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Multi-task Incentive Contract with Specific Task Ability

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Abstract

Based on traditional H-M principal-agent model and introduction of specific task ability, optimal incentive model with the specific task ability is established. the first best contract under symmetric information and the second best contract under asymmetric information are given. And the better specific task ability is, the higher incentive intensity is; the higher the risk aversion level and uncertainty are, the lower incentive intensity is.

Key words

Multi-task, Principal-agent, Incentive Contract.

1. Introduction

Principal-agent theory is one of the most important contact theory in 30 years which is developed through research on *asymmetric information* and *incentive* problem inner-enterprise by lots of economists. It studies principal-agent relationships in the aspect other than general micro-economics and has a better explanation on some organization phenomena than traditional micro-economics. Its core task is how consignor design the best contact to incentive agent and direct agent to design the salary incentive mechanism to make agent work with consignor's will under *asymmetric informat*ion and interest conflict. Holmstrom & Milgrom (1991) [1-2] extended traditional single task to multi-task and give an analyzing frame of multi-task principal-agent. After that lots of scholars studied and expanded with this theory [3-8].

In this paper, the specific task ability is introduced, hypothesis of agent has same ability to different tasks in traditional model is expended and multi-task principal-agent model with specific task ability is established. After solving the model, the first best contract under symmetric information and the second best contract under asymmetric information are given and impact analysis of relative incentive intensity and effort intensity is done.

2. Model

2.1 Basic Hypothesis

H1: (e_1, e_2) is effort vector of the agent, where e_1 , e_2 are effort level of the agent is conducting the different tasks. The principal can observe the agent's effort level but the agent's effort intensity.

H2: the ability of the agent in different tasks are different, so introduce the specific task ability, so the cost function is, $C(e_1, e_2) = \frac{1}{2}\varphi_1 e_1^2 + \frac{1}{2}\varphi_2 e_2^2$, where $C(e_1, e_2)$ is strictly incremental convex function which satisfies $\partial C / \partial e_i > 0$ and $\partial^2 C / \partial e_i^2 > 0$, with i=1, 2.

H3: output of two tasks of the agent are $S_1 = e_1 + \theta_1$ and $S_2 = e_2 + \theta_2$, where θ_1 , θ_2 are random disturbance terms which $\theta_1 : N(0, \sigma_1^2)$, $\theta_2 : N(0, \sigma_2^2)$ and $\sigma_1^2 > 0$, $\sigma_2^2 > 0$ define the degree of uncertainty.

H4: the principal provide the following linear salary incentive contact to incentive the agent $w=\alpha+b_1S_1+b_2S_2$, where α is fixed salary, b_1 , b_2 are extra payment of agent.

H5: the principal is risk neutral while agent is risk aversion. The agent has constant absolute risk aversion utility function, which is $\mu = -e^{-p[w-C]}$, where $\rho = -\frac{\mu''}{\mu'} > 0$ is absolute risk aversion coefficient and reservation utility is \overline{u} .

2.2 The Revenue

(1) The principal revenue

Based on H5, the principal is risk neutral, so the expected income of the principal is

$$E(S_1 + S_2) = E(e_1 + \theta_1 + e_2 + \theta_2) = e_1 + e_2$$

So the expected net income of principal is

$$E(S_1 + S_2) - E(w) = E(e_1 + \theta_1 + e_2 + \theta_2) - E(\alpha + b_1S_1 + b_2S_2) = (1 - b_1)e_1 + (1 - b_2)e_2 - \alpha$$

(2) The agent revenue

The expected income of the agent is

 $E(w) = E(\alpha + b_1S_1 + b_2S_2) = \alpha + b_1e_1 + b_2e_2$

Based on hypothesis 5, the agent is risk aversion, so the certainty equivalence is

$$CE = \alpha + b_1 e_1 + b_2 e_2 - \frac{1}{2} \varphi_1 e_1^2 - \frac{1}{2} \varphi_2 e_2^2 - \frac{1}{2} \rho b_1^2 \sigma_1^2 - \frac{1}{2} \rho b_2^2 \sigma_2^2$$

2.3 Model Establishment

Under symmetric information, the principal can clearly observe the effort level of the agent, so the first best incentive model is

$$\max_{\bar{\alpha},\bar{b}_{1},\bar{b}_{2},\bar{e}_{1},\bar{e}_{2}}(1-b_{1})e_{1} + (1-b_{2})e_{2} - \alpha$$

st.(1) α + $b_{1}e_{1}$ + $b_{2}e_{2} - \frac{1}{2}\varphi_{1}e_{1}^{2} - \frac{1}{2}\varphi_{2}e_{2}^{2} - \frac{1}{2}\rho b_{1}^{2}\sigma_{1}^{2} - \frac{1}{2}\rho b_{2}^{2}\sigma_{2}^{2} \ge \overline{u}(IR)$,

where (1) is individual rationality constraint.

Under asymmetric information, the principal cannot clearly observe the effort level of agent, so the second best incentive model is

$$\begin{aligned} \max_{\bar{\alpha},\bar{b}_{1},\bar{b}_{2},\bar{e}_{1},\bar{e}_{2}}(1-b_{1})e_{1}+(1-b_{2})e_{2}-\alpha \\ st.(1)\alpha+b_{1}e_{1}+b_{2}e_{2}-\frac{1}{2}\varphi_{1}e_{1}^{2}-\frac{1}{2}\varphi_{2}e_{2}^{2}-\frac{1}{2}\rho b_{1}^{2}\sigma_{1}^{2}-\frac{1}{2}\rho b_{2}^{2}\sigma_{2}^{2}\geq\overline{u}(IR) \\ (2)(e_{1},e_{2})\in\arg\max_{e_{1},e_{2}}\alpha+b_{1}e_{1}+b_{2}e_{2}-\frac{1}{2}\varphi_{1}e_{1}^{2}-\frac{1}{2}\varphi_{2}e_{2}^{2}-\frac{1}{2}\rho b_{1}^{2}\sigma_{1}^{2}-\frac{1}{2}\rho b_{2}^{2}\sigma_{2}^{2}(IC) \\ ,\end{aligned}$$

where (2) is incentive compatible constraint.

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3. Model Solve

3.1 The First Best Contract

Proposition 3.1 Under symmetric information, the first best contract that principal can provide to the agent is fixed-wage system.

Proof. Under symmetric information, the first best contract that principal can provide is solution of the following optimal model;

$$\max_{\bar{\alpha},\bar{b}_{1},\bar{b}_{2},\bar{e}_{1},\bar{e}_{2}}(1-b_{1})e_{1} + (1-b_{2})e_{2} - \alpha$$

st.(1) α + $b_{1}e_{1}$ + $b_{2}e_{2} - \frac{1}{2}\varphi_{1}e_{1}^{2} - \frac{1}{2}\varphi_{2}e_{2}^{2} - \frac{1}{2}\rho b_{1}^{2}\sigma_{1}^{2} - \frac{1}{2}\rho b_{2}^{2}\sigma_{2}^{2} \ge \overline{u}(IR)$

When individual rationality constraint take equal sign,

$$\alpha = \overline{u} - b_1 e_1 - b_2 e_2 + \frac{1}{2} \varphi_1 e_1^2 + \frac{1}{2} \varphi_2 e_2^2 + \frac{1}{2} \rho b_1^2 \sigma_1^2 + \frac{1}{2} \rho b_2^2 \sigma_2^2$$
(1)

Then

$$U_{p} = e_{1} + e_{2} - \frac{1}{2}\varphi_{1}e_{1}^{2} - \frac{1}{2}\varphi_{2}e_{2}^{2} - \frac{1}{2}\rho b_{1}^{2}\sigma_{1}^{2} - \frac{1}{2}\rho b_{2}^{2}\sigma_{2}^{2} - \overline{u}$$

So we have

$$\max_{b_1, b_2, e_1, e_2} e_1 + e_2 - \frac{1}{2} \varphi_1 e_1^2 - \frac{1}{2} \varphi_2 e_2^2 - \frac{1}{2} \rho b_1^2 \sigma_1^2 - \frac{1}{2} \rho b_2^2 \sigma_2^2 - \overline{u}$$
(2)

From the first order condition of incentive intensity b_1 , b_2 of equation (2), we have

$$-\frac{1}{2}\rho\sigma_{1}^{2}\cdot 2b_{1}=0, -\frac{1}{2}\rho\sigma_{2}^{2}\cdot 2b_{2}=0$$

So

$$\overline{b}_1 = 0, \ \overline{b}_2 = 0 \tag{3}$$

From the first order condition of effort intensity e_1 , e_2 of equation (2)

$$1 - \varphi_1 e_1 = 0, \quad 1 - \varphi_2 e_2 = 0$$

So

$$\overline{e}_1 = \frac{1}{\varphi_1}, \quad \overline{e}_2 = \frac{1}{\varphi_2} \quad , \tag{4}$$

substitute the equation (3) and (4) into (1), then

$$\alpha = \overline{u} + \frac{1}{2}\varphi_{1}e_{1}^{2} + \frac{1}{2}\varphi_{2}e_{2}^{2}$$
$$= \overline{u} + \frac{1}{2\varphi_{1}} + \frac{1}{2\varphi_{2}}$$

3.2 The Second Best Incentive Contact

Proposition 3.2 Under asymmetric information, the second best contract that principal can provide to the agent is

$$\overline{w} = \overline{\alpha} + \overline{b}_1 S_1 + \overline{b}_2 S_2,$$

where the incentive coefficients are

$$\overline{b}_{1} = \frac{\frac{1}{\varphi_{1}}}{\frac{1}{\varphi_{1}} + \rho \sigma_{1}^{2}}, \quad \overline{b}_{2} = \frac{\frac{1}{\varphi_{2}}}{\frac{1}{\varphi_{2}} + \rho \sigma_{2}^{2}},$$

The effort intensity are

$$\overline{e}_{1} = \frac{\frac{1}{\varphi_{1}}}{\varphi_{1}(\frac{1}{\varphi_{1}} + \rho\sigma_{1}^{2})}, \quad \overline{e}_{2} = \frac{\frac{1}{\varphi_{2}}}{\varphi_{2}(\frac{1}{\varphi_{2}} + \rho\sigma_{2}^{2})}$$

The fixed salary is,

$$\overline{\alpha} = \overline{u} - \frac{1}{2\varphi_1}\overline{b_1}^2 - \frac{1}{2\varphi_2}\overline{b_2}^2 + \frac{1}{2}\rho\overline{b_1}^2\sigma_1^2 + \frac{1}{2}\rho\overline{b_2}^2\sigma_2^2$$

Proof. Under asymmetric information, the second best contract that principal can provide is solution of the following optimal model;

$$\max_{\bar{\alpha},\bar{b}_{1},\bar{b}_{2},\bar{e}_{1},\bar{e}_{2}} (1-b_{1})e_{1} + (1-b_{2})e_{2} - \alpha$$

$$st.(1)\alpha + b_{1}e_{1} + b_{2}e_{2} - \frac{1}{2}\varphi_{1}e_{1}^{2} - \frac{1}{2}\varphi_{2}e_{2}^{2} - \frac{1}{2}\rho b_{1}^{2}\sigma_{1}^{2} - \frac{1}{2}\rho b_{2}^{2}\sigma_{2}^{2} \ge \bar{u}(IR)$$

$$(2)(e_{1},e_{2}) \in \arg\max_{e_{1},e_{2}} \alpha + b_{1}e_{1} + b_{2}e_{2} - \frac{1}{2}\varphi_{1}e_{1}^{2} - \frac{1}{2}\varphi_{2}e_{2}^{2} - \frac{1}{2}\rho b_{1}^{2}\sigma_{1}^{2} - \frac{1}{2}\rho b_{2}^{2}\sigma_{2}^{2}(IC)$$

from the first order condition of incentive compatible constraint in the model, we have

$$e_1 = \frac{b_1}{\varphi_1}, e_2 = \frac{b_2}{\varphi_2}$$
 (5)

Take the equation (5) into the individual rationality constraint of the model

$$\overline{\alpha} = \overline{u} - b_1 e_1 - b_2 e_2 + \frac{1}{2} \varphi_1 e_1^2 + \frac{1}{2} \varphi_2 e_2^2 + \frac{1}{2} \rho b_1^2 \sigma_1^2 + \frac{1}{2} \rho b_2^2 \sigma_2^2$$
(6)

substitute the equation (5) and (6) into the objective function

$$\max_{b_1, b_2} \frac{b_1}{\varphi_1} + \frac{b_2}{\varphi_2} - \overline{u} - \frac{1}{2\varphi_1} b_1^2 - \frac{1}{2\varphi_2} b_2^2 - \frac{1}{2} \rho b_1^2 \sigma_1^2 - \frac{1}{2} \rho b_2^2 \sigma_2^2$$
(7)

from the first order condition of the incentive intensity b_1 , b_2 of the equation (7), then

$$\frac{1}{\varphi_{1}} - b_{1} \frac{1}{\varphi_{1}} - \rho b_{1} \sigma_{1}^{2} = 0 \quad \frac{1}{\varphi_{2}} - b_{2} \frac{1}{\varphi_{2}} - \rho b_{2} \sigma_{2}^{2} = 0$$
,
so
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$$\overline{b}_{1} = \frac{\overline{\phi}_{1}}{\frac{1}{\phi_{1}} + \rho \sigma_{1}^{2}}, \quad \overline{b}_{2} = \frac{\overline{\phi}_{2}}{\frac{1}{\phi_{2}} + \rho \sigma_{2}^{2}}, \quad \overline{b}_{2} = \frac{\overline{\phi}_{2}}{\frac{1}{\phi_{2}} + \rho \sigma_{2}^{2}}$$

Substitute the incentive intensity b_1 , b_2 into α , e_1 , e_2 and get

$$\overline{e}_{1} = \frac{\frac{1}{\varphi_{1}}}{\varphi_{1}(\frac{1}{\varphi_{1}} + \rho\sigma_{1}^{2})}, \quad \overline{e}_{2} = \frac{\frac{1}{\varphi_{2}}}{\varphi_{2}(\frac{1}{\varphi_{2}} + \rho\sigma_{2}^{2})},$$

so the fixed salary is

$$\overline{\alpha} = \overline{u} - \frac{1}{2\varphi_1}\overline{b_1}^2 - \frac{1}{2\varphi_2}\overline{b_2}^2 + \frac{1}{2}\rho\overline{b_1}^2\sigma_1^2 + \frac{1}{2}\rho\overline{b_2}^2\sigma_2^2$$

4. Comparative Static Analysis

Corollary 4.1 The better specific ability is, the higher incentive intensity is. Namely

$$\frac{\partial L}{\partial \varphi_1} < 0, \quad \frac{\partial L}{\partial \varphi_2} > 0$$

Proof. From the Proposition 1 and 2, we get

$$\frac{\partial L}{\partial \varphi_{1}} = \frac{-\left[\frac{\rho \sigma_{1}^{2}}{\varphi_{1}^{2} \varphi_{2}^{2}} + \frac{\rho^{2} \sigma_{1}^{2} \sigma_{2}^{2}}{\varphi_{1}^{2} \varphi_{2}}\right]}{\left[\frac{1}{\varphi_{1} \varphi_{2}} + \frac{1}{\varphi_{2}} \rho \sigma_{1}^{2}\right]^{2}}$$

$$\frac{\partial L}{\partial \varphi_2} = \frac{\left[\frac{\rho \sigma_2^2}{\varphi_1^2 \varphi_2^2} + \frac{\rho^2 \sigma_1^2 \sigma_2^2}{\varphi_1 \varphi_2^2}\right]}{\left[\frac{1}{\varphi_1 \varphi_2} + \frac{1}{\varphi_2} \rho \sigma_1^2\right]^2}$$

Since the variables $\varphi_1 > 0$, $\varphi_2 > 0$, $\rho > 0$, $\sigma_1^2 > 0$, $\sigma_2^2 > 0$, so

$$\frac{\partial L}{\partial \varphi_{1}} = \frac{-\left[\frac{\rho \sigma_{1}^{2}}{\varphi_{1}^{2} \varphi_{2}^{2}} + \frac{\rho^{2} \sigma_{1}^{2} \sigma_{2}^{2}}{\varphi_{1}^{2} \varphi_{2}}\right]}{\left[\frac{1}{\varphi_{1} \varphi_{2}} + \frac{1}{\varphi_{2}} \rho \sigma_{1}^{2}\right]^{2}} < 0$$
$$\frac{\partial L}{\partial \varphi_{2}} = \frac{\left[\frac{\rho \sigma_{2}^{2}}{\varphi_{1}^{2} \varphi_{2}^{2}} + \frac{\rho^{2} \sigma_{1}^{2} \sigma_{2}^{2}}{\varphi_{1} \varphi_{2}^{2}}\right]}{\left[\frac{1}{\varphi_{1} \varphi_{2}} + \frac{1}{\varphi_{2}} \rho \sigma_{1}^{2}\right]^{2}} > 0$$

Corollary 4.2 The higher risk aversion level is, the lower incentive intensity is. Namely

when
$$\varphi_2 \sigma_2^2 \cdot \varphi_1 \sigma_1^2 > 0$$
, we have $\frac{\partial L}{\partial \rho} > 0$
and when $\varphi_2 \sigma_2^2 \cdot \varphi_1 \sigma_1^2 < 0$, we have $\frac{\partial L}{\partial \rho} < 0$

Proof. From the Proposition 1 and 2, we get

$$\frac{\partial L}{\partial \rho} = \frac{\varphi_2 \sigma_2^2 - \varphi_1 \sigma_1^2}{\frac{1}{\varphi_1 \varphi_2} + \frac{1}{\varphi_2} \rho \sigma_1^2} \cdot \frac{1}{\varphi_1^2 \varphi_2^2}$$

as $\phi_1 > 0$, $\phi_2 > 0$, $\rho > 0$, $\sigma_1^2 > 0$, $\sigma_2^2 > 0$ So when $\phi_2 \sigma_2^2 - \phi_1 \sigma_1^2 > 0$, we have

$$\frac{\partial L}{\partial \rho} = \frac{\varphi_2 \sigma_2^2 - \varphi_1 \sigma_1^2}{\frac{1}{\varphi_1 \varphi_2} + \frac{1}{\varphi_2} \rho \sigma_1^2} \cdot \frac{1}{\varphi_1^2 \varphi_2^2} > 0$$

When
$$\varphi_2 \sigma_2^2 - \varphi_1 \sigma_1^2 < 0$$
,

$$\frac{\partial L}{\partial \rho} = \frac{\varphi_2 \sigma_2^2 - \varphi_1 \sigma_1^2}{\frac{1}{\varphi_1 \varphi_2} + \frac{1}{\varphi_2} \rho \sigma_1^2} \cdot \frac{1}{\varphi_1^2 \varphi_2^2} < 0$$

Corollary 4.3 The higher the uncertainty of specific task is, the lower incentive intensity is. Namely

$$\frac{\partial L}{\partial \sigma_1^2} < 0 , \frac{\partial L}{\partial \sigma_2^2} > 0$$

Proof. From the Proposition 1 and 2,

$$L = \frac{\overline{b_{1}}}{\overline{b_{2}}} = \frac{\frac{1}{\varphi_{1}\varphi_{2}} + \frac{1}{\varphi_{1}}\rho\sigma_{2}^{2}}{\frac{1}{\varphi_{1}\varphi_{2}} + \frac{1}{\varphi_{2}}\rho\sigma_{1}^{2}}$$

so

$$\frac{\partial L}{\partial \sigma_{1}^{2}} = \frac{-\left[\frac{\rho}{\varphi_{1}\varphi_{2}^{2}} + \frac{\rho^{2}\sigma_{2}^{2}}{\varphi_{1}\varphi_{2}}\right]}{\left[\frac{1}{\varphi_{1}\varphi_{2}} + \frac{1}{\varphi_{2}}\rho\sigma_{1}^{2}\right]^{2}}, \frac{\partial L}{\partial \sigma_{2}^{2}} = \frac{-\left[\frac{\rho}{\varphi_{1}^{2}\varphi_{2}} + \frac{\rho^{2}\sigma_{1}^{2}}{\varphi_{1}\varphi_{2}}\right]}{\left[\frac{1}{\varphi_{1}\varphi_{2}} + \frac{1}{\varphi_{2}}\rho\sigma_{1}^{2}\right]^{2}}$$

As
$$\phi_1 > 0$$
, $\phi_2 > 0$, $\rho > 0$, $\sigma_1^2 > 0$, $\sigma_2^2 > 0$

so

$$\frac{\partial L}{\partial \sigma_{1}^{2}} = \frac{-\left[\frac{\rho}{\varphi_{1}\varphi_{2}^{2}} + \frac{\rho^{2}\sigma_{2}^{2}}{\varphi_{1}\varphi_{2}}\right]}{\left[\frac{1}{\varphi_{1}\varphi_{2}} + \frac{1}{\varphi_{2}}\rho\sigma_{1}^{2}\right]^{2}} < 0, \quad \frac{\partial L}{\partial \sigma_{2}^{2}} = \frac{-\left[\frac{\rho}{\varphi_{1}^{2}\varphi_{2}} + \frac{\rho^{2}\sigma_{1}^{2}}{\varphi_{1}\varphi_{2}}\right]}{\left[\frac{1}{\varphi_{1}\varphi_{2}} + \frac{1}{\varphi_{2}}\rho\sigma_{1}^{2}\right]^{2}} > 0$$

Corollary4.4 Under asymmetric information, the agency cost is

$$F_{p} = \frac{\rho^{2}(\sigma_{1}^{2})^{2} + \frac{1}{\varphi_{1}}\rho\sigma_{1}^{2}}{2\varphi_{1}(\frac{1}{\varphi_{1}} + \rho\sigma_{1}^{2})^{2}} + \frac{\rho^{2}(\sigma_{2}^{2})^{2} + \frac{1}{\varphi_{2}}\rho\sigma_{2}^{2}}{2\varphi_{2}(\frac{1}{\varphi_{2}} + \rho\sigma_{2}^{2})^{2}}$$

Proof. Under symmetric information, expected net revenue of the principal is

$$e_{1}+e_{2}-\frac{1}{2}\varphi_{1}e_{1}^{2}-\frac{1}{2}\varphi_{2}e_{2}^{2}-\frac{1}{2}\rho b_{1}^{2}\sigma_{1}^{2}-\frac{1}{2}\rho b_{2}^{2}\sigma_{2}^{2}-\overline{u}$$

So the maximum is

$$U_{p} = e_{1} + e_{2} - \frac{1}{2}\varphi_{1}e_{1}^{2} - \frac{1}{2}\varphi_{2}e_{2}^{2} - \frac{1}{2}\rho b_{1}^{2}\sigma_{1}^{2} - \frac{1}{2}\rho b_{2}^{2}\sigma_{2}^{2} - \overline{u}$$
$$= \overline{u} + \frac{1}{2\varphi_{1}} + \frac{1}{2\varphi_{2}}$$

And Under asymmetric information, expected net revenue of the principal is

$$U_{p} = \frac{b_{1}}{\varphi_{1}} + \frac{b_{2}}{\varphi_{2}} - \overline{u} - \frac{1}{2\varphi_{1}}b_{1}^{2} - \frac{1}{2\varphi_{2}}b_{2}^{2} - \frac{1}{2}\rho b_{1}^{2}\sigma_{1}^{2} - \frac{1}{2}\rho b_{2}^{2}\sigma_{2}^{2}$$

So the maximum is

$$U_{p}^{"} = \frac{(\frac{1}{\varphi_{1}})^{2} + \frac{1}{\varphi_{1}}\rho\sigma_{1}^{2}}{2\varphi_{1}(\frac{1}{\varphi_{1}} + \rho\sigma_{1}^{2})^{2}} + \frac{(\frac{1}{\varphi_{2}})^{2} + \frac{1}{\varphi_{2}}\rho\sigma_{2}^{2}}{2\varphi_{2}(\frac{1}{\varphi_{2}} + \rho\sigma_{2}^{2})^{2}} - \frac{1}{\mu}$$

So the agency cost is

$$\begin{split} F_{p} &= U_{p} - U_{p} \\ &= \left[\overline{u} + \frac{1}{2\varphi_{1}} + \frac{1}{2\varphi_{2}} \right] - \left[\frac{\left(\frac{1}{\varphi_{1}}\right)^{2} + \frac{1}{\varphi_{1}}\rho\sigma_{1}^{2}}{2\varphi_{1}\left(\frac{1}{\varphi_{1}} + \rho\sigma_{1}^{2}\right)^{2}} + \frac{\left(\frac{1}{\varphi_{2}}\right)^{2} + \frac{1}{\varphi_{2}}\rho\sigma_{2}^{2}}{2\varphi_{2}\left(\frac{1}{\varphi_{2}} + \rho\sigma_{2}^{2}\right)^{2}} - \overline{\mu} \right] \\ &= \frac{\rho^{2}(\sigma_{1}^{2})^{2} + \frac{1}{\varphi_{1}}\rho\sigma_{1}^{2}}{2\varphi_{1}\left(\frac{1}{\varphi_{1}} + \rho\sigma_{1}^{2}\right)^{2}} + \frac{\rho^{2}(\sigma_{2}^{2})^{2} + \frac{1}{\varphi_{2}}\rho\sigma_{2}^{2}}{2\varphi_{2}\left(\frac{1}{\varphi_{2}} + \rho\sigma_{2}^{2}\right)^{2}} \end{split}$$

Conclusion

Traditional principal-agent model is based on that the agent has the same ability in different tasks. However, in this paper traditional model is extended by introducing the specific task ability. The model considers the different ability in different tasks of the agent, loses the assumed conditions and puts the specific task ability variable into the cost function. The effect of the specific task ability to the cost is emphasized which makes the model is closer to the reality.

In this paper, based on traditional H-M principal-agent model and introduction of the specific task ability, the optimal incentive model with the specific task ability is established. The first best contract under symmetric information and the second best contract under asymmetric information are given by solving the model. And the better the specific task ability is, the higher incentive intensity is; the higher the risk aversion level and uncertainty are, the lower incentive intensity is.

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