a, \max\{b, c\}\} = \max\{a, b, c\} = \max\{\max\{a, b\}, c\} = (a \vee b) \vee c$$

$$a \wedge (b \wedge c) = \min\{a, \min\{b, c\}\} = \min\{a, b, c\} = \min\{\min\{a, b\}, c\} = (a \wedge b) \wedge c;$$

Absorption law:

$$a \wedge (a \vee b) = \min\{a, \max\{a, b\}\} = a,$$

$$a \vee (a \wedge b) = \max\{a, \min\{a, b\}\} = a;$$

And meets the rules: $\forall a, b \in P_{X^A,X^B}$, $a \le b \Leftrightarrow a \vee b = \max\{a, b\} = b$.

So by the theorem 2, $(P_{X^A,X^B}, \vee, \wedge)$ is a lattice.

(2) In lattice $(P_{X^A,X^B}, \vee, \wedge)$, for $\forall a, b, c \in P_{X^A,X^B}$, there is

$$a \wedge (b \vee c) = \min\{a, \max\{b, c\}\} = \max\{\min\{a, b\}, \min\{a, c\}\} = (a \wedge b) \vee (a \wedge c)$$

So by the definition 3, the lattice $(P_{X^A,X^B}, \vee, \wedge)$ is a distributive lattice.

Definition 10

We call the lattice $(P_{X^A,X^B},\vee,\wedge)$ is the probability lattice in the random decision formal context.

Algorithm 11

The algorithm of probability lattice in the random decision formal context:

Let (U,A,I,B,J) is a random decision formal context, of which $U=\{x_1,x_2,\cdots,x_n\}$ is an object set and each $x_i(i\le n)$ is called an object; For the condition attribute set $A=\{a_1,a_2,\cdots,a_m\}$, each $a_j(j\le m)$ is called an condition attribute; For the decision attribute set $B=\{b_1,b_2,\cdots,b_k\}$, each $b_q(q\le k)$ is called a decision attribute; Let I is the binary relationship from U to $A, I\subseteq U\otimes A$. If $(x,a)\in I$, it means x has a condition attribute a; If $(x,a)\notin I$, it means x has no condition attribute a. Let J is the binary relationship from U to $B, J\subseteq U\otimes B$. If $(x,b)\in J$, it means the x may make decision b; If $(x,b)\notin J$, it means the x not make decision b.

Step 1. Find out the condition attribute space $X^A=(A,\cap,\cup)$ and the decision attribute space $X^B=(B,\cap,\cup)$.

Step 2. Find out the probability set in the random decision formal context:

$$P_{X^A,X^B}=\{p_{s,t}(x)\,|\,\forall x\in U,\forall s\in X^A,\forall t\in X^B,(x,s)\in I,(x,t)\in J\}$$

Step 3. Obtain the probability lattice in the random decision formal context $(P_{X^A,X^B},\vee,\wedge)$ and come to a solution.

3. The application case

Example 2.

The relationship between the occurrence of the disease and the clinical performance is not a certainty. When a doctor visits a patient, it is often based on his knowledge or experience, (for example, his previous clinical record), and the clinical manifestation of the patient to make a preliminary decision about whether or not the patient had a particular disease. Of course, patients with the same clinical manifestations should be diagnosed with the same disease. This is a subjective probability, like a doctor who points out that a patient has an operation under certain conditions, "The success rate is 50%."

Suppose set U is for a group of patients, $U=\{x_1,x_2,x_3,x_4,x_5\}$. Set A is for a collection of disease symptoms provided by the patient (conditional properties), A={have a headache, a stuffy nose, a lot of snot}$=\{a_1,a_2,a_3\}$. Set B is for a collection of initial judgment results (decision properties), B={rhinitis}$=\{b_1\}$. The $p_{a_i,b_j}(x_k)$ is the probability of x_k to make the correct diagnosis b_j, when disease symptoms a_i occurs.

Table 2. the random decision formal context of example 2

	Condition attributes A			Decision attributes B
U	Has a headache a_1	A stuffy nose a_2	A lot of snot a_3	Rhinitis b_1
x_1	1	1	0	0.6
x_2	0	1	1	0.1
x_3	1	0	1	0.3
x_4	0	1	0	0.1
x_5	0	0	1	0.1

The solution is:

By the algorithm 11 of probability lattice in the random decision formal context, we get the following:

Step 1. The condition attribute space:

$$X^A=(A,\cap,\cup)$$

$$=\{a_1,a_2,a_3,a_1\cap a_2,a_1\cap a_3,a_2\cap a_3,a_1\cap a_2\cap a_3,a_1\cup a_2,a_1\cup a_3,a_2\cup a_3,a_1\cup a_2\cup a_3\}$$

The decision attribute space:

$X^B = (B, \cap, \cup) = \{b_1\}$.

Step 2. The probability set in the random decision formal context:

$$P_{X^A,X^B} = \{p_{s,t}(x) \mid \forall x \in U, \forall s \in X^A, \forall t \in X^B, (x,s) \in I, (x,t) \in J\}$$

$$= \{p_{a_2,b_1}(x_4), p_{a_3,b_1}(x_5), p_{a_1 \cap a_2,b_1}(x_1), p_{a_1 \cap a_3,b_1}(x_3), p_{a_2 \cap a_3,b_1}(x_2)\}$$

$$= \{0.1, 0.1, 0.6, 0.3, 0.1\}$$

Step 3. Let the "$\leq$" is the less than or equal to in real number field R. Let for $\forall p_1, p_2 \in P_{X^A,X^B}$, there are $p_1 \vee p_2 = \max\{p_1, p_2\}$, $p_1 \wedge p_2 = \min\{p_1, p_2\}$. We get the probability lattice in the random decision formal context: $(P_{X^A,X^B}, \vee, \wedge)$.

Therefore, we come to the solutions:

(1) Because of $p_{a_1 \cap a_2,b_1}(x_1) \wedge p_{a_1 \cap a_3,b_1}(x_3) = \min\{0.6, 0.3\} = 0.3$, according to disease symptoms a_1, the probability of making correct diagnosis is only 0.3;

Because of $p_{a_2,b_1}(x_4) \wedge p_{a_1 \cap a_2,b_1}(x_1) \wedge p_{a_2 \cap a_3,b_1}(x_2) = \min\{0.1, 0.6, 0.1\} = 0.1$, according to disease symptoms a_2, the probability of making correct diagnosis is only 0.1;

Because of $p_{a_3,b_1}(x_5) \wedge p_{a_1 \cap a_3,b_1}(x_3) \wedge p_{a_2 \cap a_3,b_1}(x_2) = \min\{0.1, 0.3, 0.1\} = 0.1$, according to disease symptoms a_3, the probability of making correct diagnosis is only 0.1;

Because of $p_{a_1 \cap a_2,b_1}(x_1) = 0.6$, according to disease symptoms $a_1 a_2$, the probability of making correct diagnosis is only 0.6;

Because of $p_{a_1 \cap a_3,b_1}(x_3) = 0.3$, according to disease symptoms $a_1 a_3$, the probability of making correct diagnosis is only 0.3;

Because of $p_{a_2 \cap a_3,b_1}(x_2) = 0.1$, according to disease symptoms $a_2 a_3$, the probability of making correct diagnosis is only 0.1;

(2) Having both disease symptoms $a_1 a_2$ is an important basis for diagnosing patients with rhinitis;

(3) When you have disease symptoms a_1, you need to be alert for rhinitis.

Example 3.

In recent years, with the rapid economic growth, China's ports have been developing rapidly. In 2016, seven of the world's top 10 ports came from China. There are many studies on the effect of shipping safety and port operation. The factors affecting the port are mainly climatic factors (such as storm surge, fog, ice and snow, high temperature, etc.), environmental factors, and other factors (such as policy). Among them, the bad weather factors will affect the normal operation of the port, thus making it uncertain to complete the transportation task on time. According to past experience, in a certain season, it is different probability for different ports to be able to complete the transport task on time, in different bad weather conditions. How to choose the proper port in order to transport the task as timely as possible, under the influence of various adverse weather conditions?

Suppose set U is for a group of cargos, $U=\{x_1,x_2,x_3,x_4,x_5\}$. Set A is for a collection of the weather factors (conditional properties), A={typhoon, high temperature, others factors of weather (e.g. policy)}$=\{a_1,a_2,a_3\}$. Set B is for a collection of ports (decision properties), B={port b_1, port b_2, port b_3}. The $p_{a_i,b_j}(x_k)$ is the probability of x_k to complete the transportation task on time for choosing port b_j, when bad weather factors a_i occurs.

Table 3, the random decision formal context of example 3

	Condition attributes A			Decision attributes B		
U	Typhoon a_1	High temperature a_2	Other factors of weather (e.g. policy) a_3	Port b_1	Port b_2	Port b_3
x_1	1	0	0	0.9	0.95	0.8
x_2	1	1	0	0.82	0.83	0.78
x_3	0	1	1	0.76	0.8	0.98
x_4	0	1	0	0.93	0.97	0.96
x_5	0	0	1	0.78	0.80	0.99

The solution is:

By the algorithm 11 of probability lattice in the random decision formal context, we get the following:

Step 1. The condition attribute space:

$$X^A = (A, \cap, \cup)$$

$$= \{a_1, a_2, a_3, a_1 \cap a_2, a_1 \cap a_3, a_2 \cap a_3, a_1 \cap a_2 \cap a_3, a_1 \cup a_2, a_1 \cup a_3, a_2 \cup a_3, a_1 \cup a_2 \cup a_3\}.$$

The decision attribute space:

$$X^B = (B, \cap, \cup) = \{b_1, b_2, b_3, b_1 \cap b_2, b_1 \cap b_3, b_2 \cap b_3, b_1 \cap b_2 \cap b_3, b_1 \cup b_2, b_1 \cup b_3, b_2 \cup b_3, b_1 \cup b_2 \cup b_3\}.$$

Step 2. The probability set in the random decision formal context:

$$P_{X^A, X^B} = \{p_{s,t}(x) \mid \forall x \in U, \forall s \in X^A, \forall t \in X^B, (x,s) \in I, (x,t) \in J\}$$

$$= \{p_{a_1,b_1}(x_1), p_{a_1,b_2}(x_1), p_{a_1,b_3}(x_1), p_{a_1 \cap a_2, b_1}(x_2), p_{a_1 \cap a_2, b_2}(x_2), p_{a_1 \cap a_2, b_3}(x_2), p_{a_2 \cap a_3, b_1}(x_3), p_{a_2 \cap a_3, b_2}(x_3),$$

$$p_{a_2 \cap a_3, b_3}(x_3), p_{a_2,b_1}(x_4), p_{a_2,b_2}(x_4), p_{a_2,b_3}(x_4), p_{a_3,b_1}(x_5), p_{a_3,b_2}(x_5), p_{a_3,b_3}(x_5)\}$$

$$= \{0.9, 0.95, 0.8, 0.82, 0.83, 0.78, 0.76, 0.8, 0.98, 0.93, 0.97, 0.96, 0.78, 0.80, 0.99\}.$$

Step 3. Let the "$\le$" is the less than or equal to in real number field R. Let for $\forall p_1, p_2 \in P_{X^A, X^B}$, there are $p_1 \vee p_2 = \max\{p_1, p_2\}$, $p_1 \wedge p_2 = \min\{p_1, p_2\}$. We get the probability lattice in the random decision formal context: $(P_{X^A, X^B}, \vee, \wedge)$.

Therefore, we come to the solutions:

(1) Because of

$$p_{a_1,b_1}(x_1) \wedge p_{a_1,b_2}(x_1) \wedge p_{a_1,b_3}(x_1) \wedge p_{a_1 \cap a_2, b_1}(x_2) \wedge p_{a_1 \cap a_2, b_2}(x_2) \wedge p_{a_1 \cap a_2, b_3}(x_2)$$

$$= \min\{0.9, 0.95, 0.8, 0.82, 0.83, 0.78\} = 0.78$$

So, when a bad weather typhoon a_1 occurs, it should be warned that the worst option is port b_3, because the probability that we can complete the transportation task on time is just 0.78;

Because of

$$p_{a_1,b_1}(x_1) \vee p_{a_1,b_2}(x_1) \vee p_{a_1,b_3}(x_1) \vee p_{a_1 \cap a_2, b_1}(x_2) \vee p_{a_1 \cap a_2, b_2}(x_2) \vee p_{a_1 \cap a_2, b_3}(x_2)$$

$$= \max\{0.9, 0.95, 0.8, 0.82, 0.83, 0.78\} = 0.95$$

So, when a bad weather typhoon a_1 occurs, the most optimistic option is port b_2, because the probability that we can complete the transportation task on time is 0.95.

(2) Because of

$$p_{a_1 \cap a_2, b_1}(x_2) \wedge p_{a_1 \cap a_2, b_2}(x_2) \wedge p_{a_1 \cap a_2, b_3}(x_2) \wedge p_{a_2 \cap a_3, b_1}(x_3) \wedge p_{a_2 \cap a_3, b_2}(x_3)$$

$$\wedge p_{a_2 \cap a_3, b_3}(x_3) \wedge p_{a_2, b_1}(x_4) \wedge p_{a_2, b_2}(x_4) \wedge p_{a_2, b_3}(x_4)\}$$

$$= \min\{0.82, 0.83, 0.78, 0.76, 0.8, 0.98, 0.93, 0.97, 0.96\} = 0.76$$

So, when a bad weather high temperature a_2 occurs, it should be warned that the worst option is port b_1, because the probability that we can complete the transportation task on time is just 0.76;

Because of

$$p_{a_1 \cap a_2, b_1}(x_2) \vee p_{a_1 \cap a_2, b_2}(x_2) \vee p_{a_1 \cap a_2, b_3}(x_2) \vee p_{a_2 \cap a_3, b_1}(x_3) \vee p_{a_2 \cap a_3, b_2}(x_3)$$

$$\vee p_{a_2 \cap a_3, b_3}(x_3) \vee p_{a_2, b_1}(x_4) \vee p_{a_2, b_2}(x_4) \vee p_{a_2, b_3}(x_4)\}$$

$$= \max\{0.82, 0.83, 0.78, 0.76, 0.8, 0.98, 0.93, 0.97, 0.96\} = 0.98$$

So, when a bad weather high temperature a_2 occurs, the most optimistic option is port b_3, because the probability that we can complete the transportation task on time is 0.98.

(3) Because of

$$p_{a_2 \cap a_3, b_1}(x_3) \wedge p_{a_2 \cap a_3, b_2}(x_3) \wedge p_{a_2 \cap a_3, b_3}(x_3) \wedge p_{a_3, b_1}(x_5) \wedge p_{a_3, b_2}(x_5) \wedge p_{a_3, b_3}(x_5)$$

$$= \min\{0.76, 0.8, 0.98, 0.78, 0.80, 0.99\} = 0.76$$

So, when other factors of weather (e.g. policy) a_3 occurs, it should be warned that the worst option is port b_1, because the probability that we can complete the transportation task on time is just 0.76;

Because of

$$p_{a_2 \cap a_3, b_1}(x_3) \vee p_{a_2 \cap a_3, b_2}(x_3) \vee p_{a_2 \cap a_3, b_3}(x_3) \vee p_{a_3, b_1}(x_5) \vee p_{a_3, b_2}(x_5) \vee p_{a_3, b_3}(x_5)$$

$$= \max\{0.76, 0.8, 0.98, 0.78, 0.80, 0.99\} = 0.99$$

So, when other factors of weather (e.g. policy) a_3 occur, the most optimistic option is port

b_3, because the probability that we can complete the transportation task on time is 0.99.

(4) Because of

$$p_{a_1 \cap a_2, b_1}(x_2) \wedge p_{a_1 \cap a_2, b_2}(x_2) \wedge p_{a_1 \cap a_2, b_3}(x_2)$$
$$= \min\{0.82, 0.83, 0.78\} = 0.78$$

So, when both typhoon and high temperature a_1a_2 occurs together, it should be warned that the worst option is port b_3, because the probability that we can complete the transportation task on time is just 0.78;

Because of

$$p_{a_1 \cap a_2, b_1}(x_2) \vee p_{a_1 \cap a_2, b_2}(x_2) \vee p_{a_1 \cap a_2, b_3}(x_2)$$
$$= \max\{0.82, 0.83, 0.78\} = 0.83$$

So, when both typhoon and high temperature a_1a_2 occur together, the most optimistic option is port b_2, because the probability that we can complete the transportation task on time is 0.83.

(5) Because of

$$p_{a_2 \cap a_3, b_1}(x_3) \wedge p_{a_2 \cap a_3, b_2}(x_3) \wedge p_{a_2 \cap a_3, b_3}(x_3)$$
$$= \min\{0.76, 0.8, 0.98\} = 0.76$$

So, when both high temperature and other factors of weather (e.g. policy) a_2a_3 occur together, it should be warned that the worst option is port b_1, because the probability that we can complete the transportation task on time is just 0.76;

Because of

$$p_{a_2 \cap a_3, b_1}(x_3) \vee p_{a_2 \cap a_3, b_2}(x_3) \vee p_{a_2 \cap a_3, b_3}(x_3)$$
$$= \max\{0.76, 0.8, 0.98\} = 0.98$$

So, when both high temperature and other factors of weather (e.g. policy) a_2a_3 occur together, the most optimistic option is port b_3, because the probability that we can complete the transportation task on time is 0.98.

Of course, in the end, we should still combine other factors, such as transportation costs and shipping periods, to consider which route and port to choose.

Conclusion

The results of this paper are theorem 9 and algorithm 11. In this paper, we constructed the probability lattice model in the random decision formal context $(P_{X^A,X^B},\vee,\wedge)$, and solve the risk decision problem and the problems of stochastic concept of random phenomena in nature. Also, the research on the probability lattice has yet to be continued. How to perfect and better use this model to solve practical problems and a better probability lattice algorithm is the next step.

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References

1. Feng C., Liang J., The solution of the more general traveling salesman problem. AMSE Journals –Series: Advances A, vol. 51, no. 1, pp. 27-40, 2014.
2. Zhang W., Qiu G., Uncertain decision based on rough set [M]. Beijing: Tsinghua University Press, pp. 185-231, 2005.
3. Liu B., Li Y., The conceptual lattice building principle and algorithm in the context of stochastic decision making[J]. Computer science. vol. 40, no. 6A, pp. 90-119, June 2013.
4. Li H., Basis of general Lattice theory [M]. Northwestern Polytechnic University Press, pp. 17- 46, 2012.
5. Wang L., Liu X., Concept analysis via rough set and AFS algebra[J]. Information Sciences, vol. 178, no. 21, pp. 4125-4137, 2008.
6. Qiu G., Approaches to reasoning in decision making based on variable threshold concept lattices[J]. Systems Engineering-Theory and Practice, vol. 30, no. 6, pp. 1092-1098, 2010.
7. Qu K., Zhai Y., Liang J., et al, Study of decision implications based on formal concept analysis[J]. International Journal of General Systems, vol. 36, no. 2, pp.147-156, 2007.

8. Wu W., Leung Y., Mi J., Granular computing and knowledge reduction in formal contexts[J]. IEEE Transactions on Knowledge and Data Engineering, vol. 21, no. 10, pp.1461-1474, 2009.
9. Yang L., Xu Y., Fuzzy concept based on lattice value logic[J]. Fuzzy Systems and Mathematics, vol. 23, no. 5, pp.15-20, 2009.
10. Yang H., Leung Y., Shao W., Rule acquisition and attribute reduction in real decision formal contexts [J]. Soft computing, vol. 15, no. 6, pp. 1115-1128, 2011.
11. Li J., Mei C., Lv Y., Knowledge reduction in real decision formal contexts [J]. Information Sciences, vol. 189, pp. 191- 207, 2012.
12. Li J., Mei C., Lv Y., Incomplete decision contexts: Approximate concept construction, rule acquisition and knowledge reduction [J]. International Journal of Approximate Reasoning, vol. 54, no. 1, pp.149-165, 2013.